

**Table of Laplace and Z-transforms**

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	$z^{-k}$
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	$e^{-at}$	$e^{-akT}$	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	$t$	$kT$	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	$t^2$	$(kT)^2$	$\frac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	$t^3$	$(kT)^3$	$\frac{T^3 z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{(1 - e^{-aT})z^{-1}}{(1 - z^{-1})(1 - e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1 - e^{-aT}z^{-1})(1 - e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	$te^{-at}$	$kTe^{-akT}$	$\frac{Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1 - (1+aT)e^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1 + e^{-aT}z^{-1})z^{-1}}{(1 - e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT - 1 + e^{-aT}) + (1 - e^{-aT} - aTe^{-aT})z^{-1}]z^{-1}}{(1 - z^{-1})^2(1 - e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1 - z^{-1} \cos \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1 - 2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1 - e^{-aT} z^{-1} \cos \omega T}{1 - 2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
18.	–	–	$a^k$	$\frac{1}{1 - az^{-1}}$
19.	–	–	$a^k$ $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1 - az^{-1}}$
20.	–	–	$ka^{k-1}$	$\frac{z^{-1}}{(1 - az^{-1})^2}$
21.	–	–	$k^2 a^{k-1}$	$\frac{z^{-1}(1 + az^{-1})}{(1 - az^{-1})^3}$
22.	–	–	$k^3 a^{k-1}$	$\frac{z^{-1}(1 + 4az^{-1} + a^2 z^{-2})}{(1 - az^{-1})^4}$
23.	–	–	$k^4 a^{k-1}$	$\frac{z^{-1}(1 + 11az^{-1} + 11a^2 z^{-2} + a^3 z^{-3})}{(1 - az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1 + az^{-1}}$

$x(t) = 0$  for  $t < 0$

$x(kT) = x(k) = 0$  for  $k < 0$

Unless otherwise noted,  $k = 0, 1, 2, 3, \dots$

## Definition of the Z-transform

$$\mathcal{Z}\{x(k)\} = X(z) = \sum_{k=0}^{\infty} x(k)z^{-k}$$

## Important properties and theorems of the Z-transform

	$x(t)$ or $x(k)$	$Z\{x(t)\}$ or $Z\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t) + bx_2(t)$	$aX_1(z) + bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z) - zx(0)$
4.	$x(t+2T)$	$z^2X(z) - z^2x(0) - zx(T)$
5.	$x(k+2)$	$z^2X(z) - z^2x(0) - zx(1)$
6.	$x(t+kT)$	$z^kX(z) - z^kx(0) - z^{k-1}x(T) - \dots - zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z) - z^kx(0) - z^{k-1}x(1) - \dots - zx(k1-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz \frac{d}{dz} X(z)$
11.	$kx(k)$	$-z \frac{d}{dz} X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^a)$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z \frac{d}{dz} X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z \rightarrow \infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z \rightarrow 1} [(1-z^{-1})X(z)]$ if $(1-z^{-1})X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k) = x(k) - x(k-1)$	$(1-z^{-1})X(z)$
19.	$\Delta x(k) = x(k+1) - x(k)$	$(z-1)X(z) - zx(0)$
20.	$\sum_{k=0}^n x(k)$	$\frac{1}{1-z^{-1}} X(z)$
21.	$\frac{\partial}{\partial a} x(t, a)$	$\frac{\partial}{\partial a} X(z, a)$
22.	$k^m x(k)$	$\left(-z \frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^n x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$