The force on the elevator each time it comes to a rest, besides the friction force, is equal to the slope of the potential energy curve at the points where the total energy line touches the potential energy curve. The condition for the elevator to stop is that, at a point where its velocity is zero, the friction force is equal to the resultant of the gravitational and elastic forces. If, at one of the points where the potential energy curve and the total energy line meet, the slope of the potential energy curve is lower than or equal to the maximum static friction force, the condition is fulfilled, and the elevator will end its journey at the first point where this occurs (y_F on Fig. 2).

This method, when presented to our students, seemed to please them much more than the usual differential equation treatment. They understood and successfully applied it rather easily. It can be used in many situations, every time the potential energy and the dry friction forces are known.

Computing the Balmer wavelengths with a hand calculator

Miriam Sidran
The Bernard M. Baruch College, CUNY, New York, New York 10010

I would like to describe how a student's question in my general physics class (for premedical students) led spontaneously to a class project which turned into an exhilarating experience for us all.

It is a fact often stated in elementary physics textbooks that the wavelength λ of any hydrogen spectral line may be exactly calculated by means of the formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_{lower}^2} - \frac{1}{n_{higher}^2} \right) = \frac{E(n_{higher}) - E(n_{lower})}{hc} \quad (1)$$

This formula is easily derived from Bohr theory. In this expression, n_{higher} and n_{lower} are the principal quantum numbers of the initial and final orbits, respectively, of the electron whose transition gives rise to the line emission. $E(n_{higher})$ and $E(n_{lower})$ are the energies of the corresponding orbits, h is Planck's constant, and c is the speed of light in vacuum. The Rydberg constant R_H is given in units of m^{-1} by the expression:

$$R_H = 2\pi^2 m_r k^2 e^4/h^3 c; \qquad m_r = Mm/(M+m), \quad (2)$$

in which e is the electron charge, k the constant of proportionality in Coulomb's law, m the electron mass, M the proton mass, and m_r is the reduced mass of the electron orbiting the proton.

The students in my class were surprised to learn that the Balmer Series wavelengths could be exactly calculated using such a simple formula as Eq. (1). An enterprising student tried to verify this "allegation" using his hand calculator. He retained 8 significant digits in each input constant in Eq. (2), in order to guarantee accuracy.* He found that the wavelengths he computed for the first four Balmer lines were larger than the observed values listed in the text; the "error" in each case amounted to several Angstrom units. I then assigned this as a homework problem, and by our next meeting, all of the students had repeated the calculation and obtained the same results.

It was noted that the errors in question were not constant, nor were they a constant percentage of the wavelengths; they did however increase with increasing wavelength. I suggested that we list the possible reasons for these errors, and assign a volunteer to explore each reason. We agreed to limit the references used in this study to those handbooks and elementary/intermediate textbooks which were available in the department library. We further agreed that no student should spend too much time on this project, and that if they could not account for the errors within two weeks, I would explain them.

The first volunteer looked up the measured wavelength values in a handbook, and reported that they differed slightly from those in the text; however, these differences were much smaller than the errors. The second student looked up the most recent values of all the constants in Eq. (2). Using these revised data, the students recalculated the wavelengths. They were disappointed to find that the errors did not vanish as they had hoped.

At this point they strongly challenged the claim that Eq. (1) really gives exact values for the wavelengths. To support their challenge, they quoted several textbooks which stated that the wavelengths derived from Eq. (1) were "very close" to the observed wavelengths, differing from them by "only" a few Angstrom units. Furthermore, a student had come across the Dirac fine structure formula for correcting $E(n)$ in Eq. (1) for the effect of electron spin. The class speculated that this might represent the correction to Eq. (1) which they were seeking. However, these correction terms turned out to be an order of magnitude smaller than our errors. Thus the problem was not yet

*With the exception of k, the input constants of Eq. (2) are given to eight digits in Harvey White, *Modern College Physics*, 6th edition (Van Nostrand, 1972) on the page facing the rear inside cover. The eight-digit value of k was calculated from $k = 10^{-7}c^2$.

solved, although our list of possible reasons for the errors had not been fully explored. Since the deadline for a solution was approaching, I suggested that we re-examine the problem step by step, from the beginning.

This time the students noticed that the errors recalculated with the revised data were a constant fraction, 0.00029 of the corresponding wavelengths. Although they could not explain this fact, they were encouraged by discovering it. Two days before the deadline, the answer finally emerged. A triumphant student found two references which described the decrease in wavelength for radiation traveling from vacuum into air. The index of refraction for air was given as 1.00029, which exactly accounted for the errors of 0.00029λ.

$$\lambda_{\text{vacuum}} = \lambda_{\text{air}} (1.00029) \tag{3}$$

On the basis of this new evidence, the students were able to reason correctly that the errors arose because Eq. (1) gives the wavelengths in vacuum, while the laboratory measurements were made in air. Thus the mystery was finally solved.

The students were jubilant at their success, and their faith in Eq. (1) was restored. One said she felt like a member of a scientific research team. Another said it was like being part of a detective story. A third criticized the textbook for giving incomplete information which could lead to such errors when checked by students with hand calculators. But all agreed that they had enjoyed the shared project, and that they would never forget the Bohr Atom.

Geometrical representation of an electrical circuit

William G. Delinger
Northern Arizona University, Flagstaff, Arizona 86011

Sometimes diagrams or graphs provide a means of solving an electrical circuit problem, or at least, quickly estimating parameters in the design of electrical circuits. Also they may show relationships between parameters which would not be immediately obvious from just the mathematical equations. Some graphical techniques—for example, load lines for tube or transistor circuits—are well known. In teaching physics the author has used a geometrical representation which does not seem, however, to be as familiar.

Consider the resistors R_1 and R_2 connected in parallel as shown in Fig. 1. The equivalent resistance R of the combination can be calculated from the reciprocal addition equation $1/R = 1/R_1 + 1/R_2$ or $R = R_1R_2/(R_1 + R_2)$. This equivalent resistance can also be determined graphically by constructing the diagram shown in Fig. 2. To do this, draw the base line AC of some convenient length and draw the two perpendicular lines AF and CD scaled to represent the resistance values of R_1 and R_2. Then form an "X" by drawing lines AD and FC. The perpendicular distance BE represents the value of the resistance R. One can see immediately the resistance R will always be less than the smaller of the two resistance values. And in particular, if the resistors R_1 and R_2 are equal, the value of R will be one-half of one of the resistances.

The formal equivalence between the diagram shown in Fig. 2 and the parallel resistance equation can be demonstrated by the following geometrical argument: The triangle BCE is similar to triangle ACF and the triangle ABE is similar to triangle ACD. Since corresponding sides of similar triangles are proportional, $R/R_1 = BC/AC$, $R/R_2 = AB/AC$. If these two equations are added together and the fact $AC = AB + BC$ is used, one obtains $1/R = 1/R_1 + 1/R_2$.

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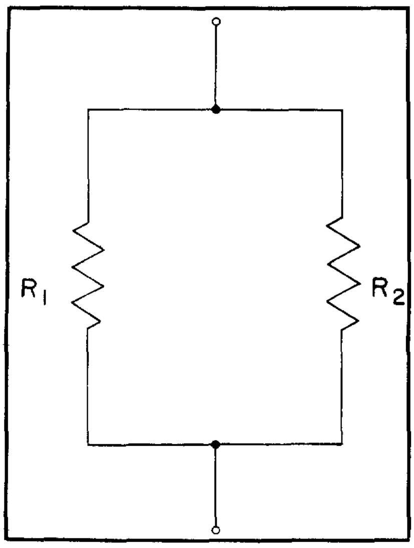

Fig. 1. Circuit diagram of two resistors connected in parallel.

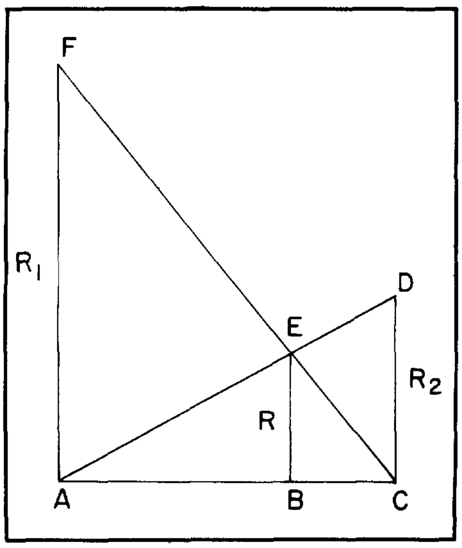

Fig. 2. Geometric representation of two resistors connected in parallel.