→ Aula de Exercícios (23/08)

1) Revisão dos conceitos

a) Unidades

Comprimento/Distância: 

| régua
| laser
| ⋮

SI: $[x] = m$ $\qquad 1\,cm = 10^{-2}\,m$

$1\,Km = 10^{3}\,m.$

Tempo:  – | relógio
| laser
⋮

SI: $[t] = s$

$1\,hora = 3600\,s$

$1\,dia = 24 \cdot 3600\,s = 86{,}400\,s$

Velocidade: SI: $[v] = \frac{m}{s}$, $1\frac{Km}{h} = 3{,}6\,\frac{m}{s}$

b) Equações horárias: função $\underbrace{\mathbb{R}}_{t} \rightarrow \underbrace{\mathbb{R}^3}_{\substack{\vec{x}(t) \\ \vec{v}(t) \\ \vec{a}(t) \\ \vdots}}$ $\quad f(t)$

Ex: $x(t) = at + b$

$x(t) = Ae^{-Kt}$

$v(t) = \sum_n A_n t^n$

⋮

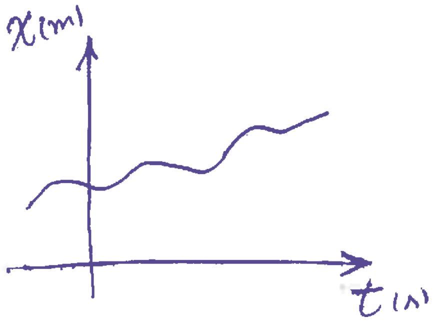
$\vec{x}(t) = at\,\hat{i} + bt^2\,\hat{j}$

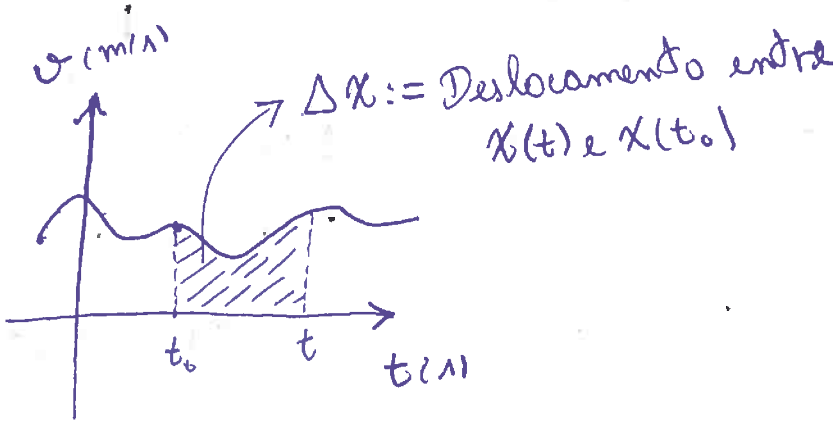
C) Posição, velocidade e aceleração (escalar)

$$x(t), \quad v(t) := \frac{dx(t)}{dt} = \dot{x}(t) \longrightarrow x(t) = x(t_0) + \int_{t_0}^{t_0} v(t)\,dt$$

$$a(t) := \frac{d^2x(t)}{dt} = \ddot{x}(t)$$

D) Gráficos

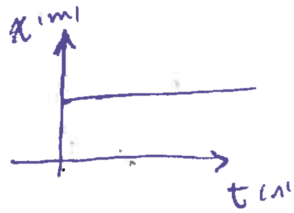




Velocidade média $:= \dfrac{\Delta x}{\Delta t}$

E) Tipos de Movimento:

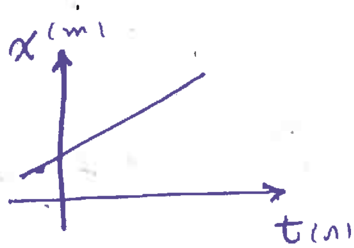
(i) $v(t) = 0 \to a(t) = 0, \quad x(t) = C$



(ii) $v(t) = v_0 = cte \to a(t) = 0; \quad x(t) = x_0 + v_0 t$

$\Downarrow$

movimento uniforme



Exercícios propostos: 12, 16, 18, 19, 27, 29, 30, 32, 34, 35 e 52.

12. a) 

$r = 10cm$

$r = 10 \cdot 10^{-2} m$

$r = 10^{-1} m$

$R_\odot \approx 10^9 m \quad R_T \approx 10^7 m$

$r_\odot — R_\odot \qquad 10^{-1} m — 10^9 m$

$r_T — R_T \qquad r_T — 10^7 m$

b) $\left.\begin{array}{l} d — D \\ r_\odot — R_\odot \end{array}\right\} \rightarrow d = \frac{D}{R_\odot} r_\odot = \frac{10^{11}}{10^9} \cdot 10^{-1} m$

$10^9 r_T = 10^6 \rightarrow r_T = 10^{-3} m$

$d = 10^{11-1-9} = 10m$

16. $x(t) = (3t^2 - 2t + 3)\ m$

$v = \dot{x} = 6t - 2 \rightarrow v(t) = 6t - 2$

a) $v_m = \frac{\Delta x}{\Delta t} \Rightarrow v_m(2s \rightarrow 3s) = \frac{x(3) - x(2)}{3 - 2} = x(3) - x(2)$
$= 24 - 11 = 13 m/s$

b) $v(2) = 6 \cdot 2 - 2 = 10 m/s$

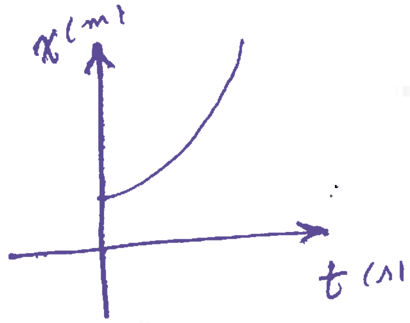
$v(3) = 6 \cdot 3 - 2 = 16 m/s$

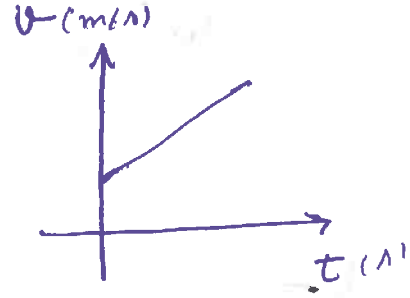
d) $a = \frac{dv}{dt} = \ddot{x} = 6 m/s^2$

c) $a_m = \frac{\Delta v}{\Delta t} \Rightarrow a_m(2s \rightarrow 3s) = \frac{16 - 10}{3 - 1} = 6 m/s^2$

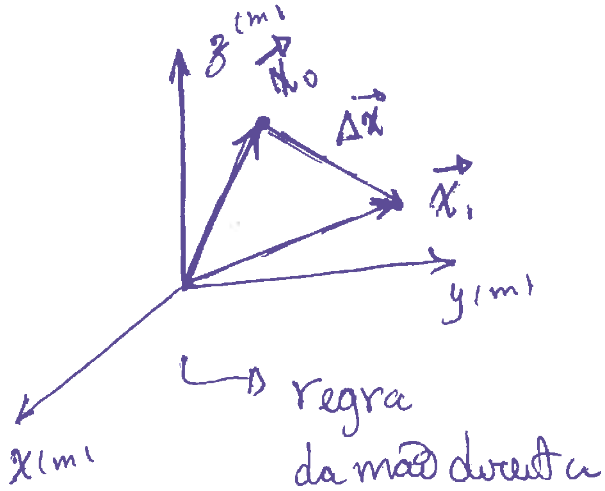
(iii) $v(t) = v_0 + at \rightarrow a(t) = a = cte$

$$x(t) = x_0 + v_0 t + \frac{at^2}{2}$$





F) Vetores em $\mathbb{R}^3$



Soma de Vetores

$\vec{a} = (\alpha, \beta, \gamma)$

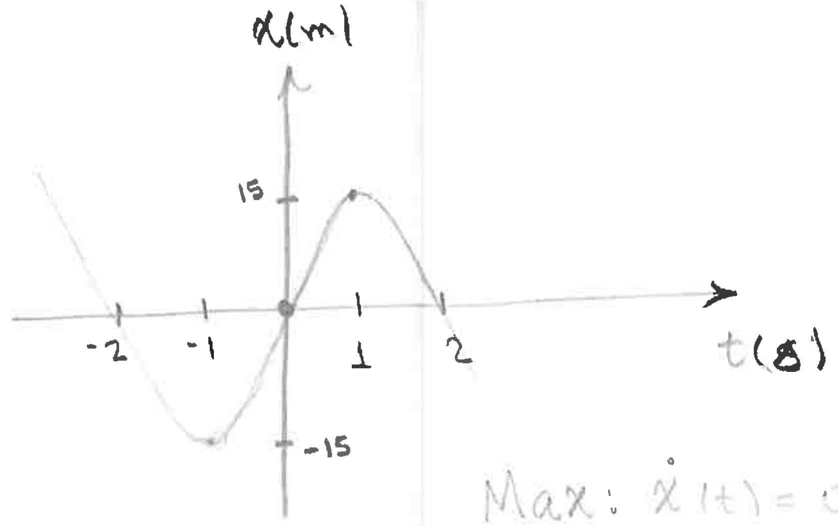
$\vec{b} = (\xi, \varphi, \lambda)$

$\vec{a} \pm \vec{b} = (\alpha \pm \xi, \beta \pm \varphi, \gamma \pm \lambda)$

Produto Escalar

$\vec{a} \cdot \vec{b} = \alpha \cdot \xi + \beta \cdot \varphi + \gamma \cdot \lambda$

d)



t	x
0	0
1	15
-1	-15
2	0
-2	0

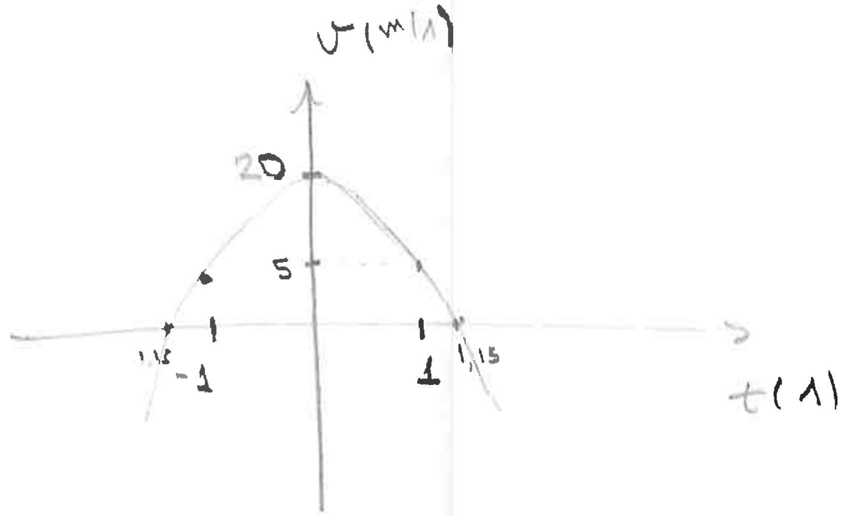
Max: $\dot{x}(t) = 0$

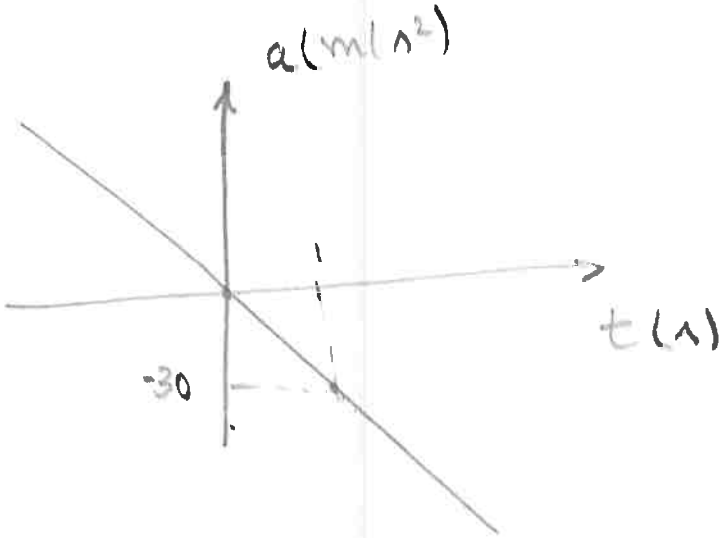
Min

$\downarrow$

$v(t) = 0 \rightarrow t_{max} = \sqrt{\frac{20}{15}}$

$t_{min} = -\sqrt{\frac{20}{15}}$





18. $\Delta x \overset{N}{=}$ Área Do gráfico $= \triangle + \square + \triangle + \square$

$$= \frac{8 \cdot 2}{2} + 8 \cdot 8 + \frac{4 \cdot 2}{2} + 4 \cdot 4 = 8 + 64 + 4 + 16 = 92$$

$$\Delta x = 92\,m$$

19. $v(t) = at + b$ $\qquad a = \frac{\Delta v}{\Delta t} = \frac{-50-50}{10} = -10\,\frac{m}{s^2}$

$v(t) = 50 - 10t$ $\qquad b = v(t=0) = 50$

$$\hookrightarrow x = x_0 + \int_{t_0}^{t} v(t)\,dt$$

$$x(t) = 0 + \int_0^t (50 - 10t)\,dt = 50t - 5t^2$$

$$a(t) = \frac{dv}{dt} = -10\,m/s$$

27. $x(t) = 20t - 5t^3$

a) $v(t) = 20 - 15t^2 \rightarrow$ p/ $v(t_a) = 20 - 15t_a^2 = 0$

$\rightarrow 15t_a^2 = 20 \rightarrow t_a^2 = \frac{20}{15} \rightarrow t_a = \pm\sqrt{\frac{20}{15}} \cong \pm 1,15$

b) $a(t) = -30t \rightarrow a(t_b) = 0 \rightarrow t_b = 0$

c) $a(t) < 0$ p/ $\forall\, t > 0$, se $t < 0 \rightarrow a > 0$

29. a) CL ⇒ $\bar{v}_{CL} = 10\,m/s$

BR ⇒ $\bar{v}_{BR} = \dfrac{42,19\,km}{2h\,10min} = \dfrac{42,19\cdot \cancel{10^3}\,m}{7,8\cdot \cancel{10^3}\,s} \cong 5,41\,\dfrac{m}{s}$

1h — 60 min
t — 10 min

$t = \dfrac{1}{6}\,h = \dfrac{1}{6}\cdot 3600\,s$
$= 600\,s$

$2h\,10min = 2\cdot 3600 + 600$
$= 7800\,s$

b) $x_{CL} = \bar{v}_{CL}\cdot t \rightarrow t = \dfrac{x_{CL}}{\bar{v}_{CL}} = \dfrac{42,19\cdot 10^{\cancel{3}\,2}\,\cancel{m}}{\cancel{10}\,\cancel{m}/s}$

$\boxed{t = 4219\,s}$

30. $\Delta x = v\,\Delta t = 90\,\dfrac{km}{h}\cdot 0,5\,s = \dfrac{90}{3,6}\,\dfrac{m}{s}\cdot 0,5\,s$
$= 12,5\,m$

32. trem 1: $v_{0_1} = 72\,\dfrac{km}{h} = 20\,\dfrac{m}{s}$ $\quad a_1 = -1,0\,m/s^2$

$\longrightarrow$

$v_1(t) = 20 - t$

trem 2: $v_{0_2} = -144\,\dfrac{km}{h} = -40\,\dfrac{m}{s}$ $\quad a_2 = +1,0\,m/s^2$

$\longleftarrow$

$v_2(t) = -40 + t$

$$x_1(t) = 0 + \int_0^t v_1\,dt = 20t - \frac{t^2}{2}$$

$$x_2(t) = 950 + \int_0^t v_2\,dt = -40t + \frac{t^2}{2} + 950$$

Colidem se $x_1(t) = x_2(t)$

$$20t - \frac{t^2}{2} = -40t + \frac{t^2}{2} + 950$$

$$t^2 - 60t + 950 = 0 \rightarrow \text{se } \exists \text{ solução } \in \mathbb{R} \rightarrow \text{Colidem}$$

$$\Delta = (60)^2 - 4 \cdot 950 =$$

$$3600 - 3800 = -200 < 0 \rightarrow \nexists\, t \in \mathbb{R} \rightarrow \tilde{N} \text{ colidem}$$

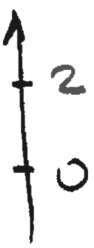
35. a) $v = v_0 + gt = 10t$

$y = -3 + 5t^2$

$y(t_a) = 0 \rightarrow t_a = \pm\sqrt{\frac{5}{3}}$

$\rightarrow v = 10\sqrt{\frac{5}{3}}$ m/s

b) quando $y = 2 \rightarrow v_{min} = 0$



$v(t) = v_0 - gt = v_0 - 10t$

$y = y_0 + v_0 t - \frac{gt^2}{2} = v_0 t - 5t^2$

$t = \frac{v_0}{10}$ p/ y=2

$\frac{v_0^2}{10} - \frac{5v_0^2}{100} = 2$

$10v_0^2 - 5v_0^2 = 200$

$v_0^2 = \frac{200}{5} = 40$

$v_0 = \sqrt{40} = 6{,}32$ m/s

c) $a_M = \frac{v_0 - 0}{0{,}02} = 316$ m/s²

34.

$\uparrow v = 20 m/s$ $\quad y(t) = y_0 + v_0 t - \frac{gt^2}{2}$

$$y(t) = 20t - 5t^2$$

a) $y(t) = 0$ (retorno ao chão)

$20t - 5t^2 = 0$

$t = 0$ ou $20 - 5t = 0 \rightarrow t = 4 s$

b) $y_{max} \rightarrow \dot{y} = 0 \rightarrow 20 - 10t = 0$

$t = 2 s$

$y_{max} = y(2) = 20 \cdot 2 - 5 \cdot 4 = 40 - 20 = 20 m$

c) $15 = 20t - 5t^2$

$t^2 - 4t + 3 = 0 \qquad \Delta = 16 - 12 = 4$

$t = \frac{4 \pm 2}{2} \rightarrow t_1 = 1 s$

$t_2 = 3 s$

→ Lois Lane e Super-homem

Consegue resgatar? O que precisamos saber para ter certeza?

Posição LL e SH, $\vec{x}(t)_{LL}$ $\vec{x}_{SH}(t)$

LL: queda-livre

$$\vec{x}(t) = \left(0, 0, h - \frac{gt^2}{2}\right)$$

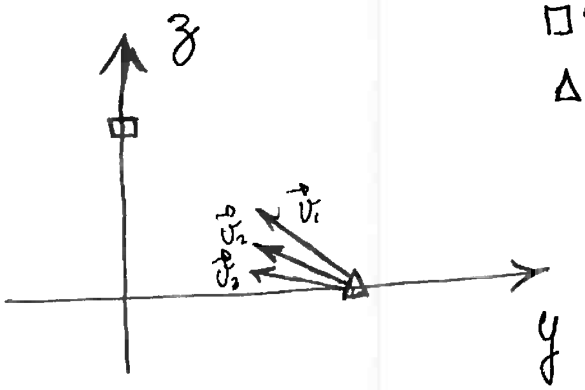
SH:

$$\vec{x}(0) = (x, y, 0)$$

$$\vec{x}(t) = (x(t), y(t), z(t))$$

fixar $x(t) = 0$

Início:



□: LL

Δ: SH

$$\vec{v}_{SH}(t) = (0, \dot{y}_{SH}, \dot{z}_{SH})$$

$$\left\{ \begin{array}{l} tg\theta = \dfrac{\dot{z}_{SH}}{\dot{y}_{SH}} \\ |v_{SH}|^2 = \dot{y}_{SH}^2 + \dot{z}_{SH}^2 \end{array} \right.$$

Supor que $\dot{y}_{SH} = v_y = cte$ e $\dot{z}_{SH} = v_z = cte$

$$\vec{x}_{SH} = (0,\ y - v_y t,\ v_z t)$$

$$\begin{cases} y - v_y t = 0 \rightarrow t = y/v_y \\ h - \dfrac{gt^2}{2} = v_z t \rightarrow \dfrac{g}{2}t^2 + v_z t - h = 0 \end{cases}$$

$$\Delta = v_z^2 + 4 \cdot \frac{gh}{2} = v_z^2 + 2gh$$

$$t = \frac{-v_z \pm \sqrt{v_z^2 + 2gh}}{g}$$

$$\frac{y}{v_y} = \frac{v_z\left(-1 \pm \sqrt{1 + 2gh/v_z^2}\right)}{g}$$

Supor que $\dot{y}_{SH} = v_y = cte$ e $\dot{z}_{SH}(t) = at$

$$\vec{x}_{SH} = (0,\ y - vt,\ \frac{a}{2}t^2)$$

$$\begin{cases} y - v_y t = 0 \longrightarrow v_y t = y \rightarrow v_y = y/t \\ h - \dfrac{gt^2}{2} = \dfrac{a}{2}t^2 \longrightarrow 2h = (a+g)t^2 \end{cases}$$

$$\hookrightarrow t^2 = \frac{2h}{(a+g)} \rightarrow t = \sqrt{\frac{2h}{a+g}}$$

$$\boxed{v_y = y\sqrt{\frac{a+g}{2h}}}$$