Review article

Fatigue of structures and materials in the 20th century and the state of the art ☆

J. Schijve *

Delft University of Technology, Faculty of Aerospace Engineering, Kluyverweg 1, 2629HS Delft, The Netherlands

Abstract

The paper surveys the historical development of scientific and engineering knowledge about fatigue of materials and structures in the 20th century. This includes fatigue as a material phenomenon, prediction models for fatigue properties of structures, and load spectra. The review leads to an inventory of the present state of the art. Some final remarks follow in an epilogue.
© 2003 Elsevier Science Ltd. All rights reserved.

Keywords: Fatigue mechanism; Fatigue properties; Prediction; Load spectra; History

Contents

 * Tel.: +1-31-15-3695-194.
    E-mail address: J.Schijve@lr.tudelft.nl (J. Schijve).

Nomenclature

CA	Constant amplitude
VA	Variable amplitude
OL	Overload
S_f	Fatigue limit

---

1. Introduction

An evaluation of fatigue of structures and materials in the 20th century raises the question what happened in the 19th century? The answer is that fatigue of structures became evident as a by-product of the industrial revolution in the 19th century. In some more detail, it was recognized as a fracture phenomenon occurring after a large numbers of load cycles where a single load of the same magnitude would not do any harm. Fatigue failures were frequently associated with steam engines, locomotives and pumps. In the 19th century, it was considered to be mysterious that a fatigue fracture did not show visible plastic deformation. Systematic fatigue tests were done at a few laboratories, notably by August Wöhler. It was recognized that small radii in the geometry of the structure should be avoided. Fatigue was considered to be an engineering problem, but the fatigue phenomenon occurring in the material was still largely in the dark. Some people thought that fatigue implied a change from a fibrous to a crystalline, brittle structure in view of the absence of visible plastic deformation.

A fundamental step regarding fatigue as a material problem was made in the beginning of the 20th century by Ewing and Humfrey in 1903 [1]. They carried out a microscopic investigation which showed that fatigue crack nuclei start as microcracks in slip bands. Much more evidence about fatigue as a material phenomenon was going to follow in the 20th century.

Fatigue as a technical problem became evident around the middle of the 19th century. About 100 years later, in the middle of the 20th century, the development of fatigue problems were reviewed in two historical papers by Peterson in 1950 [2] and Timoshenko in 1954 [3]. Both authors were already well-known for important publications. Peterson reviewed the discussion on fatigue problems during meetings of the Institution of Mechan-ical Engineers at Birmingham held just before 1850. He also mentioned historical ideas about fatigue as a material phenomenon and the microscopic studies carried out by Gough and co-workers and others around 1930. Crack initiation occurred in slip bands and (quoting Peterson) "*one or more of these minute sources starts to spread and this develops into a gross crack which, in general, meanders through the grains in zig-zag fashion in an average direction normal to the direction of tensile stresses. It should be remembered, however, that although the fractured surface generally follows a normal stress field, the microscopic source of failure is due to shear*". Peterson also refers to the concept of the 'endurance limit', as already defined by Wöhler. In this paper the endurance limit is generally referred to as the fatigue limit which is an important material property for various engineering predictions on fatigue.

Timoshenko in his review discussed the significance of stress distributions and emphasized stress concentrations around notches. According to Timoshenko, the importance was recognized by design engineers around the end of the 19th century, and the knowledge was further refined in the beginning of the 20th century. Timoshenko referred to the significance of theoretical stress analysis employing complex variables (Kolosov, Inglis, Mushkelisvili, Savin and others). But he considered experimental studies on stress distributions and stress concentrations to be of prime importance. He mentioned several developments on strain measurements, basically by using mechanical displacement meters, strain gauges and photo-elastic models. A famous book published in 1950 was Handbook of Experimental Stress Analysis by Hetényi [4]. Timoshenko thought that great progress had been made. He also raised the question "*how does a high, localized stress weaken a machine part in service? This important question can be satisfac-*

torily answered only on the basis of an experimental investigation".

The above résumé of developments before 1950 now seems to be 'old stuff', primarily because substantial improvements of our present knowledge about fatigue occurred in the second half of the 20th century. The improvements became possible due to the development of essentially new experimental facilities, computers and numerical stress analysis. However, some basic concepts remained, such as that fatigue in metallic materials is due to cyclic slip, and stress concentrations contribute to a reduced fatigue endurance. One other characteristic issue of a more philosophical nature also remained, the question of whether fatigue is a material problem or an engineering problem, or both in some integrated way? The present paper primarily covers developments in the second half of the previous century. It is not the purpose to summarize all noteworthy happenings in a historical sequence, also because informative reviews about the history of 'fatigue' have been presented in the last decades of the 20th century, e.g. by Mann [5], Schütz [6], Smith [7] and others. Moreover, collections of significant publications have been compiled [8,9]. The emphasis in this paper will be on how the present knowledge was acquired. The development of fatigue problems of structures and materials in the 20th century was fundamentally affected by milestone happenings, important discoveries, and various concepts of understanding fatigue phenomena. Furthermore, the approach to solving fatigue problems and the philosophy on the significance of fatigue problems is of great interest.

The efforts spent on fatigue investigations in the 20th century is tremendous, as illustrated by numerous publications. John Mann [10] published books with references to fatigue. Later he continued this work to arrive at about 100 000 references in the 20th century compared to less than 100 in the 19th century. The large number of publications raises an obvious question. Is the problem so difficult and complex, or were we not clever enough to eliminate fatigue problems of our industrial products? Various conferences on fatigue of structures and materials are already planned for the forthcoming years of the 21st century implying that the fatigue problem is apparently not yet fully solved. If the problem still exists after 100 years in the previous century, there is something to be explained.

In a recent textbook [11] the author has used the picture shown in Fig. 1 to survey prediction problems associated with fatigue properties of structures. The predictions are the output of a number of procedures and Fig. 1 presents the scenario of the various aspects involved. The input problems occur in three categories: (i) design work, (ii) basic information used for the predictions, and (iii) fatigue load spectra to which the structure is subjected. Each of the categories contains a number of separate problems, which again can be subdivided into spe-
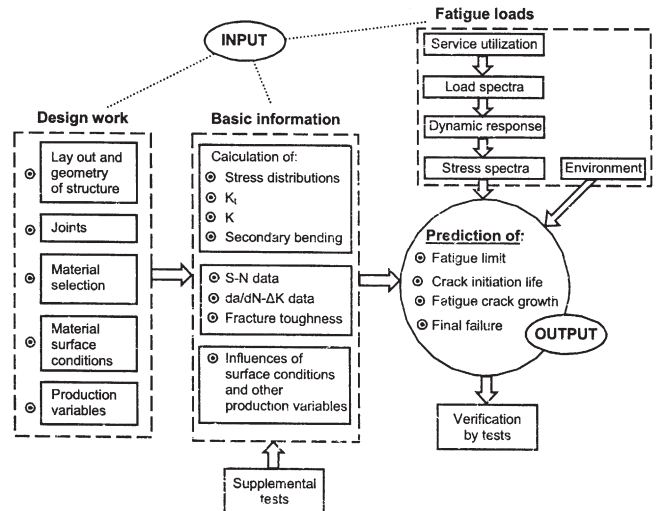

Fig. 1.   Survey of the various aspects of fatigue of structures [11].

cific aspects, e.g. 'joints' cover welded joints, bolted joints, riveted joints, adhesively bonded joints. Fig. 1 illustrates that the full problem can be very complex depending on the structural design, type of material, production variables, load spectra and environment. Prediction models are presented in the literature and software is commercially available. The prediction of the fatigue performance of a structure is the result of many steps of the procedures adopted, and in general a number of plausible assumptions is involved. It implies that the accuracy of the final result can be limited, the more so if statistical variables also have to be considered. The reliability of the prediction should be carefully evaluated, which requires a profound judgement, and also so-called engineering judgement, experience and intuition. It has persistently been emphasized in Ref. [11] that physical understanding of the fatigue phenomena is essential for the evaluation of fatigue predictions. A designer cannot simply rely on the validity of equations. Behind an equation is a physical model and the question is whether the model is physically relevant for the problem considered. This implies that each topic in Fig. 1 should also be a relevant subject for research, and the number of variables which can affect the fatigue behavior of a structure is large. Without some satisfactory understanding of aspects involved, predictions on fatigue become inconceivable. In this paper, it will be summarized how the understanding in the previous century has been improved, sometimes as a qualitative concept, and in other cases also quantitatively. It should already be said here that qualitative understanding can be very important, even if a strictly quantitative analysis is not yet possible. The major topics discussed in the following sections are associated with: (i) material fatigue as a physical phenomenon (Section 2), (ii) the S-N curve and the fatigue limit (Section 3), (iii) prediction of fatigue properties (Section 4), and (iv) fatigue

load spectra in service (Section 5). These topics are first discussed to see the development of the knowledge about fatigue of structures and materials in the 20th century. Afterwards, the text covers an evaluation of the present understanding also in relation to the engineering significance (Section 6). The paper is concluded with some general remarks about the present state of the art and expectations for the 21st century (Section 7).

2. Fatigue of materials as a physical phenomenon

2.1. Fatigue crack initiation

As said before, fatigue damage in steel in the 19th century was associated with a mysterious crystallizing of a fibrous structure. It was not yet defined in physical terms. In the first half of the 20th century, cyclic slip was considered to be essential for microcrack initiation. Cracks, even microcracks, imply decohesion in the material and should thus be considered to be damage. But is cyclic slip also damage, and what about cyclic strain hardening in slip bands? In the thirties, Gough [12] postulated that fatigue crack initiation is a consequence of exceeding the limit of local strain hardening. The idea was adopted by Orowan in 1939 [13] who argued that the local exhaustion of ductility leads to a localized increase of the stress and ultimately to cracking. This concept was used in 1953 by Head [14] in a model for obtaining an equation for fatigue crack growth.

An important question about the ductility exhaustion theory is how cracking occurs on an atomic level. Stroh [15] analyzed the stress field around a piled-up group of dislocations. According to him, the local stress can become sufficiently high to cause local cleavage. However, it was difficult to see why high local stresses can not be relaxed near the material surface by plastic deformation in a basically ductile material. The ductility exhausting theory did not become a credible crack initiation model, the more so since the detection of striations in the late 1950s [16,17] indicated that crack extension occurred in a cycle-by-cycle sequence, and not in jumps after intervals of cycles required for an increasing strain-hardening mechanism.

In the 1950s, the knowledge of dislocations had been well developed. Cyclic slip was associated with cyclic dislocation movements. It is not surprising that people tried to explain the initiation and crack growth in terms of creating crevices in the material or intrusions into the material surface as a result of some specific dislocation mobilities. Interesting dislocation models were proposed in the 1950s, noteworthy by Cottrell and Hull, based on intersecting slip systems [18], and by Mott, based on generation of vacancies [19]. Microscopic observations were made to see whether the proposed models for crack initiation and crack growth were in agreement with a

model. Several papers of historical interest were collected in 1957 [20] and 1959 [21] respectively. The microscopic work of Forsyth [22] on extrusions and intrusions in slip bands should be mentioned, see Fig. 2. Similar figures have been used by several authors to discuss basic aspects of the fatigue crack initiation process. Three fundamental aspects are: the significance of the free material surface, the irreversibility of cyclic slip, and environmental effects on microcrack initiation.

Microcracks usually start at the free surface of the material,[1] also in unnotched specimens with a nominally homogeneous stress distribution tested under cyclic tension. The restraint on cyclic slip is lower than inside the material because of the free surface at one side of the surface material. Furthermore, microcracks start more easily in slip bands with slip displacements normal to the material surface [23] which seems to be logical when looking at Fig. 2. It still remains to be questioned why cyclic slip is not reversible. Already in the 1950s, it was understood that there are two reasons for non-reversibility. One argument is that (cyclic) strain hardening occurs which implies that not all dislocations return to their original position. Another important aspect is the interaction with the environment. A slip step at the free surface implies that fresh material is exposed to the environment. In a non-inert environment, most technical materials are rapidly covered with a thin oxide layer, or some chemisorption of foreign atoms of the environment occurs. An exact reversibility of slip is then prevented. *A valid and important conclusion is that fatigue crack initiation is a surface phenomenon.*

In the 1950s, microscopical investigations were still made with the optical microscope. It implies that crack nucleation is observed on the surface where it indeed occurs. As soon as cracks are growing into the material away from the free surface, only the ends of the crack front can be observed at that free surface. It is questionable whether that information is representative for the growth process inside the material, a problem sometimes overlooked. Microscopic observations on crack growth inside the material require that cross-sections of a speci-
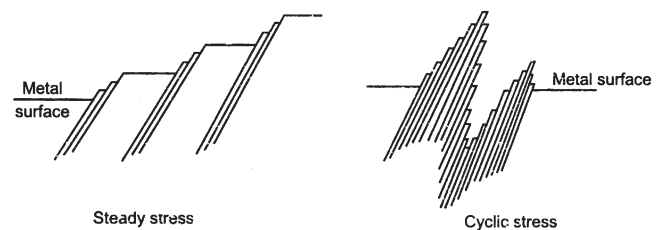

Fig. 2.   Geometry of slip at the material surface according to Forsyth [16].

---

[1] Microcrack initiation in certain alloys can also start at inclusions close to the surface, and even more subsurface due to residual stress distributions.

men are made. Several investigations employing sectioning were made in the 1950s and before. These showed that in most materials fatigue cracks are growing transcrystalline. Although the fatigue fractures looked rather flat as viewed by the unaided eye, it turned out that the crack growth path under the microscope could be rather irregular depending on the type of material. In materials with a low stacking fault energy (e.g. Cu- and Ni-alloys), cross slip is difficult and as a result cyclic slip bands are narrow and straight. Crack growth on a microscale occurs in straight segments along these bands. In materials with a high stacking fault energy (e.g. Al-alloys) cross slip is easy. Moreover, in the Al crystal lattice there are many slip systems which can easily be activated. As a consequence, slip lines are wider and can be rather wavy. Crack growth on a micro scale does not suggest that it occurs along crystallographic planes. As a result, fatigue on a microscale can be significantly different for different materials. The behavior is structure-sensitive, depending on the crystal structure (fcc, bcc, or hexagonal), elastic anisotropy of the crystalline structure, grain size, texture, and dislocation obstacles (e.g. pearlite bands in steel, precipitated zones in Al-alloys, twins, etc.). An extensive survey of the material fatigue phenomenon was recently presented in a book by Suresh [24].

2.2. Fractographic observations

The description of the fatigue mechanism in different materials was studied in the 1950s and in the following decades. A significant experimental milestone was the introduction of the electron microscope (EM), originally the transmission electron microscope (TEM) in the 1950s, and later the scanning electron microscope (SEM) in the 1970s. Microscopic investigations in the TEM are more laborious than in the SEM because either a replica of the fracture surface must be made, or a thin foil of the material. The thin foil technique is destructive and does not show the fatigue fracture surface, but information on the material structure can be obtained, such as forming of subgrains under cyclic loading. The thin foil technique requires a good deal of experimental expertise.

Investigations of fatigue fracture surfaces in the SEM are now a rather well standardized experimental option, which can indicate where the fatigue fracture started, and in which directions it was growing.[2] A fundamental observation was made with the electron microscope around 1960. Fractographic pictures revealed striations which could be correlated with individual load cycles.

By mixing of small and large load cycles in a fatigue test the occurrence of one striation per load cycle was proven by Ryder [17]. An example is shown in Fig. 3. The striations are supposed to be remainders of microplastic deformations at the crack tip, but the mechanism can be different for different materials. Several models for forming striations were proposed in the literature, two early ones in 1967 by Pelloux and Laird, respectively [25]. Because of microplasticity at the crack tip and the crack extension mechanism in a cycle, it should be expected that the profile of striations depends on the type of material. Terms such as ductile and brittle striations were adopted [22]. Striations could not be observed in all materials, at least not equally clearly. Moreover, the visibility of striations also depends on the severity of load cycles. At very low stress amplitudes it may be difficult to see striations although fractographic indications were obtained which showed that crack growth still occurred in a kind of a cycle-by-cycle sequence [26].

Striations have also shown that the crack front is not simply a single straight line as usually assumed in fracture mechanics analysis. Noteworthy observations on this problem were made by Bowles in the late 1970s [27,28] who developed a vacuum infiltration method to obtain a plastic casting of the entire crack. The casting could then be studied in the electron microscope. An example is shown in Fig. 4 which illustrates that the crack front is indeed a curved line and the crack tip is rounded.

Macroscopic shear lips, see Fig. 5, were well known for aluminium alloys from the early 1960s [29], but they were also observed on fatigue cracks in other materials [30–32]. The width of the shear lips increased for faster fatigue crack growth, and finally a full transition from a tensile mode fatigue crack to a shear mode fatigue crack
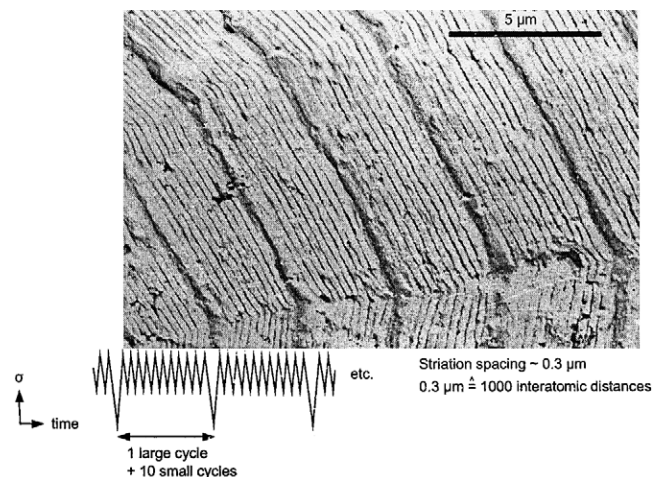

Fig. 3. Correspondence between striations and load cycles during fatigue crack growth in an Al-alloy specimen (picture Nat. Aerospace Lab., NLR, Amsterdam).

---

[2] Unfortunately, fractographic observations are not always made and reported in publications on fatigue tests although it is essential information of the test results.
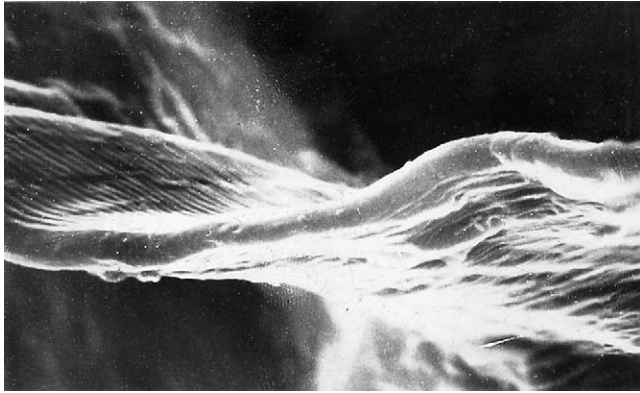
Fig. 4. SEM picture of a plastic casting of a fatigue crack (2024-T3 Al-alloy). Technique developed by Bowles [27]. Curved crack front and striations visible at the upper and lower fracture surface. Width of picture 16 μm.
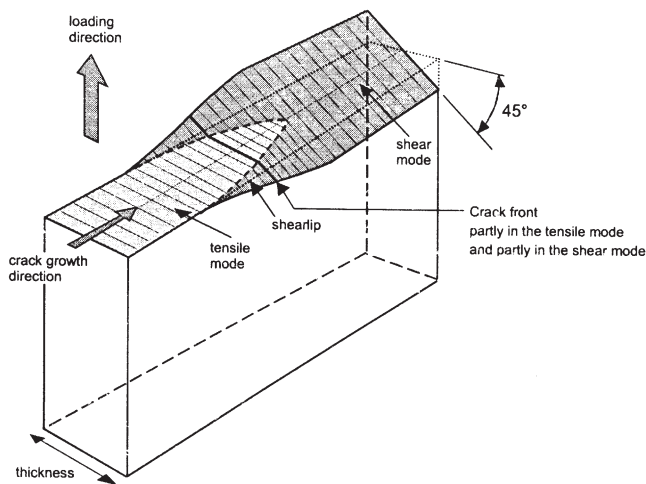

Fig. 5. Fatigue crack growth with a transition from tensile mode to shear mode.

can occur. The shear lips are a surface phenomenon because crack growth in the shear mode is not so constrained in the thickness direction. Shear lips are a macroscopic deviation from a mode-I crack assumed in a fracture mechanics analysis.

Fatigue cracks in thick sections can be largely in the tensile mode (mode I) because shear lips are then relatively small. However, the topography of the tensile mode area observed in the electron microscope indicates a more or less tortuous surface although it looks rather flat if viewed with the unaided eye. Large magnifications clearly show that the fracture surface on a microlevel is not at all a nicely flat area. It is a rather irregular surface going up and down in some random way depending on the microstructure of the material. It has also been shown for aluminium alloys that the roughness of the fracture surface depends on the environment [33]. An inert environment increased the surface roughness whereas an aggressive environment (salt water) promoted a more

smooth fracture surface. Similarly, shear lips were narrower in an aggressive environment and wider in an inert environment. These trends were associated with the idea that an aggressive environment stimulates tensile-decohesion at the crack tip, whereas an inert environment promotes shear decohesion. It should be understood that the crack extension in a cycle (i.e. the crack growth rate) depends on the crack growth resistance of the material, but also on the crack driving force which is different if deviations of the pure mode I crack geometry are present, e.g. shear lips and fracture tortuosity.

2.3. More about fatigue crack growth

In the 1950s, many investigators mentioned how early in the fatigue life they could observe microcracks. Since then it was clear that the fatigue life under cyclic loading consisted of two phases, the crack initiation life followed by a crack growth period until failure. This can be represented in a block diagram, see Fig. 6. The crack initiation period may cover a large percentage of the fatigue life under high-cycle fatigue, i.e. under stress amplitudes just above the fatigue limit. But for larger stress amplitudes the crack growth period can be a substantial portion of the fatigue life. *A special problem involved is how to define the transition from the initiation period to the crack growth period.*

It was in the early 1960s that the stress intensity factor was introduced for the correlation between the crack growth rate, da/dN, and the stress intensity factor range, ΔK. The first paper was published in 1961 by Paris, Gomez and Anderson [34], and it turned out to be a milestone publication. They adopted the K-value from the analysis of the stress field around the tip of a crack as proposed by Irwin [35] in 1957, another milestone of the application of fracture mechanics. The well-known general equation in polar coordinates for the stress distribution around the crack tip is:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f(\theta_{ij}) \tag{1}$$

with K as the stress intensity factor and the polar coordinates r and θ (see Fig. 7). Eq. (1) is an asymptotic solution which is valid for small values of r only, i.e. r≪a with 'a' as the crack length. The stress intensity factor is given by:
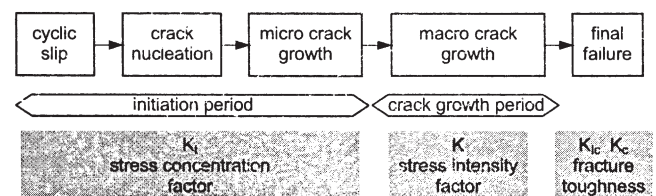

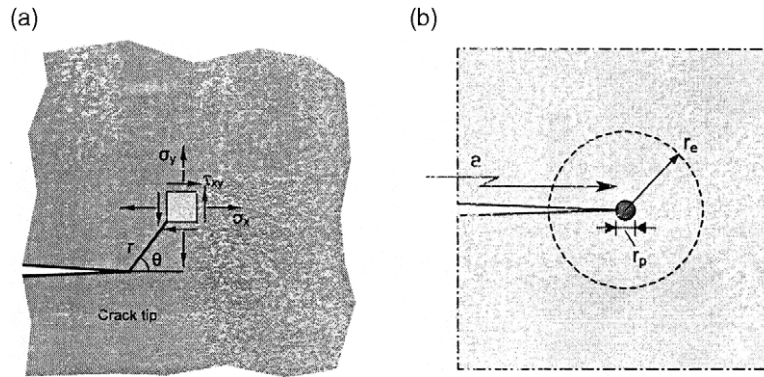Fig. 6. Different phases of the fatigue life and relevant factors.

(a) (b)

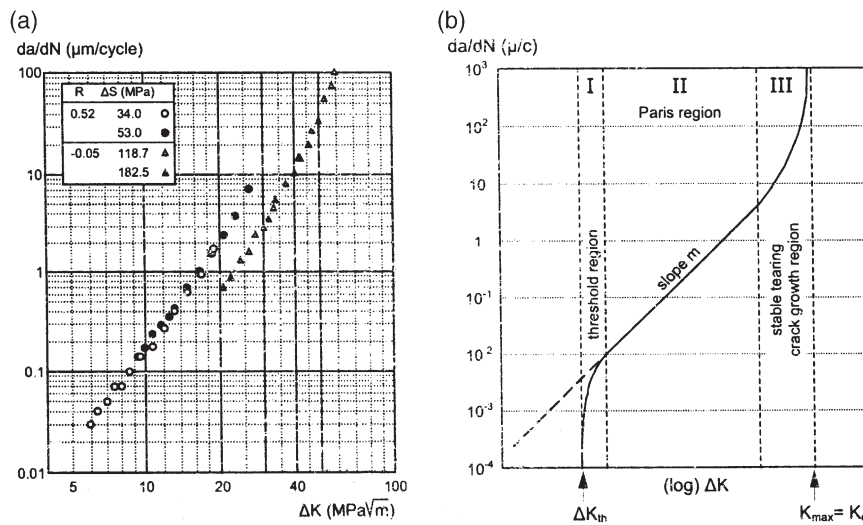Fig. 7.   (a) Crack tip with polar coordinates. (b) K-dominated zone.

(a) (b)

Fig. 8.   (a) Crack growth results presented as da/dN-K data. (b) Three regions of crack growth.

$$K = \beta S \sqrt{\pi a} \qquad (2)$$

with β as the geometry factor. The results of the crack growth tests of Paris et al. were expressed in terms of da/dN as a function of ΔK on a double log scale,[3] see Fig. 8a, which shows a linear relation between log(da/dN) and log(ΔK). Many more crack growth tests carried out later indicated the same trend which led to the well-known Paris equation:

$$da/dN = C\Delta K^m \qquad (3)$$

with C and m as experimentally obtained constants. The equation is a formal description of results of a fatigue crack growth experiment. At the same time, it must be recognized that fatigue crack growth is subjected to physical laws. In general terms, something is driving the crack extension mechanism which is called the crack-driving force. This force is associated with the ΔK-value.

The stress intensity factor is related to the strain energy release rate, i.e. the strain energy in the material which is available for producing crack extension. The relation to be found in textbooks is:

$$\frac{dU}{da} = \frac{K^2}{E*} \qquad (4)$$

with $E* = E$ (Young's modulus) for plane stress, and $E* = E/(1-v^2)$ for plane strain (v = Poisson's ratio). The strain energy looks like a characteristic variable for energy balances. The material response (da/dN) is characterized in Eq. (3), but the experimental constants C and m are not easily associated with physical properties of the material. However, the crack growth rate obtained is representing the crack growth resistance of the material.

Already by the 1960s it was clear that the correlation of da/dN and ΔK depends on the stress ratio R. This could be expected because an increased mean stress for a constant ΔS should give a faster crack growth while the R-value is also increased. Furthermore, results of crack growth tests indicated systematic deviations of the

---

[3] The enthusiasm to present the results in this new format was sometimes so large that authors unfortunately did not mention the range of crack sizes covered in the experiments.

Paris equation at relatively high and low ΔK-values. It has led to the definition of three regions in da/dN-ΔK graphs, regions I, II and III respectively, see Fig. 8b. Obvious questions are associated with the vertical asymptotes at the lower ΔK boundary of region I and the upper ΔK boundary of region III. The latter boundary appears to be logical because if K_{max} exceeds the fracture toughness (either K_c or K_{Ic}) a quasi-static failure will occur and fatigue crack growth is no longer possible. It still should be recognized that the K_{max} value causing specimen failure in the last cycle of a fatigue crack growth experiment may well be different from K_c or K_{Ic} measured in a fracture toughness test.

From the point of view of fracture mechanics, the occurrence of a lower boundary in region I is not so obvious. As long as a K-value can be defined for the tip of a crack, a singular stress field should be present and micro-plasticity at the tip of the crack should occur. So, why should the crack not grow any more; for which physical reason should there be a threshold ΔK-value (ΔK_{th}).

New ideas on ΔK_{th} were associated with observations on so-called small cracks. These cracks occur as microcracks in the beginning of the fatigue life starting at the material surface or just subsurface. The first relevant paper was published by Pearson [36] in 1975 who observed that small surface cracks were growing much faster than large macro cracks at nominally similar ΔK-values. It was confirmed in several investigations that microcracks could grow at low ΔK-values, whereas macrocracks did not grow at these low ΔK-values where $\Delta K < \Delta K_{th}$. Illustrative data of Wanhill are shown in Fig. 9 [37]. The small-crack problem became a well recognized subject for further research. Various crack growth barriers offered by the material structure (e.g. grain boundaries, pearlite in steel, phase boundaries in general) could be significant for microcracks [38] whereas they were less relevant to macrocrack growth. As a result, considerable scatter was observed in microcrack growth rates, see Fig. 9. Moreover, the barriers affecting microcrack growth could be quite different for different materials. Although proposals for fracture mechanics predictions of the growth of microcracks were presented in the literature, the publications about this issue in the last decades of the previous century were not always convincing. Actually, it should be recognized that the K-concept for such small cracks in a crystalline material becomes questionable. The plastic zone is a slip band and its size is not small compared to the crack length of the microcrack. They may even have a similar size.

Another question about the ΔK_{th} concept applies to macrocracks. Why do large cracks stop growing if $\Delta K < \Delta K_{th}$? A formal answer to this question is because the crack driving force does not exceed the crack growth resistance of the material. At low K-values the crack
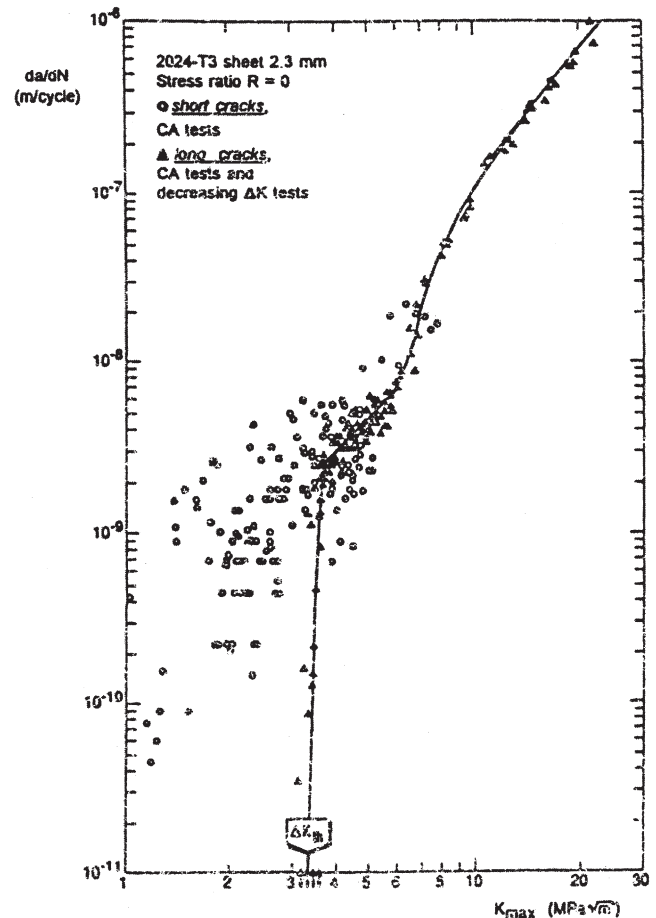

Fig. 9.   Crack growth results of Wanhill for large cracks and small microcracks [37].

driving force is low which affects the crack front microgeometry, the crack front becomes more tortuous, and also the crack closure mechanism is changing [39]. It may then occur that the crack driving force is just no longer capable of producing further crack growth.

A concept to be discussed here is the occurrence of crack closure, and more specifically plasticity induced crack closure. In the late sixties [40,41], Elber observed that the tip of a growing fatigue crack in an Al-alloy sheet specimen (2024-T3) could be closed at a positive stress (tensile stress).[4] Crack opening turned out to be a non-linear function of the applied stress, see Fig. 10. During loading from $S = 0$ to $S = S_{op}$ the crack opening displacement (COD) is a non-linear function of the applied stress. For $S > S_{op}$ the behavior is linear with a

---

[4] Elber carried out a crack growth test on a specimen with a central crack. After substantial crack growth, but before failure, he wanted to open the crack by saw cutting of the unfailed ligaments. After he (himself) cut the first ligament, he observed that the specimen was distorted in a strange way. Elber understood that this had to be due to plastic deformation left in the wake of the fatigue crack. He then made his well-known crack closure measurements (Fig. 10) which led to Eq. (7).
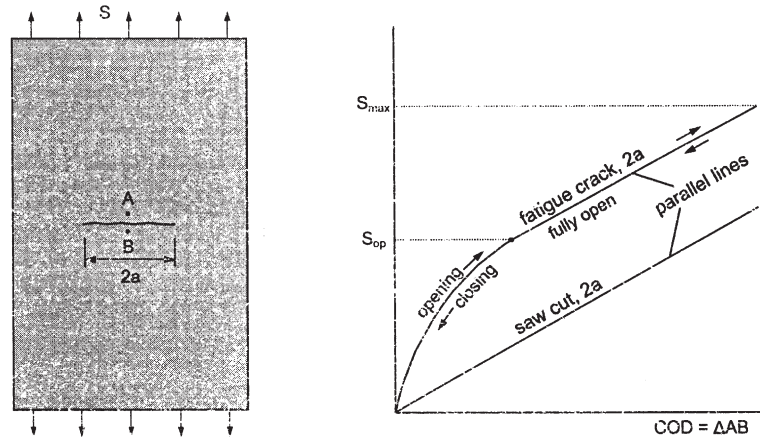
Fig. 10.   Measurement of the crack opening displacement (COD) showing the occurrence of plasticity induced crack closure at a positive stress according to Elber [40,41].

slope corresponding to the specimen compliance with a fully opened crack. The same non-linear response was observed during unloading. During the non-linear behavior the crack is partly or fully closed due to plastic deformation left in the wake of the growing crack. Elber argued that a load cycle is only effective in driving the growth of a fatigue crack if the crack tip is fully open. He defined the effective ΔS and ΔK as:

$$\Delta S_{eff} = S_{max} - S_{op} \text{ and } \Delta K_{eff} = \beta \Delta S_{eff} \sqrt{\pi a} \quad (5)$$

(β is the geometry factor). He then assumed that the crack growth rate is a function of ΔK_{eff} only.

$$da/dN = f(\Delta K_{eff}) \quad (6)$$

Elber found that the crack opening stress level depends on the stress ratio for which he proposed the relation:

$$U = \frac{\Delta S_{eff}}{\Delta S} = f(R) = 0.5 + 0.4R \text{ (for 2024} \quad (7)$$

$$- T3 \text{ Al} - \text{alloy})$$

This relation is an empirical result. Moreover, Elber proposed that the relation should be independent of the crack length. The Elber approach was carried on in later investigations, partly because it was attractive to present crack growth data of a material for various R-values by just one single curve according to Eq. (6). It turned out that the relation in Eq. (7) could be significantly different for other materials which is not surprising because the cyclic plastic behavior depends on the type of material. In the 1980s, the crack closure concept was much welcomed by investigators on crack growth models for fatigue under VA loading [42].

3. The S-N curve and the fatigue limit

3.1. Aspects of the S-N curve

Wöhler had already carried out experiments to obtain S-N curves in the 19th century. For a long time such curves were labeled as a Wöhler curve instead of the now more frequently used term S-N curve. In the 20th century numerous fatigue tests were carried out to produce large numbers of S-N curves. In the beginning, rotating beam tests on unnotched specimens were popular because of the more simple experimental facilities available in the early decades. The significance of testing notched specimens was recognized, especially by engineers. Fatigue-testing machines for loading in tension, torsion and bending were available before 1940. The excitation of cyclic loads occurred by mechanical or hydraulic systems. High frequencies were obtained with resonance machines. Fatigue became more and more recognized as a problem to be considered for various small and large industrial products in view of economical reasons. Apart from basic research on unnotched specimens, many test series were also carried out on notched specimens. The tests were performed with a constant mean stress and a constant stress amplitude (referred to as constant-amplitude tests, or CA tests). The S-N curves should give useful information about the notch and size effect on the fatigue life and the fatigue limit. Initially, the fatigue life N was plotted on a logarithmic scale in the horizontal direction, and the stress amplitude on a linear scale in the vertical direction. Many curves can be found in the literature and collections of these curves have been published as data banks while commercial software also contains this type of information.

For low stress amplitudes, the S-N curve exhibited a

lower limit which implies that fatigue failures did not occur after high numbers of load cycles, see Fig. 11. The horizontal asymptote of the S-N curve is called the fatigue limit (in some publications the name endurance limit is used). The fatigue limit is of practical interest for many structures which are subjected to millions of load cycles in service while fatigue failures are unacceptable. The fatigue limit is considered in more detail later.

At the upper side of the S-N curve (large stress amplitudes) another horizontal asymptote appears to be present. If failure did not occur in the first cycle, the fatigue life could be several hundreds of cycles. Such fatigue tests were not easily carried out on older fatigue machines because adjusting the correct load amplitude required too many cycles. A real breakthrough for fatigue testing equipment occurred in the 1950s and 1960s when closed-loop fatigue machines were introduced employing a feedback signal from the specimen to monitor the load on the specimen. With this technique, the fatigue load could be adjusted by a computer-controlled system. Furthermore, if S-N curves were plotted on a double-logarithmic scale, the curves became approximately linear (the Basquin relation).

The interest for short fatigue lives is relevant for structures with a load spectrum of small numbers of severe load cycles only (e.g. high-pressure vessels). In the 1960s, this has led to ϵ-N curves. Instead of applying a stress amplitude to a specimen, a constant strain amplitude is maintained in the critical section of the specimen. The problem area was designated as 'low cycle fatigue' (Fig. 11) which actually implies that macro plastic deformation occurs in every cycle. It turned out that the ϵ-N curve in the low-cycle regime, again plotted on a double log scale, was a linear function:

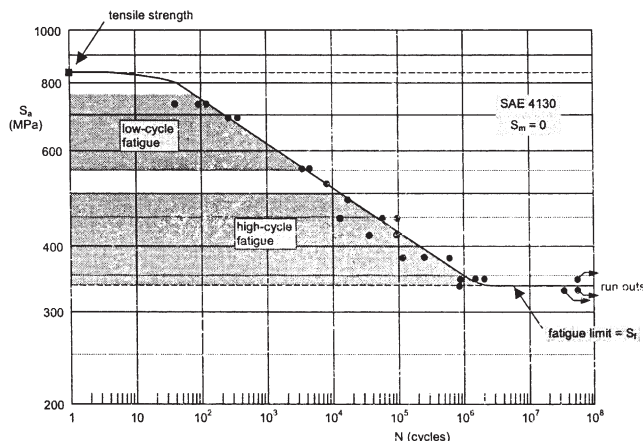$$\epsilon_a N^\beta = \text{constant} = C \tag{8}$$

Fig. 11. Fatigue test results of unnotched specimens of a low alloy steel (NACA TN 2324, 1951). Regions of low-cycle and high-cycle fatigue.

The equation is known as the Coffin–Manson relation [43,44].

At lower stress amplitudes macro plastic deformation does not occur and the fatigue phenomenon is labeled as 'high-cycle fatigue', see Fig. 11. It is assumed that cyclic deformations on a macro scale are still elastic.

Questions to be considered with respect to the S-N curve are: (i) Is the fatigue phenomenon under high-cycle and low-cycle fatigue still a similar phenomenon, and (ii) what is the physical meaning of the fatigue limit? With respect to the first question, it should be pointed out that crack initiation and crack growth are both significant for the fatigue life. The crack initiation is easily affected by a number of material surface conditions (surface roughness, surface damage, surface treatments, soft surface layers, surface residual stress). These conditions are important for high-cycle fatigue and the fatigue limit, but they are generally overruled during low-cycle fatigue where plastic deformations at the material surface will occur anyway. Moreover, macro crack growth under low-cycle fatigue is rather limited because small cracks will already induce complete failure at the high stress levels. This implies that low-cycle fatigue is important for special problems where large plastic deformations cannot be avoided. Low-cycle fatigue problems are also often associated with high-temperature applications. However, during high-cycle fatigue significant macrocrack growth occurs. At still lower stress amplitudes the physical meaning and the engineering significance of the fatigue limit have to be considered.

3.2. The fatigue limit

The formal definition of the fatigue limit (see footnote 5) appears to be rather obvious. It is the stress amplitude for which the fatigue life becomes infinite in view of the asymptotic character of the S-N curve. However, fatigue tests must be terminated after a long testing time. If it occurs after 10^7 cycles (see papers in ref. [45]), the no-failure stress level need not be a fatigue limit. Actually, it was also labeled as an endurance limit associated with a certain high number of cycles.

From an engineering point of view, it appears to be more logical to define the fatigue limit as the highest stress amplitude for which failure does not occur after high numbers of load cycles. This definition should cover situations where fatigue failures are unacceptable, e.g. in various cases of machinery. Design stress levels must then remain below the fatigue limit which emphasizes the significance of the fatigue limit as a material property.

The two definitions do not refer to physical aspects of the fatigue phenomenon. A more physically based definition should be associated with microcracks. If microcracks are not initiated, a fatigue failures does not

occur. However, it is possible that cyclic slip does occur at stress amplitudes just below the fatigue limit. A few microcracks can be initiated but microcrack growth may be stopped. Non-propagating small cracks got some interest in the late 1950s and early 1960s. Noteworthy experiments were carried out by Frost and associates [46] on notched specimens with high K_t-values. Actually, in such cases crack growth can stop because the crack driving force is not large enough to continue crack growth away from the material surface while the stress amplitude was sufficient to nucleate a small crack at the free surface. But non-propagating microcracks could also occur in unnotched specimens at stress amplitudes slightly below the fatigue limit due to crack growth barriers, e.g. grain boundaries, or just because of insufficient cyclic slip for further growth of the microcrack. It then seems that the fatigue limit should be defined as the threshold stress amplitude to take care of microcrack nucleation and subsequent growth to a macrocrack. Predictions on the fatigue limit of components is an important engineering problem which implies that the physical concept of this property should be understood. This is necessary for evaluating prediction models on the effect of the notch geometry on the fatigue limit (effects of K_t, notch size and stress gradients).

Because the fatigue limit is an important material property from an engineering point of view, the experimental technique to determine this property was an important topic in the previous century. Fatigue tests to determine a fatigue limit must be carried on to high numbers of cycles. Such tests are very time consuming and thus expensive, especially if a reasonably large number of tests is carried out in view of information about the statistical variability of the fatigue limit. Statistical procedures for doing so have been standardized, for instance the Staircase method and the Probit method [47]. Because of the problem of testing time, it is not surprising that investigators have been looking for a physical relation between the fatigue limit and some other material properties which can be measured in a short time. At best, it could be hoped that some emission effect of the material is associated with the microcrack initiation process. However, from the previous discussion on the fatigue phenomenon with respect to crack initiation and crack growth barriers of non-propagating microcracks it must be concluded that such correlations are illusive. An approximate determination of the fatigue limit can be done with a small number of specimens using so-called step tests in which the stress amplitude is increased in small steps [48].

4. Predictions and fatigue damage

4.1. The engineering need for prediction models

The prediction of fatigue properties of structures and avoiding structural fatigue were recognized as engineer-

ing problems in the early decades of the 20th century. It was understood that high stress concentrations could be harmful and should be avoided. The significance of stress concentration factors was known before 1950 and designers realized that the fatigue performance of a structure was dependent on improved detail design. The title of a book by Heywood 'Designing against fatigue' [49] was characteristic for the engineering fatigue problem. Various models were developed for the prediction of notch and size effects. Initially, the aim was to derive fatigue properties of notched elements from fatigue properties on unnotched specimens. The proposed models included a good deal of empirism. One specific goal was to predict the fatigue limit, an important fatigue property for many products of the industry. In the 1960s and afterwards a need was also felt to predict fatigue crack growth, especially for aircraft structures in view of fail-safe properties, service inspections and safety in general. But it was also a problem for other structures, such as welded structures and pressure vessels. Prediction models on crack growth were much stimulated by the introduction of the stress intensity factor. Still another fatigue problem was associated with load spectra containing load cycles of various magnitudes, or in other words, fatigue under variable-amplitude (VA) loading. If fatigue cycles above the fatigue limit occur, crack initiation can not be avoided and a finite life is possible. A need for predictions on fatigue under VA loading was present. Several prediction problems can thus be defined. A practical problem also associated with VA-loading was the question of how long old structures could still be used without running into fatigue problems. In the second half of the previous century, this question was raised for old bridges, quite often bridges built in the 19th century. The question was whether fatigue problems should be anticipated or whether the bridge should be replaced by a new one. Bridges were often more intensively loaded by heavy traffic than previously expected in the design process long ago. A similar problem occurs for old aircraft. Some aircraft of military fleets were designed and built in the 1960s, but these aircraft are still meeting the present performance requirements. For economical reasons, some aircraft types are planned to be used up until 2040. Also, several civil passenger aircraft are used beyond the life time for which they were originally designed (often 20 years). The term 'aging aircraft' has been introduced for such aircraft. Some aircraft with fatigue problems in the 1980s for which special regulations were introduced were labeled as 'geriatric' aircraft [50].

Questions emerging from the above picture are associated with the reliability and accuracy of prediction models and the physical concept of fatigue damage. Prediction problems can be defined in two categories. The similarity concept (sometimes called the similitude concept) is characteristic for the first category. Fatigue

damage accumulation is at the base of the second category.

4.2. Predictions based on the similarity of conditions (CA-loading)

The physical argument of the similarity approach is:

Similar conditions, applied to similar systems, should produce the same consequences.

This physical principle is the basis of many predictions of properties of materials and structures. It can also be applied to fatigue prediction problems. At the same time, it should be realized that this physical principle does not necessarily imply that the physical mechanism of the fatigue phenomenon should be understood.

For fatigue of notched elements the similarity concept implies: similar stress cycles applied to an unnotched specimen and to the material at the root of a notch in a notched specimen will give the same crack initiation life. It is not essential to know how the initiation occurs. Of course the requirement of similar systems implies that the unnotched and notched specimen should be of the same material. However, some other aspects which may violate the similarity are easily recognized. A significant aspect is that the stress cycle in the unnotched specimen is present in a large volume of the material with a relatively large area of surface material. In the notched specimen, the stress cycle of the peak stress (K_t·nominal stress) at the notch root is present in a relatively small volume of the material with a relatively small surface area. More differences can be mentioned, e.g. the surface roughness of the material is not necessarily the same for the unnotched and notched specimen because of different ways of machining the specimens.

Neuber published a famous book on calculations of stresses around notches in 1937 [51]. He found it rather disappointing to see that the reduction factor of the fatigue limit of notched steel specimens was much less than the K_t-value obtained with his calculations. Empirical equations were then presented in the literature [52–54] to account for the deficiencies of the similarity approach. An important variable in these equations is the root radius of the notch in order to account for stress gradient effects in the notched elements. Although the approach of the empirical equations was not fully rational [11], reasonable estimates of the fatigue limit of notched elements could be obtained.

The similarity concept for the prediction of fatigue crack growth is different from the concept for the notch problem. It now reads:

The same ΔK-cycle applied to different cracks should give the same crack growth rate.

In other words, if the same ΔK applies to a crack in a specimen and to a crack in a structure, the same da/dN should be obtained. This similarity concept was replaced later by requiring similar ΔK_{eff}-cycles in order to account for the stress ratio effect associated with crack closure as discussed before. Also in the case of fatigue crack growth it is not really necessary to know the physical crack extension mechanism, e.g. whether it occurs by shear decohesion or tensile decohesion. But in this case it may also be questioned whether the same ΔK cycle is a valid argument to prescribe similar conditions applied to the same system. The system in this case is the material around the crack tip. It is well-known that the K-value does not describe the stress distribution far away from the crack tip, but more interestingly, this is also true for the crack tip plastic zone, i.e. in the close proximity of the crack tip where the fatigue crack extension occurs. Eq. (1) is no longer valid in the plastic zone, and the plasticity will cause some stress redistribution. Around the crack tip, a zone with radius r_e is considered in Fig. 7b. The size of this zone is selected to be significantly larger than the plastic zone ($r_e > r_p$) but still considerably smaller than the crack length ($r_e > a$). As a result, the plastic deformation at the crack tip will have a marginal effect on the stress distribution at a distance r_e from the crack tip (de Saint Venant's principle). As a consequence K can still be characteristic for the stresses applied to the r_e-zone, which is called *the K-dominated zone* [55]. This zone is the system to be considered. Similar ΔK-cycles on the system then imply similar stresses on the K-dominated zones. As a consequence the same small crack tip plastic zones are obtained, and similar crack growth rates, da/dN, may be expected.

Apparently ΔK can be used for the prediction of the crack growth rate in a structure if da/dN for similar ΔK-values is known from experiments on crack growth specimens. This fracture mechanics application has extensively been explored and confirmed in many cases in the last decades of the previous century. Handbooks with K-values for various geometries were published [56–58] and data are also included in commercial software. Moreover, K-values for part through cracks and cracks with curved crack fronts can be calculated with modern FE techniques.

Limitations occur for high values of K_{max} and ΔK when plastic zones at the crack tip are large. Because the K-concept is essentially based on elastic material behavior, an elastic K-dominated zone is no longer a realistic concept.

Shear lips occur at the free surface of the material during fatigue crack growth, (see Fig. 5). Again this need not upset the similarity concept for crack growth predictions if a K-dominated zone can still be assumed to be relevant. The shear lips will then be present in the same way in both systems, the specimen and the structure. It

has been shown that predictions can still be reasonably accurate.

4.3. Predictions based on fatigue damage accumulation (VA-loading)

As early as 1923, Pålmgren [59] published the hypothesis which is now generally known as the Miner-rule or the linear cumulative damage hypothesis. According to Pålmgren applying n_i times a cycle with a stress amplitude $S_{a,i}$ and a corresponding fatigue life N_i is equivalent to consuming a portion of n_i/N_i of the fatigue life. Failure occurs when 100% of the fatigue life is consumed. Failure thus occurs when:

$$\sum \frac{n_i}{N_i} = 1. \tag{9}$$

Pålmgren did not give any physical derivation for the rule. He was in need for an estimate of the fatigue life under VA-loading, and he adopted the most simple assumption for fatigue damage accumulation. Langer, being unaware of the Pålmgren paper, postulated the same rule in 1937 [60], but he introduced a refinement by dividing the fatigue life into two phases: a crack initiation life followed by a crack growth life until failure. According to Langer, Eq. (9) must be applied separately to both periods with N_i being the crack initiation life in the first period and the crack growth life in the second period. Miner in 1945 [61] referred to the publication of Langer, but he restricted his analysis to the crack initiation life of small specimens tacitly assuming that the fatigue life until failure could be considered to be approximately the crack initiation life. Miner introduced the idea that fatigue damage is the consequence of work absorbed by the material which was assumed to be proportional to the number of cycles. That is why the Miner rule, Eq. (9), is called the linear cumulative damage rule. It is important to note that this concept describes the fatigue damage quantitatively with one single parameter. If the damage parameter is denoted by D, it implies:

$$D = n/N. \tag{10}$$

The Miner concept implies that this linear function (see Fig. 12a) is applicable to any cyclic stress level. Later it was suggested that the function could be non-linear, see Fig. 12b.

$$D = f\left(\frac{n}{N}\right) \tag{11}$$

This might appear to be more realistic. However, it was not always realized that a non-linear damage equation, if the equation is independent of the cyclic stress, still leads to the Miner rule of Eq. (9) [62]. Only if $D = f(n/N)$ depends on the stress cycle, see the two curves in Fig. 12c, deviations from the Miner rule are possible.

The single damage parameter definition still offers another problem. Negative damage is impossible. Already in the 1950s it was known that a single high load (frequently labeled as an overload, OL) can increase the fatigue life at a lower stress level and thus is causing negative damage in terms of life time consumed. However, a damage curve with negative damage is physically unacceptable. Going from negative damage to later positive damage implies passing the zero damage level corresponding to the pristine condition present before the test was started. This cannot be true.

It was understood in the 1950s that an overload in tension applied to a notched specimen could lead to plastic deformation at the root of the notch. As a result, compressive residual stresses are introduced which delays crack initiation and subsequent crack growth.[5] Clarence Smith around 1960 [64] suggested that the Miner rule could still be adopted after a high load if the residual stress was accounted for by a change of the mean stress at the critical location at the notch. Later in the early 1970s [65,66] the strain history prediction model was developed which appeared to be promising for low-cyclic fatigue under VA loading when plasticity occurred in many cycles of a load–time history. With this model the strain history at the root of the notch was derived from the load history by using an analytical relations between the plastic strain and the stress as proposed by Neuber for notch root plasticity [67]. However, the Miner rule was still adopted for calculating the accumulated fatigue damage. Instead of S-N curves $\epsilon_a - N$ were used, i.e. the Coffin–Manson relation discussed earlier.

An important short-coming of the Miner rule for VA life predictions is associated with load cycles of amplitudes below the fatigue limit. According to the Miner rule, such cycles are non-damaging because in n/N the value of $N = \infty$. This is physically unacceptable for a VA load history. Cycles with amplitudes larger that the fatigue limit can initiate a fatigue crack, and later cycles with amplitudes below the fatigue limit can propagate the crack and thus become damaging. A simple procedure to account for this phenomenon is to extrapolate the S-N curve below the fatigue limit. A noteworthy proposal was made by Haibach [68]. He started from the idea that the S-N curve is a linear function in a double-logarithmic plot, which is known as the Basquin equation:

$$S_a^k \cdot N = \text{constant} \tag{12}$$

with the empirical constant k (slope factor) as the negative inverse slope of the $\log(S_a)/\log(N)$ plot. Haibach proposed to extend the S-N curve with a second linear

---

[5] Already in 1943 Heyer [63] attributed the increased fatigue life after application of a high preload to compressive residual stresses. His paper in German was written during World War II.
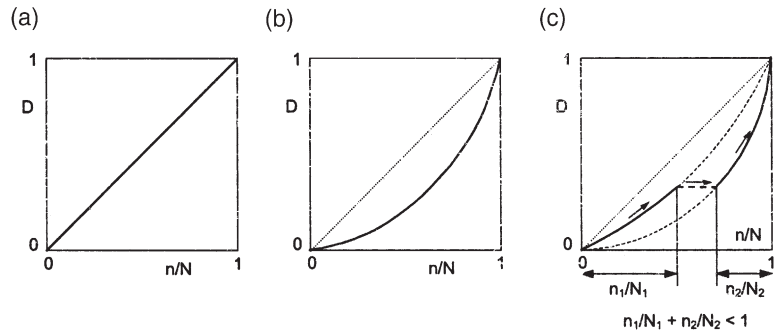
Fig. 12.   Fatigue damage functions if the damage is defined by a single parameter D.

part as shown in Fig. 13 with a slope depending on k of the first part (slope factor k′ = 2k + 1). Another suggestion to extend the S-N curve was made for welded joints [69] as also indicated in Fig. 13.

Although the extended S-N curves may give a better prediction of the fatigue life, and anyway a more conservative one, it does not imply that the Miner rule is given a physical derivation. Numerous test series have shown that statistically similar VA load histories can give significantly different fatigue lives, whereas the Miner rule predicts the same life. Already around 1960 this was obvious from so-called programme fatigue tests on notched specimens with different load sequences. Illustrative results of Hardrath et al. [70,71] are shown in Fig. 14. According to the Miner rule, all sequences should have given the same fatigue life instead of variations with factors of about 3. Moreover, large differences from $\Sigma n/N = 1$ were observed. Many more similar deficiencies were obtained later, especially for high strength materials, see the surveys in Refs. [72,73].

4.3.1. Fatigue damage description

Shortcomings of the Miner rule are primarily due to the primitive fatigue damage concept adopted. It implies that fatigue damage is indicated by one single damage parameter. In reality, all changes inside the material induced by fatigue load cycles are part of the fatigue damage. Those changes were studied in various publications. In principle, a description of fatigue damage
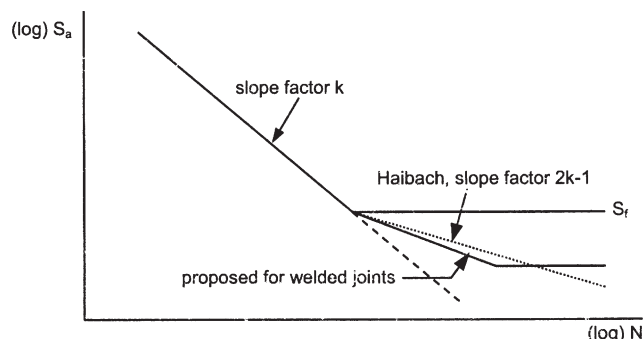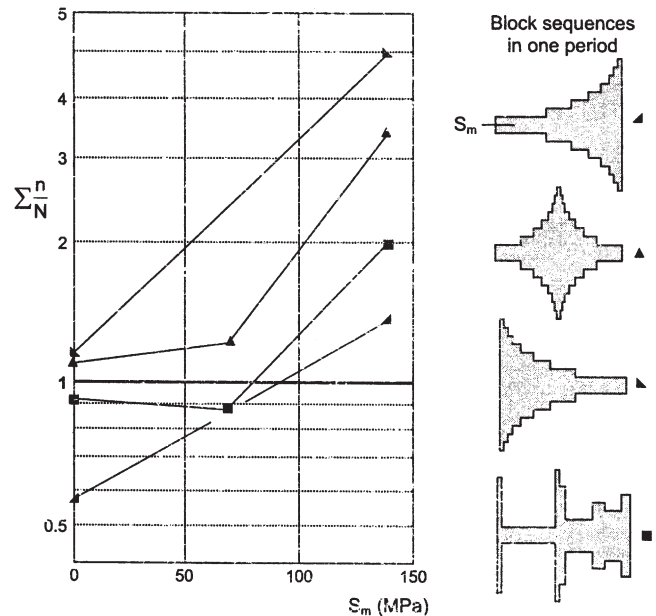

Fig. 14.   The effect of overloads (OL) on fatigue crack growth in sheet specimens of the 2024-T3 Al-alloy [74].

should cover several features affected by the fatigue load, such as:

- micro- and macro-plastic deformation
- residual stress
- the size and the shape of fatigue cracks (crack geometry)
- fractographic features (crack path in the material structure, shear lips)
- plastic deformation in the wake of the crack (crack closure).

Spatial distributions of these aspects are involved [6]

Fig. 13.   S-N curve extrapolated below the fatigue limit.

_____

[6] It is noteworthy that the crack tip plastic zone is frequently assumed to have only one characteristic dimension, which is the width of the zone in the crack growth direction. In several materials the zone has a butterfly shape. Furthermore, the size of the plastic zone is supposed to be equal to a constant times $(K_{max}/S_{0.2})^2$. The choice of $S_{0.2}$ is arbitrary. If $S_{0.1}$ is used the calculated size is larger.

which implies that a quantitative description is not a realistic option, although the meaning of the aspects is qualitatively understood. In the literature, rather drastic simplifications have been made to arrive at improved prediction rules for fatigue damage accumulation. The success is limited for fatigue life predictions for VA loading. However, for fatigue crack growth (macrocracks) some progress has been made, although simplifications were still necessary.

4.3.2. Fatigue crack growth under VA loading

Fatigue crack growth (macrocracks) under variable amplitude loading was extensively investigated in numerous test programmes. The cumulative damage problem is then associated with the extension of the cracks in subsequent cycles of the load–time history. The promising application of the K-concept to predictions on fatigue crack growth under CA loading was drastically upset by the first experiments with overloads (OL's) in CA test carried out around 1960 [74,75]. Illustrative results are given in Fig. 15. Three OL's in this figure induced highly retarded crack growth. Originally this was attributed to compressive residual stress in the crack tip plastic zone of an OL, while later is was related to crack closure in the plastic zone leading to smaller effective stress ranges. Actually, there is no contradiction between the two explanations because both are associated with the plastic deformation in the crack tip zone.

The large effect of high loads on fatigue crack growth has stimulated much research, both experimental investigations as well as analytical studies. The significance of crack tip plasticity was easily recognized and it obviously suggested that the plastic zone size must be important for crack growth retardations. Because the plastic zone size depends on the state of stress (plane strain or plane stress or intermediate situations), the retardation of the growth of a through crack after an OL

should depend on the material thickness and the material yield stress. This has been amply confirmed by experimental results, starting in the mid-seventies (e.g. Refs. [76,77]). More detailed observations also indicated that the maximum retardation of the crack growth rate after an overload did not occur immediately but required some penetration of the crack tip into the OL plastic zone (so-called delayed crack growth retardation).

The first analytical models of crack growth under VA loading were based on plastic zone sizes. The Willenborg model [78] and the Wheeler model [79] are two notable examples published in the early 1970s. They are now considered to be rather primitive. A second generation of crack growth prediction models for VA loading was based on plasticity induced crack closure. The Elber crack closure concept was used and the stress opening stress level S_{op} had to be predicted in a cycle-by-cycle calculation. Crack growth is then predicted from:

$$a = a_0 + \sum \Delta a_i \text{ with } \Delta a_i = (da/dN)_i = f(\Delta K_{eff,i})$$

$$\text{and } \Delta K_{eff,i} = C_i(S_{max,i} - S_{op,i})\sqrt{\pi a_i}$$

$$(13)$$

The crack extension Δa_i in cycle i is a function of K_{eff} in that cycle. The value of $\Delta K_{eff,i}$ depends on the imposed $S_{max,i}$ of the load history and the predicted $S_{op,i}$. The geometry factor C_i depends on the current crack length a_i. The crack opening stress level $S_{op,i}$ must be predicted by a crack growth model taking care of the effect of plastic deformation left in the wake of the crack as a residue from previous load cycles. As an illustration, Fig. 16 gives a sample of a VA load history with varying S_{op}-values. Some models for plasticity induced crack closure under VA loading were proposed in the literature in the 1980s (for a survey see [42,80]). The most well-known model is the CORPUS model of Ary de Koning [81]. In this model, the effect of the previous load history on S_{op} is physically associated with humps on the surface of the fatigue crack as remainders of previous high loads. Several details of the model include the transition of plane strain to plane stress and the amplification effect
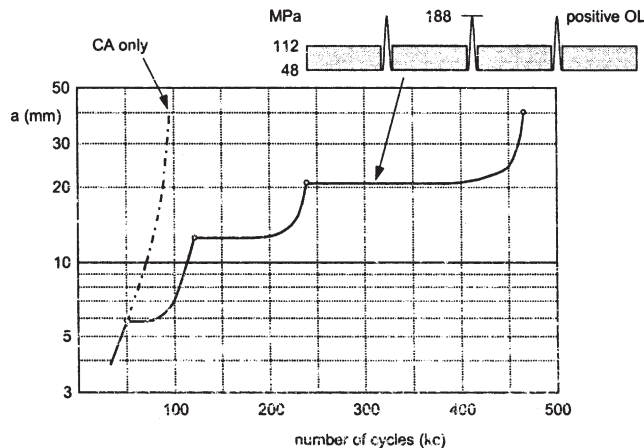

Fig. 15. Results of block-programme fatigue tests of NASA [71]. Notched specimens, $K_t = 4$, material 7075-T6. Different block sequences and effect of mean stress.
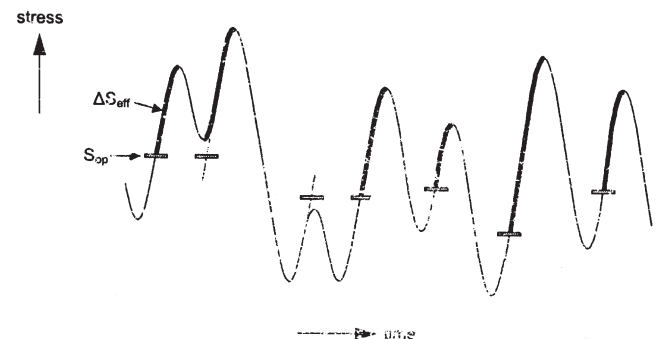

Fig. 16. Variable-amplitude loading with a cycle-by-cycle variation of the crack opening stress.

of frequently occurring high loads. The CORPUS model includes some empirical constants and fairly accurate crack growth predictions for rather complex flight-simulation load histories were obtained [82]. An example of such a load history is shown in Fig. 17.

The crack opening stress, S_{op}, in the CORPUS model was still obtained with an empirical relation employing previous maxima and minima of the load history, a relation with a similar character as U(R) discussed before. In a following generation of crack growth prediction models, crack closure was still considered to be the leading mechanism to arrive at effective stress range. However, S_{op} was no longer obtained from an empirical function. The Dugdale strip yield model [83] was adopted to calculate the plastic deformation in the crack tip zone and the plastic deformation left in the wake of the crack. Algorithms were developed [84–88] to calculate the plastic deformations and to determine the crack opening displacements from which the crack opening stress level is obtained. The models are rather complex, due to the non-linear material behavior, reversed plasticity under compressive stress, and the iterative character of the calculations. Although these strip yield models are more realistic, the problem of the plane stress to plane strain transition is still present. Moreover, it remains difficult to cover some aspects such as the 3D character of crack closure. Accurate predictions thus remain problematic, the more so for part through cracks.

5. Load spectra

Information on the load–time history of a structure in service is collected in load spectra. As indicated in Fig. 1, a load spectrum is a necessary input for the prediction of fatigue properties of a structure or service-simulation fatigue tests. In spite of this link, the literature on load spectra suggests the topic to be more or less a separate discipline. Developments on load spectra problems

started in aeronautics because of some early fatigue disasters. In Germany a Lufthansa aircraft crashed in 1927 with six fatalities. The accident was due to a fatigue failure [5]. It started significant fatigue research in the 1930s, notably by Gassner [89] and Teichmann [90]. It was understood that aircraft wings were dynamically loaded during flying in turbulent air which resulted in numerous load cycles with quite variable amplitudes (gust loads). The need for measuring these loads was well recognized. A strain measurement technique was developed for this purpose in the early 1930s [91]. The load–time history was scratched with a diamond on glass and the history was analyzed under the microscope, an advanced technique for that time. Teichmann recognized that a statistical description of the load–time history could be done in different ways. Considering the maxima and minima as the relevant data of a load–time history, the statistical data could be restricted to counting these peak values in specified intervals, which leads to a one-dimensional spectrum (1D). However, Teichmann also defined a statistical counting of ranges between successive maxima and minima and considering different mean values of these ranges, which then leads to a two-dimensional spectrum (2D). Teichmann was aware of the fact that information on the sequence of the loads was lost in the counted distribution of occurrences. Gassner introduced the so-called programme fatigue test [89], see Fig. 18, a fatigue test with a programmed sequences of load cycles with different amplitudes representing the amplitudes occurring in service. A more realistic simulation of gust loads on an aircraft wing as illustrated by Fig. 19 was still impossible with the existing fatigue test machines at that time.

In later years, other load measurement techniques were developed as well as statistical evaluation methods for the data obtained. Again most of the developments were initiated for aircraft fatigue problems. A well-known technique in the 1950s and afterwards was based on measuring accelerations in the centre of gravity of the aircraft from which the loads on the structure were calculated. A counting accelerometer, also called 'the
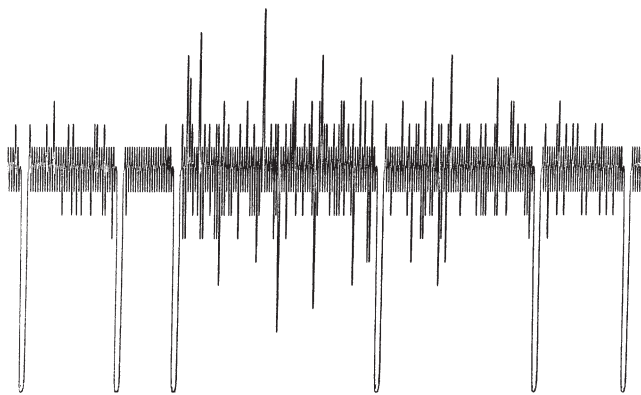

Fig. 17. Sample of a load history applied in flight-simulation fatigue tests. Five flights are shown with gust loads corresponding to different weather conditions.
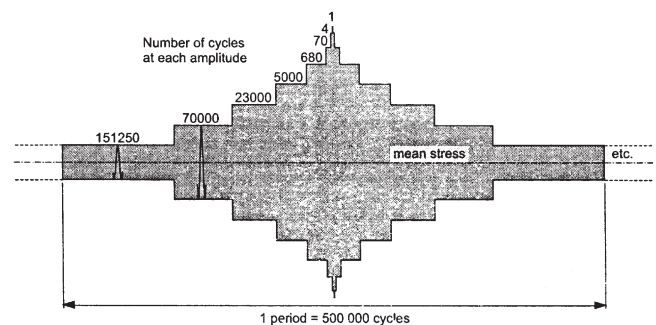

Fig. 18. Block-programme fatigue test introduced by Gassner [89]. CA-load cycles in each block. Blocks in a low-high-low sequence of the amplitude.
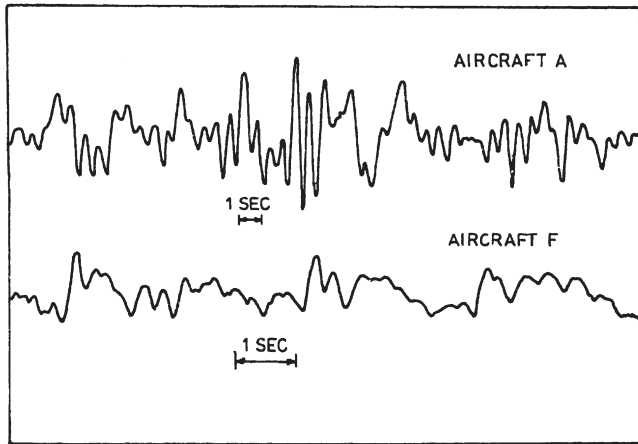
Fig. 19.   Strain gage records of wing bending of two different aircraft flying in turbulent air [21].

fatigue meter' was developed by the Royal Aircraft Establishment in the UK [92]. Exceedings of a number of preset acceleration levels were counted by this apparatus and an example of results is given in Fig. 20. Such load spectra could be used for fatigue calculations, but also for a comparison of the service severity of different aircraft in the same fleet or aircraft of different fleets.[7]

The calculation of loads in fatigue critical components starting from accelerations measured in the centre of gravity of the aircraft was not always realistic or sufficiently accurate. It then became more appropriate to measure the load history on a fatigue critical component itself by strain gages. This technique was available for
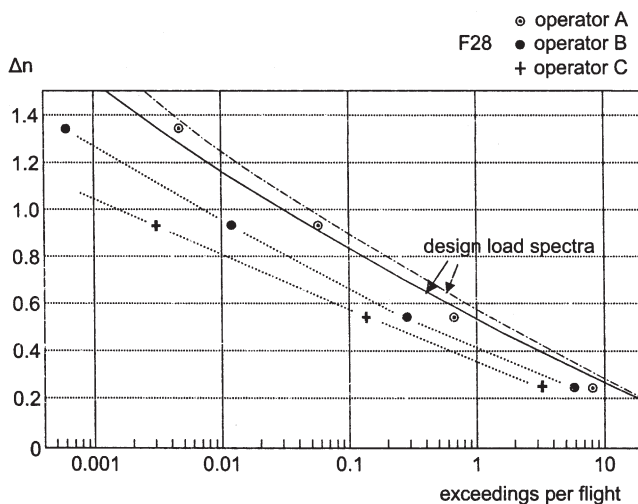

Fig. 20.   Gust load spectra measured with counting accelerometers compared to the design load spectrum of the Fokker F-28 [93].

a long time, but it was adopted more abundantly in the last decades of the previous century when strain gage techniques were developed with a long-term reliability under various service conditions. At the same time, dedicated computer techniques for a statistical analysis of strain gage signals and data storage were also significantly improved [94]. Counting results of occurrences of peak values or ranges could be retrieved in matrix format.

An important development regarding load–time history analysis should be mentioned here. It is associated with small load ranges occurring in between larger ranges, see Fig. 21. In principle, load ranges can be counted as shown in Fig. 21a, but that would imply that the large range AD is not counted. In the so-called rainflow counting method proposed by Endo [95], the counting occurs as shown in Fig. 21b and c. It implies that a smaller range in between a larger range is counted and removed after which the counting is continued[8] and the larger range is counted later. The counting method is described in an ASTM standard [97]. Although the rainflow counting method is not based on an exact physical concept to account for fatigue damage accumulation, it may well be expected to give a more realistic representation of the severity of load–time histories.

Measurements of service load spectra of aircraft were originally made to see what happened to the structure of the aircraft. In later years, such measurements became desirable for other purposes. Full-scale fatigue tests were carried out on newly designed aircraft types in order to be sure that unexpected fatigue events will not affect the safety of the aircraft. Especially after two Comet aircraft flying at cruising altitude exploded in 1954 due to fatigue cracks, such full-scale tests were frequently carried out. Those tests in the 1950s and the 1960s were partly carried out in a realistic way, i.e. on the full aircraft structure including pressurization cycles on the fuselage. However, the loads on the wing were a simplified simulation of what occurred in service. The gust loads on the wing were CA cycles which were moreover the same in all flights. Based on a Miner-rule damage calculation these cycles were supposed to have the same fatigue damaging contribution as the random gust loads in service. Unfortunately, the Miner rule is essentially unreliable for such calculations. The introduction of electro-hydraulic closed-loop systems in full-scale tests enabled a more realistic simulation of service load–time histories. A sample of such a flight-simulation load history applied to the wing structure of the Fokker F-28 aircraft in 1968 was shown in Fig. 17. Already in the early 1960s, Branger in Switzerland carried out realistic

[7] Operator C is flying the aircraft close to the design spectrum used by Fokker. The load spectrum for operator C indicates a significantly milder spectrum. This operator having one aircraft only was using the F-28 as an executive aircraft, and executives do not fly during stormy weather.

[8] A similar concept of removing small intermediate ranges was described by Anne Burns in 1956 [96]. The Strain Range Counter developed by the Vickers aircraft industry was counting in accordance with this method.
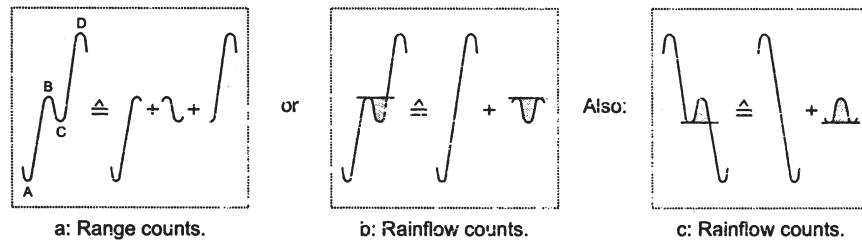
Fig. 21.   Intermediate load reversal as part of a larger range. Principle of the rainflow counting.

flight-simulation tests on a fighter aircraft structure [98]. In the last decades of the previous century, complex service-simulation fatigue tests were carried out, not only on aircraft structures, but on various types of structures for which a satisfactory endurance had to be shown.

By the end of the previous century, techniques and apparatus for measuring load–time histories had been developed to a high level of perfection. It can even be done by wireless systems on moving vehicles. It is also possible to use load–time measurements as input signals for simulation tests. In the automotive industry, this can be attractive, because it opens the possibility for a virtual reality simulation in the laboratory. This is done not only for detecting fatigue problems in a new structure, but more generally to check the functional reliability of new structures. In summary, load spectra, if they cannot be calculated, can now be measured without great difficulties. The question remains whether the possibility of using a structure beyond the operational design specifications (maybe abuse) should be taken into account.

6. Evaluation of the present state of the art

In the first decades of the 20th century, the scientific analysis of fatigue problems was started partly by microscopic investigations and for another part by studies on the analysis of stress concentrations. From an engineering point of view, the experience of dealing with fatigue problems in the beginning of the previous century was still a matter of trial and error. In later decades, continued research clearly indicated that the number of variables that can affect the fatigue strength and the fatigue life of a structure is large. In principle, it is correct to consider fatigue as a phenomenon characterized by microcrack initiation, crack growth as an invisible microcrack and later as a visible macrocrack, which finally leads to complete failure. Unfortunately, this concept does not mean that the fatigue phenomenon occurs in the same way in all metallic materials. On the contrary, the fatigue mechanism in detail can be considerably different for various materials. The initiation of microcracks and the initial growth of these cracks depends highly on the conditions of the material surface. Well-known features involved are the surface quality

depending on production variables, residual stress, special surface treatments, environmental conditions, etc. Moreover, the variety of engineering structures for which fatigue can be a problem is also extensive. In view of the large number of variables and structures, the picture looks rather complex. *Apparently there are many different fatigue problems which all have their own specific characteristics and practical significance.*

The purpose of this section is to summarize the major aspects of the present state of the art about fatigue of structures and materials. In view of so many different fatigue problems this cannot be done in great detail, but major characteristics are indicated. As a result of numerous fatigue research programmes in the second half of the previous century, various effects of variables and their significance for practical applications are reasonably well-known. Moreover, most of these trends are understood in terms of physical arguments, at least in a qualitative way. However, the quantitative description is generally a weak point responsible for uncertainties if predictions on fatigue properties have to be made. In spite of this, we know and understand in many cases how fatigue properties of structures can be improved and how deleterious influences can be avoided. *Designing against fatigue with the present knowledge is possible.*

In view of the prospects of fatigue problems in the new century (the present 21st century) it seems to be useful to list some major gains and shortcomings emerging from the developments in the last decades of the previous century. Important progress was associated with the developments of new equipment and computers:

- The electron microscope significantly contributed to the present knowledge about fatigue, noteworthy by fractographic observations.
- The closed-loop fatigue machines controlled by computers have revolutionarily enlarged the possibilities for various kinds of fatigue tests, noteworthy for complex service-simulation fatigue tests.
- Measuring techniques to obtain information about load–time histories in service and statistical techniques to evaluate the data of such measurements were developed.
- Computers have significantly increased the potential of stress analysis for obtaining stress distributions in

components, and more specifically to obtain values of the stress concentration factor (K_t) and the stress intensity factor (K).

It should be realized that this list is mainly concerned with modern tools to tackle problems on fundamental questions about fatigue properties. However, the tools have enabled some noteworthy developments with respect to the following topics:

- As a result of numerous microscopic investigations, including fractographic observations, the concept 'fatigue damage' in physical terms has obtained a much better basis.
- The introduction of 'fracture mechanics' has led to significant possibilities for the prediction of fatigue crack growth under CA loading. The application of fracture mechanics requires that the fracture mechanism is known and understood.
- Observations on the crack closure concept have opened various approaches to fatigue crack growth problems and to understand load history effects under VA loading.

Problem areas can be indicated where experience in the previous century has revealed weaknesses of the procedures for fatigue predictions. Four topics are briefly addressed:

- Predictions on the fatigue limit as affected by the notch effect, size effect and surface quality
- Life predictions for CA loading
- Predictions on the fatigue strength of joints
- Life predictions for VA loading.

Major problems are associated with the accuracy of prediction. Furthermore, if the reliability of the predictions is not satisfactory, the alternative is to carry out relevant fatigue tests. Even then it can be a problem to know whether the experiment will give reliable and worthwhile information.

6.1. Prediction of the fatigue limit

This problem is of interest for many types of dynamically loaded components subjected to large numbers of cycles (high-cycle fatigue) whereby fatigue failures are undesirable. It was noted previously that rigorous predictions starting from fatigue data for unnotched material cannot be made, but empirical equations have been proposed which may lead to reasonable estimates of the fatigue limit. Such estimates are supposed to be average values of S_f. If fatigue failures cannot be tolerated, a safety factor must be applied in view of scatter of this property, and perhaps also because of uncertainties of the magnitude of the cyclic load. This is a delicate prob-

lem which is usual for selecting safety factors.[9] One problem is that the distribution function of the fatigue limit of the component is unknown [99] Scatter of the fatigue limit can be studied by performing many tests. However, it must be realized that the statistical information obtained applies to the conditions of the laboratory test series, which in general is not valid for the components produced in large quantities by the industry. Moreover, the statistical information for a non-failure criterion of a large population is another problem associated with the lower tail of the probability of failure distribution function. Physical arguments to assume a function for this purpose at very low probabilities are not available. The selection of a safety factor then becomes a matter of a judicious choice taking into account the consequences of an highly unlikely failure with an incidental character. Potential reasons why such a failure might still happen have to be considered, e.g. the possibility of surface damage as a starting point for a fatigue crack.

6.2. Predictions of the fatigue life under CA loading

In various structures, the occurrence of fatigue cracks is accepted because it would not be economical to design for an infinite life. Low design stress levels would then be necessary which leads to oversized heavy structures. But a sufficient fatigue life should be present before the occurrence of cracks necessitates repair or replacement, and thus prediction problems are still involved.

If the geometry of notches in fatigue critical parts of a structure are open notches (holes, grooves, fillets), K_t-values may be available or can be calculated with FE techniques. A finite life has to be predicted both for the crack initiation period and the crack growth period. The prediction of the crack initiation life is difficult. The application of fracture mechanics is questionable for this period. As an alternative it has been proposed to assume some initial crack-like defect and to predict the entire life as a crack growth life starting from this defect to failure. It implies that the crack initiation period is ignored in this approach. If the initial defect is assumed to be not very small, the prediction result may be conservative, but any quantitative accuracy cannot be guaranteed.

From a mechanistic point of view, the initiation period and the crack growth period require different prediction models. This problem is complicated in view of defining the moment of the transition from the initiation period to the crack growth period. Actually, it must be admitted that a rigorously and physically satisfactory solution of this problem is not available.

---

[9] An extensive survey about scatter of the fatigue strength observed in numerous test series was recently published by Adenstedt [100].

Where accurate predictions are questionable, an experimental analysis and safety factors have to be considered. Experiments should preferably be carried out on the component itself to avoid possible notch-, size- and surface-finish effects. Safety factors can be applied to the fatigue life obtained in the experiments. But it is also possible to carry out the experiment at a higher stress level than the design stress level, which is also introducing a safety factor. Safety factors to be used are again a matter of a judicious choice considering all sources of possible scatter and unknown circumstances. One topic not yet mentioned is the influence of the environment in service, such as the effect of humid and salty air, or even sea water. It is important to know whether the material used is sensitive to such environmental influences. This knowledge may be obtained from laboratory investigation, while practical experience is instructive.

6.3. Predictions on the fatigue strength of joints

Finite life predictions for joints are generally considered to be a problem where elementary concepts can not be used in a fully rational way. However, joints are most significant in various structures. Welded joints are abundantly used in many large steel and aluminium structures. Numerous riveted joints occur in aircraft structures. Bolted joints are applied in many structures where dismounting of the joint is required. Lug type connections (single-pin loaded hole) are necessary in various structures with moving components. Unfortunately, the fatigue properties of joints can be rather critical for a number of reasons: high K_t-values, fretting corrosion, eccentricities in the joint inducing secondary bending, defects and unfavourable geometries in welded joints, etc. Because of these complexities simple calculation procedures for calculating fatigue properties of joints cannot be formulated. The similarity concept based on stress concentration factors is not conceivable. This has led to various life estimation procedures starting from fatigue test data of similar joints of the same material. Local parameters for the characteristic features of a joint are defined and used for estimating the fatigue strength. It is still a kind of a similarity approach. If data on similar joints are not available, additional experiments should be recommended. Various aspects of fatigue strength assessments based on local parameters were recently surveyed by Radai [101]. In spite of a limited accuracy when estimating fatigue properties of joints, a qualitative understanding of load transmission in joints and the associated empirical trends was developed in the previous century. It implies that designing against fatigue of joints is quite possible.

6.4. Fatigue damage accumulation under VA loading

The prediction of the fatigue life and crack growth under VA loading drew much interest in the previous century. Many structures in service are subjected to a load spectrum of cycles with different amplitudes. As discussed earlier, the simple Miner rule is in serious conflict with the present understanding of fatigue damage accumulation, but a fully rational alternative is not available. Actually, it should be concluded from the present knowledge that an accurate life prediction rule for VA loading will not be obtained in the near future. In this context, it should be recognized that the Miner rule is supposed to predict a fatigue life from S-N curve data. Apart from the question whether such curves are not always available, the Miner rule is indeed a large extrapolation step, i.e. from CA-loading data to VA-load histories. As long as another realistic rule is not available, we can only hope that the Miner rule will give a kind of an estimate of the fatigue life for VA-loading. Extrapolation of S-N curves below the fatigue limit discussed previously (Fig. 13) should always be used, but it does not imply that the rule becomes an accurate and rational prediction rule. It is a practical attempt to save the usefulness of the Miner rule for obtaining a rough estimate of the fatigue life.

With respect to the prediction of crack growth under VA loading, some progress has been made in the previous century. An extensive survey was recently published by Skorupa [102]. Also in the case of crack growth, the problem of accurate predictions can not be considered to be satisfactorily solved. Apparently, accurate predictions on both fatigue life and crack growth are still beyond the present state of the art. It then is not surprising that the best information on fatigue properties should come from realistic service-simulation fatigue tests. Fortunately, complex load–time histories which can be generated by a computer programme can also be applied to closed loop load actuators. It therefore appears that it is no longer a problem to carry out service-simulation fatigue tests. They can be carried out in the laboratory at reasonably high frequencies, say 10 Hz and also faster. As a consequence, these tests need not take much time. Actually, a determination of an S-N curve to carry out a Miner calculation is much more demanding in time and numbers of specimens, apart from the fact that the calculation can not claim to be a realistic approach. However, it should also be recognized that a service-simulation fatigue test as simulation of the history in service is not free from questions. First, a service-simulation test is still an accelerated test because the service life must be expressed in years instead of testing days in the laboratory. If time-dependent influences are important, think of corrosion, the test results should be interpreted with care in view of such effects. Secondly, it has to be known what the service load–time history is. Moreover, only one load–time history is simulated in a test. Usually, a conservative history will be used. But the problem-setting requires a profound understanding of what is going on in service, what is significant for

the simulation in the test, and how important are differences with the real environment and usage of the structure in service.

6.5. Some 'smart' ideas

The need for new life predictions of old structures is heavily felt for both civil and military aircraft, but also for other non-aeronautical structures. The best approach is to carry out service-simulation tests on fatigue critical parts taken from an old structure, a procedure which has been adopted for aging aircraft [103] in view of safety considerations. Economic arguments can also be important, for instance for the question of whether old bridges should be replaced. Load spectra measurements are of paramount importance which should be followed by fatigue tests on representative test articles with a realistic simulation of the load–time history occurring in service. This is not so much a smart idea for an old problem, but more a practical solution.

From time to time, relatively simple devices were proposed to measure the 'life consumption' of a structure. In view of aging structures, it has been proposed to bond small specimens with a very low thickness and a crack starter notch to a fatigue critical aircraft component [104]. Fatigue crack growth in these gages will then occur and the crack length is used as a measure for the fatigue damage experience of the structure. The correlation between this indication of the crack gage and the fatigue damage done to the structure is an obvious problem. It could be attempted to calibrate the crack gage in the full-scale test on the aircraft structure. Actually, if bonding something to a structure is a feasible option, the best solution is to use strain gages which can produce more relevant data about the load spectrum.

Another solution to find early fatigue cracks is to build in a self-warning system. In the 1970s, an interesting system was adopted for helicopter blades [105]. A blade failure is always catastrophic. The main beam of the blades consisted of a hollow extrusion which was closed at the ends and filled with an inert gas (nitrogen) with a slight over-pressure. If a fatigue crack penetrated through the wall thickness the pressure was lost and the color indication of a pressure-gauge at the root of the blade changed from yellow into red. A walk-around inspection was sufficient to know whether cracks were present. Several cracks have been found this way[10] with

---

[10] A similar warning was occasionally observed on the fuselage of the 747. Leakage of cabin air at cruising altitude due to a fuselage crack was easily balanced by the aircraft pressurization system. Room temperature cabin air escaped through the crack into the -55°C environment, and condensation of water vapor on the crack edges occurred immediately. A brown staining residue of nicotine was decorating the crack. The cracks were easily visible and have been found by aircraft personnel during walk around inspections.

a single exception [105]. The concept could be labeled as a smart structure.

Nowadays one of the new terms is 'smart materials' which can give indications of local failures by wire type systems. This concept seems to be promising for structures of composite materials, but it is not yet sure whether they have a similar potential for metallic structures. After all, designers, and also the user of a structure, do not want to be warned of fatigue problems. They prefer to have a structure which is free from cracks and the nuisance of repairs. But of course, smart ideas should be studied for their potential usefulness [106].

It is not surprising that ideas about removing fatigue damage from a structure have been proposed through the years. Before 1950 it was sometimes thought that fatigue damage could be neutralized by prolonged load cycling below the fatigue limit. Also a heat treatment to annihilate the fatigue damage was considered. With the present knowledge of the fatigue phenomenon and fatigue damage it can be concluded that such proposals are illusive. However, the idea to remove fatigue damage was considered already in the 1950s. Based on the idea that early fatigue damage consists of microcracks in a thin surface layer, removal of this layer can also remove the damage. This option has been adopted by aircraft operators. If small cracks occur in bolt holes, the hole is reamed to a larger diameter and a larger bolt is inserted. Actually, this procedure has the character of a repair, but without a good understanding of fatigue cracking in the structure a clever solution cannot be introduced. Confirmation by experiments must be recommended.

7. Epilogue

In the beginning of the 20th century, the knowledge about fatigue was still in its infancy, with respect to the fatigue phenomenon occurring in the material and the engineering aspects. During the 20th century, and predominantly in the second half of this century, numerous research programmes on fatigue problems were carried out. Research was stimulated by the need for information on designing against fatigue, as well as by scientific interest to understand the fatigue phenomenon. Moreover, significant developments were possible by fundamental improvements of techniques for calculations, experiments and measurements. Important stimuli for fatigue research also came from catastrophic accidents due to fatigue problems. In general, the purpose of designing against fatigue is to avoid fatigue problems for reasons of safety, economy, durability and liability. Various aspects are involved which are not solved by engineering science, but which should be considered by the community in which we are living (see a recent paper by Miller [107]).

As a result of much work on fatigue problems in the

20th century, an extremely large number of papers, reports and books was published. Several impressions emerge from these publications. First, the variety of practical fatigue problems is large. For instance, fatigue of a welded offshore structure in the open sea is entirely different from fatigue of a crank shaft of an engine, or fatigue of a pressurized aircraft fuselage. A second impression is that these different problems can now be well defined in terms of relevant conditions. Third, from a physical point of view the influences of the conditions on the fatigue behavior are reasonably well understood thanks to extensive laboratory research. However, the character of the understanding is primarily qualitative. The corollary of the qualitative understanding is the following conclusion: A quantitative prediction of fatigue properties of a structure cannot be given because of the qualitative understanding of the complexity of fatigue damage accumulation. Accurate fatigue predictions which designers prefer are still subjected to uncertainties. As a consequence, safety factors on predictions are required, and these factors have to be judiciously chosen, based on experience, information on data and spectra, knowledge of the governing conditions, statistical variations, and consequences of fatigue failures. The limited accuracy of predictions also emphasizes the significance of realistic simulation tests. With the present knowledge, the relevance of realistic testing can be well judged.

Although the turn of the century is a matter of numbering years, it is good to consider problems which should be further investigated in the 21st century. Looking back on the experience of the last decades of the previous century, the limited accuracy of predictions is a significant issue. This is especially true for prediction models which employ basic material data obtained on simple specimens. Stress analysis need not be a real obstacle, but it is the fatigue behavior of structural elements which cannot be realistically modeled in sufficient detail. In general, fatigue models imply a large extrapolation step from simple tests to a real structure. This is understood by various industries which now perform service-simulation fatigue experiments close to the reality of the structures in service. As a result, the extrapolation step is drastically reduced although it remains a simulation within the constraints of experimental possibilities. Evaluation and validation of such tests on components still require a profound understanding of the fatigue behavior in service. If something is to be learned from these arguments, it is that *education of understanding fatigue scenarios of structures must be strongly advised.*

In certain cases, it may be possible that conservative fatigue predictions can be made, not accurate but conservative, and that the results indicate that an undesirable fatigue performance need not be feared (i.e. predictions on 'fitness-for-purpose'). Workshop experience often reveals that frequently asked questions (FAQ's) are: please give us material fatigue data and equations. However, without understanding the physical reality of the full prediction scenario, the risk is that a calculation is carried out instead of a trustworthy prediction. *Again there is a case for education.*

Another important lesson of the previous century is that designing against fatigue is possible. A considerable gain of knowledge on the influences of various design and production aspects has been achieved. Also here, *education on designing against fatigue is required.*

With respect to research topics to be continued, the tools used for predictions should be further investigated. It was said before that there are many different fatigue problems. Each problem has its own characteristic aspects, and it is certainly worthwhile to go into greater depth to understand the phenomena involved. Unfortunately, the picture of financial support for research is changing. Decision-makers require that economical profits should be clear before the research is started (which in fact is a contradictio in terminis). One can only hope that universities and research laboratories will find the funds for basic research. Some examples of topics deserving further research can be mentioned: predictions of the crack initiation life under biaxial loading, fatigue under variable-amplitude loading, corrosion fatigue, fatigue of complex joints, fatigue of new materials, inspection techniques for detecting minute fatigue cracks. In addition, industries come with new products and new materials, and the durability including fatigue will be a matter of concern for research. *There is still some work for the future.*

References

[1] Ewing JA, Humfrey JCW. The fracture of metals under repeated alternations of stress. Phil Trans Roy Soc 1903;A200:241–50.
[2] Peterson RE. Discussion of a century ago concerning the nature of fatigue, and review of some of the subsequent researches concerning the mechanism of fatigue. In: ASTM Bulletin, 164. American Society for Testing and Materials; 1950. p. 50–6.
[3] Timoshenko S. Stress concentration in the history of strength of materials. The William M. Murray Lecture. Proc Soc Exp Stress Analysis (SESA) 1954;12:1–12.
[4] Hetényi M. Handbook of experimental stress analysis. New York: John Wiley; 1950.
[5] Mann JY. Aircraft fatigue—with particular emphasis on Australian operations and research. In: Proc 12th Symp of the Int Committee on Aeronautical Fatigue (ICAF). Centre d'Essais Aéronautique de Toulouse; 1983.
[6] Schütz W. A history of fatigue. Engng Fract Mech 1996;54:263–300.
[7] Smith RA. Fatigue crack growth, 30 years of progress. Pergamon; 1986.
[8] Sanfor RJ, editor. Selected papers on foundations of linear elastic fracture mechanics. SEM Classic papers, Vol. CP1, SPIE Milestone series, Vol. MS 137. 1997.
[9] Hanewinckel D, Zenner H. Fatigue strength, a collection of fac-

simile, historical papers until 1950. Clausthal: Technical University; 1989.

[10] Mann JY. Bibliography on the fatigue of materials, components and structures. Vol. 1–4, covering 1838–1969. Pergamon Press; 1970–1990.

[11] Schijve J. Fatigue of structures and materials. Dordrecht, Boston: Kluwer Academic Publ; 2001.

[12] Gough HJ. Crystalline structure in relation to failure of metals, especially by fatigue. Proc ASTM, American Society for Testing and Materials 1933;33:3–114.

[13] Orowan E. Theory of the fatigue of metals. Proc Roy Soc 1939;A171:79–106.

[14] Head AK. The growth of fatigue cracks. Phil Mag 1953;44:925–38.

[15] Stroh AN. The formation of cracks as a result of plastic flow. Proc Roy Soc 1955;A223:404.

[16] Forsyth PJE. The application of 'fractography' to fatigue failure investigations. Roy Aircraft Est, Tech Note Met 1957;257.

[17] Ryder DA. Some quantitative information obtained from the examination of fatigue fracture surfaces. Roy Aircraft Est, Tech Note Met 1958;288.

[18] Cottrell AH, Hull D. Extrusion and intrusion by cyclic slip in copper. Proc Roy Soc 1957;A242:211–3.

[19] Mott NF. A theory of the origin of fatigue cracks. Acta Metall 1958;6:195–7.

[20] A discussion on work-hardening and fatigue in metals. Proc Roy Soc Math Phys Sci 1957;242:1229.

[21] Averbach BL, Felbeck DK, Hahn GT, Thomas DA. In: Proc Int Conf on the Atomic Mechanisms of Fracture. Tech Press, MIT. John Wiley & Sons; 1959.

[22] Forsyth PJE. The physical basis of metal fatigue. London: Blackie and Son; 1969.

[23] Backofen WA. Formation of slip-band cracks in fatigue. In: Proc Int Conf on the Atomic Mechanisms of Fracture. Tech Press, MIT. John Wiley & Sons; 1959. p. 435–49.

[24] Suresh S. Fatigue of materials, 2nd ed. Cambridge: Cambridge University Press; 1998.

[25] Fatigue crack propagation, ASTM STP 415. Philadelphia, PA: American Society for Testing and Materials; 1967. p.131–80 and 505–35.

[26] Schijve J. The significance of fractography for investigations of fatigue crack growth under variable- amplitude loading. Fatigue Fract Engng Mater Struct 1999;22:87–99.

[27] Bowles CQ. The role of environment, frequency and wave shape during fatigue crack growth in aluminum alloys. Doctor Thesis, Delft University of Technology; 1978.

[28] Bowles CQ, Schijve J. Crack tip geometry for fatigue cracks grown in air and vacuum. In: ASTM 811. Philadelphia, PA: American Society for Testing and Materials; 1983. p. 400–26.

[29] Broek D, de Rijk P, Sevenhuysen PJ. The transition of fatigue cracks in alclad sheet, TR M2100. Amsterdam: Nat Aerospace Laboratory NLR; 1962.

[30] Lindley TC, Richards CE. The relevance of crack closure to fatigue crack propagation. Mater Sci Eng 1974;14:281–93.

[31] Veer FA. The effect of shear lips, loading transition and test frequency on constant K and constant amplitude fatigue tests. Doctor Thesis, Delft University of Technology; 1993.

[32] Zuidema J. Square and slant fatigue crack growth. Doctor Thesis, Delft University of Technology; 1995.

[33] Schijve J, Arkema WJ. Crack closure and the environmental effect on fatigue crack growth, Report LR-217. Delft University of Technology; 1976.

[34] Paris PC, Gomez MP, Anderson WE. A rational analytical theory of fatigue. The Trend of Engineering 1961;13:9–14.

[35] Irwin GR. Analysis of stresses and strains near the end of a crack traversing a plate. Trans ASME J Appl Mech 1957;24:361–4.

[36] Pearson S. Initiation of fatigue cracks in commercial aluminium alloys and the subsequent propagation of very short cracks. Engng Fract Mech 1975;7:235–47.

[37] Wanhill RJH. Durability analysis using short and long fatigue crack growth data. In: Aircraft damage assessment and repair. Barton, Australia: Institute of Engineering; 1991.

[38] Miller KJ. The three thresholds for fatigue crack propagation. In: Fatigue and Fracture Mechanics, 27th vol. ASTM STP 1296. Philadelphia, PA: American Society for Testing and Materials; 1997. p. 267–86.

[39] Schijve J. Fatigue crack closure observations and technical significance. In: Mechanics of fatigue crack closure. ASTM STP 982. Philadelphia, PA: American Society for Testing and Materials; 1988. p. 5–34.

[40] Elber W. Fatigue crack propagation. PhD Thesis, University New South Wales, Australia; 1968.

[41] Elber W. The significance of fatigue crack closure. In: Damage tolerance in aircraft structures, ASTM STP 486. Philadelphia, PA: American Society for Testing and Materials; 1971. p. 230–42.

[42] Schijve J. Fatigue crack growth under variable-amplitude loading. In: Fatigue and fracture, ASM handbook, vol. 19. ASM International; 1996. p. 110–33.

[43] Coffin Jr. LF. Low cycle fatigue—a review. Appl Mater Res 1962;1:129.

[44] Manson SS, Hirschberg MH. Fatigue behavior in strain cycling in the low- and intermediate-cycle range. In: Fatigue an Interdisciplinary Approach. Syracuse University Press; 1964. p. 133–78.

[45] Stanzl-Tschegg S, Mayer H. Fatigue in the very high cycle regime. In: Proc Int Conf, July 2001, Inst Meteorology and Physics. Vienna: University of Agricultural Science; 2001.

[46] Frost NE, Marsh KJ, Pook LP. Metal fatigue. Oxford: Clarendon; 1974.

[47] A guide for fatigue testing and the statistical analysis of fatigue data. ASTM STP No.91-A. Philadelphia, PA: American Society for Testing and Materials; 1963.

[48] Bellows RS, Muju S, Nicholas T. Validation of the step test method for generating Haigh diagrams for Ti-6Al-4V. Int J Fracture 1999;21:687–97.

[49] Heywood RB. Designing against fatigue. London: Chapman and Hall; 1962.

[50] Ramsden JM. The geriatric problem. Flight International 1977;22 Oct:1201–7.

[51] Neuber H. Kerbspannungslehre. Berlin: Springer; 1937 (Translated: Theory of Notch Stresses. Ann Arbor, Michigan: JW Edwards; 1946).

[52] Peterson RE. Stress concentration factors. New York: John Wiley & Sons; 1974.

[53] Kuhn P, Hardrath HF. An engineering method for estimating notch-size effect in fatigue tests of steel. NACA TN 1952;2805.

[54] Siebel E, Stieler M. Significance of dissimilar stress distributions for cyclic loading. ZVDI 1955;97:146–8 (in German).

[55] Schijve J. Fundamental aspects of predictions on fatigue crack growth under variable-amplitude loading. In: Blom AF, Beevers CJ, editors. Proc. Conference of the University of Birmingham. Theoretical concepts and numerical analysis of fatigue. EMAS; 1992. p. 111–30.

[56] Rooke DP, Cartwright DJ. Stress intensity factors. London: Her Majesty's Stationery Office; 1976.

[57] Tada H, Paris PC, Irwin GR. The stress analysis handbook, 2nd ed. St Louis: Paris Productions Inc.; 1985.

[58] Murakami Y, editor. Stress intensity factors handbook. Oxford: Pergamon; 1987/1993.

[59] Pålmgren A. The endurance of ball bearings. Z Ver Deut Ing 1923;68:339–41 (in German).

[60] Langer BF. Fatigue failure from stress cycles of varying amplitude. J Appl Mech 1937;4:A160–A2.

[61] Miner MA. Cumulative damage in fatigue. J Appl Mech 1945;12:A159–A64.

[62] Schijve J. Some remarks on the cumulative damage concept. In: Minutes 4th Conference of the Int Committee on Aero Fatigue (ICAF), Zürich; 1956. Paper 2. 1956.

[63] Heyer K. Multi-step tests on structural elements, 152. Bericht: Lilienthal-Gesellschaft für Luftfahrtforschung; 1943 (in German).

[64] Smith CR. Fatigue-service life prediction based on tests at constant stress levels. Proc. Soc Exp Stress Analysis (SESA) 1958;16(1):9.

[65] Martin JF, Topper TH, Sinclair GM. Computer based simulation of cyclic stress-strain behavior with applications to fatigue. Mater Res Stand 1971;11(23-28):50.

[66] Wetzel RM, editor. Fatigue under complex loading: Analysis and experiments. Advances in Engineering, Vol. 7. Society of Automotive Engineering; 1977.

[67] Neuber H. Theory of stress concentration for shear strained prismatic bodies with arbitrary nonlinear stress-strain law. Trans ASME J Appl Mech 1961;28:544–50.

[68] Haibach E. Modified linear damage accumulation hypothesis accounting for a decreasing fatigue strength during increasing fatigue damage, Report TM Nr. 50. Darmstadt: Laboratorium für Betriebsfestigkeit, LBF; 1970 (in German).

[69] European Prestandard. European Committee for Standardization, Brussels, May 1998. ENV 1999-2.

[70] Hardrath HF, Naumann EC. Variable-amplitude fatigue tests on aluminum alloy specimens. In: ASTM STP 274. Philadelphia, PA: American Society for Testing and Materials; 1960. p. 125–38.

[71] Naumann EC, Hardrath HR, Guthrie EC. Axial load fatigue tests of -T3 and 7075-T6 aluminum alloy sheet specimens under constant- and variable-amplitude loads, NASA Report TN D-212, 1959.

[72] Schijve J. The accumulation of fatigue damage in aircraft materials and structures, AGARDograph No.157. Advisory Group for Aerospace Research and & Development; 1972.

[73] Schijve J. The significance of flight simulation fatigue tests. In: Proc 13th ICAF Symposium Pisa. Durability and damage tolerance in aircraft design. EMAS; 1985. p. 71–170.

[74] Schijve J. Fatigue crack propagation in light alloy sheet material and structures. In: Advances in aeronautical sciences, vol. 3. Pergamon Press; 1961. p. 387–408.

[75] Hudson CM, Hardrath HF. Effects of changing stress amplitude on the rate of fatigue crack propagation in two aluminum alloys. NASA TN D-960, 1961.

[76] Mills WJ, Hertzberg RW. The effect of sheet thickness on fatigue crack retardation in 2024-T3 aluminum alloy. Engng Fract Mech 1975;7:705–11.

[77] Petrak GS. Strength level effects on fatigue crack growth and retardation. Engng Fract Mech 1974;6:725–33.

[78] Willenborg J, Engle RM, Wood HA. A crack growth retardation model using an effective stress concept. Air Force Flight Dynamic Laboratory, Dayton, Report AFFDL-TR71-1; 1971.

[79] Wheeler OE. Spectrum loading and crack growth. ASME Trans J Basic Engng 1972;94:181–6.

[80] Padmadinata UH. Investigation of crack-closure prediction models for fatigue in aluminum sheet under flight-simulation loading. Doctor Thesis, Delft University of Technology; 1990.

[81] de Koning AU. A simple crack closure model for prediction of fatigue crack growth rates under variable-amplitude loading. In: ASTM STP 743. Philadelphia, PA: American Society for Testing and Materials; 1981. p. 63–85.

[82] Padmadinata UH, Schijve J. Prediction of fatigue crack growth under flight-simulation loading with the modified CORPUS model. In: Harris CE, editor. Advanced structural integrity methods for airframe durability and damage tolerance. NASA Conf Publ; 1994. p. 547–62.

[83] Dugdale DS. Yielding of steel sheets containing slits. J Mech Phys Solids 1960;8:100–4.

[84] Dill HD, Saff CR. Spectrum crack growth prediction method based on crack growth surface displacement and contact analysis. In: ASTM STP 595. Philadelphia, PA: American Society for Testing and Materials; 1976. p. 306–19.

[85] Dill HD, Saff CR, Potter JM. Effects of fighter attack spectrum and crack growth. In: ASTM STP 714. Philadelphia, PA: American Society for Testing and Materials; 1980. p. 205–17.

[86] Newman Jr. JC. A crack-closure model for predicting fatigue crack growth under aircraft spectrum loading. In: ASTM STP 748. Philadelphia, PA: American Society for Testing and Materials; 1981. p. 53–84.

[87] Dougherty DJ, de Koning AU, Hillberry BM. Modelling hgh crack growth rates under variable amplitude loading. In: ASTM STP 1122. Philadelphia, PA: American Society for Testing and Materials; 1992. p. 214–33.

[88] Wang GS, Blom AF. A strip model for fatigue crack growth predictions under general load conditions. Engng Fract Mech 1991;40:507–33.

[89] Gassner E. Strength experiments under cyclic loading in aircraft structures. Luftwissen 1939;6:61–4 (in German).

[90] Teichmann A. Basic consideration on fatigue durability in service (in German). Jahrbuch der Deutschen Luftfahrtforschung 1941;1:467–71.

[91] Seewald F. Measurements with a glass scratching recorder of the German Institute for Aeronautics. Maschinenbau 1931;10:725–7 (in German).

[92] Taylor J. Measurements of gust loads in aircraft. J Roy Aero Soc 1953;57:78–88.

[93] Data of the Fokker Aircraft Industry, 1980.

[94] Proc. 20th Symp. Aircraft integrated monitoring systems 2000. Tech University Münich, Faculty Maschinenwesen, 2000.

[95] Murakami Y, editor. The rainflow method in fatigue, The Tatsuo Endo memorial volume. Oxford: Butterworth-Heinemann; 1992.

[96] Burns A. Fatigue loadings in flight: Loads in the tailplane and fin of a Varsity. London: ARC Technical Report CP 256; 1956.

[97] Standard practices for cycle counting in fatigue analysis. ASTM Standard E 1049-90. American Society for Testing and Materials 1990.

[98] Branger J. The full-scale fatigue test on the DH-112 Venom. Eidgenossische Flugzeugwerk Emmen 1964; Report F&W S-162.

[99] Schijve J. Fatigue predictions and scatter. Fatigue Fract Engng Mater Struct 1994;17:381–96.

[100] Adenstedt R. Scatter of the fatigue strength (in German). Doctor Thesis, Technical University of Clausthal, 2001.

[101] Radai D. Review of fatigue strength assessment of nonwelded and welded structures based on local parameters. Int J Fatigue 1996;18:153–70.

[102] Skorupa M. Load interaction effects during fatigue crack growth under variable amplitude loading— A literature review. Part I: Empirical trends, Part II: Qualitative interpretation. Fatigue Fract Engng Mater Struct 2001;21/22:987–1006.

[103] Goranson UG. Continuous airworthiness of aging aircraft. In: Proc 2nd Annual International Conference on Aging Aircraft. Baltimore: Federal Aviation Adm US Dept of Trans; 1989. p. 61–89.

[104] Saff CR, Holloway DR. Evaluation of crack growth gages for service life tracking. In: 13th Conf, ASTM STP 743. Fracture mechanics. American Society for Testing and Materials; 1981. p. 623–40.

[105] Wanhill RJH, de Graaf EAB, Delil AAM. Significance of a rotor blade failure for fleet operation, inspection, maintenance, design and certification. In: Fifth European rotorcraft and powered lift aircraft forum, Amsterdam, 4–7 September 1979. Paper 38.

[106] Flatau AG, Chong KP. Dynamic smart material and structural systems. Engng Struct 2002;24:261–70.

[107] Miller KJ. Structural integrity—whose responsibility? The Institution of Mechanical Engineers, The 36th John Player Memorial Lecture, 2001.