Relatório de Resoluções

O código de cada membro pode ser consultado a seguir:

x_{04}: Beatriz Chessa	x_{11}: Luca Monaco
x_{05}: José Soares Jr.	x_{15}: Rodrigo Melendez
x_{06}: Maurício Damião	x_{18}: Matheus Cardoso
x_{08}: Pedro Lopes Silva	x_{20}: Gustavo Zequini
x_{09}: Rafael Maddalena	

Resolução (‖ Questão: 3.2.1 ‖ Relator: $\mathbf{x}_{08}$ ‖ Revisor: $\mathbf{x}_{06}$ ‖)

Find the value of Y for the case when $Y = C + 150$ and $C = 600 + 0.9Y$ in the model of Example 1. Verify that formula (**) gives the same result.

Fórmula (**) $Y = \frac{a}{1-b} + \frac{1}{1-b} \cdot \bar{I}$

Ainda, segundo o exemplo 1 temos que $Y = C + \bar{I}$, de modo que de acordo com esse exercício $\bar{I} = 150$ e $C = bY + a$. Assim sendo, de acordo com esse exercício $a = 600$ e $b = 0,9$.

Resolução:

$Y = C + 150 = 600 + 0,9Y + 150 = 0,9Y + 750$

$Y = 0,9Y + 750 \Rightarrow Y - 0,9Y = 750 \Rightarrow 0,1Y = 750 \Rightarrow Y = 7500.$

Se usarmos a fórmula (**) e substituirmos os valores de a, b e $\bar{I}$ teremos:

$Y = \frac{a}{1-b} + \frac{1}{1-b} \cdot \bar{I} = \frac{600}{1-0,9} + \frac{1}{1-0,9} \cdot 150 = \frac{600}{0,1} + \frac{1}{0,1} \cdot 150 = 6000 + 1500 = 7500$

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Resolução (‖ Questão: 3.2.2 ‖ Relator: $\mathbf{x}_{09}$ ‖ Revisor: $\mathbf{x}_{08}$ ‖)

Resolva as seguintes equações para x

a) $\dfrac{1}{ax} + \dfrac{1}{bx} = 2$

$$\frac{1}{ax} + \frac{1}{bx} = 2 \iff \frac{abx}{ax} + \frac{abx}{bx} = 2abx \iff b + a = 2abx \iff \frac{b+a}{2ab} = \frac{2abx}{2ab} \iff \frac{b}{2ab} + \frac{a}{2ab} = x \iff \frac{1}{2a} + \frac{1}{2b} = x$$

b) $\dfrac{ax+b}{cx+d} = A$

$$\frac{ax+b}{cx+d} = A \iff \frac{(cx+d)(ax+b)}{cx+d} = A(cx+d) \iff ax + b = Acx + Ad \iff b - Ad = Acx - ax \iff b - Ad = x(Ac - a) \iff \frac{b - Ad}{Ac - a} = \frac{x(Ac-a)}{Ac-a} \iff \frac{b-Ad}{Ac-a} = x$$

c) $\frac{1}{2}px^{-1/2} - w = 0$

$\frac{1}{2}px^{-1/2} - w = 0 \iff px^{-1/2} = 2w \iff x^{-1/2} = \frac{2w}{p} \iff x = \left(\frac{2w}{p}\right)^{-2} \iff x = \frac{p^2}{4w^2}$

d) $\sqrt{1+x} + \frac{ax}{\sqrt{1+x}} = 0$

$\sqrt{1+x} + \frac{ax}{\sqrt{1+x}} = 0 \iff (\sqrt{1+x})(\sqrt{1+x}) + (\sqrt{1+x})\frac{ax}{\sqrt{1+x}} = 0(\sqrt{1+x}) \iff 1+x+ax = 0 \iff$
$x(a+1) = -1 \iff x = \frac{-1}{a+1}$

e) $a^2x^2 - b^2 = 0$

$a^2x^2 - b^2 = 0 \iff a^2x^2 = b^2 \iff x^2 = \frac{b^2}{a^2} \iff \sqrt{x^2} = \frac{\sqrt{b^2}}{\sqrt{a^2}} \iff x = \pm\frac{b}{a}$

f) $(3+a^2)^x = 1$

$(3+a^2)^x = 1 \iff (3+a^2)^x = (3+a^2)^0 \iff x = 0$

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Resolução (‖ Questão: 3.2.3 ‖ Relator: $\mathbf{x}_{11}$ ‖ Revisor: $\mathbf{x}_{09}$ ‖)

3. Solve the following equations for the indicated variables:

a)$q = 0.15p + 0.14$ for p (supply of rice in India);

$$q = 0,15p - 0,14 \tag{1}$$
$$p = \frac{q - 0,14}{0,15} \tag{2}$$
$$p = \frac{q - \frac{14}{100}}{\frac{15}{100}} \tag{3}$$
$$p = \frac{100 \cdot q}{100} \cdot \frac{100}{15} \tag{4}$$
$$p = \frac{100 \cdot q - 14}{15} \tag{5}$$
$$p = \frac{20q}{3} - \frac{14}{15} \tag{6}$$

b)$S = \alpha + \beta P$ for P (supply function);

$$S = \alpha + \beta P \tag{7}$$
$$S - \alpha = \beta P \tag{8}$$
$$P = \frac{S - \alpha}{\beta} \tag{9}$$

c)$A = \frac{1}{2} \cdot gh$ for g (the area of a triangle);

$$A = \frac{1}{2} \cdot gh \tag{10}$$
$$\frac{2A}{h} = g \tag{11}$$

d)$V = \frac{4}{3} \cdot \pi \cdot r^3$ for r (the volume of a ball);

$$V = \frac{4}{3} \cdot \pi \cdot r^3 \tag{12}$$
$$\frac{3V}{4} = \pi r^3 \tag{13}$$
$$\frac{3V}{4\pi} = r^3 \tag{14}$$
$$(\frac{3V}{4\pi})^{\frac{1}{3}} = r \tag{15}$$

e)$A \cdot (K)^\alpha \cdot L^\beta = Y_0$ for L (production function);

$$A \cdot (K)^\alpha \cdot L^\beta = Y_0 \tag{16}$$
$$\frac{Y_0}{AK^\alpha} = L^\beta \tag{17}$$
$$L = (\frac{Y_0}{AK^\alpha})^{\frac{1}{\beta}} \tag{18}$$

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Resolução (‖ Questão: 3.2.4 ‖ Relator: $\mathbf{x}_{15}$ ‖ Revisor: $\mathbf{x}_{11}$ ‖)

Solve the following equations for the indicated variables:

(a) $\alpha x - a = \beta x - b$ for x

$\alpha x - a = \beta x - b \Leftrightarrow \alpha x - \beta x = a - b \Leftrightarrow x(\alpha - \beta) = a - b \Leftrightarrow x = \frac{a-b}{\alpha-\beta}$

(b) $\sqrt{pq} - 3q = 5$ for p

$\sqrt{pq} - 3q = 5 \Leftrightarrow \sqrt{pq} = 5 + 3q \Leftrightarrow pq = 25 + 30q + 9q^2 \Leftrightarrow p = \frac{25}{q} + 30 + 9q$

(c) $Y = 94 + 0.2(Y - (20 + 0.5Y))$ for Y

$Y = 94 + 0.2(Y - (20 + 0.5Y)) \Leftrightarrow Y = 94 + 0.2(Y - 20 - 0.5Y) \Leftrightarrow Y = 94 + 0.2Y - 4 - 0.1Y \Leftrightarrow 0.9Y = 90 \Leftrightarrow Y = \frac{90}{0.9} \Leftrightarrow Y = 100$

(d) $K^{\frac{1}{2}}(\frac{1}{2}\frac{r}{w}K)^{\frac{1}{4}} = Q$ for K

$K^{\frac{1}{2}}(\frac{1}{2}\frac{r}{w}K)^{\frac{1}{4}} = Q \Leftrightarrow K^{\frac{1}{2}}K^{\frac{1}{4}}\frac{1}{2^{\frac{1}{4}}}\frac{r^{\frac{1}{4}}}{w^{\frac{1}{4}}} = Q \Leftrightarrow K^{\frac{3}{4}} = Q2^{\frac{1}{4}}w^{\frac{1}{4}}\frac{1}{r^{\frac{1}{4}}} \Leftrightarrow K^{\frac{3}{4}} = Q.(\frac{2w}{r})^{\frac{1}{4}} \Leftrightarrow K = Q^{\frac{4}{3}}(\frac{2w}{r})^{\frac{1}{3}}$

(e) $\dfrac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{4}}}{\frac{1}{4}L^{-\frac{3}{4}}K^{\frac{1}{2}}} = \frac{r}{w}$ for L

$\dfrac{\frac{1}{2}K^{-\frac{1}{2}}L^{\frac{1}{4}}}{\frac{1}{4}L^{-\frac{3}{4}}K^{\frac{1}{2}}} = \frac{r}{w} \Leftrightarrow \frac{1}{2}.4K^{-\frac{1}{2}}K^{-\frac{1}{2}}L^{\frac{1}{4}}L^{\frac{3}{4}} = \frac{r}{w} \Leftrightarrow 2K^{-1}L = \frac{r}{w} \Leftrightarrow L = \frac{rK}{2w}$

(f) $\frac{1}{2}pK^{-\frac{1}{4}}(\frac{1}{2}\frac{r}{w})^{\frac{1}{4}} = r$ for K

$$\frac{1}{2}pK^{-\frac{1}{4}}(\frac{1}{2}\frac{r}{w})^{\frac{1}{4}} = r \Leftrightarrow K^{-\frac{1}{4}} = \frac{2r}{p(\frac{1}{2}\frac{r}{w})^{\frac{1}{4}}} \Leftrightarrow K = \left(\frac{2r}{p(\frac{1}{2}\frac{r}{w})^{\frac{1}{4}}}\right)^{-4} \Leftrightarrow K = \left(\frac{p(\frac{1}{2}\frac{r}{w})^{-\frac{1}{4}}}{2r}\right)^{4}$$

$$\Leftrightarrow K = \frac{p^4 \frac{1}{2}\frac{r}{w}}{2^4 r^4} \Leftrightarrow K = \frac{p^4 r}{2^4.2.r^4 w} \Leftrightarrow K = \frac{p^4}{2^5.r^3 w}$$

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Resolução (‖ Questão: 3.2.5 ‖ Relator: $\mathbf{x}_{18}$ ‖ Revisor: $\mathbf{x}_{15}$ ‖)

Solve the following equations for the indicated variables:

a) $\frac{1}{s} + \frac{1}{T} = t$, for s

Para $s \neq 0$, $T \neq 0$ e $t \neq 0$

$\frac{1}{s} + \frac{1}{T} = t \iff 1 + \frac{s}{T} = \frac{s}{t} \iff s(\frac{1}{T} - \frac{1}{t}) = -1 \iff s = -(\frac{1}{T} - \frac{1}{t})^{-1} \iff s = (\frac{1}{t} - \frac{1}{T})^{-1}$

b) $\sqrt{KLM} - \alpha L = B$, for M

Para $KLM \geq 0$

$\sqrt{KLM} - \alpha L = B \iff KLM = (B + \alpha L)^2 \iff M = \frac{(B+\alpha L)^2}{KL}$

c) $\frac{x-2y+xz}{x-z} = 4y$, for z

Para $x \neq z$ e $x \neq -4y$

$\frac{x-2y+xz}{x-z} = 4y \iff x + xz - 2y = 4yx - 4yz \iff x - 2y - 4xy = -xz - 4yz \iff 4xy + 2y - x = z(x + 4y) \iff z = \frac{4xy+2y-x}{x+4y}$

d) $V = C(1 - \frac{T}{N})$, for T

Para $C \neq 0$ e $N \neq 0$

$V = C(1 - \frac{T}{N}) \iff \frac{V}{C} = 1 - \frac{T}{N} \iff T = N(1 - \frac{V}{C})$

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