# A Remark on Didactic Transposition Theory

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*With reference to a historical study on the relation between the production and distribution of mathematical knowledge, using calculus as an example, some assumptions in didactic transposition theory, as introduced by Yves Chevallard, are discussed. Given the prominent status of this theory, the paper intends to initiate a debate that could help lifting it out from its relative isolation within mathematics education as a research domain.*

## Introduction

The relation between mathematical practices in educational institutions and other mathematical practices has been a key concern for mathematics education. The practice of academic mathematics at universities is still meant to be a model for conceptualising school mathematics in post compulsory mathematics, even though there are other conceptions of curriculum. Elements of this practice are selected and 'transformed' for the purpose of teaching.

A theory that has been concerned with this process and has gained considerable attention, also in the Nordic community of mathematics education, is the "didactic transposition theory" (Chevallard, 1985; 1991). This theory has been employed also in subject areas other than mathematics, has initiated an extended research programme (Bosch & Gascon, 2006), but has also met critique (see e.g. Beitone, Decugis, Dollo, & Rodrigues, 2004, pp. 62-69; Freudenthal, 1986). The aim of this paper is to discuss and challenge some basic premises of didactic transposition theory by way of an example, based on a historical study of the didactic transposition of a specific body of mathematical knowledge, the calculus (Klisinska, 2009). The discussion points to the role of the pedagogic discourse for the social construction of mathematical knowledge. By this we attempt to draw attention to possible productive interactions of the didactic transposition theory with other theorising employed in mathematics education.

## The didactic transposition

The first comprehensive outline of the theory of didactic transposition is found in Chevallard (1985), with an extended edition in Chevallard (1991) and it was presented at the first *Ecole d'été de didactique des mathématiques* in 1980 (Chevallard, 1991, p. 7). The term 'transposition didactique' was used earlier (Chevallard, 1978). It aims at producing a scientific analysis of 'didactic systems' and is based on the assumption that the knowledge set up as a teaching object ('savoir enseigné'), normally has a pre-existence as *scholarly knowledge* ('savoir savant'), that is "a *body* of knowledge, not knowledge in itself", ranging from "genuinely scholarly bodies of knowledge to scholarly-like or even pseudo-scholarly ones" (Chevallard, 1992a, p. 228). "For many bodies of knowledge taught at school the integrated whole required existed outside school. School mathematics, for example, has essentially evolved from mathematicians' mathematics" (Chevallard, 1989, p. 57). Thus, the mathematics taught at school has been generated outside school and moved ('transposed') to school by a series of adaptations before being accepted for teaching.

According to Chevallard (1991, p. 43) there is often an immense difference between these objects of knowledge. He defines the didactic transposition as the work done during the transformation from *scholarly knowledge* via *knowledge to be taught* and the actual *knowledge taught* to *learnt knowledge* (see e.g. Bosch & Gascon, 2006). The first step in this sequence of transformations of knowledge is taking place in the *noosphere*, a non-structured set of experts, educators, politicians, curriculum developers, recommendations to teachers, textbooks etc. Analysing the 'knowledge to be taught' through the agents and materials from the noosphere reveals the conditions and constraints under which it is constituted.

The analysis of the didactician aims at making visible the difference between the transposed (taught) object and the scholarly object, a difference not spontaneously perceived by the teacher. In addition, while ruled by norms and values attached to the educational institution, the teacher does not always take responsibility of the epistemological consequences of this difference. There is an *illusion of transparency*, a feeling that the knowledge to be taught is not to be questioned, which may lead to an 'epistemological rupture' of the knowledge objects (Chevallard, 1991, p. 42-43). As to the notion of transparency, Chevallard refers to Bourdieu, Chamboredon, and Passeron (1973). In their critique of naïve sociology, they propose that the transparent unmediated, common sense knowledge about the facts of social life are hard to overcome because of common metaphors lurking in language. Chevallard links the illusion of transparency to the teachers' conception of the supposedly mathematical facts thought at school. The difference between the teacher and didactician resembles, then, the difference between the holder of naïve sociological knowledge and the sociologist.

The didactic transposition implies a *textualisation* of knowledge, as well as a *depersonalisation*, thus producing an objectification possible to be made public and to form a basis for social control of the learners by developing systems for testing (Chevallard, 1991, pp. 61-62). The 'text of knowledge' produced thus serves as a norm for knowledge and for what it means to know, as well as for the progression of knowledge, authorising didactical choices.

Knowledge is “living” in institutions. Chevallard (1989, 1992b) distinguishes “knowledge in use” or “practical knowledge” from teachable knowledge. Based on Bourdieu’s notion of “practical logic”, he makes the distinction between “institutional systems of acquaintance (connaissance)” and knowledge (savoir) and suggests that in order to become teachable the former have to be transformed into knowledge. Such “body of knowledge”, must be produced before it can be taught or “utilised” and not only “put to use” (1992b, p. 162).

A question that can be analysed from the perspective of didactic transposition is what different institutions define as legitimate knowledge. An essential difference between research mathematics and school mathematics is seen in the principles that govern knowledge growth. In the scholarly field, a problem is the driving force of knowledge construction, while in teaching the progression is run by the contrast between the old and the new objects (Chevallard, 1991). “Bodies of knowledge are, with a few exceptions, not designed to be taught, but to be used. To teach a body of knowledge is thus a highly artificial enterprise” (Chevallard, 1989, p. 56). Hence, research-type mathematical behaviours and attitudes are difficult, if not impossible, to obtain in the mathematical practice of classrooms. This becomes clear when, for example, proving theorems is at issue.

**Intellectual roots of the theory**

The idea of a didactic transposition of scholarly knowledge was adapted and elaborated by Chevallard from the sociologist Michel Verret (1975), who emphasised that knowledge can not be taught in the way it was produced in the scientific community: the ‘transmission didactique’ induces a selection as it privileges the success, continuity, and synthesis of knowledge, not typical characteristics of the production of knowledge (pp. 140-141). Due to the separation of subjects in teaching institutions, and the need for evaluation, a didactic transposition process is defined by decomposition, depersonalisation and development of a detailed teaching sequence of knowledge (pp. 146-147). These three notions are used by Chevallard. The process of transposition presupposes that knowledge to be taught is clearly defined and open to social control.

Other intellectual roots are mostly mentioned as sources of inspiration rather than in the form of specific references and often remain implicit. Ideas concerning the need of a ‘transmission didactique’ to make teaching possible were expressed already by Auguste Comte (1852) in his *Catéchisme positiviste.* When discussing the teaching of religion, he provides the following argumentation for his choice of the dialogue as the format of his didactic text (pp. 10-11):

> A discourse, then, which is in the full sense didactic, ought to differ essentially from a simple logical discourse, in which the thinker freely follows his own course, paying no attention to the natural conditions of all communication. [1]

To avoid the tedious logical elaborations of a lecture, Comte argues for dialogue:

> One should use a dialogic format, appropriate in any true communication, for explaining such notions that are at the same time important and mature enough […] Far from indicating an excusable negligence just towards secondary matters, this format, being well institutionalised, instead constitutes the only mode of exposition which is truly didactic: it suits equally well all levels of intelligence. [2]

He concludes that (our emphasis)

> this transformation for the purposes of teaching (orig. *transformation didactique*) is only practicable where the doctrines are sufficiently worked out for us to be able to distinctly compare the different methods of expanding them as a whole and to easily foresee the objections which they will naturally elicit. [3]

This statement highlights that it is only for the teaching of an elaborated body of knowledge that Comte sees a need of a 'transformation didactique', which changes the principles of the discourse, to make it conceivable for the intended audience. In the dialogue outlined in his book, which can be seen as a textbook in religion, the scholarly knowledge (to use the term of Chevallard) is presented by a sanctioned representative of the field in focus, a priest. In his presentation of the didactic transposition, Chevallard does not refer to Comte, though he mentions Comte when discussing other issues (Chevallard, 1991, pp. 215, 218).

Verret was concerned with teaching at university level, while Chevallard uses examples mainly from secondary school mathematics in France. This is emphasised by Beitone et al. (2004, p. 57) who claim that didactic transposition theory is even more useful today than in the mid 70's for analysing university teaching. Both Verret and Chevallard also discuss different types of knowledge and which types can be seen as scholarly knowledge, and note that certain kinds of practical and tacit knowledge are not, and cannot, be taught outside their own fields of practice.

**Critique of the theory**

Soon after the first publication of Chevallard's (1985) exposition of his theory, Hans Freudenthal (1991) presented a critical review of the book, written in French. While appreciating the eloquence of the language, though wondering what is hidden between the serious and the ironic, Freudenthal questions the whole idea of a didactic transposition mainly because of the vagueness of the term 'savoir' (knowledge), and in particular the term 'savoir savant' (translated to 'scholarly knowledge' by Chevallard). What is the scholarly knowledge that is transposed, for example, into the teaching of arithmetic or algebra at elementary school levels? From the example given by Chevallard in the text (further elaborated with M-A. Johsua in the 1991 edition), Freudenthal concludes, ironically, that scholarly knowledge must refer to the 'good mathematics' produced by some great mathematicians from history, now to be transposed to the level of understanding of the youth (Freudenthal, 1986, p. 325). He points out that school mathematics and the students of today are concerned with technological aspects of knowing how to do rather than learning transposed versions of more or less ancient scholarly mathematical knowledge, which has only a marginal influence in the modern technological culture.

After this and other critique of his theory, Chevallard notes that "Experience shows that the theory of didactic transposition is an easy prey to misunderstanding" (1989, p. 51). According to Beitone et al. (2004), the theory has been challenged by three main points of critique. One argument states that scholarly knowledge is the source of knowledge for teaching and cannot have different value or character depending on the institution that handles it. As the transposition process in the theory is conceptualised as 'degrading' knowledge, it should be avoided by the participants (such as teachers) by not taking part in it. According to the second argument, knowledge as treated in school is not a simple derivation of scholarly knowledge with a logic imposed by academic mathematics. The view of knowledge as described by the didactic transposition theory has to be seen as an expression of elitism not in line with modern pedagogy. In particular, school subjects not similar to mathematics and science in terms of knowledge principles remain out of the scope of the theory, as for example the teaching of language. Finally, reference knowledge for teaching in school also has other origins than in scholarly fields, such as different kinds of tacit knowledge involved in social practices. This is, for example, the case in school curriculum conceptions underpinned by ethnomathematics, or in university mathematics courses for some traditionally non-academic vocations (such as nurses). However, the issue has been recognized by Chevallard (1989).

As to the possibility of identifying a clearly delineated body of scholarly knowledge as a blueprint for judging its didactic transposition as legitimate, Chevallard recognizes a difficulty:

> In most cases […] a given body of knowledge will appear only in fragments. […] The first step in establishing some body of knowledge as teachable knowledge therefore consists in making it into a *body* of knowledge, i.e., into an organized and more or less integrated whole (ibid., p. 57).

This quote points to a possible influence of the distribution of knowledge on the production of knowledge, an issue that questions the premises of the didactic transposition theory. Further, it suggests the pre-existence of the "body of knowledge" that has been produced outside the teaching institution. These issues were highlighted in a study with a focus on the didactic transposition of proof, which used the Fundamental Theorem of Calculus as an example (Klisinska, 2009).

## The didactic transposition of calculus

In the development of the mathematical sub-area today known as calculus, the Fundamental Theorem of Calculus (FTC) played a key role in linking integration and differentiation. Through a historical study Klisinska (ibid.) investigated the dynamic between the adaption of the FTC and the calculus for teaching and its establishment as part of the scholarly knowledge. In an interview study, she also investigated how mathematicians at universities (producers of mathematical knowledge) interpret the FTC and how they see this area when they act as 'transposers' of this body of knowledge for teaching. Our exposition below will refer to the outcomes of the historical study only. [4]

The development of the statements connected with the FTC, which initiated the institutionalisation of a new body of knowledge, was studied with reference to original works of researchers and classical works about the history of calculus. By institutionalisation we refer to a process of crystallisation of a specific discourse. We take the regularity by which significations are recognised as belonging to the distinctive discourse of a practice, and the extent of the stability by which these significations are employed, as an indication of their institutionalisation. In Chevallard's (1992, p. 144) terms, this is the process by which an object (of knowledge) comes into existence for an institution, and he states that this presupposes a recognised denomination. "Institution" here does not necessarily refer to a formal organisation, but there must at least be some alliance amongst a group. Our conception of institutionalisation is in line with that of Chevallard. For an expansion of the notion of institution, which in his theorising is a "primitive term", see Hardy (2009).

As indicators of institutionalisation of the FTC, reference to a sub-area or to a proposition with a common name and textbook or handbook appearances were considered. Textbooks were separated from research publications by their intention to address an audience with less specialized knowledge in the area to which the sub-area belongs. Only some examples will be discussed here (for further details on the sources and the methodology, see Klisinska, 2009).

Classical outlines of the history of mathematics commonly trace the ideas of calculus back to ancient Greece. It is also common to refer to Leibniz and Newton as "inventors" of the modern calculus. However, Leibniz and Newton did not invent the same calculus, and did not set out calculus as a well-defined sub-area of mathematics as they differed in problems studied, approaches taken, and methods and notations used (Boyer, 1959; Baron, 1987). The early development of the limit concept [5] was crucial for the process of institutionalisation of the calculus. By using limits as the basis for definitions, Cauchy's work established new criteria by integrating definitions and proofs with applicable methods. In the 19$^{th}$ century the formal ε-δ definition of limit by Weierstrass, the definition of the Riemann-integral and a set theoretic definition of function were added in accordance with the development of the criteria for legitimate knowledge.

Throughout the history of calculus, the institutionalisation of knowledge for the purpose of teaching was one driving force for the change of knowledge criteria. However, in the historical development, it is not easy to differentiate between criteria for the producers and distributers of knowledge. While the first developments in calculus were communicated entirely within the field of knowledge production through personal communication, textbooks for the wider distribution of calculus soon appeared. The first printed textbook in differential calculus appeared in 1696, written by de l'Hospital with the help of Johann Bernoulli. From the introduction (1716 edition) it becomes clear that the name "Calcul integral" was already in use. Thus, by having a specific name it had gained an 'official' status as a specific part of knowledge to which one could easily refer. However, what was signified by this name changed considerably.

Cauchy's *Cours d'analyse* from 1821 and *Résumé* from 1823, written for The *École Polytéchnique* in Paris, were the first textbooks in which calculus appeared as an integrated body of knowledge with clear borders towards other mathematical areas. It included a proof of a proposition (with no name) very similar to what now is called the FTC. The textbook can be seen as an attempt to provide access for a wider audience to a knowledge based on the same criteria as those promoted by the producers of mathematical knowledge.

Also in other early textbooks the propositions related to what is now called FTC are not named, but in the *Course d'analyse mathématiques* from 1902 by Goursat, translated into English already in 1904 and widely spread, the expression "fundamental theorem" is used for the fact that "every continuous function f(x) is the derivative of some other function". In the textbook *An introduction to the summation of differences of a function* by Groat, printed in 1902, the expression "the fundamental theorem of the integral calculus" appears, as well as the more short "fundamental theorem". In *The theory of functions of a real variable & the theory of Fourier series*, published in 1907 by Hobson, one chapter has the title "The fundamental theorem of the integral calculus for the Lebesgue integral". That the name of the theorem serves as a chapter title and is extended to a more general application indicates a strong level of institutionalisation. Wiener refers several times to "the fundamental theorem of the calculus" in *Fourier transforms in the complex domain* from 1934. That this name became institutionalised is evident from the classical book *What is mathematics?* from 1941, where Courant and Robbins write (p. 436):

> There is no separate differential calculus and integral calculus, but only one *calculus*. It was the great achievement of Leibniz and Newton to have first clearly recognized and exploited this *fundamental theorem of the calculus*.

The development of the calculus shows that the process of institutionalisation of a body of knowledge has to be seen in relation to the practices of publication and education.

## Discussion

As the FTC and its proof, in the version that has become institutionalised, links two different fields of investigation, it can be attributed a systematising function. Cauchy's further systematisation by means of introducing a set of basic concepts for an outline of the theory was developed in the context of the teaching at the École Polytéchnique. This clearly shows that there is a dynamic relation between the production of knowledge and its transmission in the development of knowledge criteria. For example, reference to Cauchy's scholarly work is commonly done by drawing on his textbooks. That his textbooks became popular and that his exposition of the calculus became generally adopted (Boyer, 1959, pp. 282-283) can be explained by the combination of its influence on the producers of knowledge through applying criteria for knowledge that became internally socially shared, and the relative autonomy of teaching that accounts for its distribution. Action as 'producer' and at the same time as 'transposer' affects both the unmediated and the pedagogic discourse.

There are other prominent examples of a dynamic relationship between the development of 'knowledge for teaching' and the intention of re-organising and re-describing a set of related outcomes of research in different sub-areas for the purpose of presenting it in a coherent way. Felix Klein's *Elementarmathematik vom höheren Standpunkte aus* from 1908 is an example of a work that provided insights for both teachers of mathematics and researching mathematicians. The same year Godfrey Harold Hardy published his *A course of pure mathematics* (reprinted in many new editions), which "was intended to help reform mathematics teaching in the UK" and more specifically to prepare students to study mathematics at university [6]. It is not easy to locate such publications in relation to their role for a didactic transposition of scholarly knowledge. Transposition might include a 're-systematisation' of knowledge, for example, from a hierarchical structure of embedded specialised theories into an organisation by shared techniques within different specialised areas. This is often the case when the outcomes of basic research are to be used in applied research.

The necessity for a didactic transposition assumes a separation between the producers of knowledge and teachers. If this separation includes a division of labour, the producers of knowledge are not the ones responsible for a transposition of the outcomes of research into knowledge for teaching. Several examples from the history of calculus (Bernoulli-l'Hospital, Lagrange, Cauchy) show that such a division of labour did not always exist. Prominent researchers in the area worked as 'transposers' of the knowledge produced by themselves for the purpose of the very introduction into that area. In this case, the transposition is not achieved by another group of agents, who belong to the 'noosphere' in the theory of didactic transposition. In the situation during the time of Cauchy the social base for the noosphere was different than today as there was no independent textbook industry and no massification of higher education. In addition, the relatively low degree of specialisation of the discipline reduces the gap between levels of mathematical knowledge in terms of a hierarchical knowledge organisation. The time-span, after which a piece of scholarly knowledge becomes a potential object for teaching, depends on the level of specialisation of the knowledge to be taught in relation to the goals of the teaching institution. For example, Lebesgue integration soon entered advanced university courses, but still is not the standard approach in introductory calculus courses. The textbook by Hobson, published in 1907, includes the Lebesgue integral that was published in 1904.

The historical study also shows that it is hard to find a distinct point in time when the calculus as a delineated body of knowledge has become institutionalised. Consequently, the scholarly body of knowledge named 'calculus', which could be the starting point for a didactic transposition, cannot easily be identified. The investigation of the names for the FTC used in research publications and textbooks shows that different names were used for similar versions of *the* FTC, but also that the same name was used for different versions of it, or no specific name was used. There is no distinct transposition of a clearly identifiable piece of scholarly knowledge, but a series of re-descriptions. The institutionalisation seems to be happening within the 'knowledge for teaching' rather than within the scholarly knowledge. An example of this is Cauchy and his *Cours d'analyse* from 1821 and the follow up in the *Résumé* from 1823, which was to meet the new demands arising in higher education after the French revolution, even explicitly at "the urging of several of his colleagues" (Katz, 2004, p. 432). Our example shows that the original scholarly body of knowledge that is to be the starting point for a didactic transposition is not as fixed, organised and stable as suggested by the theory. It is indeed hard to trace back the original elements of different mathematical discourses from the field of knowledge production that are manifested in a curriculum. [7]

The relation between the mathematical knowledge in institutions that are not only established for the purpose of education, and its forms developed for apprenticeship into the discipline, is but one facet of the life of knowledge in society ("la vie des savoirs dans la société"; Chevallard, 1991, p. 210), which is the focus of concern for Chevallard and the reason for naming the further elaboration of his theory the 'anthropological theory of didactic' (ATD; see Bosch & Gascon, 2006). Even if this name does not suggest so, the theory introduces a programme that sets out to develop a sociology of (mathematical) knowledge by studying 'didactic systems'. However, scholars working within the ATD have not, to our knowledge, engaged in a critical discussion of its basic principles and relations to other frameworks such as Activity Theory or to Bernstein's work that is concerned with the production, reproduction and distribution of knowledge, and in particular with the process of knowledge recontextualisation. From the viewpoint of Bernstein's theory (e.g. 2000), pedagogic discourse is defined by the fact that it recontextualises a practice by moving it from its original site in order to use it for a different purpose. As the discourse moves from its original site (to become a pedagogic discourse), there is always space in which ideology can play. An ideologically transformed discourse is not the same discourse anymore. This is reminiscent of the description of 'didactic transposition' accomplished in the 'noosphere' in the ATD, though the ideological transformation of knowledge is not a major concern, neither is its distribution in relation to social structures and their (re-)production through education. Hence, we see a potential for productive interaction of the ATD with other languages of description used in mathematics education research by relating it to theories that share some of the (often implicit) intellectual roots of the didactic transposition theory.

## Notes

1. "Le discours pleinement didactique devrait donc différer essentiellement du simple discours logique, où le penseur suit librement sa propre marche, sans aucun égard aux conditions naturelles d'une communication quelconque."

2. "On réserve la forme dialogique, propre à toute vraie communication, pour expliquer les conceptions qui sont à la fois assez importantes et assez mûries. […] Loin d'indiquer une négligence excusable seulement envers les cas secondaires, cette forme, quand elle est bien instituée, constitue, au contraire, le seul mode d'exposition qui soit vraiment didactique: il convient également à toutes les intelligences."

3. "cette transformation didactique ne devient réalisable qu'envers des doctrines assez élaborées pour qu'on puisse nettement comparer les diverses manières d'exposer leur ensemble, et prévoir aisément les objections qu'elles devront susciter."

4. For the full report, see Klisinska (2009).

5. For example in the influential textbooks *Théorie des functions analytiques* from 1797 by Lagrange and *Traité élémentaire* from 1802 by Lacroix.

6. Wikipedia online: <en.wikipedia.org/wiki/A_Course_of_Pure_Mathematics>

7. When discussing transformations of academic mathematics for school mathematics, Ernest (2006) makes a similar general claim that "for any particular theory or area of mathematics, there is no fixed or unique mathematical theory formulation as a source. There are always multiple formulations by different mathematicians and groups of mathematicians constructed and published at different times" (p. 73). He also adds that for topics in school mathematics which today are no longer topics of research mathematics, at the time when they were, the academic texts written (with Stevin's *Decimal fractions* as one example) "were both advanced academic treatises for scholars, as well as teaching texts" (p. 73).

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