Diagnostic tests 4: likelihood ratios

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Notes

Statistics Notes
Diagnostic tests 4: likelihood ratios

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The properties of a diagnostic or screening test are often described using sensitivity and specificity or predictive values, as described in previous Notes.[1] [2] Likelihood ratios are alternative statistics for summarising diagnostic accuracy, which have several particularly powerful properties that make them more useful clinically than other statistics.[3]

Each test result has its own likelihood ratio, which summarises how many times more (or less) likely patients with the disease are to have that particular result than patients without the disease. More formally, it is the ratio of the probability of the specific test result in people who do have the disease to the probability in people who do not.

A likelihood ratio greater than 1 indicates that the test result is associated with the presence of the disease, whereas a likelihood ratio less than 1 indicates that the test result is associated with the absence of disease. The further likelihood ratios are from 1 the stronger the evidence for the presence or absence of disease. Likelihood ratios above 10 and below 0.1 are considered to provide strong evidence to rule in or rule out diagnoses respectively in most circumstances.[4] When tests report results as being either positive or negative the two likelihood ratios are called the positive likelihood ratio and the negative likelihood ratio.

The table shows the results of a study of the value of a history of smoking in diagnosing obstructive airway disease.[5] Smoking history was categorised into four groups according to pack years smoked (packs per day × years smoked). The likelihood ratio for each category is calculated by dividing the percentage of patients with obstructive airway disease in that category by the percentage without the disease in that category. For example, among patients with the disease 28% had 40+ smoking pack years compared with just 1.4% of patients without the disease. The likelihood ratio is thus $28.4/1.4 = 20.3$. A smoking history of more than 40 pack years is strongly predictive of a diagnosis of obstructive airway disease as the likelihood ratio is substantially higher than 10. Although never smoking or smoking less than 20 pack years both point to not having obstructive airway disease, their likelihood ratios are not small enough to rule out the disease with confidence.

> **Calculation of post-test probabilities using likelihood ratios**
>
> Pretest probability $= p_1 = 0.1$
>   pretest odds $= p_1/(1-p_1) = 0.1/0.9 = 0.11$
>   post-test odds $=$ pretest odds×likelihood ratio
>   post-test odds $= o_2 = 0.11 \times 20.43 = 2.27$
> Post-test probability $= o_2/(1+o_2) = 2.27/3.37 = 0.69$

Likelihood ratios are ratios of probabilities, and can be treated in the same way as risk ratios for the purposes of calculating confidence intervals.[6]
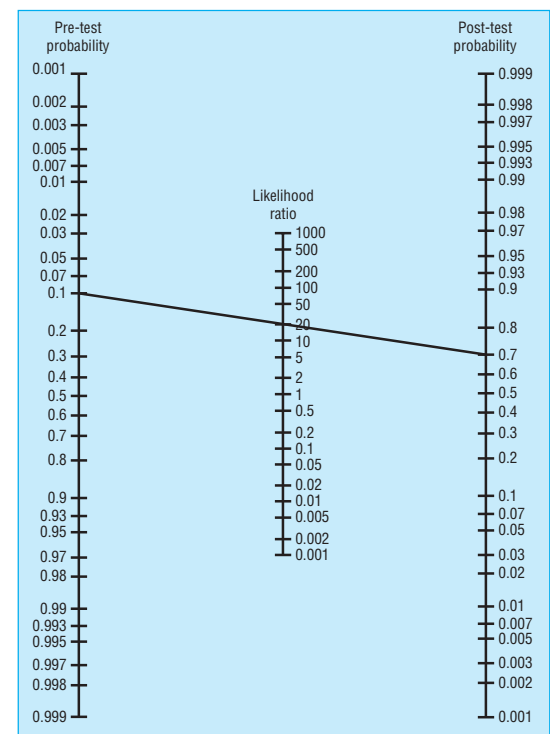
For a test with only two outcomes, likelihood ratios can be calculated directly from sensitivities and specificities.[1] For example, if smoking habit is dichotomised as above or below 40 pack years, the sensitivity is 28.4% (42/148) and specificity 98.6% (142/144). The positive likelihood ratio is the proportion with obstructive airway disease who smoked more than 40 pack years (sensitivity) divided by the proportion without disease who smoked more than 40 pack years (1–specificity), $28.4/1.4 = 20.3$, as before. The negative likelihood ratio is the proportion with disease who smoked less than 40 pack years (1–sensitivity) divided by the proportion without disease who smoked less than 40 pack years (specificity), $71.6/98.6 = 0.73$. However, unlike sensitivity and specificity, computation of likelihood ratios does not require dichotomisation of

Likelihood ratios are ratios of probabilities, and can be treated in the same way as risk ratios for the purposes of calculating confidence intervals[6]

Smoking habit (pack years)	Obstructive airway disease		Likelihood ratio	95% CI
	Yes (n (%))	No (n (%))		
≥40	42 (28.4)	2 (1.4)	(42/148)/(2/144)=20.4	5.04 to 82.8
20-40	25 (16.9)	24 (16.7)	(25/148)/(24/144)=1.01	0.61 to 1.69
0-20	29 (19.6)	51 (35.4)	(29/148)/51/144)=0.55	0.37 to 0.82
Never smoked or smoked for <1 yr	52 (35.1)	67 (46.5)	(52/148)/67/144)=0.76	0.57 to 1.00
Total	148 (100)	144 (100)		

Use of Fagan's nomogram for calculating post-test probabilities[7]

test results. Forcing dichotomisation on multicategory test results may discard useful diagnostic information.

Likelihood ratios can be used to help adapt the results of a study to your patients. To do this they make use of a mathematical relationship known as Bayes theorem that describes how a diagnostic finding changes our knowledge of the probability of abnormality.[3] The post-test odds that the patient has the disease are estimated by multiplying the pretest odds by the likelihood ratio. The use of odds rather than risks makes the calculation slightly complex (box) but a nomogram can be used to avoid having to make conversions between odds and probabilities (figure).[7] Both the figure and the box illustrate how a prior probability of obstructive airway disease of 0.1 (based, say, on presenting features) is updated to a probability of 0.7 with the knowledge that the patient had smoked for more than 40 pack years.

In clinical practice it is essential to know how a particular test result predicts the risk of abnormality. Sensitivities and specificities[1] do not do this: they describe how abnormality (or normality) predicts particular test results. Predictive values[2] do give probabilities of abnormality for particular test results, but depend on the prevalence of abnormality in the study sample and can rarely be generalised beyond the study (except when the study is based on a suitable random sample, as is sometimes the case for population screening studies). Likelihood ratios provide a solution as they can be used to calculate the probability of abnormality, while adapting for varying prior probabilities of the chance of abnormality from different contexts.

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A memorable patient

Living history

I was finally settling down at my desk when the pager bleeped: it was the outpatients' department. An extra patient had been added to the afternoon list—would I see him?

The patient was a slightly built man in his 60s. He had brought recent documentation from another hospital. I asked about his presenting complaint.

"Well, I'll try, but I wasn't aware of everything that happened. That's why I've brought my wife—she was with me at the time."

This was turning out to be one of those perfect neurological consultations: documents from another hospital, a witness account, an articulate patient. The only question would be whether it was seizure, syncope, or transient ischaemic attack. As we went through his medical history, I studied his records and for the first time noticed the phrase "Tetralogy of Fallot."

"Yes, my lifelong diagnosis," he smiled. "I was operated on."

I saw the faint dusky blue colour of his lips. "Blalock-Taussig shunt?" I asked, as dim memories from medical school somehow came back into focus.

"Yes, twice. By Blalock." He paused. "In those days the operation was only done at Johns Hopkins—all the patients went there. I remember my second operation well. I was 12 at the time."

"And the doctors?" I asked.

"Yes, especially Dr Taussig. She would come around with her entourage every so often. Deaf as a post, she was."

"What? Taussig deaf?"

"Yes. She had an amplifier attached to her stethoscope to examine patients."

I asked to examine him, only too aware that it was more for my benefit than his.

A half smile suggested that he had read my thoughts: "Of course."

To even my unpractised technique, his cardiovascular signs were a museum piece: absent left subclavian pulse, big arciform scars on the anterior chest created by the surgeon who had saved his life, central cyanosis, right-sided systolic murmurs, loud pulmonary valve closure sound (iatrogenic pulmonary hypertension, I reasoned), pulsatile liver—all these and undoubtedly more noted by the physician who first understood his condition.

That night, I read about his doctors. Helen Taussig indeed had had substantial hearing impairment, a disability that would have meant the end of a career in cardiology for a less able clinician. I also learnt of the greater challenges of sexual prejudice that she fought and overcame all her life. I learnt about Alfred Blalock, the young doctor denied a residency at Johns Hopkins only to be invited back in his later years to head its surgical unit.

The experiences of a life in medicine are sometimes overwhelming. For weeks, I reflected on the perspectives opened to me by this unassuming patient. The curious irony of a man with a life threatening condition who had outlived his saviours; the extraordinary vision of his all-too-human doctors; the opportunity to witness history played out in the course of a half-hour consultation. And memories were jogged, too: the words of my former professor of medicine, who showed us cases of "Fallot's" and first told us about Taussig, the woman cardiologist; the portrait of Blalock adorning a surgical lecture theatre in medical college.

(And my patient's neurological examination and investigations? Non-contributory. I still don't know whether it was seizure, syncope, or transient ischaemic attack.)

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