Research article

Design of a heuristic environment-friendly road pricing scheme for traffic emission control under uncertainty

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ABSTRACT

Road transportation is one of the main sources of atmospheric emissions in many countries and areas. Road pricing, is not only effective for urban transportation management, but also helpful in reducing the negative externalities caused by transportation. In this study, an inexact two-phase minimal emission programming (TMEM) model is proposed for design of the environment-friendly toll scheme with an acceptable road network performance. Through introduction of fuzzy stochastic programming, multiple uncertainties involved in vehicle emission evaluation are dealt with; the Traffic Performance Index (TPI) based constraints are incorporated to reflect the decision-maker’s requirements for network congestion management. The solution method is proposed for generating the range of fuzzy stochastic objectives. An optimal toll scheme associated with the minimal emission based flow pattern is obtained through searching for a set of the best and the worst optimal solutions. A numerical experiment and a real-world road network in Beijing of China are used to illustrate the application of the developed method. In the case study, the toll scheme is obtained at the desired congestion level. The effects of emission and congestion abatement are analyzed under different policy scenarios. The proposed TMEM method can generate the toll scheme with obvious improvements in total emission reduction and congestion mitigation.

1. Introduction

Road transportation is one of the main sources of atmospheric emissions in many countries and areas (Mitchell and Milne, 2005). For example, in US, more than 75% of carbon monoxide (CO) and 60% of nitrogen oxides (NOx) were emitted from on- and off-road vehicles in 2012 (Gately et al., 2017). Generally, vehicular emissions account for 40–80% of air quality problems in the megacities in developing countries (Timilsina and Dulal, 2011; Weiss et al., 2011). In China, transportation sector is now the major source of CO emission. Additionally, vehicles contribute nearly 70% of NOx and more than 25% of VOCs emissions (Wu et al., 2017). A study from Oberholzer-Gee and Weck-Hannemann (2002) showed that under some circumstances, people were even more interested in cleaner air than in congestion relief. Thus, both transportation emission control and traffic congestion reduction, have become some of the major considerations in urban life.

The pricing strategy as a market-based economic approach is not only effective for travel demand management (TDM) but also helpful in reducing negative externalities caused by transportation (Anas and Lindsey, 2011). Recently, recognizing that an improvement on travel time may lead paradoxically to an increase of emissions without any change in the travel demand (Nagurney, 2000), more researches focus on the road pricing design with environmental considerations (Afandizadeh and Abdulmanaf, 2016; Szeto et al., 2012). However, design of such a toll scheme may be complicated when vehicle emissions are characterized by uncertainties. In addition, the traffic indicators used for congestion assessment lack an effective link to the environment-friendly toll strategies in the real-world transportation system management practice (Beliakov et al., 2018; Younes and Boukerche, 2015). These existing issues place the problems of the pricing scheme design beyond the conventional optimization approaches, which should be considered.

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Affected by various factors (e.g., acceleration, deceleration, and braking, as well as drivers’ preferences), uncertainties exist in the process of traffic emission estimation (Labib et al., 2018; Liu et al., 2007; Pandian et al., 2009). For example, even the distribution of vehicle fleet with the same average link speed is known, its emissions may be different under various scenarios (e.g., driving uphill and downhill). In reality, vehicle emissions usually fluctuate within a certain range, which can hardly be evaluated as deterministic values (Wang et al., 2008). However, most previous studies used either highly simplified functions (e.g., the emission function that considers the speed as the most key factor of resulting in emission variations) or the simple integration of state-of-the-art models (e.g., MOVES) for emission estimation (Chen and Yang, 2012; Yang et al., 2014). The uncertainties in terms of emission estimations are usually associated with combinations of randomness and vagueness, and may further intensify the complexities of the study problems. In addition, the conventional road pricing models mainly focus on the uncertainties in terms of the travel demand and travel time. Unfortunately, environmental uncertainties coupled in the toll design models have received little attention. Therefore, there is an urgent need to develop innovative optimization techniques and to investigate pricing schemes under emission uncertainties.

Moreover, the assessment of the impact of the pricing scheme implementation on congestion mitigation depends on appropriate network evaluation indicators. Generally, the system total travel time (TTT) is always calculated in association with a traffic network model for this purpose (Yang et al., 2010). Nevertheless, the local authorities can hardly adjust the traffic guidance strategies to improve the network performance according to the TTT investigation. Meanwhile, various indexes are developed for road performance/congestion evaluation; however, these indexes are only used for traffic performance descriptions (Belakov et al., 2018; Younes and Boukerche, 2015). Inadequate attention has been paid to the interconnection with the development of policy to improve congestion. Therefore, it is challenging to integrate an appropriate indicator into the optimization framework of pricing design. Thus, the toll scheme can influence the performance of traffic networks directly; the feedback of system evaluated by the indicator can also result in adjustment of the management strategies.

Between the system emissions and the traffic states, the exiting potential compromises will lead to an optimal output with minimal total emissions and acceptable network performance. Consequently, it can result in shifts from the existing pricing modeling approaches to integration of the inexact emission evaluation method, the network performance indicator, as well as a variety of components in terms of system constraints into a general modeling framework. Therefore, the objective of this study is to develop a traffic performance index (TPI) based inexact two-phase minimal emission programming approach for road toll design. In detail, fuzzy-random programming will be introduced for dealing with uncertainties involved in emission factors (EFs); the TPI will be calculated to evaluate the traffic congestion degree in constraints for network performance management; a two-phase pricing model framework will be formulated; and the case study will illustrate the theoretical and practical applications of the developed method.

2. Literature review

For the pricing theory, it was first proposed by Pigou and Knight through investigation of a congested road and the expression of some ideas about externalities and optimal congestion charges in the 1920s. Afterwards, the road pricing issues have widely attracted researchers (Jiang et al., 2016; Lin and Yu, 2008; Xiao-Jun et al., 2018). For example, Palma and Lindsey (2011) reviewed the methods and technologies for road congestion pricing. Other special topics have been discussed as well, such as path differentiated (Zangui et al., 2015), link-based (Yin and Lawphongpanich, 2006), cordon-based (Meng and Liu, 2012), area-based (Yang et al., 2014), and nonlinear pricing (Lawphongpanich and Yin, 2011). Furthermore, pricing schemes have been applied to several central congested urban areas in some countries to regulate traffic demand, such as Singapore, London, Stockholm, Berlin, and Milan (Dias et al., 2016; Gibson and Carnovale, 2015; Holman et al., 2015; Noordegraaf et al., 2014). Recently, increasing attentions have been paid to the emission related road pricing design (Afandizadeh and Abdulmanafi, 2016; Anas and Lindsey, 2011; Szeto et al., 2012; Wang et al., 2015).

The uncertainties involved in traffic emission estimations have been widely discussed for decades. This ensures that the emission abatement related management policy can be designed and implemented appropriately (Franco et al., 2013; Lv et al., 2011; Smit et al., 2010). Some researchers summarized various factors that may result in uncertainties involved in vehicle emission estimation processes, such as vehicle operation conditions (i.e., acceleration, deceleration, and braking), vehicle types, fuel consumption, speed, as well as drivers’ preferences (Labib et al., 2018; Liu et al., 2007; Pandian et al., 2009). Meanwhile, the other researchers well addressed the inherent uncertainties in the vehicle EF database and the emission inventory (Pan et al., 2016; Shen et al., 2015; Wang et al., 2008; Yan et al., 2014). Furthermore, from the viewpoint of the transportation network optimization, conventional road pricing models mainly focused on the uncertainties in terms of travel demands and travel time through stochastic and fuzzy programming methods (Lee et al., 2012; Li et al., 2012; Lv et al., 2015; Nikolova and Stier-Moses, 2014; Sumalee and Xu, 2011). Unlike the above studies, traffic emissions are usually characterized by multiple uncertainties due to a variety of influencing factors, which will no doubt present challenges to the existing programming methods.

Furthermore, under some real-world decision-making situations, the hybrid uncertainties with both fuzziness and randomness may exist simultaneously; correspondingly, the fuzzy random variable (FRV) can be introduced to quantify such complexity. The concept of FRV was introduced by Kwakernaak (1978, 1979) and Puri and Ralescu (1986). The emergence of the fuzzy-random variable/parameter makes the combination of randomness and fuzziness more persuasive. Gil et al. (2006) reviewed the development of FRVs, including the concepts, modeling approaches, as well as the applications. The FRV has been extensively studied in terms of the solution approaches (Katagiri et al., 2017; Ozha et al., 2014; Ren, 2018; Sakawa et al., 2012), and the applications to inventory control (Khan and Dey, 2017; Soni and Joshi, 2015), transportation planning (Ozha et al., 2014; Wei et al., 2015; Yan et al., 2017) and environmental management (Kong et al., 2017; Xu et al., 2013). Among various types of FRVs, the LR-FVR is commonly used in practice for the fuzzy-stochastic system; for example, Wang et al. (2016) proposed the LR-FRV arithmetic operations and applied it to reliability analysis. However, few studies have reported the application of FRVs to tackling the environmental uncertainties in the toll design problems.

As for the road network performance evaluation, various indicators have been developed by different agencies/companies all over the world (Bian et al., 2016; Yang et al., 2010). For example, the Roadway Congestion Index (RCI) by the Texas Transportation Institute is calculated based on traffic density; the INRIX index is developed based on traffic speed; the TOMTOM Traffic Index and the AutoNavi Index are measured with travel time delay compared with the free flow situation; and several comprehensive indexes are used in some Chinese cities (e.g., Beijing, Guangzhou Shenzhen and Shanghai). Comparatively, the comprehensive index can address a regional evaluation of network traffic states rather than focusing on road segments (Wang et al., 2018). For example, the TPI is proposed by Beijing Traffic Management Bureau (BTMB). The TPI has obvious advantages in traffic states evaluation from the network perspective and classification of road segments (BTMB, 2011). Thus, it is desired for the comprehensive evaluation indicator to be incorporated into the environment-friendly road pricing models.
3. Modeling formulation

3.1. MTPI model for optimal network performance evaluation

Definition 1. Based on the TPI method, link \( a \) with road class \( j \) can be defined as the congested link as follows (BTMB, 2011):

\[
v_{a} \leq \sum_{j} \tilde{s}_{a,j} \times \bar{\delta}_{j}, \quad \forall a
\]

(1)

where \( j \) represents road class, and \( j = 1, 2, 3, 4 \) representing the expressway, the arterial road, the secondary arterial road and the branch road, respectively. \( a \) stands for a link, and \( a = 1, 2, \ldots, N \). \( \bar{\delta}_{j} \) is the average speed on link \( a \). \( \bar{s}_{a,j} \) stands for the threshold speed for road class \( j \). \( \tilde{s}_{a,j} \) indicates (0 or 1) which road class link \( a \) belongs to. For any link \( a \), we have \( \sum_{j} \tilde{s}_{a,j} = 1 \).

Definition 2. Definition 1 can also be expressed through the traffic flow \( x_{a} \) on arc \( a \) (\( a \in A \)), which is as follows:

\[
x_{a} \geq s_{a} \left[ \frac{1}{0.15 \left( \frac{x_{a}}{t_{a}} \times \sum_{j} \tilde{s}_{a,j} \times \bar{\delta}_{j} \right)} - 1 \right]^{\frac{1}{2}}, \quad \forall a
\]

(2)

where \( L_{a} \) is the length of link \( a \), \( t_{a} \) is the travel time of link \( a \), which can be determined by the US Bureau of Public Roads with the link time function using the form \( t_{l}(x_{a}) = \tilde{\mu}(1 + 0.15(x_{a}/\bar{s}_{j}))^{\gamma} \) (BPR function) (Roads, 1964); \( \tilde{\mu} \) and \( s_{a} \), are parameters representing the free-flow travel time (in minutes) and the capacity (vehicles per hour) on link \( a \), respectively.

Since \( v_{a} = L_{a}/t_{a}, \forall j \), Inequality (1) can also be presented as \( L_{a}/t_{a} \leq \sum_{j} \tilde{s}_{a,j} \times \bar{\delta}_{j}, \forall a \). Thus, according to the BPR function, we have Equation (2).

Definition 3. The TPI corresponding to the network performance/congestion level is identified in Table 1. The road congested distance proportion (CDP) of \( \rho \) is calculated by the following equations:

\[
\rho = \sum_{j=1}^{4} \left( \frac{\sum_{a \in A} \tilde{s}_{a,j} L_{a} x_{a}}{T(x_{a})} \times \frac{\sum_{a \in A} \tilde{s}_{a,j} k_{a}(\tilde{s}_{a,j} L_{a})}{\sum_{a \in A} \tilde{s}_{a,j} L_{a}} \right)
\]

(3a)

where the total link number \( N = \sum_{j=1}^{4} N_{j} = N_{1} + N_{2} + N_{3} + N_{4} \). \( T(x_{a}) \) is the vehicle kilometers traveled (VKT) within a road network, and \( x_{a} \) is the set of origin-destination traffic flows on arc \( a \).

Let \( \mu_{a} = s_{a} \left[ \frac{1}{0.15 \left( \frac{x_{a}}{t_{a}} \times \sum_{j} \tilde{s}_{a,j} \times \bar{\delta}_{j} \right)} - 1 \right]^{\frac{1}{2}}, \forall a \); then, \( k_{a} \) representing the congestion index can be stated as:

\[
k_{a} = \begin{cases} 
1, & \text{if } x_{a} \geq \mu_{a} \\
0, & \text{if } x_{a} < \mu_{a} 
\end{cases}
\]

(3b)

Thus, when \( \bar{\delta}_{j} \) is known, \( \mu_{a} \) can be determined on link \( a \) with road class \( j \) accordingly.

### Table 1

<table>
<thead>
<tr>
<th>Congested distance proportion (CDP), ( \rho ) (%)</th>
<th>TPI</th>
<th>Performance/Congestion level</th>
<th>Network Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>([0,4])</td>
<td>[0,2]</td>
<td>I</td>
<td>Very smooth</td>
</tr>
<tr>
<td>([5,8])</td>
<td>(2,4)</td>
<td>II</td>
<td>Smooth</td>
</tr>
<tr>
<td>([8,11])</td>
<td>(4,6)</td>
<td>III</td>
<td>Lightly congested</td>
</tr>
<tr>
<td>([11,14])</td>
<td>(6,8)</td>
<td>IV</td>
<td>Moderately congested</td>
</tr>
<tr>
<td>([14,24])</td>
<td>(8,10)</td>
<td>V</td>
<td>Severely congested</td>
</tr>
<tr>
<td>(&gt;24)</td>
<td>10</td>
<td>V</td>
<td>Severely congested</td>
</tr>
</tbody>
</table>

Therefore, for a given road network, the optimal network operation level can be obtained through solving the following minimum TPI (MTPI) model:

\[
\min \sum_{j=1}^{4} \left( \frac{\sum_{a \in A} \tilde{s}_{a,j} L_{a} x_{a}}{T(x_{a})} \times \frac{\sum_{a \in A} \tilde{s}_{a,j} k_{a}(\tilde{s}_{a,j} L_{a})}{\sum_{a \in A} \tilde{s}_{a,j} L_{a}} \right) \quad (4a)
\]

S.T.

\[
k_{a} = \begin{cases} 
1, & x_{a} \geq \mu_{a} \\
0, & x_{a} < \mu_{a} 
\end{cases}
\]

(4b)

\[
x \in A \quad (4c)
\]

where \( A = \left\{ x | \sum_{p \in P} f^{w}_{p} = d^{w}_{p} f^{w}_{p} \geq 0, \forall p \in P^{w}, w \in W_{a}, x_{a} \right\} \) is the feasible region of above problem. \( W_{a} \) is the set of origin-destination (OD) pairs. \( A \) is the set of links of the road network; \( d^{w}_{p} \) represents the travel demand for the OD pair (\( w \in W_{a} \)); \( P^{w} \) is the set of all routes between OD pairs; \( \delta^{w}_{a} \) is a binary variable (0 or 1), which indicates whether route \( p \) uses link \( a \in A \); \( f^{w}_{p} \) is the amount of flow on route \( p \); and \( x_{a} \) is a variable that represents the traffic flow on arc \( a \).

Theorem 1. Model (4) can be transformed into the equivalent form as follows:

\[
\min \sum_{j=1}^{4} \left( \frac{\sum_{a \in A} \tilde{s}_{a,j} L_{a} x_{a}}{T(x_{a})} \times \frac{\sum_{a \in A} \tilde{s}_{a,j} k_{a}(\tilde{s}_{a,j} L_{a})}{\sum_{a \in A} \tilde{s}_{a,j} L_{a}} \right) \quad (5a)
\]

S.T.

\[
(k_{a} - \theta)(x_{a} - \mu_{a}) \geq 0 \quad (5b)
\]

\[
x \in A \quad (5c)
\]

Proof. Based on the notion of \( k_{a} \) (\( k_{a} \in [0,1], a = 1, 2, \ldots, N \), \( \forall a \)), for any \( \theta \) which belongs to \( 0 \) or \( 1 \), the relationship described by Inequality (4b) can be guaranteed. Thus, Inequality (5b) can be used to replace (4b), and Model (5) is then equivalent to Model (4).

3.2. Development of TPI based TMEP model

Definition 4. When a road network performance level \( c \) is set, the CDP range of \( [\tilde{c}_{c}, \bar{c}_{c}] \) could be determined according to Table 1. Thus, the TPI related constraints can be expressed as the following inequalities:

\[
\sum_{j=1}^{4} \left( \frac{\sum_{a \in A} \tilde{s}_{a,j} L_{a} x_{a}}{T(x_{a})} \times \frac{\sum_{a \in A} \tilde{s}_{a,j} k_{a}(\tilde{s}_{a,j} L_{a})}{\sum_{a \in A} \tilde{s}_{a,j} L_{a}} \right) \leq \bar{c}_{c} \quad (6a)
\]

\[
(k_{a} - \tilde{c}_{c})(x_{a} - \mu_{a}) \geq 0 \quad \forall \tilde{c}_{c} \leq k_{a} x_{a} \geq \tilde{c}_{c} \mu_{a} + k_{a} \mu_{a} - \theta \mu_{a} \quad (6b)
\]

\[
k_{a} \in [0,1], \forall a \quad (6c)
\]

To integrate the above constraints into the road pricing modeling framework for network performance management, a two-phase minimal emission programming (TMEP) method can be formulated while achieving the equilibrium of the road network. For the first-phase submodel, the objective is to minimize the total emission of the road network and the decision variable is the road link flow. For the second-phase one, the objective is to optimize the charging spots and the optimal pricing scheme can be obtained simultaneously.

Thus, the first-phase submodel (minimum emissions) of the TMEP method can be expressed as follows:

\[
\min \sum_{a \in A} \epsilon_{a} x_{a} \quad (7a)
\]
S.T.

\[ x \in \lambda \]  

(7b)

where \( e_a \) is the EF of link \( a \).

Subsequently, the second-phase submodel is described as follows:

\[ \min \sum_{a \in \lambda} c_a \]  

(7c)

S.T.

\[ \sum_{a \in \lambda} (\ell_a(x^*_a) + e_a)x^*_a = \sum_{w \in W} d_w \lambda_w \]  

(7d)

\[ \sum_{a \in \lambda} (\ell_a(x^*_a) + e_a)\delta_{r} \geq \lambda_{w}, \ r \in R_w \]  

(7e)

\[ 0 \leq \delta_e \leq \delta_{\max} \]  

(7f)

\[ \delta_e \in [0,1] \]  

(7g)

where \( x^*_a \) is the optimal solution from the first phase submodel; \( \delta_e \) is the charges (\( \delta_e > 0 \)) on link \( a \), which is measured in unit of time; \( \delta_{\max} \) is the maximum allowable charge on link \( a \); \( w \) denotes the minimum generalized travel cost between the OD pairs; and \( \delta_e \) is a binary variable (0 or 1), indicating whether it can be charged for link \( a \).

### 3.3. TMEP model under emission uncertainty

Since the uncertainties in terms of vehicle emissions are caused by multiple factors, it may result in the EFs with different positive and negative deviations, respectively. Fig. 1 presents an example of vehicle emission investigation based on the empirical data. Fig. 1a shows that the uncertain vehicle emissions on the link generally increase with travel time \( t(x(a)) \). Accordingly, the EFs are estimated based on different travel time (Fig. 1b). It is shown that the EFs fluctuate within different ranges under each \( t(x(a)) \) through repeated tests; the positive and negative deviations have different distributional characteristics. Apparently, the uncertainties complicate the expressions of the EFs. As presented as possibility distribution with probability distribution functions (PDFs), the LR-FRV has advantages in handling the distributional differences rather than the PDF. Thus, the uncertain EF is presented as a LR-FRV and we have Definition 5.

**Definition 5.** For \( \forall \omega \in \Omega \), the membership function \( (\mu L_{\omega} L_{\omega}(x) / \mu R_{\omega} R_{\omega}(x)) \) of a LR-FRV \( (\overline{L_{\omega}(x)}) \) is expressed as follows (Gil et al., 2006):

\[
\mu L_{\omega} L_{\omega}(x) = \frac{L((\overline{L_{\omega}(x)} - x) / \beta_{L_{\omega}})}{ \beta_{L_{\omega}} > 0, \ \beta_{R_{\omega}} > 0, \ \chi_{L_{\omega}} > 0, \ \chi_{R_{\omega}} > 0} 
\]

(8)

where the functions of \( L \) and \( R \) (\( L, R: [0, \infty) \rightarrow [0,1] \)) represent two continuous non-increasing functions, respectively (\( L(0) = R(0) = 1 \); \( \overline{L_{\omega}(x)} \) is the random variable with the maximum membership value; and \( \beta_{L_{\omega}} \) and \( \chi_{L_{\omega}} \) denote the left and the right widths, respectively. Thus, the LR-FRV can also be denoted as \( \overline{L_{\omega}(x)} = (\overline{L_{\omega}(x)}, \beta_{L_{\omega}}, \chi_{L_{\omega}}) \), which can be shown in Fig. 2.

Therefore, the TMEP model under the emission uncertainty can be formulated as follows:

The first-phase submodel:

\[
\min \sum_{a \in \lambda} c_a x_a \]  

(9a)

S.T.

\[
\left( \frac{\sum_{a \in \lambda} \delta_{a} L_{\omega} L_{\omega}(x_a) x_a}{T(x_a)} \right) \left( \frac{\sum_{a \in \lambda} \delta_{a} L_{\omega} L_{\omega}(x_a) x_a}{T(x_a)} \right) \leq \mu \]  

(9b)

\[
(k_a - \delta_e)(x_a - \mu) \geq 0 \]  

(9c)

\[
k_a \in [0,1], \ \forall a \]  

(9d)

(Traffic assignment conditions)

\[ x \in \lambda \]  

(9e)

The second-phase submodel:

\[
\min \sum_{a \in \lambda} c_a x_a \]  

(9f)

S.T.

( Relationship between link flows and toll)
\[
\sum_{a \in A} (\ell_a(x^*_a) + t_a)x^*_a = \sum_{a \in W} d_a \lambda_w
\]  
(9g)

\[
\sum_{a \in A} (\ell_a(x^*_a) + t_a) \delta_r \geq \lambda_w, \quad r \in R_w
\]  
(9h)

(Upper and lower limits of road toll)

\[0 \leq t_a \leq \alpha_r \alpha_{\max}\]
(0-1 decision variables)

\[\alpha_r \in [0,1]\]
(9i)

where \(\ell_a\) is the EF with fuzzy-random feature on link \(a\).

According to the concept of the LR-FRV, for a given \(\alpha \in [0,1]\), the fuzzy random objective function in model (9) can be transformed by the corresponding \(\alpha\)-level set (Lahandjula and Gupta, 1996):

\[
\min \sum_{a \in A} \left[\bar{\ell}_{a,a}^x \ell_{a}^d\right] x_a
\]
(10)

S.T. \(\alpha\)

In model (10), \(\bar{\ell}_{a,a}^x\) and \(\ell_{a}^d\) are random boundaries at the \(\alpha\)-cut level \((\bar{\ell}_{a,a}^x \leq \ell_{a}^d)\). According to the expectation function based stochastic programming method (Lan and Zhou, 2016), \(\bar{\ell}_{a,a}^x\) and \(\ell_{a}^d\) can be estimated with the expression of \(\bar{\ell}_{a,a}^x(\omega) = \beta_a L_a^x(\sigma)\) and \(\ell_{a}^d(\omega) = \chi_a R_a^x(\sigma)\), respectively; where \(L_a^x(\sigma) = \sup\{x | L_a(x) \geq \sigma\}\) and \(R_a^x(\sigma) = \inf\{x | R_a(x) \geq \sigma\}\) are the pseudo-inverse functions of \(L_a(x)\) and \(R_a(x)\), respectively. In the study, \(\bar{\ell}_{a,a}^x(\omega)\) and \(\ell_{a}^d(\omega)\) are the expectations of the random boundaries of EF on link \(a\), which have approximate positive correlation with link travel time \(t_a(x_a)\) (Alexopoulos et al., 1993; Yin and Lawphongpanich, 2006). According to the BPR function, the above expressions can be further presented as follows:

\[
\bar{\ell}_{a,a}^x(\omega) - \beta_a L_a^x(\sigma) = d_{a}^{f(1)} + x_{a}^{d(1,2)} - \beta_a L_a^x(\sigma)
\]  
(11a)
where $d_a^{(1)}$ and $d_a^{(2)}$ are parameters which can be estimated based on the real-world database.

When $x_\alpha \geq 0$, $\sum_{a \in A} \bar{d}_a x_a$ and $\sum_{a \in A} \bar{d}_a x_a$ are called the best-optimal and the worst-optimal objectives, respectively. The second-phase submodel has the same form as objective (9f) associated with the constraint set of (9g) to (9j).

### 3.4. Solution method

Accordingly, the solution method for the TMEP model is proposed. The first-phase model is transformed into two submodels according to the $\alpha$-cut level based fuzzy programming algorithm. The two submodels with the best-optimal objective of $\sum_{a \in A} \bar{d}_a x_a$ and the worst-optimal objective of $\sum_{a \in A} \bar{d}_a x_a$ are two stochastic programming models, which can be transformed to the approximated deterministic models through either the expectation-based stochastic programming or Monte Carlo simulation techniques. In detail, in the expectation-based stochastic programming method, random variables are approximately substituted by their expectations; in the Monte Carlo simulation techniques, the original problem can be converted into a sample mean approximation problem by random sampling. Both the methods can convert the original two stochastic programming submodels into deterministic 0–1 mixed integer non-linear programming models. Thus, a heuristic solution approach using Genetic Algorithm (GA) can be developed to find a set of Pareto-optimal solutions for the two submodels, respectively (Goldberg, 1989). Consequently, the traffic flow solutions of $X^{1,\alpha}$ and $X^{1,\alpha}$ obtained from the best- and worst-optimal submodels of the first-phase model can be introduced into the second-phase model for solving the optimized charging plan.

The second-phase model is a 0–1 mixed integer programming model, which can be solved through the branch and bound algorithm. The algorithm consists of a systematic enumeration of candidate solutions by means of state space searches. In other words, the candidate solution set is considered to form a rooted tree with a complete set at the root (Gendron and Crainic, 1994; Morrison et al., 2016). The algorithm explores the branches of this tree which represent subsets of the solution set. Before enumerating the candidate solutions of a branch, the branch is checked against the estimated upper and lower bounds of the optimal solution; the branch is discarded if it cannot produce a better solution than the best one found so far by the algorithm (Nakariyakul, 2014). Finally, the flow patterns under the best- and worst-optimal conditions can be generated to form a desired toll scheme associated with the minimal emission. The detailed solution procedure is summarized in Fig. 3.

### 4. Numerical example

Consider a road network with six nodes and seven links in Fig. 4. The network has two OD pairs (1,3) and (2,4), both of which have the traffic demand of 7500 vehicles per hour. The BPR function with the form of $t_a(x_a) = t_0(1 + 0.15(x_a/s_a))^5$ is used to determine the travel time on each link. The parameters of the free-flow travel time $t_0$, the link capacity $s_a$, as well as the link length $l_a$ are listed in Table 2.

Since vehicles are responsible for most of the CO emission, especially in China (Wu et al., 2017), CO is considered an important indicator for the level of atmospheric pollution generated from vehicular traffic by many researchers (Alexopoulos et al., 1993; Ma et al., 2017; Sun et al., 2016; Yin and Lawphongpanich, 2006). To simplify our presentation, we consider only CO and estimate the EF for each link with the LR-FRV due to the inherent uncertainty. Based on the data investigation about the relationship between pollutant emissions and traffic flows, the fluctuation of the expected EF can be observed with a travel time variation, which is assumed a normally distribution. Subsequently, the lower and upper bounds represent the negative and positive deviations of the expected EF, whose distributions can be similarly generated. Since the factors that cause the deviations may be different, the random boundaries may be differently distributed. Therefore, the EF specified as a LR-FRV can be obtained associated with the parameters of $\pi$ and $\beta$. At a given $\alpha$-cut level (0.5 in the study), the expressions of $\bar{e}_{\alpha,sa}$ and $\bar{e}_{\alpha,sa}$ can be presented through equations (11a) and (11b), respectively.

The inexact TMEP model (model 9) can be solved based on a two-phase procedure. The goal of the first-phase model is to minimize the total emission considering TPI related constraints under emission uncertainties; the second-phase model aims to identify the tolled links associated with the toll levels so as that the minimum system emission can be achieved. Due to the fuzzy-random characteristics of the EF, two submodels are formulated at a given $\alpha$-cut level to demonstrate the solutions under the best- and worst-optimal conditions (BOC and WOC), respectively. Table 3 shows the flow, the average speed and the charge corresponding to each link, which are the solutions of the TMEP model. For example, for the OD pair (1,3) under the BOC, despite the toll along route 1–3 (2.84 in value of time, min), about 51% (3821.77) vehicles are distributed on this route due to less travel time. The average travel speed of link (1,3) is 20.00 km/h. In comparison, route 1-5-6-3 is free of charge, to which 3678.23 vehicles are assigned. The average travel speeds of this route are 22.09, 22.09 and 35.23 km/h for links (1, 5), (5, 6) and (6, 3), respectively, which are higher than that of route 1–3. Besides this in-depth analysis of the solutions for the TMEP under the BOC, the solutions under the WOC can be similarly interpreted based on Table 4. Generally, the total toll of the OD pair (1,3) is lower than that of the OD pair (2,4) under the BOC and the WOC.

In this study, the emission uncertainty is considered and integrated into the road pricing model. The LR-FRV can be used to describe multiple uncertainties which cannot be tackled using random variables. For example, various factors, such as road conditions (e.g., uphill and downhill), operation status (e.g., acceleration, deceleration, and braking) and driver’s behaviors, may affect the vehicle emissions, leading to the positive and negative deviations of the EFs with different distributions. Accordingly, the LR-FRV can allow us to tackle the distribution differences rather than the PDF based on the probability theory. Apparently, the LR-FRV can be employed to address more complex forms of uncertainties originated from either empirical data or models. Furthermore, a combination of the existing fuzzy and stochastic programming methods can enhance the capacities of dealing with such complexities. When two arbitrary distributions of FRV are encountered, one of the candidate methods would be the $\alpha$-cut and interval arithmetic approach (Wang et al., 2016). Fig. 5 shows the variation of the total emission at different $\alpha$-cut levels under the BOC.

### Table 2

Basic parameters of the road network.

<table>
<thead>
<tr>
<th>Link</th>
<th>(1,3)</th>
<th>(2,4)</th>
<th>(1,5)</th>
<th>(5,6)</th>
<th>(2,5)</th>
<th>(6,3)</th>
<th>(6,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$ (min)</td>
<td>8.0</td>
<td>9.0</td>
<td>2.0</td>
<td>6.0</td>
<td>3.0</td>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$s_a$ (veh/hr)</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
<td>4000</td>
<td>2000</td>
<td>2500</td>
<td>2500</td>
</tr>
<tr>
<td>$l_a$ (km)</td>
<td>8.0</td>
<td>9.0</td>
<td>2.0</td>
<td>6.0</td>
<td>3.0</td>
<td>3.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Fig. 4. The layout of the six-node road network.
Comparison of the average speeds and the related variances generated from different models.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Average speed (km/h)</th>
<th>Variance of speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>UE</td>
<td>25.96</td>
<td>59.38</td>
</tr>
<tr>
<td>SO</td>
<td>25.39</td>
<td>49.61</td>
</tr>
<tr>
<td>MTPI</td>
<td>25.09</td>
<td>45.14</td>
</tr>
<tr>
<td>UN-ME</td>
<td>BOC</td>
<td>25.37</td>
</tr>
<tr>
<td>WOC</td>
<td>25.41</td>
<td>49.90</td>
</tr>
<tr>
<td>TMEP</td>
<td>BOC</td>
<td>25.25</td>
</tr>
<tr>
<td></td>
<td>WOC</td>
<td>25.14</td>
</tr>
</tbody>
</table>

Fig. 5. Variations of the total emission at different α-cut levels.

and the WOC, respectively. It is indicated that the rising α-cut level denotes more and more narrow intervals of FRV with less left and right widths. Consequently, the total emission under the BOC increases from 80.9 to 82.4 × 10^3 g while that under the WOC decreases from 231.9 to 227.8 × 10^3 g, with the α-cut level ranging from 0.1 to 0.9; the lower and upper bounds of the total emission will become closer to each other.

For comparison, the results of numerical experiments are also obtained under different transportation management requirements. These results are generated from the system optimization (SO) model, the user equilibrium (UE) model, the minimal emission model under uncertain emissions (UN-ME), and the MTPI model (model 4). Among them, the outputs of the UE model represent the system status without policy intervention. The MTPI model to minimize the CDP can help to define the optimal performance/congestion level of the road network. The right-hand side parameters of the TPI related constraint (constraint (9b)) can be obtained based on the related level threshold in Table 1.

The TTT and the CDP generated by the five models are provided in Fig. 6. In terms of the TTT, the minimal TTT of the SO model is 6808.3 min; the UE model leads to the greatest TTT (6826.5 min). The TTT of the TMEP model ranges from 6809.8 to 6812.1 min, which is very close to that of the SO model. As for the network performance, the CDP under the SO and UE conditions is 48.6%, belonging to the severely congested road class (Level V); the CDP values of the TMEP and the MTPI models are zero. In addition, the average traffic speeds and the related variances associated with the flow distributions are shown in Table 4. It is indicated that the TPI related constraints can not only lead to a better CDP, but also affect the distribution of link speeds. Affected by the minimal emission objective and the TPI related constraints, the average speed of the TMEP method ([25.14, 25.25] km/h) is slightly higher than that of the MTPI model (with the optimal TPI objective) and less than those of the other models. The lower variance of the average speed indicates that the link speeds of the network vary within a smaller range.

The total emissions are calculated under different traffic status, respectively (Fig. 7). Due to the introduction of the TPI related constraints, the total emission of the TMEP ([81.76, 229.43] × 10^3 g) is higher than that of the UN-ME model (with the least CO emission amount) but relatively lower than those of the other models. From the perspective of traffic system management, the environment-friendly objective (i.e., the minimum emission) is not necessarily the only concerned problem by local authorities. The network performance has always become one of the primary concerns. Therefore, the policy maker will consider the outputs of the TMEP model (a balance between emission mitigation and road network performance) rather than those of the UN-ME model (the minimum emission is the single management requirement). Comparatively, in terms of the TTT, the CDP, and pollution abatement, the toll scheme of the TMEP method can guarantee better system outputs than those without intervention (i.e., by the UE model).

5. Application to pricing scheme design

In order to further illustrate the proposed TMEP method, a real-world network from the central area of Beijing, China is also introduced. Fig. 8 shows the topological graph of the study area, where the nodes are presented as the numbered circles, and the lines respecting links with directions are also numbered. In this area, the traffic flows are relatively large. Most of the traffic flows on the 2nd Ring Road exceed 200,000 standard cars per day. During the peak period, both the East and the West 2nd Ring Roads are in the serious state of congestion.

Accordingly, the road toll scheme and the link flow pattern can be obtained from the TMEP model for the study network. A total of 10 links are charged, which are Links 8, 10, 14, 37, 52, 69, 72, 125, 135 and 153, respectively. Meanwhile, the result of the UE model is also calculated for comparison, which represents the traffic status without considering the toll policy. It is indicated that the total system emission decreases from [19.221, 19.408] × 10^3 g (obtained by the UE model) to [13.552, 13.739] × 10^3 g (obtained by the TMEP method). Furthermore, the TTT significantly decreases by 29.3% through using the TMEP method (412648 min). Based on the MTPI model (model 4), the optimal congestion level can be determined as level III (lightly congested), which is used as the input of the right-hand side of constraint (9b). Consequently, the CDP value generated by the TMEP model is 10.96%, which means the road network performance can be improved from level V to level III with implementation of the toll scheme.

Table 3
Solutions under the best- and worst-optimal conditions.

<table>
<thead>
<tr>
<th>Link</th>
<th>Best-optimal condition</th>
<th>Worst-optimal condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{x}_{a}$</td>
<td>$\bar{\sigma}_{a}$</td>
</tr>
<tr>
<td>(1,3)</td>
<td>4.84</td>
<td>3821.77</td>
</tr>
<tr>
<td>(2,4)</td>
<td>5.46</td>
<td>3821.77</td>
</tr>
<tr>
<td>(1,5)</td>
<td>1.06</td>
<td>3678.23</td>
</tr>
<tr>
<td>(5,6)</td>
<td>3.26</td>
<td>7356.46</td>
</tr>
<tr>
<td>(2,5)</td>
<td>1.61</td>
<td>3678.23</td>
</tr>
<tr>
<td>(6,3)</td>
<td>0.99</td>
<td>3678.23</td>
</tr>
<tr>
<td>(6,4)</td>
<td>1.35</td>
<td>3678.23</td>
</tr>
</tbody>
</table>

Table 4
Comparison of the average speeds and the related variances generated from different models.

<table>
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<td>25.25</td>
</tr>
<tr>
<td></td>
<td>WOC</td>
<td>25.14</td>
</tr>
</tbody>
</table>
In addition, the relationship between the CDP and the link speed distribution is also investigated by using the larger-scale network. A number of flow patterns are randomly chosen from the space formed by equations (6b), (6c) and (7b), which represent the possible flow distributions under the given travel demand. The link whose speed satisfies Inequality (1) \( \sum_{\alpha} n_{\alpha} \leq \sum_{\alpha} n_{\alpha} \left( \frac{v_a}{\alpha_a \cdot \alpha_j} \right) \) is defined as the congested link (CL). Apparently, various flow patterns are corresponding to different CL numbers. Once the random samples of flow patterns are determined, the CDP values, the average link speeds with standard deviations, and the CL numbers can also be identified. The relationship between the CDP distribution and the number of congested road links is shown in Fig. 9a. In Fig. 9a, the red line covers the points with the minimal CDP values under different CL numbers. The point of the minimal CDP value is also presented with the CL number of 13. Fig. 9b shows the standard deviations of the link speeds under different CL numbers. The standard deviation points related to the minimal CDP values under different CL numbers are also connected. The standard deviation of the link speed calculated by the flow distribution of the MTPI model is 21.77 km/h. For the management purpose of network performance indicated by the TPI, the effective control measures tend to reduce the number of the congested links, which simultaneously leads to relatively lower standard deviations of link speeds. Consequently, the average link speeds tend to be evenly distributed.

6. Discussions

In this study, the proposed TMEP method has been applied to an urban road network example in Beijing, China. This TMEP can be further applied to certain megacities where strict vehicle emission standards are implemented under similar requirements, such as Singapore, London, Stockholm, Berlin, and Milan (Dias et al., 2016; Gibson and Carnovale, 2015; Holman et al., 2015; Noordegraaf et al., 2014). In practical applications, emission amounts, charging standards, road networks, and traffic flows need to be investigated and revised according to local conditions and management requirements. The uncertainty quantification method can be employed to evaluate similar fuzzy-random uncertainties in terms of typical pollutant emissions. In order to ensure the implementation of the road pricing strategy in practice, the infrastructures such as the toll collection devices (e.g., toll tag readers), can help to charge vehicles on links between OD pairs (Palma and Lindsey, 2011; Zangui et al., 2015). Besides the optimization methods, some approaches, such as cost-effective analysis (Agarwal and Kickhöfer, 2018), the emissions inventory (Liu et al., 2007; Shen et al., 2015; Wang et al., 2008), the monitoring studies (Percoco, 2013; Stopes et al., 2014), and the data-driven policy analysis (Ferreira et al., 2015; Gibson and Carnovale, 2015), have been applied to road traffic pricing to reduce emissions. These approaches can be combined with the optimization methods to make the environment-friendly pricing schemes more practical.

Overall, the TMEP method can help the users with environment-friendly consideration to determine the pricing scheme that will enforce an optimal flow pattern. The scheme can also keep an acceptable congestion level within the target range. The total emission and the network performance (measured as TPI) are significantly improved by tolls compared with the un-tolled network. The TMEP model is proposed for design of the environment-friendly pricing scheme under emission uncertainties. It can provide a linkage between pricing schemes and pre-regulated management policies. Based on fuzzy-random programming, the TMEP method can provide two extremes of the optimal total emission under the BOC and the WOC, respectively. An optimistic decision under the BOC corresponds to an advantageous
emission status with the system emission of $81.76 \times 10^3$ g; a conservative choice under the WOC corresponds to a higher system emission of $229.43 \times 10^3$ g. In comparison, the solution of the deterministic pricing model (e.g., the average case) represents one of many alternatives embedded within the solutions of the TMEP model. Although further sensitivity analysis can be undertaken to address the input uncertainties, each of such analysis can only provide one of many potential responses to the uncertain emission evaluation. Therefore, compared with the conventional programming methods, the TMEP method has enhanced abilities to reflect emission uncertainties and to provide more reliable decision supports for the local authorities.

Although the developed approach has some improvements, some unavoidable limitations can be further improved. For example, the charging scheme of the TMEP model is simply presented as $\tau$, which is measured in unit of time to make the results more general. In practical applications, the corresponding toll scheme can be converted into the expression of money based on the value of time. The conversion is related to factors such as the geographical feature, economic development level, income, and personal attitude, and changes with time (Chen and Yang, 2012; Yin and Lawphongpanich, 2006). The corresponding value of time can be investigated in the future. The study uses CO as the only indicator of pollution; additionally, the TMEP model can also be extended to multiple vehicular pollutants control. Thus, the proposed method can be applied to the estimations of other pollutant emissions. Moreover, in this study, more attention is paid to the EF expressed as the FRV. Multiple sources of uncertainties (i.e., the travel demand and/or time) and multiple presentations (i.e., the model objective and/or constraint) can be further incorporated into the modeling framework as well.

7. Conclusions

In the study, a modeling framework of the TMEP model is proposed for design of the environment-friendly pricing scheme with an acceptable road network performance. It contains two phases: a minimum emission model under uncertainty and a road pricing model through well-located charging spots. Improved upon the conventional models, the uncertain EF is addressed and presented as the LR-FRV. The management of network performance is taken into account through development of the TPI related constraints. A solution method for the TMEP

Fig. 8. Topology of the urban road network example in Beijing, China.
model is proposed to generate an optimal toll scheme corresponding to the minimal emission originated flow patterns under the best- and worst-optimal conditions.

Two different scales of road networks are used to illustrate the applications of the developed method. Both the obtained toll scheme and the optimal flow pattern are investigated at the desired congestion level; the effects of the TMEP model on emission reduction and network performance are also analyzed under different policy scenarios. The proposed TMEP method can generate the toll scheme with obvious improvements on the emission reduction and congestion mitigation, which is helpful to provide effective supports for road network management.

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