7.8 – The 180° Hybrid

Reading Assignment: *pp. 352-361*

Recall there are **two** different types of ideal **4-port** 3dB couplers: the **symmetric** solution and the **anti-symmetric** solution.

We know that the symmetric solution is the **Quadrature Hybrid.**

The anti-symmetric solution is called the **180 Degree Hybrid** (aka, ring hybrid, rat-race hybrid, Magic-T)

HO: The 180° Hybrid

The 180° Hybrid Coupler

The 180° **Hybrid Coupler** (sometimes know as the "ring", "rat-race", or "Magic-T" hybrid) is a lossless, matched and reciprocal 4-port device, with a scattering matrix of the **anti-symmetric** form (**D_1** symmetry):

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Just like the quadrature coupler, however, we find that:

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

So that the scattering matrix for this device is:

$$\mathcal{S} = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Hence, this coupler is likewise a **3dB coupler**—the power into a given port (with all other ports matched) is equally divided between two of the three output ports.

Note the relative **phase** between the outputs, however, is **dependent** on which port is the input.

For example, if the **input** is port 1 or port 3, the two signals will be **in phase**—no difference in their relative phase!

However, if the input is port 2 or port 4, the output signals will be 180° **out of phase** ($e^{j\pi} = -1$)!

An interesting application of this coupler can be seen if we place **two input signals** into the device, at ports 2 and 3 (with ports 1 and 2 terminated in matched loads). Note the signal out of port 1 would therefore be:

$$\begin{aligned} V_1^-(z) &= S_{12}\, V_2^+(z) + S_{13}\, V_3^+(z) \\ &= \frac{1}{\sqrt{2}}\left(V_3^+(z) + V_2^+(z)\right) \end{aligned}$$

while the signal out of port 4 is:

$$\begin{aligned} V_4^-(z) &= S_{42}\, V_2^+(z) + S_{43}\, V_3^+(z) \\ &= \frac{1}{\sqrt{2}}\left(V_3^+(z) - V_2^+(z)\right) \end{aligned}$$

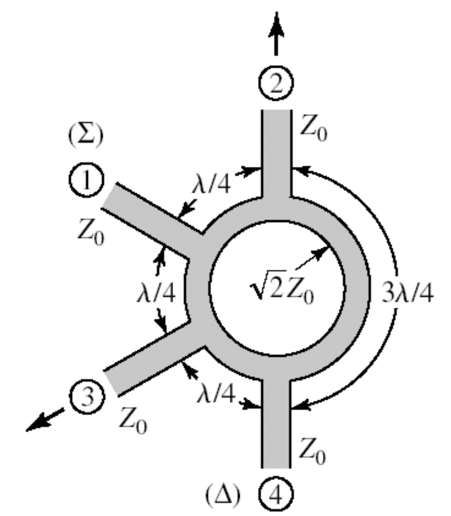
Note that the output of port 1 is proportional to the **sum** of the two inputs. Port 1 of a 180° Hybrid Coupler is thus often referred to as the **sum** (Σ) port.

Likewise, port 4 is proportional to the **difference** between the two inputs. Port 4 a 180° Hybrid Coupler is thus often referred to as the **delta** (Δ) port.

There are **many** applications where we wish to take the sum and/or difference between two signals!

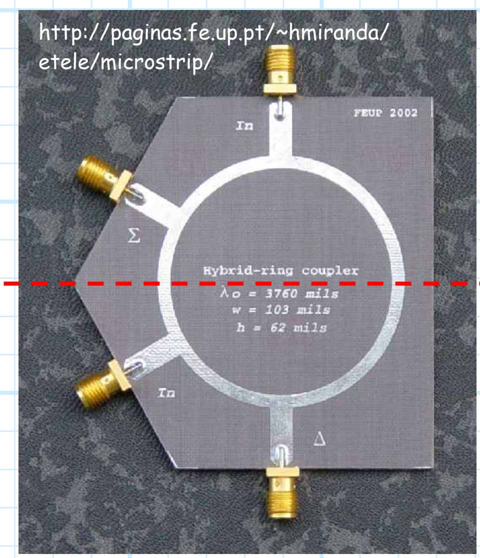
The 180° Hybrid Coupler can likewise be used in the **opposite** manner. If we have **both** the sum and difference of two signals available, we can use this device to separate the signals into their separate components!

Q: *How is this hybrid coupler constructed?*



A: Like the quadrature hybrid, it is simply made of **lengths** of transmission lines. However, unlike the quadrature hybrid, the characteristic impedance of each line is **identical** ($\sqrt{2}Z_0$), but the lengths of the lines are dissimilar.

Q: *How can we possibly analyze this mess?*



A: Note there **is** one plane of bilateral **symmetry (D_1)** in this circuit—we can use even/odd mode analysis!

However, we must perform **two** separate analysis—one using sources on ports **1 and 3**:

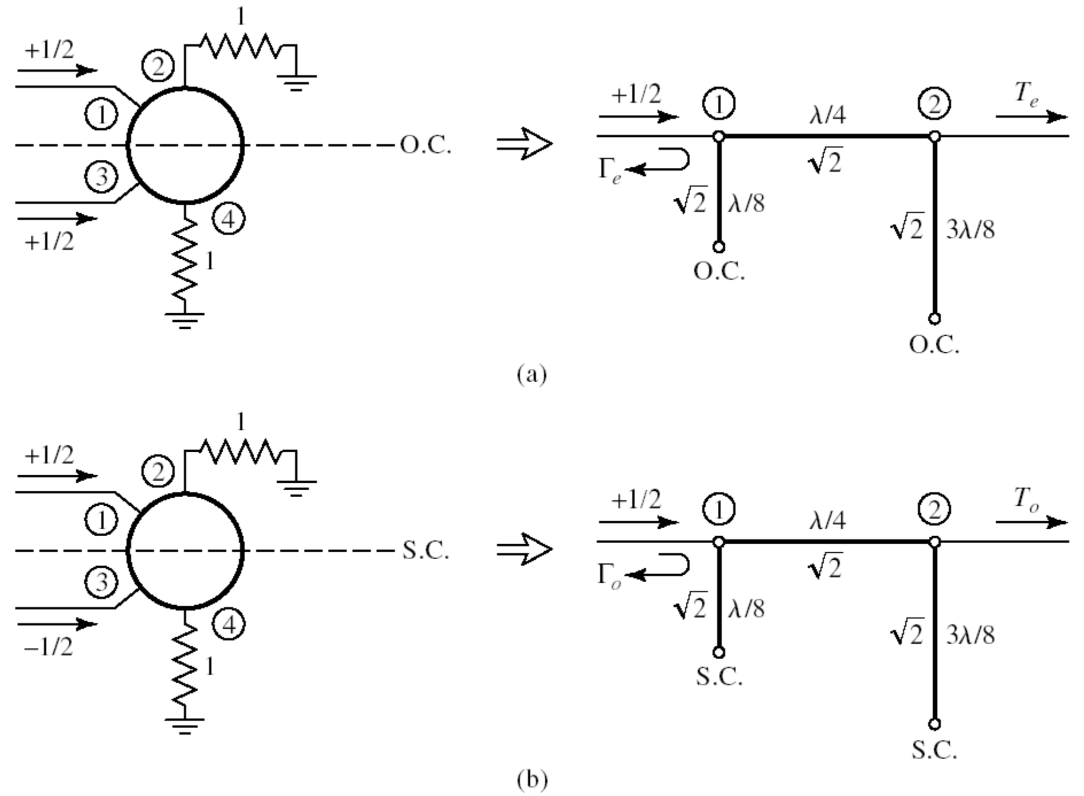


Figure 7.44 (p. 354)
Even- and odd-mode decomposition of the ring hybrid when port 1 is excited with a unit amplitude incident wave. (*a*) Even mode. (*b*) Odd mode.

While the **other** uses sources on ports **2 and 4**:

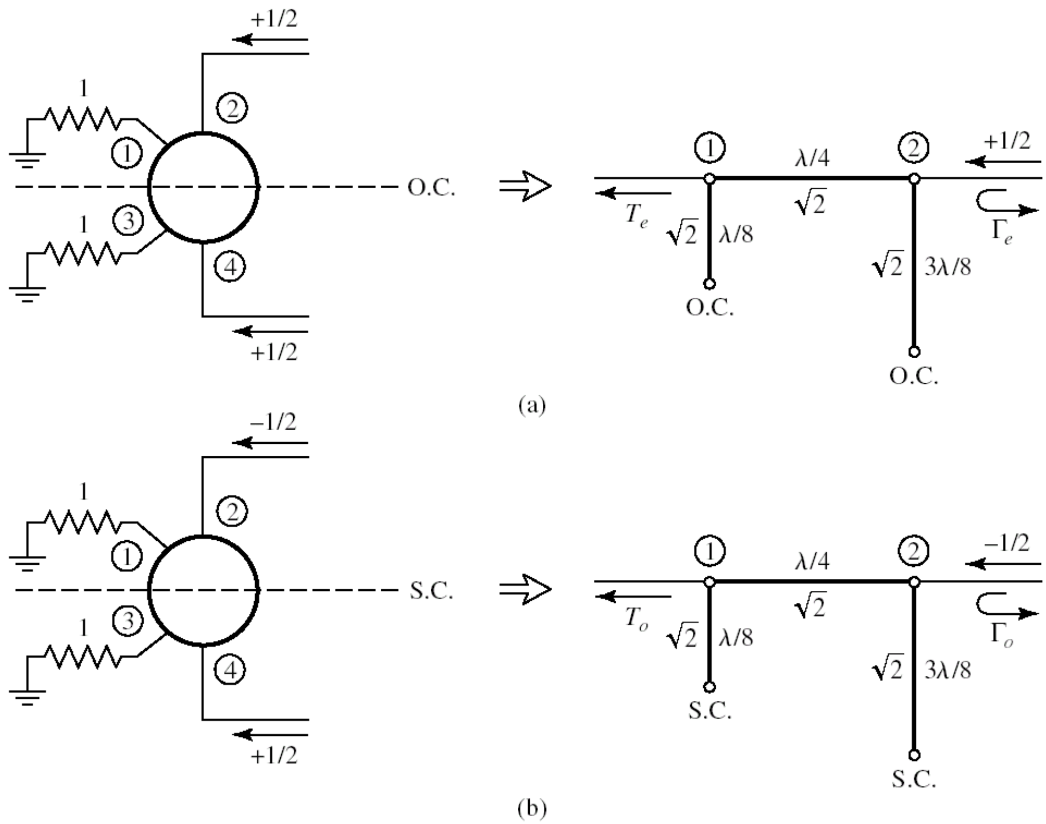


Figure 7.45 (p. 356)
Even- and odd-mode decomposition of the ring hybrid when port 4 is excited with a unit amplitude incident wave. (*a*) Even mode. (*b*) Odd mode.

Finally, because of the transmission line lengths, we find that the ring hybrid is a **narrow-band** device:

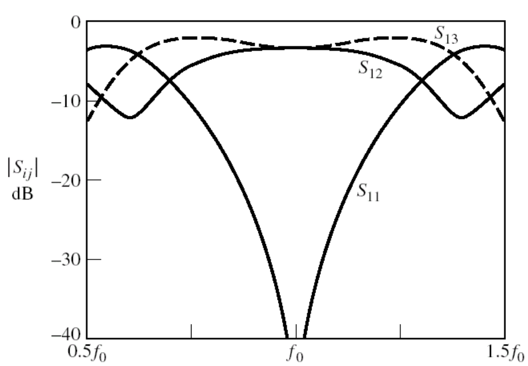


Figure 7.46 (p. 357)
S parameter magnitudes versus frequency for the ring hybrid of Example 7.9