# Measurement of $e/m_0$ using Dunnington's method—An experiment for advanced undergraduates

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In this article we report an experiment to measure $e/m_0$ using a method based on the work of Dunnington. Electrons are accelerated in an alternating electric field and subsequently deflected in a magnetic field. The measurements which are required are the frequency of the alternating accelerating voltage, the magnitude of the magnetic field at the electron orbit, and the angle through which the electrons are deflected. Using a frequency of 30 MHz the experiment yielded a value for $e/m_0$ of $(1.738 \pm 0.020)10^{11}$ C kg$^{-1}$.

## I. INTRODUCTION

1983 was the 50th anniversary of the publication of the first of F. G. Dunnington's famous series of experiments leading to a (then) accurate value for the specific charge. Following a suggestion made by E. O. Lawrence, Dunnington[1] made an accurate determinaton of $e/m_0$ by a deflection method in which electrons having been accelerated were deflected in a circular path by a magnetic field. The electrons were accelerated by an alternating voltage whose value, unlike comparable steady voltage methods did not need to be known. However the frequency was required and this, of course, could be measured very accurately.

The motivation for Dunnington's work was a substantial discrepancy between the results obtained by deflection methods and by spectroscopic methods. R. T. Birge[2] drew attention to this difference and summarized the results of various determinations prior to 1929. These discrepancies were largely resolved by Dunnington's initial experiments[1] but subsequent improvements in his techniques[3] provided a value of $e/m_0$ which was still at variance with the then improved alternative methods of specific charge determination.

In this article we report an experiment which does not attempt to reproduce the precision of Dunnington's work but provides a platform for students to acquire experience over a wide range of physics in a single experiment. Measurements of small magnetic fields ($10^{-4}$ T) and small currents (pA) have to be made while coping with the demands of radio frequency techniques and good vacuum practice.

## II. PRINCIPLE OF DUNNINGTON'S METHOD

Electrons are emitted from the filament $F$ (Fig. 1) and migrate to slit 1 where they respond to the alternating electric field. For half of any given period of the impressed voltage, e.g., 0 to $T/2$ seconds (Fig. 2), the electrons are accelerated and pass through slit 2 into a region ideally free from electric fields. The sinusoidal radio frequency voltage gives the electrons speeds from zero up to a maximum defined by $V_{max}$. The electrons move in circular orbits under the influence of a steady magnetic field **B**. The slits 2 to 5 define a circular path of radius $r$.

The application of a magnetic field **B** over a circular path of radius $r$ will select electrons of a given speed. Since the oscillator frequency $\nu$ is fixed there will be a value of $B$, say $B_0$, which selects electrons accelerated through potential $V_0$ such that they will arrive at slit 5 exactly one oscillator period $T$ after leaving slit 2. Thus on arrival at slit 6 they will all be stopped by a retarding voltage $V_0$. With this value of $B$ the electrons accelerated by other voltages will not move on the circle of radius $r$ and hence will not reach slit 5.

Consider now electrons which have been accelerated through some voltage $V$ ($V \neq V_0$). A value of the **B** field can be found for which these electrons will move in the circle of radius $r$. However since the oscillator is of fixed frequency $\nu$ these electrons will never take exactly one period to traverse from slit 2 to slit 5. Thus electrons of this kind will reach the collector and hence constitute a collector current. Experimentally $T$ and $r$ are fixed. If the **B** field is increased slowly from zero for all values of $B$ ($B \neq B_0$) a collector current will be observed. When $B = B_0$ the collector current should ideally fall to zero.

The radial force equation for an electron is

$$mv^2/r = B\,e\,v.$$

At the desired collector current minimum the electron completes the arc $\theta$ (Eq. 2) in one period $T$ so that the speed $v$ is

$$v = r\theta/T = r\,\theta\,\nu.$$

Thus

$$e/m = \theta\nu/B_0. \qquad (1)$$

$\theta$ is largely defined by the arc between slits 2 and 5 but includes a contribution due to the fact that the electrons are

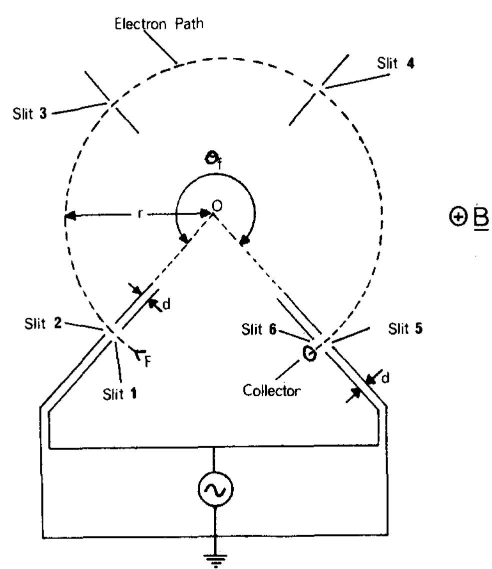


Fig. 1. Schematic diagram of apparatus.

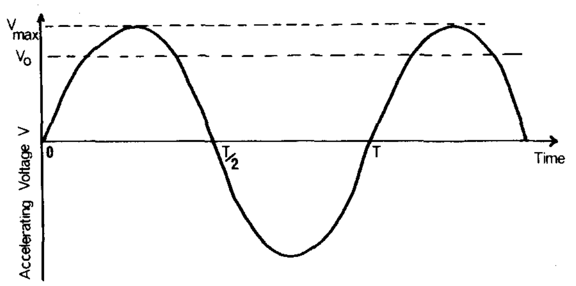

Fig. 2. Accelerating voltage as a function of time.

accelerated and decelerated over finite distances. Using a detailed analysis[3] Dunnington showed that

$$\theta = \theta_f + (4/3)d/r, \quad (2)$$

where $\theta_f$ is the major angle subtended at the center 0 by slits 2 and 5 and $d$ is the equal separation of slits 1 and 2 and slits 5 and 6. Notice that in Eq. (1) the accelerating voltage does not appear and as a result the effects of contact potentials between slits 1, 2 and 5, 6 which complicate other free electron methods, do not affect the location of the collector current minimum.

## III. EXPERIMENTAL PROCEDURES AND METHOD

The vacuum chamber design (Fig. 3) essentially followed that of Dunnington. All parts were constructed from non-magnetic materials such as brass and polytetrafluoroethylene (PTFE). The vacuum chamber was operated at a pressure of $10^{-3}$ Pa or less and this was obtained using a standard rotary/diffusion pump system. Slits 1 and 6 were formed in the walls of an open ended brass cylinder (Figs. 3 and 4) which was electrically connected to the resonant circuit by means of a copper tube suitably supported and vacuum sealed in a PTFE mount (Fig. 3). The leads to the filament entered the vacuum chamber through a glass to metal seal mounted in the end of the copper tube. The filament holder (Fig. 4) was fixed to the cylinder and the output lead from the collector was taken through an insulating seal fixed in the base of the vacuum chamber. Slits 2, 3, 4, and 5 were formed in brass plates which were attached to a central solid brass cylinder. Care was taken to ensure that these slits were properly grounded. Slits 2 and 5 were accurately positioned with respect to slits 1 and 6 by means of screws passing through insulating washers and spacers.

The region from slit 2 to slit 5 inside the vacuum chamber is required, ideally, to be free of electric fields and precautions were therefore taken to shield any parts of the radio frequency circuit which might produce a field in this region. All metallic parts not at the radio frequency potential were grounded.

The electron collector must be close to slit 6 since when near the collector current minimum, electrons emerging from slit 6 have small speeds and small radii of curvature in the **B** field. Since the collector was situated within the electron gun assembly and was therefore subject to strong radio frequency fields and collector currents of the order of picoamps were expected the collector was shielded as far as possible. A grounded conducting cylinder was used, closed at the top but open at the bottom where the collector output lead passed through the vacuum chamber base via an insulated vacuum seal. The collector current was measured using a vibrating capacitor electrometer with its own shielding positioned as close as possible to the vacuum chamber base.

The vibrating capacitor electrometer has a very high in-

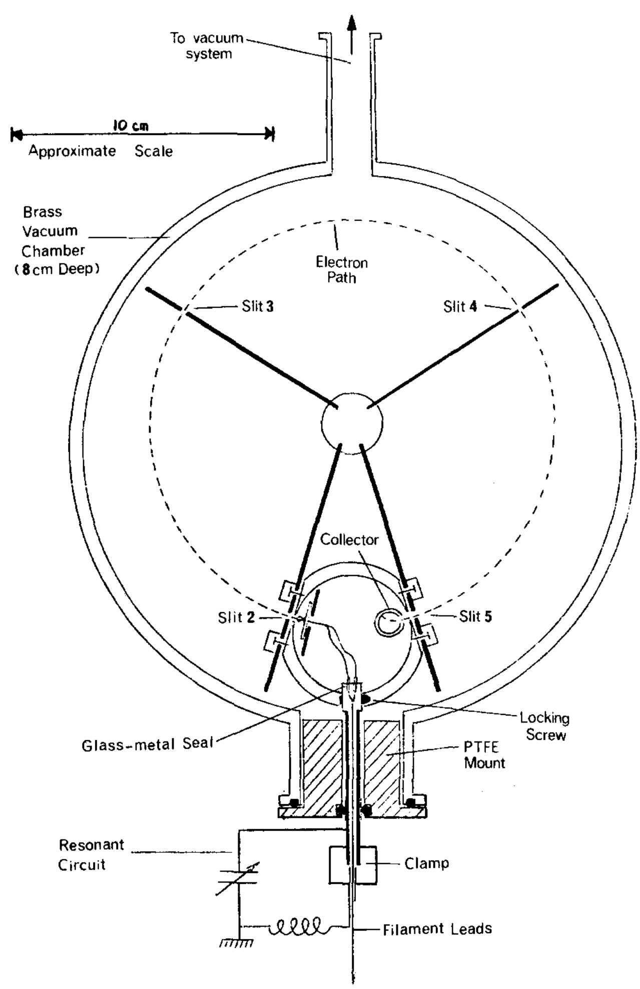

Fig. 3. Plan view of vacuum chamber.

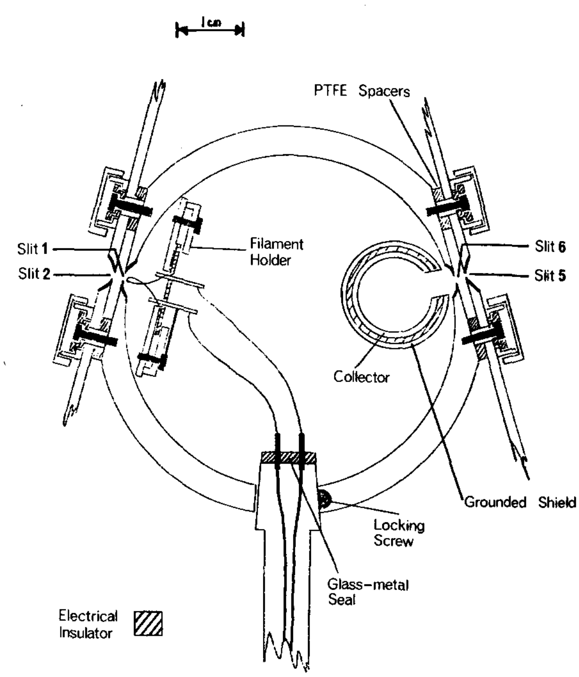

Fig. 4. Detail of gun assembly.

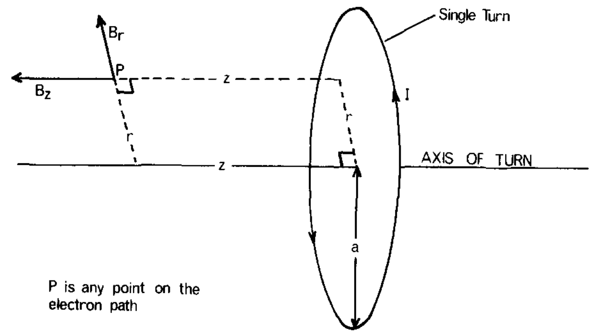


Fig. 5. Single turn from the Helmholtz coil system.

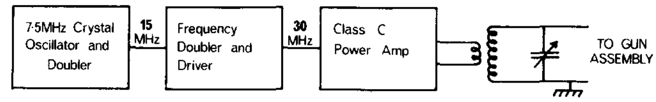


Fig. 6. Block diagram of power oscillator.

put impedance ( $> 10^{15}\,\Omega$ ). The unidirectional current from the collector flows to earth through a known fixed high value resistor built into the electrometer. The voltage signal developed across this resistor is chopped by the vibrating capacitor and the resulting alternating signal is amplified and displayed as a voltage. The collector current is calculated by dividing the displayed voltage by the value of the fixed resistor.

The configuration of electric fields in the vicinity of the collector was found to be improved by machining the brass around the slits to provide knife edges (Fig. 4). The spacing of the slits 1, 2, and 5, 6 was fixed by suitably machined insulating spacers. These spaces contributed significantly to the capacitance of the vacuum chamber which had to be sufficiently small to allow a reasonably high frequency $\nu$ (30 MHz) to be used.

The dielectric properties and dimensions of the spacers were chosen accordingly. The radio frequency voltage was applied to the accelerating plates via a resonant circuit containing an inductor and variable capacitor (Fig. 3). The inductor took the form of a coiled copper tube the exact shape of which was dictated by the required resonant frequency. This inductor tube was clamped to the copper tube entering the vacuum chamber and the filament leads were carried through both tubes.

The magnetic field was produced by means of a Helmholtz pair of coils. Each turn of the Helmholtz system was considered separately in calculating the field $B$. The field perpendicular to the plane of the electron orbit is the sum of the contributions of all "single" turns (Fig. 5). Thus for one turn

$$B_z = \frac{\mu_0 I}{2\pi}[(a+r)^2+Z^2]^{1/2}\left\{\frac{(a^2-Z^2-r^2)}{[(a-r)^2+Z^2]}E+K\right\}, \tag{3}$$

where $a$ is the turn radius, $I$ the current, $K$ and $E$ the complete elliptic integrals of the first and second kinds, respectively,[4] with arguments $k$ where

$$k^2 = 4ar/[(a+r)^2+Z^2]. \tag{4}$$

$B_z$, the component of the **B** field parallel to the axis of the single turn, was numerically integrated over all the values of $a$ and $z$ of the Helmholtz coil system, to give the field $B$ at the electron path. The axes of the orbit and that of the Helmholtz coils were made to be coincident. Direct measurement of the **B** field using a gaussmeter agreed with, but failed to improve upon, the accuracy of the calculated field. At the oscillator frequency used the local earth magnetic field was a significant correction to the field produced by the Helmholtz system. As a consequence the attitude of the vacuum chamber had to be adjusted so that the local earth field was perpendicular to the plane of the electron orbit. The magnitude of the earth field was measured using a proton magnetometer in the vicinity of the vacuum chamber and the direction ascertained by means of a dip circle.

The remaining parameter required for a calculation of the **B** field was the Helmholtz coil current. The high radio frequency power resulted in large stray radio frequency fields being produced in the vicinity of the apparatus. These were sufficiently strong to interfere with many measuring instruments and necessitated the use of accumulators to supply the filament, Helmholtz coils, and a potentiometer which was used to measure the Helmholtz coil current.

The separation of slits $d$, required in Eq. (2), was defined by the insulating spacers. The radius $r$ was taken as the average distance from the inside and outside of each slit to 0 (Fig. 1). All slits had an axial length of 1 cm and a width of 0.5 mm. Slits 3 and 4 were left with square edges. The angle $\theta_f$ is defined from the knife edges of slits 2 and 5 and was obtained from the average chord between slits 2 and 5 and the radius.

A block diagram of the radio frequency power supply is shown in Fig. 6. The output frequency from a 7.5-MHz crystal oscillator was quadrupled and driven into a class C power amplifier capable of delivering 100 W of radio frequency power. The voltage output of the amplifier could be varied continuously from 0 to 5 kV peak-to-peak. The radio frequency power was coupled to the resonant circuit via an inductor constructed from copper tube. Adjustment of the tuning capacitor in the resonant circuit allowed maximum power to be transferred to the vacuum chamber. Maximum power was detected by inserting a loop of wire, connected to a light bulb, into the coils of the inductor of the resonant circuit.

The power consumed by the resonant circuit is, neglecting other radio frequency power losses, proportional to the resistance $R$ of this circuit. Most of the $R$ arose from the physical connections between the inductor and the remainder of the circuit. $R$ was kept as low as possible by using stong mechanical connections but even so a circuit resistance of the order of one-tenth of an ohm was unavoidable. This led to a power dissipation of approximately 70 W when working at 2 kV and 30 MHz. Thus a moderately powerful oscillator is required for this experiment.

## IV. RESULTS AND DISCUSSION

The graph in Fig. 7 shows the variation of collector current with Helmholtz coil current in the region of interest. The three curves show the results obtained for filament currents of 0, 1.84, and 1.94 A. Although not shown on the graph the collector current falls sharply to zero at a coil current of 551 mA. For a sufficiently large **B** field, electrons accelerated by the peak voltage $V_{max}$ (Fig. 2) move in the circle radius $r$. If the **B** field is increased further the largest radius of curvature is less than $r$ and so no electrons are collected. The **B** field corresponding to zero collector

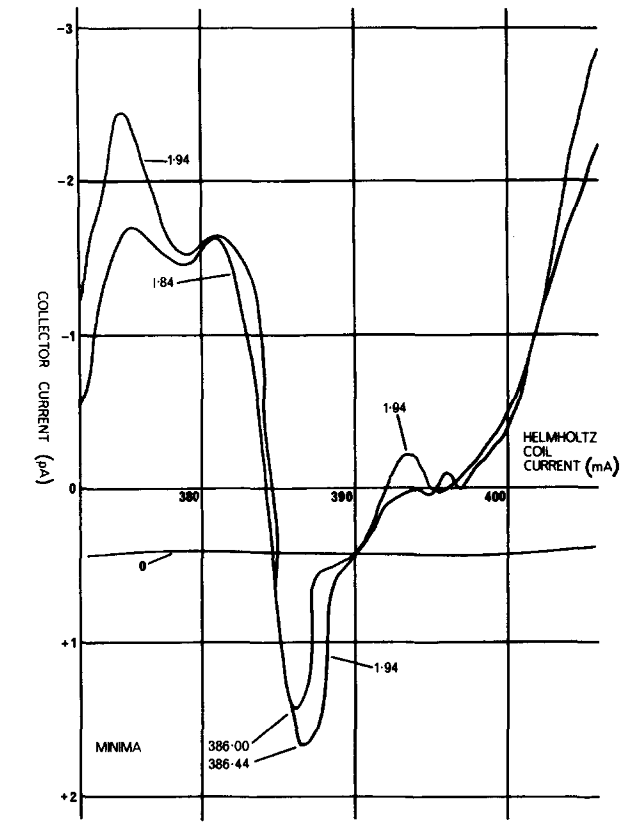


Fig. 7. Graph of collector current against Helmholtz coil current for filament currents 0, 1.84, and 1.94 A.

current with the accepted value of $e/m$ gives a value for $V_{max}$ in agreement with the measured value.

As the **B** field was increased from zero the electrometer measured an increasing negative current as would be expected from the collection of electrons. However at some point the magnitude of the negative current started to decrease and eventually the electrometer indicated a positive current. The minimum was located with the current positive. On further increase of the **B** field the magnitude of the positive current became smaller and eventually the current reversed direction and became negative once more.

The nonzero positive background current reading was observed when radio frequency voltages were applied with zero filament current and was attributed to incomplete shielding of the collector. Even after correcting for the change in zero collector current the minima were still observed with the electrometer measuring positive current. This positive current can be explained by the effects of secondary emission within the collector. Electrons which have passed through slit 6 are accelerated onto the collector surface by almost the same potential difference responsible for the deceleration up to slit 6. Electrons impinging on the collector produce secondary electrons providing the primary electron energy is above a few eV, which it always is. Thus within the collector a space charge of electrons will exist and a net loss of electrons from this space charge out of the collector gives rise to a measured positive current. Electrons near the collector aperture will, for half a cycle of the radio frequency field, experience a force accelerating them away from the collector thus providing a net loss from the collector. Away from the minimum, large numbers of electrons enter the collector and although there is a loss of electrons as described previously this loss is far outweighed by the true current arriving and hence a negative current will be measured by the electrometer. However near the minimum there are fewer electrons entering the collector and the number lost will be greater than the number gained and the measured current will be positive. This, in part, explains why the measured current goes positive. Any production of positive ions within the collector by interaction of the electrons with the residual gas will also give a contribution to the positive current.

Measurements made in the apparatus yielded the following results:

$$d = (1.2 \pm 0.1)10^{-3}\ \text{m}$$

$$r = (8.006 \pm 0.019)10^{-2}\ \text{m}$$

$$\theta_f = (5.7733 \pm 0.0075)\ \text{rad}$$

$$\nu = (30.000 \pm 0.0030)10^{6}\ \text{Hz}$$

$$B_0 = (1.001 \pm 0.012)10^{-3}\ \text{T}.$$

($B_0$ is the **B** field from the Helmholtz pair and the local earth field.)

Using these results in Eq. (1) and (2) yields the following value of $e/m_0$:

$$e/m_0 = (1.738 \pm 0.020)10^{11}\ \text{C kg}^{-1}.$$

The major source of uncertainty in the final result is the error in the **B** field. Two factors limited the accuracy to which $B_0$ may be obtained: the precision with which the magnetic field at a point may be determined and the accuracy with which the electron orbit may be located with respect to the Helmholtz coils. The remaining uncertainty in the final result is due almost entirely to the uncertainty in $\theta$ given by Eq. (2).

A relativistic correction may be considered in two parts. While the electron moves with constant speed $v$ it has a relativistic mass $m$ instead of its rest mass $m_0$ and so Eq. (1) becomes

$$e/m_0 = m\theta\nu/(m_0 B_0) = \theta\nu[1 - (r\theta\nu/c)^2]^{-1/2}/B_0, \qquad (5)$$

where $c$ is the velocity of light. The second part considers the variation of mass with speed during the acceleration and deceleration. In deriving Eq. (2) it was assumed that throughout their motion the electrons have the mass $m$; the error this introduces is insignificant.

The graph shows that the location of the collector current minimum depends on the filament current. Space charges within the apparatus may have a significant effect on the location of the minimum. The size and distribution of space charge is expected to depend upon the filament current. The effects of space charges were not explored.

Significant radio frequency currents were introduced in the Helmholtz coils due to their proximity to the resonant circuit. This made the measurement of the Helmholtz coil current difficult and necessitated the use of potentiometer, but the time dependent **B** field resulting from the induced current cannot influence the motion of electrons which are shielded by the thick brass vacuum chamber walls. All steady **B** fields will affect the motion of the electrons and since a homogeneous **B** field was required at the electron orbit care must be taken to reduce distortions to the **B** field due to stray static magnetic fields.

A further assumption used in the derivation of Eq. (2) is that all slits are of zero width. In the limit of zero slit width there is no penetration of radio frequency electric fields into the region from slit 2 to slit 5 and hence the electron speed in this region would then be constant. A series of experiments for various slit widths would allow extrapolation to zero slit width. Dunnington's work indicates that this provides another small correction. Further confinement of the electric field to the regions between the accelerating and decelerating slits could be achieved by reducing the axial length of the slits.

The most significant uncertainty after that in the **B** field came from the error in $\theta$, the major contribution provided by the uncertainty in $\theta_f$. It might be possible to use a modified optical spectrometer table to measure $\theta$ directly.

The value of $e/m_0$ should be independent of electron energy. The use of a single frequency in this work allowed the measurement of $e/m_0$ for only a single energy. An investigation of $e/m_0$ for various energies using Dunnington's method could be made using a variable frequency power oscillator.

The apparatus can be easily adapted to use a constant accelerating voltage $V$. A new collector must be positioned to intercept the electron beam immediately before slit 5 since all electrons would be stopped by the electric field existing between slits 5 and 6. A maximum collector current will then be observed when electrons move in the circle radius $r$. $V$ and $B$ are then related by

$$V \approx B^2 r^2 e/(2m) - v_0^2 m/(2e), \quad (6)$$

where $v_0$ is the initial velocity of the electron on passing through slit 1 due to thermal motion and the contact potential that may exist between the filament and slit 1. We performed this experiment in our apparatus making a series of measurements of $B$ and $V$. Using a least-squares fit of results to Eq. (6) gave a value of $v_0$ which corresponded to an electron energy of approximately 0.5 eV. Such a value for $v_0$ can have no measurable effect on the final result from Dunnington's method.

**ACKNOWLEDGMENT**

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[1]F. G. Dunnington, Phys. Rev. **43**, 404 (1933).
[2]R. T. Birge, Phys. Rev. Suppl. **1**, 1 (1929).
[3]F. G. Dunnington, Phys. Rev. **52**, 475 (1937).
[4]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965), pp. 590–591.

# Microwave–plasma interaction experiment

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A microwave–plasma interaction experiment for laboratory courses is described. The experiment employs a conventional microwave oven to generate breakdown plasmas over a pressure range of 0.02 to 36 Torr in argon. Temporally resolved measurements of forward, reflected, and transmitted microwave power are reported, as well as plasma optical emission measurements. Results indicate a uniform plasma of electron density less than $2\times10^{10}$ cm$^{-3}$ can be generated with over 300 W of peak microwave power.

## I. INTRODUCTION

In recent years, high-power microwaves have become important for ionization and heating of fusion relevant plasmas in magnetic confinement devices. In tokamak (toroidal) plasma confinement machines, microwave heating at the electron cyclotron frequency has been demonstrated to be effective in raising the electron temperature at the center of the plasma.[1] Another significant application to fusion experiments has been in the area of microwave startup,[2] in which microwave ionization generates a target plasma for subsequent heating Ohmically, by neutral beams, or radio frequency waves at the ion cyclotron frequency. In the tandem mirror (linear) type of magnetic confinement device, microwave heating is used to reduce particle losses from the ends electrostatically by modification of the ambipolar plasma potential.[3] Unmagnetized microwave discharge plasmas[4] have numerous applications, including ion sources for industrial processing and space propulsion systems.

There is clearly a need to introduce students to the important area of high-power microwave–plasma interactions. The gyrotron[5] type of microwave source used in fusion research is far too expensive and has power levels (200 kW) that are much too high for teaching-laboratory use. However, the cost of microwave ovens has recently decreased to the point where these represent an economical source of high-power microwaves (typically 600-W average at $0.33/W), making them ideal for teaching laboratories.

This article describes a high-power microwave–plasma interaction experiment developed for a plasma dynamics laboratory course offered to senior undergraduate and graduate students. This experiment provides the student with an introduction to the physics of the interaction of

  