

Mecânica Quântica I - 4302403

Respostas da 3ª lista

- 1) b) **R:** *Sol. ímpares:* Dois, para $\alpha > \hbar^2/2ma$ ou um, para $\alpha \leq \hbar^2/2ma$.
Sol. pares: Um, para qualquer $\alpha > 0$.

2) a)

$$\psi_2(x) = \left(\frac{m\omega}{4\pi\hbar}\right)^{1/4} \left(\frac{2m\omega}{\hbar}x^2 - 1\right) \exp\left(-\frac{m\omega x^2}{2\hbar}\right).$$

3) a) Notação : $\langle A \rangle_n \equiv \langle n|A|n \rangle$

$$\langle x \rangle_0 = \langle x \rangle_1 = \langle p \rangle_0 = \langle p \rangle_1 = 0.$$

$$\langle x^2 \rangle_1 = 3\langle x^2 \rangle_0 = \frac{3\hbar}{2m\omega}, \quad \langle p^2 \rangle_1 = 3\langle p^2 \rangle_0 = \frac{3}{2}m\omega\hbar.$$

c) **R:** Para ambos os estados, $\langle T \rangle + \langle V \rangle = \langle H \rangle$:

$$\langle T \rangle_1 = \langle V \rangle_1 = 3\langle T \rangle_0 = 3\langle V \rangle_0 = \frac{3}{4}\hbar\omega.$$

5) a) $A = 1/5$.

b)

$$\Psi(x, t) = \frac{1}{5} \left[3e^{-\frac{i\omega t}{2}} \psi_0(x) + 4e^{-\frac{3i\omega t}{2}} \psi_1(x) \right],$$

$$|\Psi(x, t)|^2 = \frac{1}{25} \left[9\psi_0^2(x) + 24\psi_0(x)\psi_1(x) \cos \omega t + 16\psi_1^2(x) \right],$$

onde

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}, \quad \psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{2m\omega}{\hbar}} x e^{-\frac{m\omega}{2\hbar}x^2}$$

e) $E_0 = \hbar\omega/2$ com probabilidade 9/25 e $E_1 = 3\hbar\omega/2$ com probabilidade 16/25.

6) a)

$$x_{kl} \equiv \langle k|x|l \rangle = \sqrt{\frac{\hbar}{2m\omega}} \left(\delta_{k,l+1} \sqrt{l+1} + \delta_{k,l-1} \sqrt{l} \right)$$

$$p_{kl} \equiv \langle k|p|l \rangle = i \sqrt{\frac{m\omega\hbar}{2}} \left(\delta_{k,l+1} \sqrt{l+1} - \delta_{k,l-1} \sqrt{l} \right)$$

b)

$$\langle x^2 \rangle_n = \frac{\hbar}{2m\omega} (2n+1)$$

c)

$$\frac{\langle H \rangle_n}{\langle V \rangle_n} = 2 \Rightarrow \langle T \rangle_n = \frac{\hbar\omega}{2} \left(n + \frac{1}{2} \right), \quad \langle p^2 \rangle_n = \frac{m\omega\hbar}{2} (2n+1)$$

7) a) $\langle N \rangle_z = \frac{1}{4}$

b) $e^{-\frac{i\pi}{4}} |0\rangle$

c)

$$c_n = \frac{z^n}{\sqrt{n!}} \exp\left(-\frac{|z|^2}{2}\right)$$

d) $|\langle z'|z\rangle|^2 = \exp\left(-\frac{1}{16}\right)$.

8) $\text{Prob}(H \rightarrow \frac{1}{2}\hbar\omega) = 0$, e $\text{Prob}(H \rightarrow \hbar\omega) = \frac{2\sqrt{2}}{3}$.

10) Soluções ímpares: $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$ para $n = 1, 3, 5, \dots$