

104 THE THEORY OF CHOICE: UTILITY

- a) Suppose you are exposed to a situation that results in a 50/50 chance of winning or losing \$1000. If you can buy insurance that completely removes the risk for a fee of \$125, will you buy it or take the gamble?
- b) Suppose you accept the gamble outlined in (a) and lose, so that your wealth is reduced to \$4000. If you are faced with the same gamble and have the same offer of insurance as before, will you buy the insurance the second time around?
- 4.4 Assume that you have a logarithmic utility function for wealth $U(W) = \ln(W)$ and that you are faced with a 50/50 chance of winning or losing \$1,000. How much will you pay to avoid this risk if your current level of wealth is \$10,000? How much would you pay if your level of wealth were \$1,000,000?
- 4.5 Given the exponential utility function $U(W) = -e^{-aW}$.
- a) Graph the function, assuming $a > 0$.
- b) Does the function exhibit positive marginal utility and risk aversion?
- c) Does the function have decreasing absolute risk aversion?
- d) Does the function have constant relative risk aversion?
- 4.6 What kind of utility function of wealth might be consistent with an individual gambling and paying insurance at the same time?
- 4.7 Suppose that $A > B > C > D$ and that the utilities of these alternatives satisfy $U(A) + U(D) = U(B) + U(C)$. Is it true that $U(\frac{1}{2}B + \frac{1}{2}C)$ is greater than $U(\frac{1}{2}A + \frac{1}{2}D)$ because the former has a smaller variance? Why or why not?
- 4.8 A small businesswoman faces a 10% chance of having a fire that will reduce her net worth to \$1.00, a 10% chance that fire will reduce it to \$50,000, and an 80% chance that nothing detrimental will happen, so that her business will retain its worth of \$100,000. What is the maximum amount she will pay for insurance if she has a logarithmic utility function? In other words, if $U(W) = \ln W$, compute the cost of the gamble. [Note: The insurance pays \$99,999 in the first case; \$50,000 in the second; and nothing in the third.]
- 4.9 If you are exposed to a 50/50 chance of gaining or losing \$1000 and insurance that removes the risk costs \$500, at what level of wealth will you be indifferent relative to taking the gamble or paying the insurance? That is, what is your certainty equivalent wealth? Assume your utility function is $U(W) = -W^{-1}$.
- 4.10 Consider a lottery that pays \$2 if n consecutive heads turn up in $(n + 1)$ tosses of a fair coin (i.e., the sequence of coin flips ends with the first tail). If you have a logarithmic utility function, $U(W) = \ln W$, what is the utility of the expected payoff? What is the expected utility of the payoff?
- 4.11 (Our thanks to David Pyle, University of California, Berkeley, for providing this problem.) Mr. Casadesus's current wealth consists of his home, which is worth \$50,000, and \$20,000 in savings, which are earning 7% in a savings and loan account. His (one-year) homeowner's insurance is up for renewal, and he has the following estimates of the potential losses on his house owing to fire, storm, etc., during the period covered by the renewal:

Value of Loss, \$	Probability, %
0	.98
5,000	.01
10,000	.005
50,000	.005