Lecture 2: Effective temperature of the Earth

September 30, 2003

Let us start by considering the Earth bathed in light from the Sun — see Fig.1 — and ask the question:

- What is the gross temperature of Earth?
- On what does that temperature depend?

Today we will learn that:

- $T_{SUN} = 6000K$ and it emits 'solar radiation' primarily in the visible.
- $T_{Earth} = 255K$ and it emits 'terrestrial radiation' primarily in the infra-red (IR).
- Atmosphere is essentially transparent to solar radiation
- Atmosphere is essentially opaque to IR due to its absorption by tri-atomic molecules such as H_2O, CO_2, O_3

1 The global energy balance

1.1 Emission temperature of Earth

Earth receives almost all its energy from Sun — only a small amount of geothermal heating.

Solar flux at the Earth is called the 'Solar constant', S_o — the INTENSITY of the radiation in which the Earth is bathed is:

$$S_o = 1367\ Wm^{-2}$$

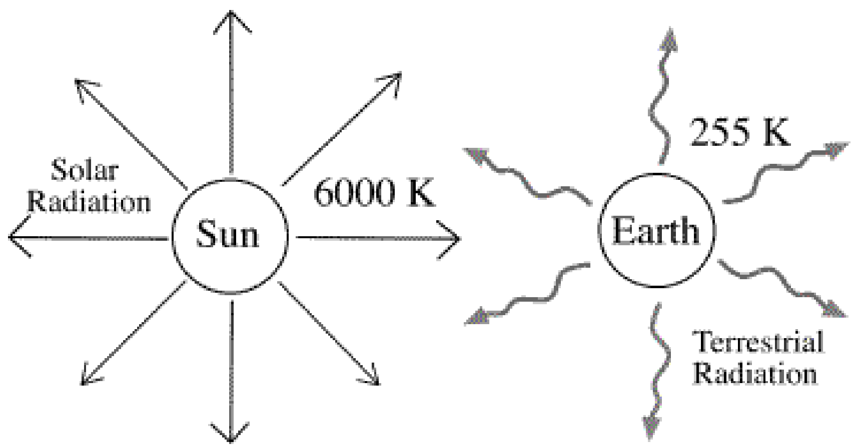

Figure 1:

S_o depends on distance of the planet from the Sun. Not really constant because of variations in Earths orbit (e.g Milancovich cycles)

The way in which radiation interacts with the atmosphere also depends on WAVELENGTH as well as the intensity. Relation between flux and wavelength is known as the spectrum.

The spectrum of solar radiation is shown in Fig.2. It peaks in the visible at a wavelength of $\lambda = 0.6\,\mu$m and decreases as λ increases and decreases. Note the colors of the rainbow — V, I, B, G, Y, O, R.

$$1 \text{ micron} = \mu\text{m} = 10^{-6} m$$

95% of all the energy lies between 0.25 and 2.5 μm — in the visible.

Why does spectrum have this pattern?

Such behavior is characteristic of the radiation emitted by *incandescent* material, as can be observed, for example, in a coal fire:

- The hottest parts of the fire are almost white and emit the most intense radiation with a wavelength that is shorter than that coming from the warm parts of the fire that glow red.
- The coldest parts of the fire do not seem to be radiating at all, but are, in fact, radiating in the infrared.

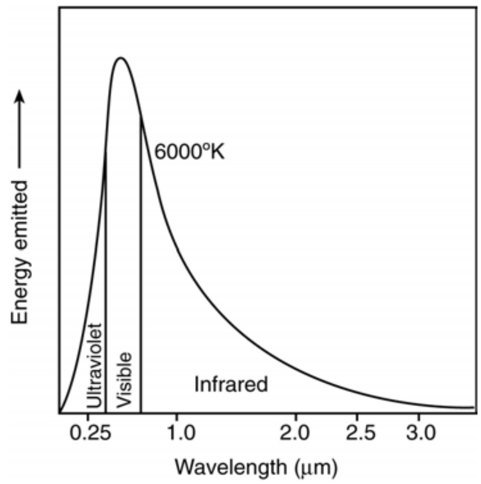

Figure 2: The energy emitted from the sun plotted against wavelength based on a black body curve with $T = T_{Sun}$. Most of the energy is in the visible and 95% of the total energy lies between 0.25 and 2.5 μm (10^{-6}m).

Experiment and theory show that the wavelength at which the intensity of radiation is a maximum, and the flux of emitted radiation depend only on the temperature of the source. The theoretical spectrum was worked out by Planck, and is known as the 'Planck' or 'blackbody'[1] spectrum.

It is plotted as a function of temperature in Fig.3.

If the observed radiation spectrum of the Sun is fitted to the black body curve, we deduce that the blackbody temperature of the sun is:

$$T_{SUN} = 6000K$$

Consider Fig.4.

$$\text{Solar flux intercepted by Earth} = S_o \pi a^2$$

Not all radiation is absorbed — a significant fraction is reflected.

$$\alpha = \frac{\text{reflected radiation}}{\text{incident radiation}}$$

[1] A black body is a theoretical construct that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black. It is also a perfect emitter of radiation.

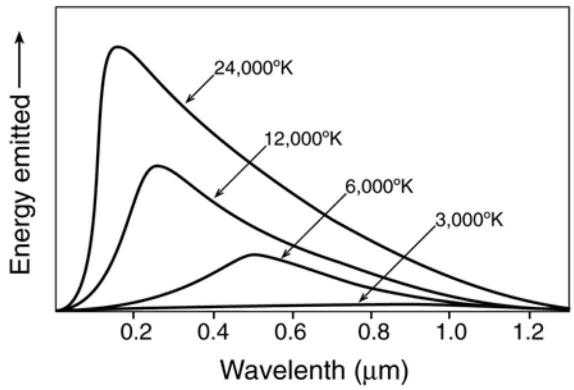

Figure 3: The energy emitted at different wavelengths for black bodies at several temperatures.

α is the albedo and depends on the nature of the reflecting surface — see Table **??**.
α large for snow, ice, cloud, desert; α low for ocean.
Earth as a whole has an albedo $\alpha_p = 0.3$.

$$\text{Absorbed radiation} = (1 - \alpha_p) S_o \pi a^2$$

What about emitted radiation?

If Earth radiates according to the Planck law then:

$$\text{emitted radiation per unit area} = \sigma T_e^4$$

where σ = Stefan-Boltzman constant ($\sigma = 5.67 \times 10^{-8} \mathrm{Wm^{-2}K^{-4}}$) and T_e is the emission (also called 'effective') temperature of the Earth.

$$\text{Emitted radiation} = \sigma T_e^4 \times 4\pi a^2$$

because the Earth is rapidly rotating.

Equating emitted to absorbed we find that:

$$T_e = \left[\frac{S_o(1 - \alpha_p)}{4\sigma} \right]^{1/4} \tag{1}$$

Note 'a', the radius of the Earth, does not appear.

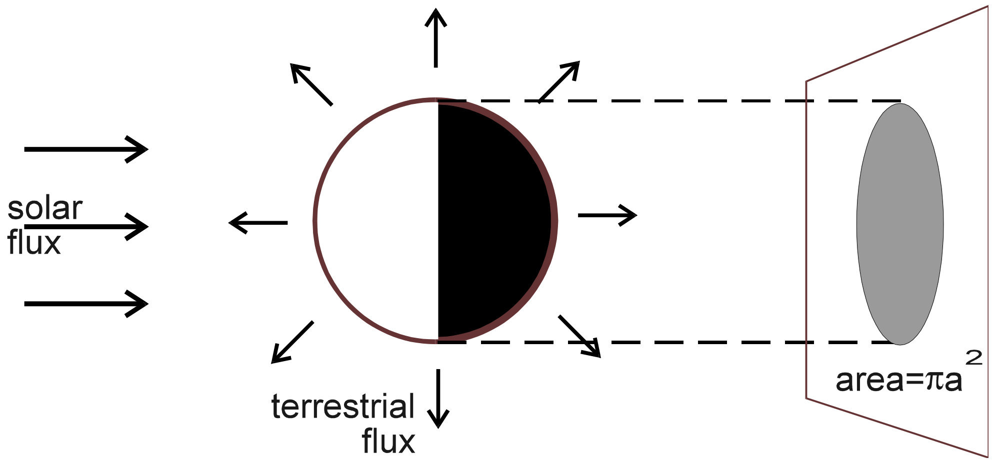

Figure 4: The spinning Earth is imagined to intercept solar flux over a disk and radiate terrestrial energy away isotropically from the sphere. Modified from Hartmann, 1994.

T_e depends only on α_p and S_o. Putting in numbers for the Earth we find that:

$$T_e = 255K$$

See Table 2.

Note T_m(measured) $\approx T_e$ except for Jupiter, where $\sim 1/2$ energy input comes from gravitational collapse.

Note also that T_s (temperature at the surface) $\neq T_e$.

1.2 Atmospheric absorption spectrum

Property of Planck radiation curve is

$$\lambda_m T = \text{constant}$$

where λ_m is the wavelength at which the Planck curve peaks — see Fig.3.

Given that $\lambda_{m_{SUN}} = 0.6\,\mu$m, $T_{SUN} = 6000K$ and $T_{Earth} = 255K$, then λ_m for the Earth is

Type of surface	Albedo
	(%)
Ocean	$2-10$
Forest	$6-18$
Cities	$14-18$
Grass	$7-25$
Soil	$10-20$
Grassland	$16-20$
Desert (sand)	$35-45$
Ice	$20-70$
Cloud (thin, thick stratus)	$30, 60-70$
Snow (old)	$40-60$
Snow (fresh)	$75-95$

Table 1: Albedos for different surfaces. Note that the albedo of clouds is highly variable and depend on the type and form.

	r	S_0	α_p	T_e	T_m	T_s	τ
	$10^9 m$	Wm^{-2}		K	K	K	Earth days
Venus	108	2632	0.77	227	230	760	243
Earth	150	1367	0.30	255	250	288	1.00
Mars	228	589	0.24	211	220	230	1.03
Jupiter	780	51	0.51	103	130	134^{cl_top}	0.41

Table 2: Properties of some of the planets. S_0 is the solar constant at a distance r from the Sun, α_p is the planetary albedo, T_e is the emission temperature computed from Eq.(1), T_m is the measured emission temperature and T_s the global mean surface temperature. The rotation period, τ, is also given in Earth days.

$$\lambda_{m_{Earth}} = \frac{6000}{255} \times 0.6\,\mu\text{m} = 14\,\mu\text{m}$$

which is in the far IR.

Thus Earth radiates back out to space in the far IR.

From Fig.5 we see that the black body spectra of the Sun and the Earth hardly overlap — this greatly simplifies our thinking about radiative transfer.

It turns out the atmosphere is largely transparent in the visible, but very opaque in the IR.

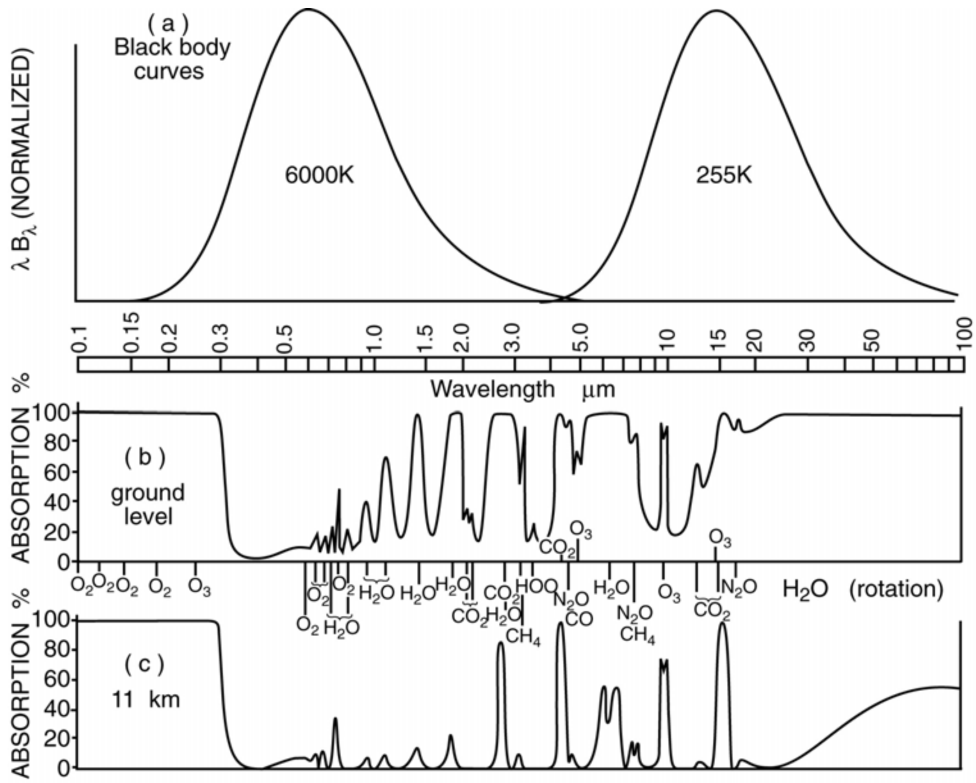

Figure 5: The normalized blackbody emission spectra,$T^{-4}\lambda B_\lambda$, for the Sun (6000K) and Earth (255K) as a function of $ln\lambda$(top) where B_λ is the black body function and λ is the wavelength. The fraction of radiation absorbed while passing from the surface to the top of the atmosphere as a function of wavelength (middle). The fraction of radiation absorbed from the tropopause (typically at a height of 11km) to the top of the atmosphere as a function of wavelength (bottom). The atmospheric molecules contributing the important absorption features at each frequency are also indicated: after Goody and Yung: "Atmospheric Radiation", Oxford Univ. Press, 1989.