

## RELATÓRIO DE RESOLUÇÕES

O código de cada membro pode ser consultado a seguir:

$x_{04}$ : Beatriz Chessa	$x_{11}$ : Luca Monaco
$x_{05}$ : José Soares Jr.	$x_{15}$ : Rodrigo Melendez
$x_{06}$ : Maurício Damião	$x_{18}$ : Matheus Cardoso
$x_{08}$ : Pedro Lopes Silva	$x_{20}$ : Gustavo Zequini
$x_{09}$ : Rafael Maddalena	

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**Resolução ( || Questão: 2.5.1 || Relator:  $x_{18}$  || Revisor:  $x_{11}$  || )**

Compute the following numbers:

a)  $\sqrt{9} = \sqrt{3 \cdot 3} = 3$

b)  $\sqrt{1600} = \sqrt{10 \cdot 10 \cdot 4 \cdot 4} = 40$

c)  $(100)^{\frac{1}{2}} = \sqrt{10 \cdot 10} = 10$

d)  $\sqrt{9 + 16} = \sqrt{25} = \sqrt{5 \cdot 5} = 5$

e)  $(36)^{\frac{1}{2}} = \sqrt{6 \cdot 6} = 6$

f)  $\sqrt{0.49} = \sqrt{(0.1)(0.1) \cdot 7 \cdot 7} = 0.7$

g)  $\sqrt{0.01} = \sqrt{(0.1)(0.1)} = 0.1$

h)  $\sqrt{\frac{1}{25}} = \sqrt{\frac{1}{5} \cdot \frac{1}{5}} = \frac{1}{5}$   
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**Resolução ( || Questão: 2.5.2 || Relator:  $x_{20}$  || Revisor:  $x_{15}$  || )** Let a and b be positive numbers.

Decide whether each “?” should be replaced by = or  $\neq$ . Justify your answer.

(a)  $\neq$ , dado que  $\sqrt{25} \cdot \sqrt{16} = 5 \cdot 4 = 20$  e  $\sqrt{16 \cdot 20} = \sqrt{400} = 20$ .

(b)  $\neq$ , dado que  $\sqrt{25} + \sqrt{16} = 5 + 4 = 9$  e  $\sqrt{16 + 25} = \sqrt{41} \neq 9$ .

(c)  $\neq$ , Como  $(a + b)^{\frac{1}{2}} = a^{\frac{1}{2}} + b^{\frac{1}{2}}$  é equivalente a  $\sqrt{(a + b)} = \sqrt{a} + \sqrt{b}$ . Foi mostrado no item (b) que esta igualdade é falsa.

(d)  $=$ , pois  $(\sqrt{a + b})^{-1} = [(a + b)^{\frac{1}{2}}]^{-1} = (a + b)^{-\frac{1}{2}}$ .

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**Resolução ( || Questão: 2.5.3 || Relator: x<sub>04</sub> || Revisor: x<sub>16</sub> || )**

3. Solve for x the following equalities:

a)  $\sqrt{x} = 9 \rightarrow x = 81$

b)  $\sqrt{x} \cdot \sqrt{4} = 4 \rightarrow \sqrt{x} = 2 \rightarrow x = 4$

c)  $\sqrt{x+2} = 25 \rightarrow x+2 = 625 \rightarrow x = 623$

d)  $\sqrt{3} \cdot \sqrt{5} = \sqrt{x} \rightarrow x = 15$

e)  $2^{2-x} = 8 \rightarrow 2^{2-x} = 2^3 \rightarrow 2-x = 3 \rightarrow x = -1$

f)  $2^x - 2^{x-1} = 4 \rightarrow (2^{x-1} \cdot 2) - 2^{x-1} = 2^2 \rightarrow 2^{x-1} \cdot (2-1) = 2^2 \rightarrow x-1 = 2 \rightarrow x = 3$

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**Resolução ( || Questão: 2.5.4 || Relator: x<sub>05</sub> || Revisor: x<sub>04</sub> || )**

(a)  $\frac{6}{\sqrt{7}} = \frac{6}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{6\sqrt{7}}{7}$

(b)  $\frac{\sqrt{32}}{\sqrt{2}} = \frac{\sqrt{32}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{32}\sqrt{2}}{2} = \frac{\sqrt{64}}{2} = 4$

(c)  $\frac{\sqrt{3}}{4\sqrt{2}} = \frac{\sqrt{3}}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}\sqrt{2}}{4 \cdot 2} = \frac{\sqrt{6}}{8}$

(d)  $\frac{\sqrt{54}-\sqrt{24}}{\sqrt{6}} = \frac{\sqrt{54}-\sqrt{24}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}\sqrt{54}-\sqrt{6}\sqrt{24}}{6} = \frac{3\sqrt{6}\sqrt{6}-2\sqrt{6}\sqrt{6}}{6} = \frac{3 \cdot 6 - 2 \cdot 6}{6} = 1$

(e)  $\frac{2}{\sqrt{3}\sqrt{8}} = \frac{2}{\sqrt{3}\sqrt{8}} \cdot \frac{\sqrt{3}\sqrt{8}}{\sqrt{3}\sqrt{8}} = \frac{2\sqrt{3}\sqrt{8}}{24} = \frac{\sqrt{3}\sqrt{8}}{12} = \frac{\sqrt{6}}{6}$

(f)  $\frac{4}{\sqrt{2y}} = \frac{4}{\sqrt{2y}} \cdot \frac{\sqrt{2y}}{\sqrt{2y}} = \frac{4\sqrt{2y}}{2y} = \frac{2\sqrt{2y}}{y}$

(g)  $\frac{x}{\sqrt{2x}} = \frac{x}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}} = \frac{x\sqrt{2x}}{2x} = \frac{\sqrt{2x}}{2}$

(h)  $\frac{x(\sqrt{x+1})}{\sqrt{x}} = \frac{x(\sqrt{x+1})}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{x\sqrt{x}(\sqrt{x+1})}{x} = \sqrt{x}(\sqrt{x+1}) = x + \sqrt{x}$

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**Resolução ( || Questão: 2.5.5 || Relator: x<sub>06</sub> || Revisor: x<sub>05</sub> || )**

5. Simplify the following expressions by making the denominators rational:

(a)  $\frac{1}{\sqrt{7}+\sqrt{5}} = \frac{1}{\sqrt{7}+\sqrt{5}} \cdot \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}^2-\sqrt{5}^2} = \frac{\sqrt{7}-\sqrt{5}}{7-5} = \frac{\sqrt{7}-\sqrt{5}}{2}$

(b)  $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})^2}{\sqrt{5}^2-\sqrt{3}^2} = \frac{\sqrt{5}^2-2\sqrt{5}\sqrt{3}+\sqrt{3}^2}{5-3} = \frac{5-2\sqrt{15}+3}{2} = \frac{2(4-\sqrt{15})}{2} = 4 - \sqrt{15}$

(c)  $\frac{x}{\sqrt{3}-2} = \frac{x}{\sqrt{3}-2} \cdot \frac{\sqrt{3}+2}{\sqrt{3}+2} = \frac{x\sqrt{3}+2x}{\sqrt{3}^2-2^2} = \frac{x\sqrt{3}+2x}{3-4} = \frac{x\sqrt{3}+2x}{-1} = -x(\sqrt{3}+2)$

(d)  $\frac{x\sqrt{y}-y\sqrt{x}}{x\sqrt{y}+y\sqrt{x}} = \frac{x\sqrt{y}-y\sqrt{x}}{x\sqrt{y}+y\sqrt{x}} \cdot \frac{x\sqrt{y}-y\sqrt{x}}{x\sqrt{y}-y\sqrt{x}} = \frac{(x\sqrt{y}-y\sqrt{x})^2}{(x\sqrt{y})^2-(y\sqrt{x})^2} = \frac{x^2y-2xy\sqrt{y}\sqrt{x}+xy^2}{x^2y-xy^2} = \frac{xy(x-2\sqrt{y}\sqrt{x}+y)}{xy(x-y)} = \frac{x-2\sqrt{y}\sqrt{x}+y}{x-y} = \frac{(\sqrt{x}-\sqrt{y})^2}{x-y}$

(e)  $\frac{h}{\sqrt{x+h}-\sqrt{x}} = \frac{h}{\sqrt{x+h}-\sqrt{x}} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{h\sqrt{x+h}+h\sqrt{x}}{\sqrt{x+h}^2-\sqrt{x}^2} = \frac{h(\sqrt{x+h}+\sqrt{x})}{x+h-x} = \sqrt{x+h} + \sqrt{x}$

$$(f) \frac{1-\sqrt{x+1}}{1+\sqrt{x+1}} = \frac{1-\sqrt{x+1}}{1+\sqrt{x+1}} \cdot \frac{1-\sqrt{x+1}}{1-\sqrt{x+1}} = \frac{(1-\sqrt{x+1})^2}{1-\sqrt{x+1}^2} = \frac{1-2\sqrt{x+1}+(x+1)}{1-x-1} = \frac{-(-2-x+2\sqrt{x+1})}{-x} = \frac{(2\sqrt{x+1}-2-x)}{x}$$

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**Resolução ( || Questão: 2.5.6 || Relator: x<sub>08</sub> || Revisor: x<sub>06</sub> || )**

Compute, without using a calculator, the following numbers:

(a)  $\sqrt[3]{125} = (125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5$

(b)  $(243)^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3$

(c)  $(-8)^{\frac{1}{3}} = ((-2)^3)^{\frac{1}{3}} = -2$

(d)  $\sqrt[3]{0.008} = ((0.2)^3)^{\frac{1}{3}} = 0.2$

(e)  $81^{\frac{1}{2}} = (9^2)^{\frac{1}{2}} = 9$

(f)  $64^{-\frac{1}{3}} = (4^3)^{-\frac{1}{3}} = \frac{1}{4}$

(g)  $16^{-2.25} = 16^{-2} \cdot (2^4)^{-\frac{1}{4}} = \frac{1}{256} \cdot \frac{1}{2} = \frac{1}{512}$

(h)  $(\frac{1}{3^{-2}})^{-2} = (9)^{-2} = \frac{1}{81}$

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**Resolução ( || Questão: 2.5.7 || Relator: x<sub>09</sub> || Revisor: x<sub>08</sub> || )**

Usando uma calculadora, encontre valores aproximados para:

a)  $\sqrt[3]{55} \approx 3,80295$

b)  $160^{1/4} \approx 3,55656$

c)  $(2,71828)^{1/5} \approx 1,22140$

d)  $(1 + 0,0001)^{10000} \approx 2,718146$

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**Resolução ( || Questão: 2.5.8 || Relator: x<sub>11</sub> || Revisor: x<sub>09</sub> || )**

The population of a nation increased from 40 million to 60 million in 12 years. What is the yearly percentage rate of growth  $p$ ?

O crescimento percentual anual será de:

$$60000000 = 40000000(1 + \frac{p}{100})^{12}$$

$$\frac{60000000}{40000000} = (1 + \frac{p}{100})^{12}$$

$$\frac{6}{4} = \frac{3}{2} = 1,5 = (1 + \frac{p}{100})^{12}$$

$$1,5^{\frac{1}{12}} = (1 + \frac{p}{100})^{12(\frac{1}{12})}$$

$$1,5^{\frac{1}{12}} = 1 + \frac{p}{100}$$

$$1,0344 - 1 = \frac{p}{100}$$

$$(0,0344)(100) = p$$

$$3,44\% = p$$

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**Resolução ( || Questão: 2.5.9 || Relator: x<sub>15</sub> || Revisor: x<sub>11</sub> || )**

Simplifique as seguintes expressões:

a)  $27x^{3p}y^{6q}z^{12r} = (3x^p y^{2q} z^{4r})^3$

b)  $\frac{(x+15)^{4/3}}{(x+15)^{5/6}} = (x+15)^{4/3-5/6} = (x+15)^{8/6-5/6} = (x+15)^{3/6} = (x+15)^{1/2} = \sqrt{(x+15)}$

c)  $\frac{8\sqrt[3]{x^2}\sqrt[4]{y}\sqrt{1/z}}{-2\sqrt[3]{x}\sqrt{y^5}\sqrt{z}} = -4x^{\frac{2}{3}-\frac{1}{3}}y^{\frac{1}{4}-\frac{5}{2}}z^{-\frac{1}{2}-\frac{1}{2}} = \frac{-4x^{\frac{1}{3}}y^{-\frac{9}{4}}}{z}$  ■

**Resolução ( || Questão: 2.5.10 || Relator: x<sub>18</sub> || Revisor: x<sub>15</sub> || )**

Simplify the following expressions, so that each contains only a single exponent:

(a)  $\left(\left(\left(a^{\frac{1}{2}}\right)^{\frac{2}{3}}\right)^{\frac{3}{4}}\right)^{\frac{4}{5}} = a^{\frac{1}{5}}$

(b)  $a^{\frac{1}{2}} \cdot a^{\frac{2}{3}} \cdot a^{\frac{3}{4}} \cdot a^{\frac{4}{5}} = a^{\frac{30+20 \cdot (2)+15 \cdot (3)+12 \cdot (4)}{60}} = a^{\frac{143}{60}}$

(c)  $\frac{[(3a)^{-1}]^{-2} \cdot (2a^{-2})^{-1}}{a^{-3}} = \frac{9a^2 \cdot (2a^{-2})^{-1} \cdot a^3}{1} = \frac{9a^2 \cdot a^2 \cdot a^3}{2} = \frac{9a^7}{2}$

(d)  $\frac{a^{\frac{1}{3}} \cdot a^{\frac{1}{12}} \cdot a^{\frac{3}{4}}}{a^{\frac{5}{12}} \cdot a^{\frac{1}{2}}} = \frac{a^{\frac{4+1+9}{12}}}{a^{\frac{5+6}{12}}} = \frac{a^{\frac{14}{12}}}{a^{\frac{11}{12}}} = a^{\frac{3}{12}} = a^{\frac{1}{4}}$

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**Resolução ( || Questão: 2.5.11 || Relator: x<sub>20</sub> || Revisor: x<sub>18</sub> || )** Which of the following equations

are valid for all  $x$  and  $y$ ?

(a)  $(2^x)^2 = 2^{x^2}$

(b)  $3^{x-3y} = \frac{3^x}{3^{3y}}$

(c)  $3^{\frac{-1}{x}} = \frac{1}{3^{1/x}}$ , with  $x \neq 0$

(d)  $5^{1/x} = \frac{1}{5^x}$ , with  $x \neq 0$

(e)  $a^{x+y} = a^x + a^y$

(f)  $2^{\sqrt{x}} \cdot 2^{\sqrt{y}} = 2^{\sqrt{x \cdot y}}$ , with  $x$  and  $y$  positives.

(a) Não é válida para todo  $x$ , pois  $(2^x)^2 \neq 2^{x^2}$ . Na verdade,  $(2^x)^2 = 2^{2x}$

(b) É válida para todo  $x$  e  $y$ .

(c) É válida para todo  $x$ .

(d) Não é válida para todo  $x$ , pois  $5^{1/x} \neq \frac{1}{5^x}$ , Na verdade  $\frac{1}{5^x} = 5^{-x}$

(e) Não é válida para todo  $x$  e  $y$ , pois  $a^{x+y} \neq a^x + a^y$ . Na verdade,  $a^{x+y} = a^x \cdot a^y$

(f) Não é válida para todo  $x$  e  $y$ , pois  $2^{\sqrt{x}} \cdot 2^{\sqrt{y}} \neq 2^{\sqrt{x \cdot y}}$ . Na verdade,  $2^{\sqrt{x}} \cdot 2^{\sqrt{y}} = 2^{\sqrt{x} + \sqrt{y}}$

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**Resolução ( || Questão: 2.5.12 || Relator: x<sub>04</sub> || Revisor: x<sub>05</sub> || )**

$$32x^{\frac{3}{2}} > 4x^3 \quad (1)$$

Dividindo ambos os lados por 4.

$$8x^{\frac{3}{2}} > x^3 \quad (2)$$

Dividindo ambos os lados por  $x^{\frac{3}{2}}$ , já que  $x > 0$ .

$$2^3 > x^3 \cdot x^{-\frac{3}{2}} = 2^3 > x^{\frac{3}{2}} \quad (3)$$

Elevando ambos os lados por  $\frac{2}{3}$

$$(2^3)^{\frac{2}{3}} > (x^{\frac{3}{2}})^{\frac{2}{3}} \quad (4)$$

Dessa forma,  $4 > x$ . Portanto, para que o processo A produza mais que o processo B, o nível de insumo tem que ser menor que 4 ■