

**Fadiga de Materiais Estruturais:  
Fundamentos e Aplicações  
Metodologia S-N  
(Strain-based Methodology)**

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# AGENDA

1. *Introdução (Introduction)*
2. *Comportamento do Material (Material Behavior)*
3. *Curvas de Fadiga  $\varepsilon$ -N (Strain-Life Curve)*
4. *Determinação das Propriedades (Fatigue Properties Determination)*
5. *Tensão Média (Mean Stress)*
6. *Fatores que Afetam a Vida à Fadiga (Factors Influencing  $\varepsilon$ -N Response)*

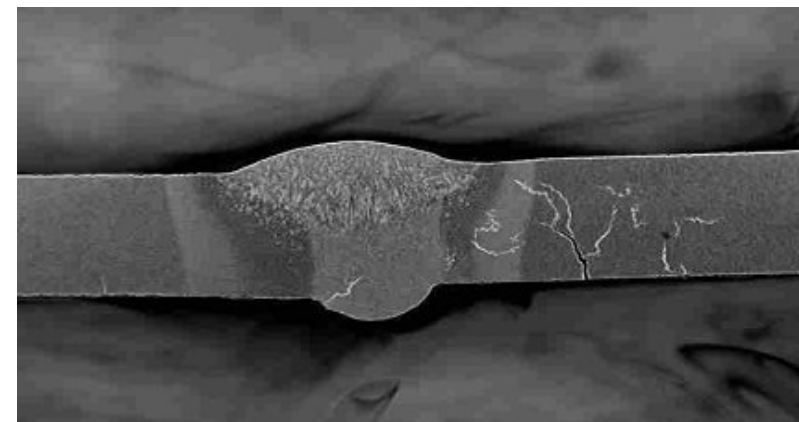
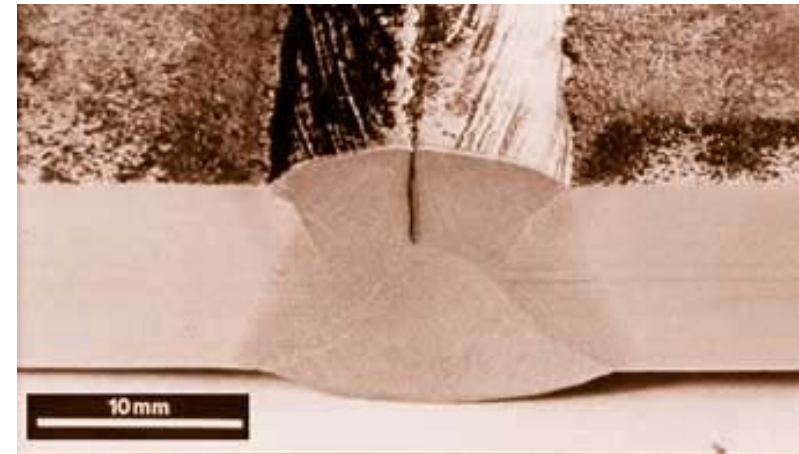
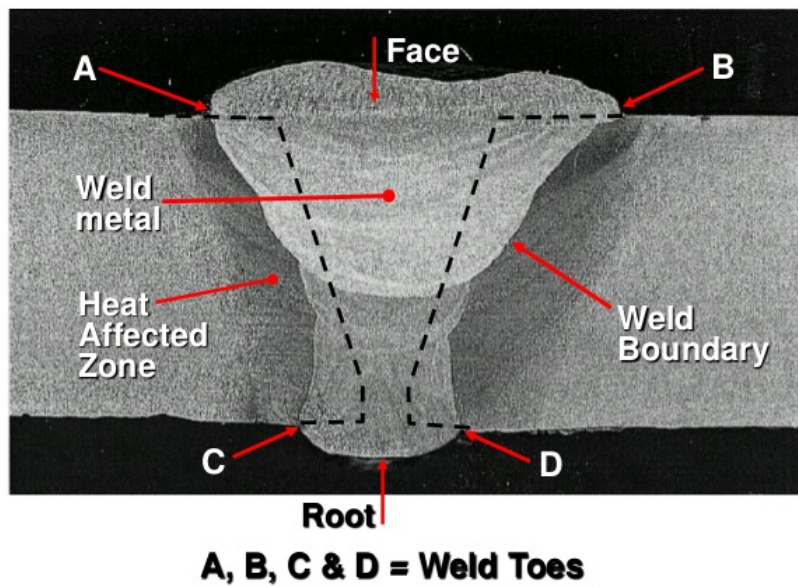
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# Introduction

- **Butt Weld – V Joint**

## Weld Zone Terminology <sup>2.5</sup>

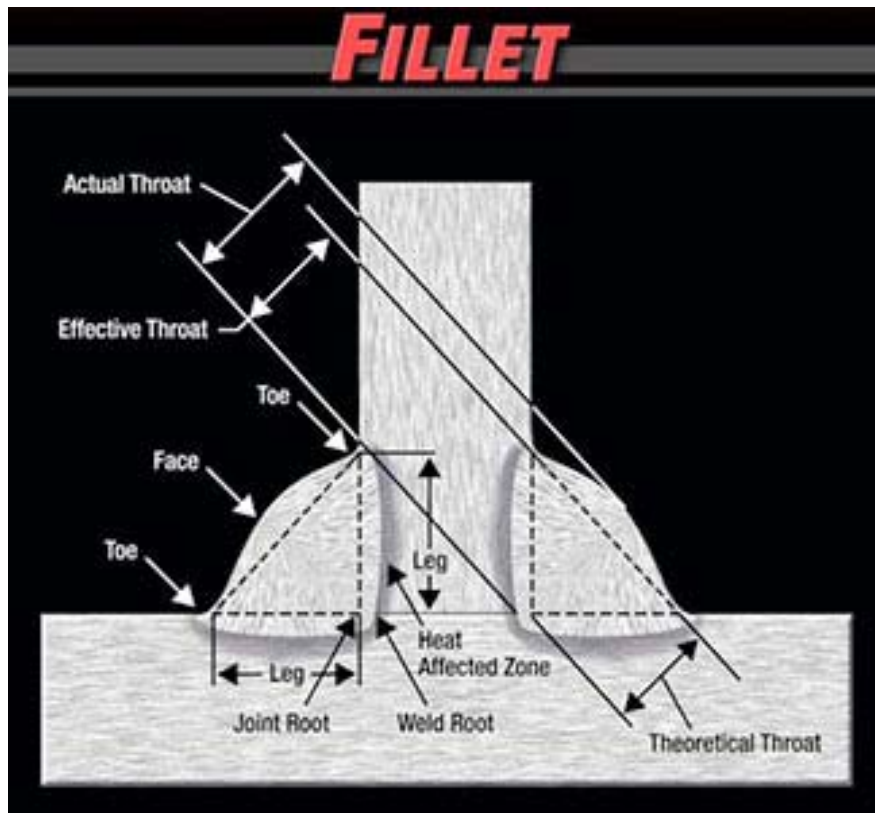


**Notches = stress raisers**

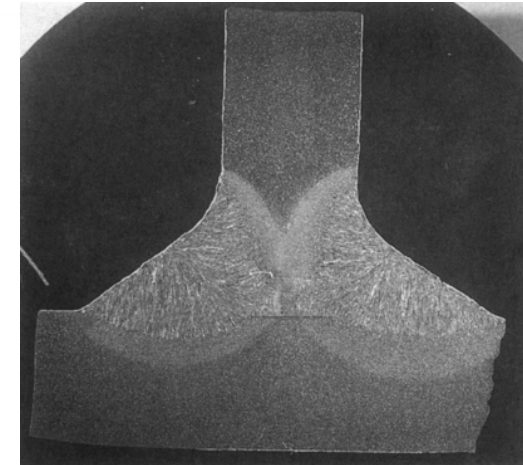


# Introduction

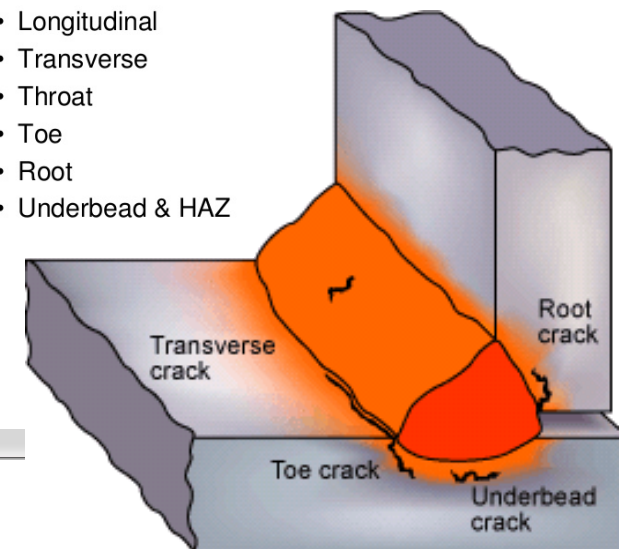
## ■ Fillet T Weld



**Notches = stress raisers**

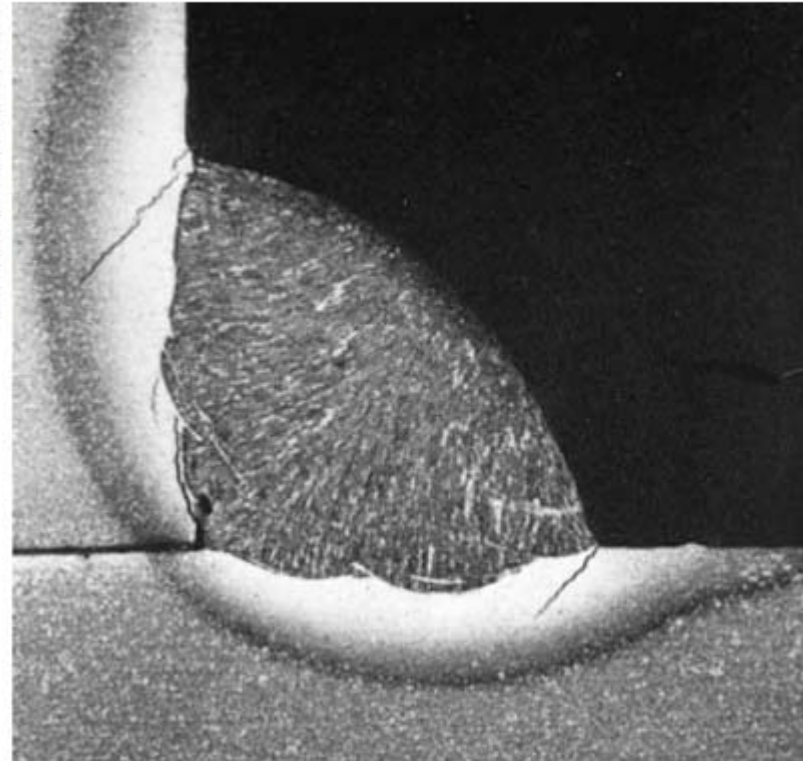
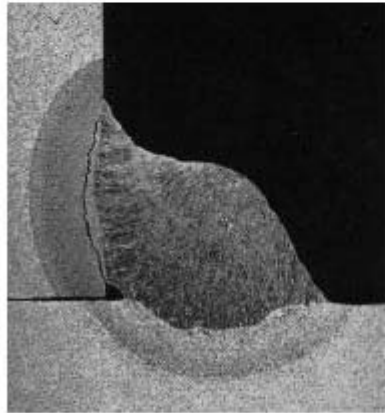
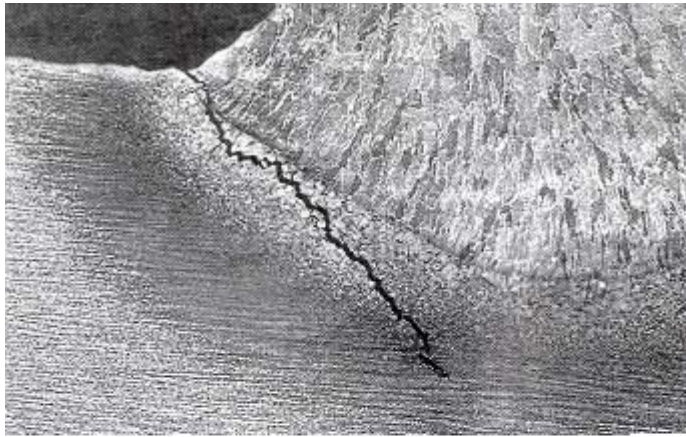


- Longitudinal
- Transverse
- Throat
- Toe
- Root
- Underbead & HAZ



# Introduction

- *Fillet T Weld*



**Notches = stress raisers**

# Opening (Notch)



**Notches = stress raisers**

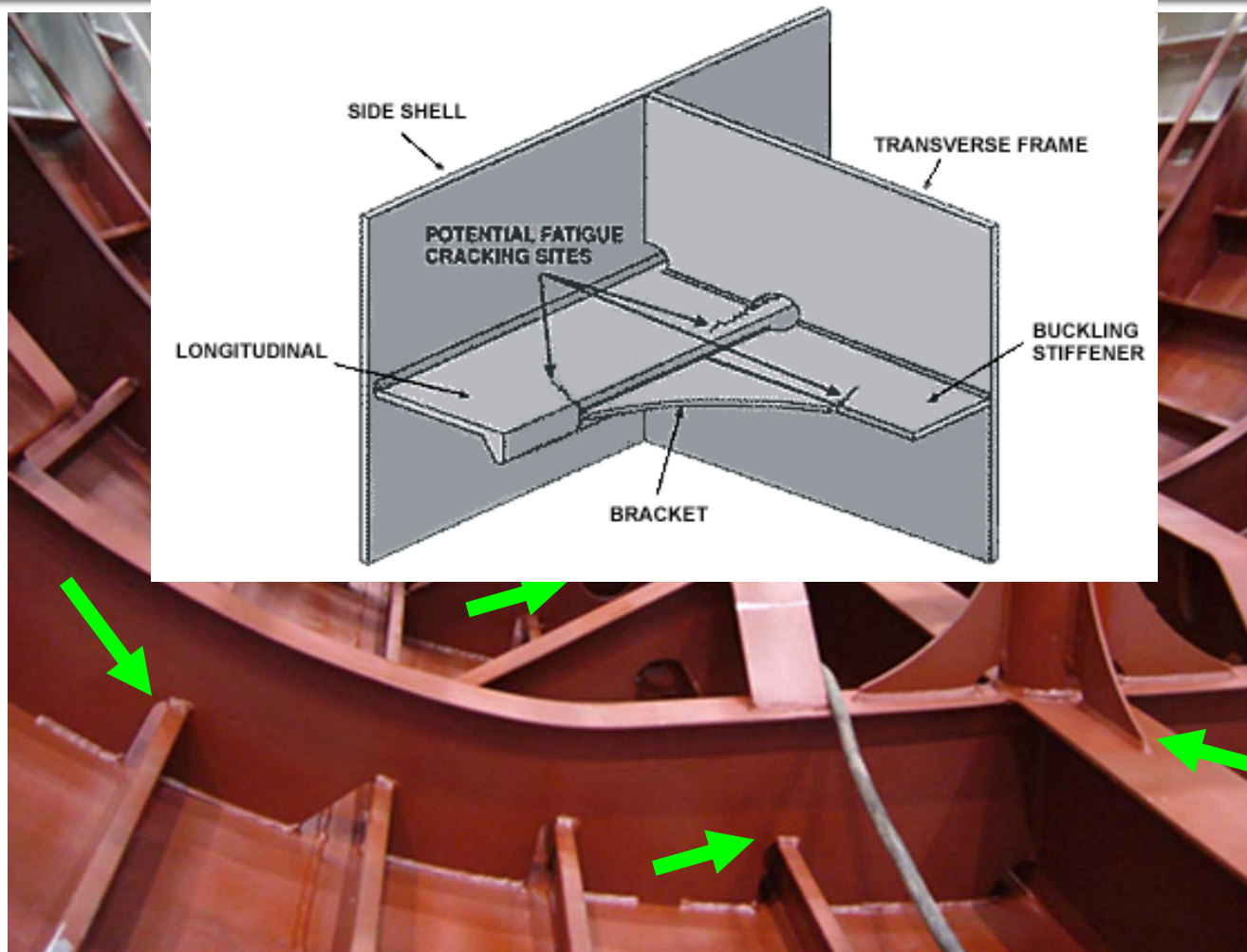


# Entalhes (Notches)



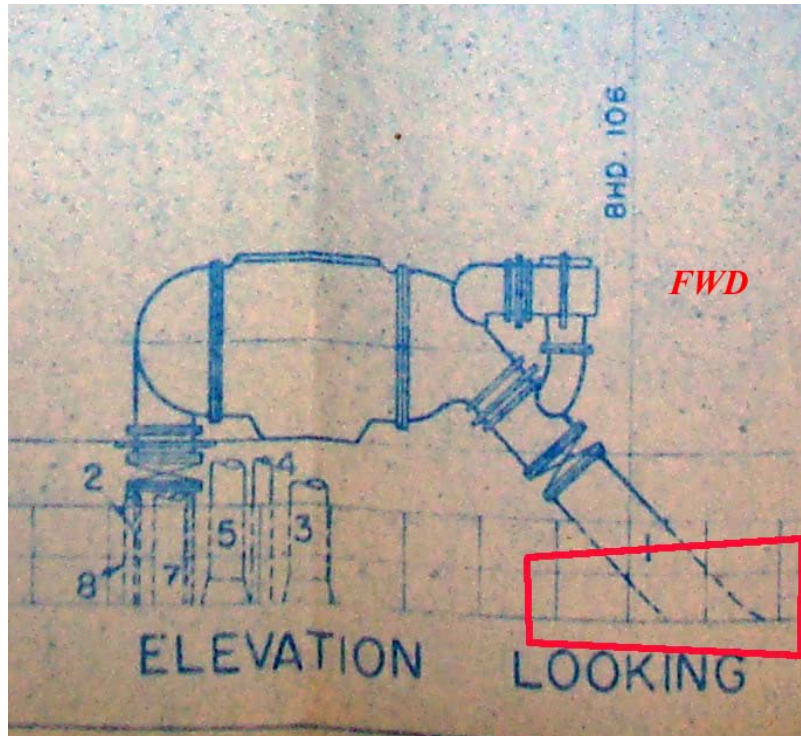


# Entalhes (Notches)



# Opening (Notch)

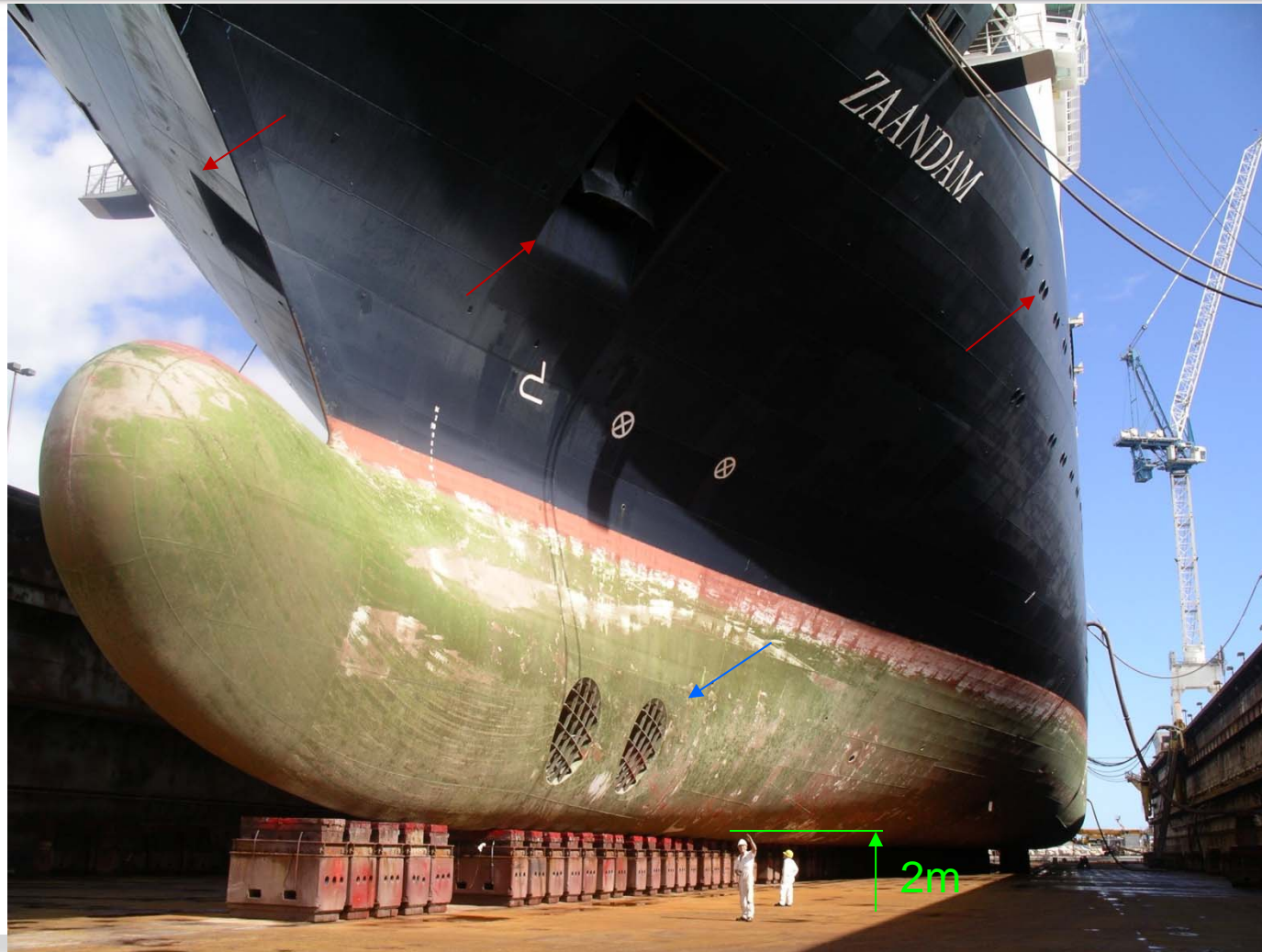
caixa de mar (sea chests )



**Sea Chest** ----A compartment through which sea water is admitted or discharged.



# Entalhes (Notches)

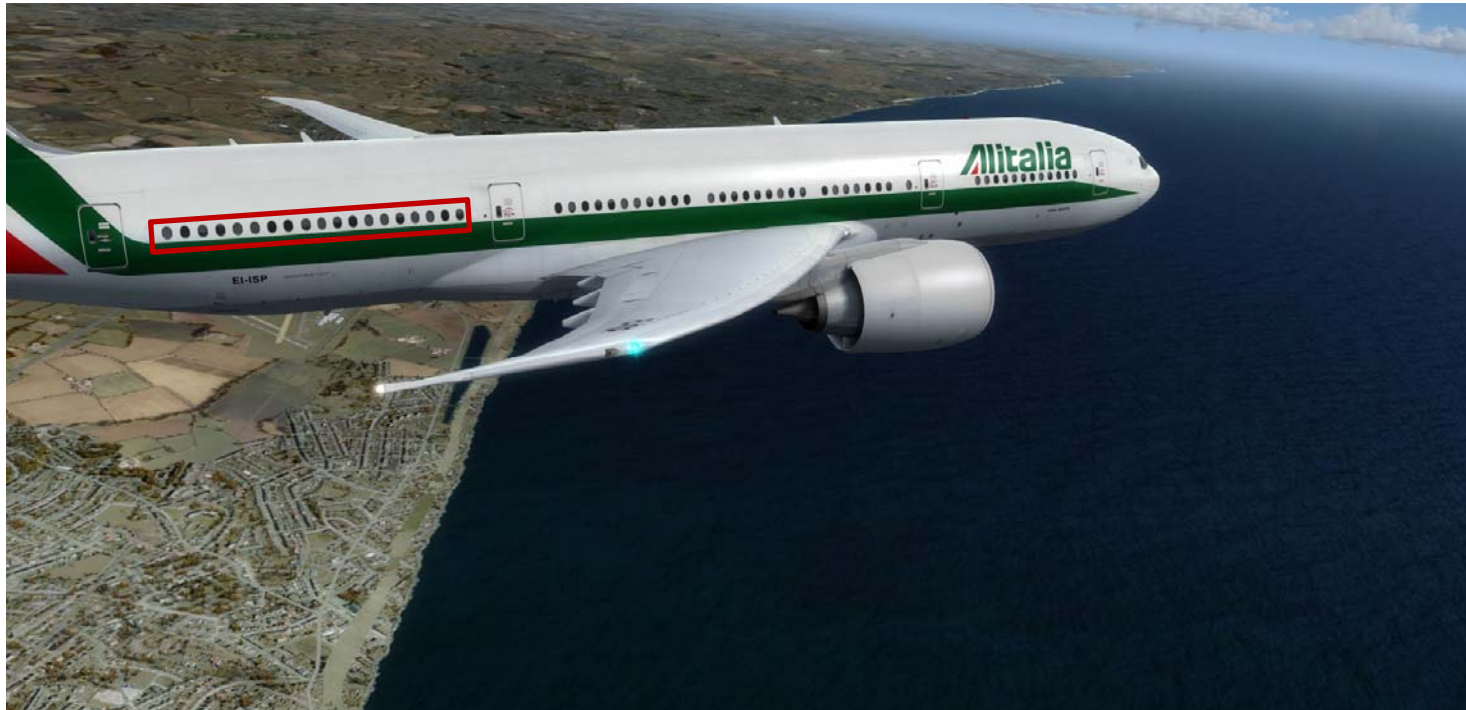




# Entalhes (Notches)

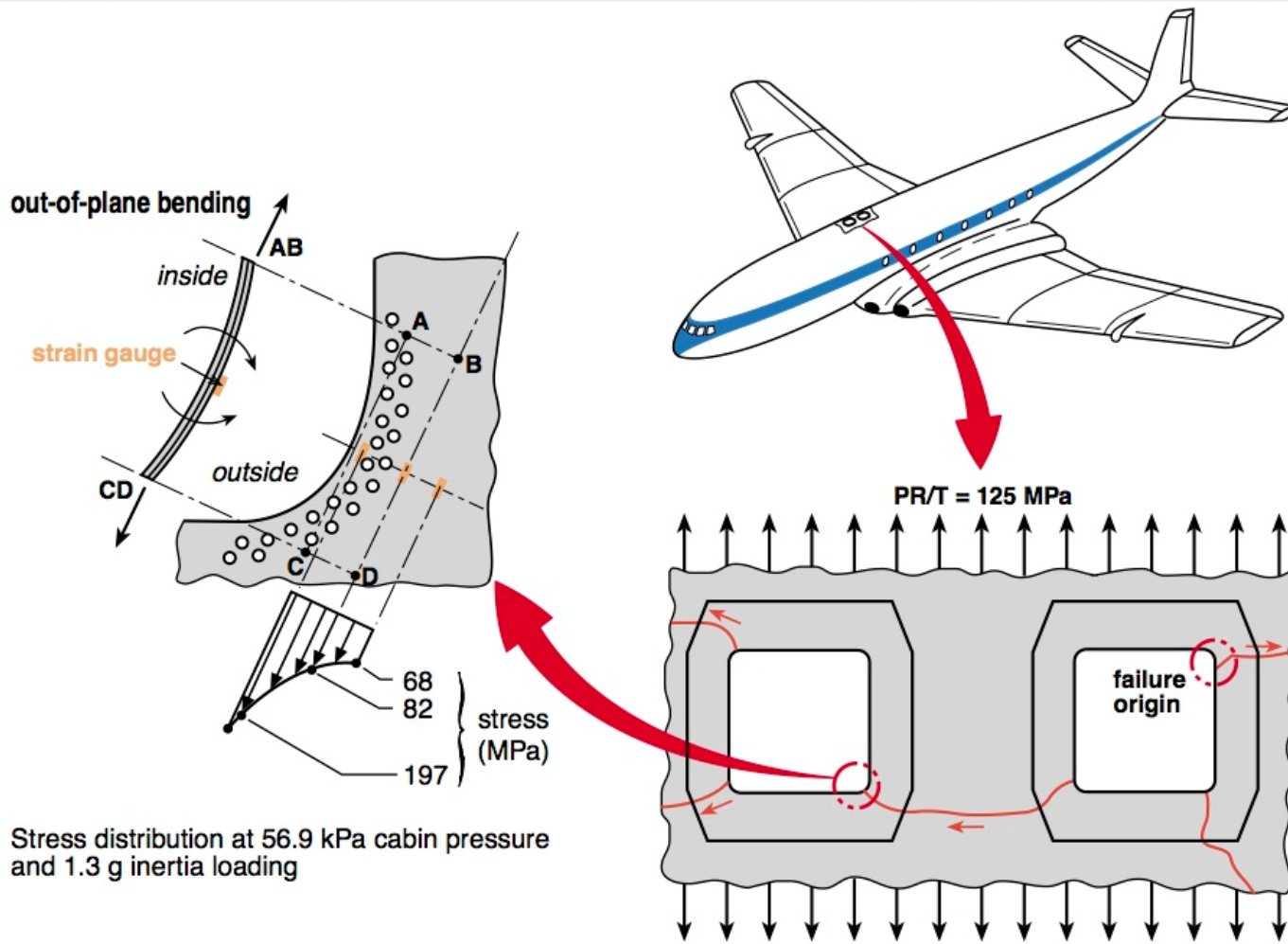


# Entalhes (Notches)





# Entalhes (Notches)

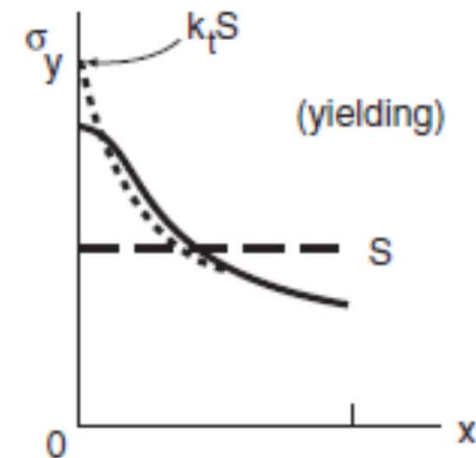
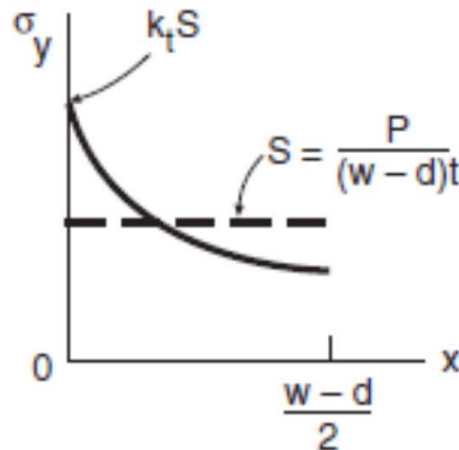
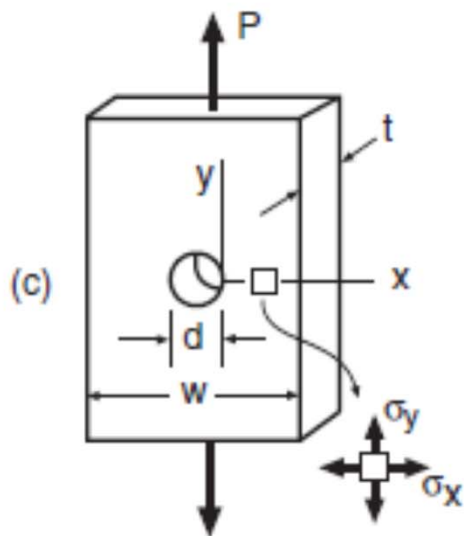




# Tensão no ponto versus Tensão Nominal

- Notched member
  - Área neta =  $(w-d)t$

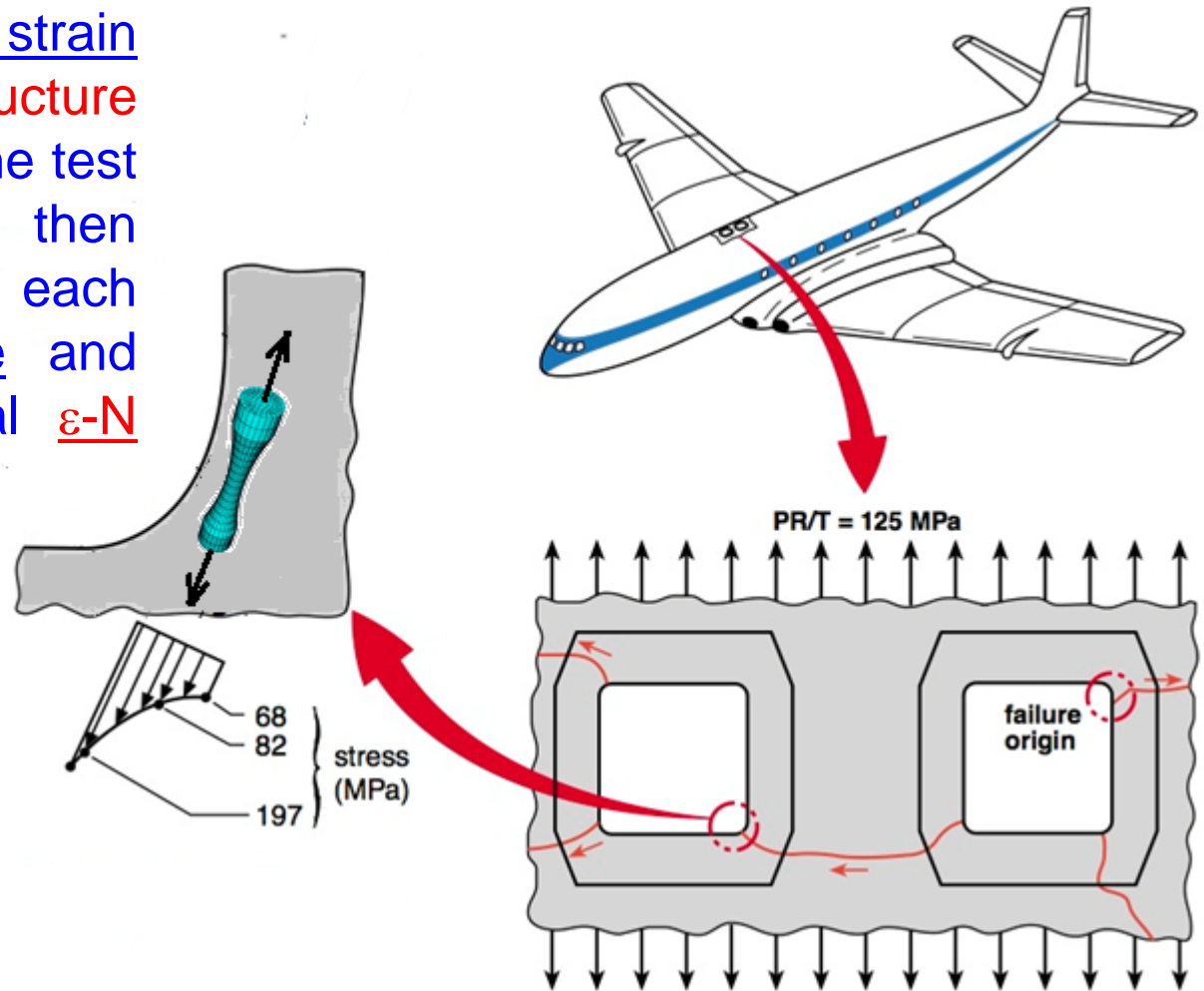
$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \longleftrightarrow S = \frac{P}{((w-d) \times t)}$$



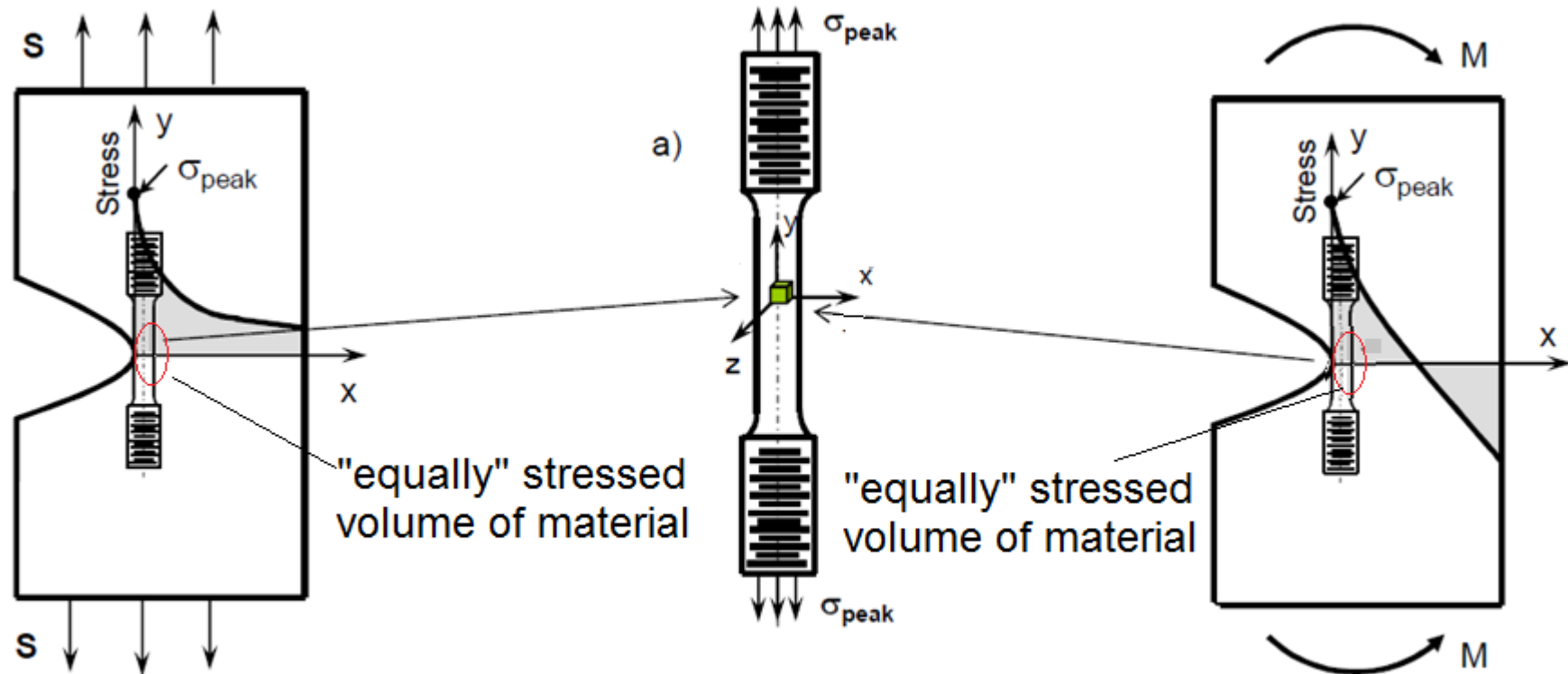
Notch= stress raiser ( holes, grooves, fillets, etc)

# Princípio de Similaridade (Similitude)

- If the local notch-tip strain history at the notched **structure** and the strain history in the test **specimen** are the same, then the **fatigue** response in each case will be the same and describe by the material  $\epsilon$ -N curve.



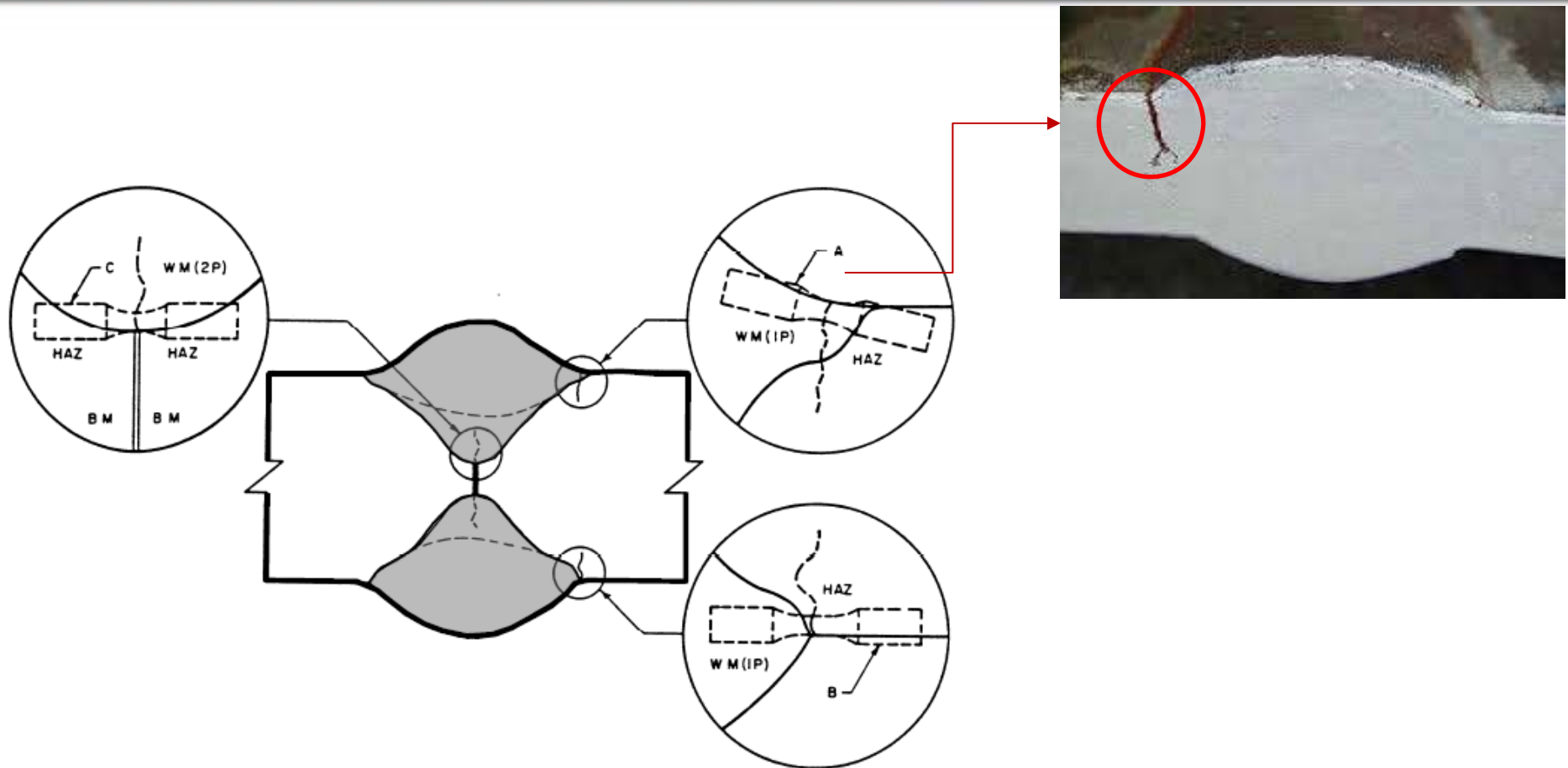
# Princípio de Similaridade (Similitude)



Smooth specimen representing notched components



# Princípio de Similaridade (Similitude)



Smooth specimen representing the critical regions of butt welded component

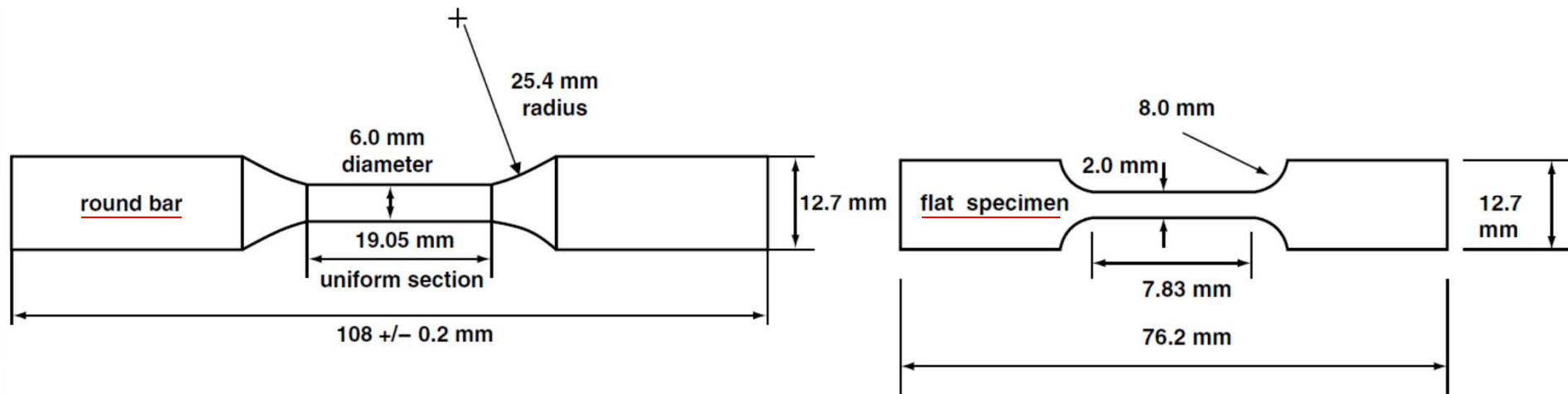
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# Monotonic Stress-Strain Behavior

## ASTM E8M

- Cylindrical specimens  $\phi \sim 6 \text{ mm} - 12 \text{ mm}$
- Flat specimens may also be used

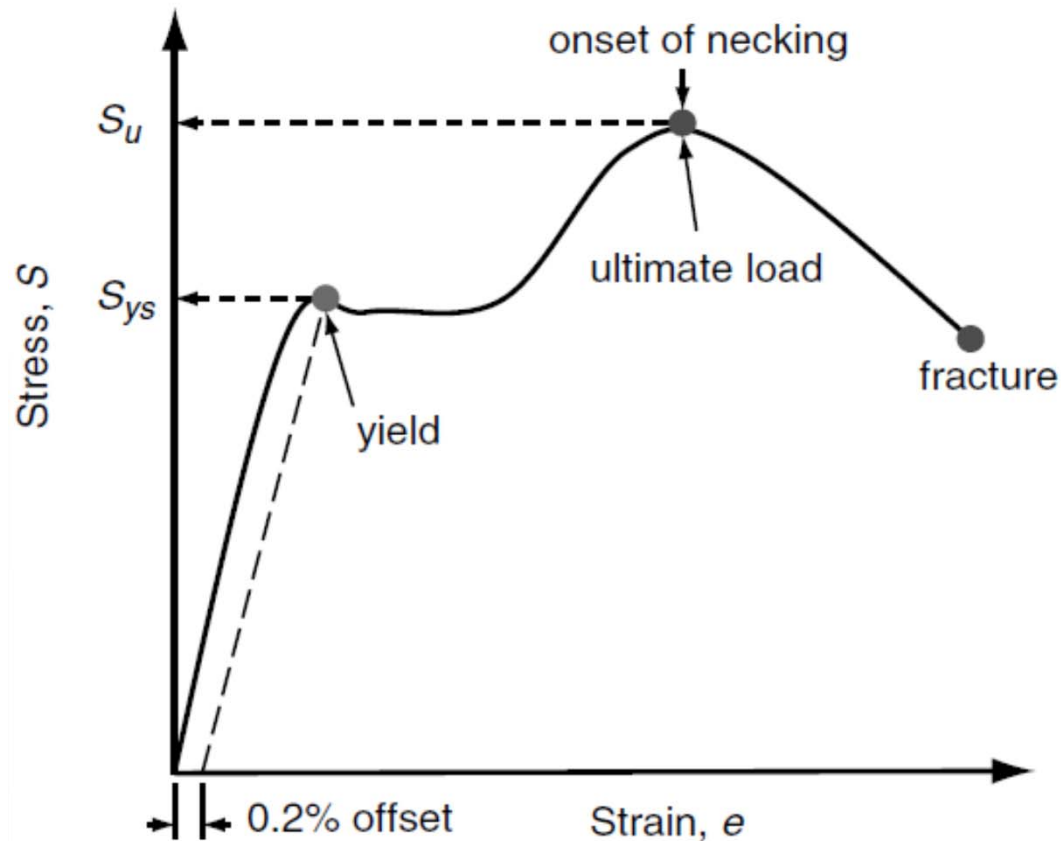




# Monotonic Stress-Strain Behavior

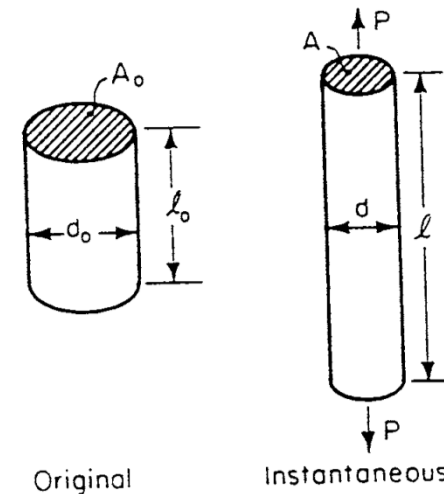
ASTM E8M

Engineering stress-strain curve



$$S = \text{engineering stress} = \frac{P}{A_0}$$

$$e = \text{engineering strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

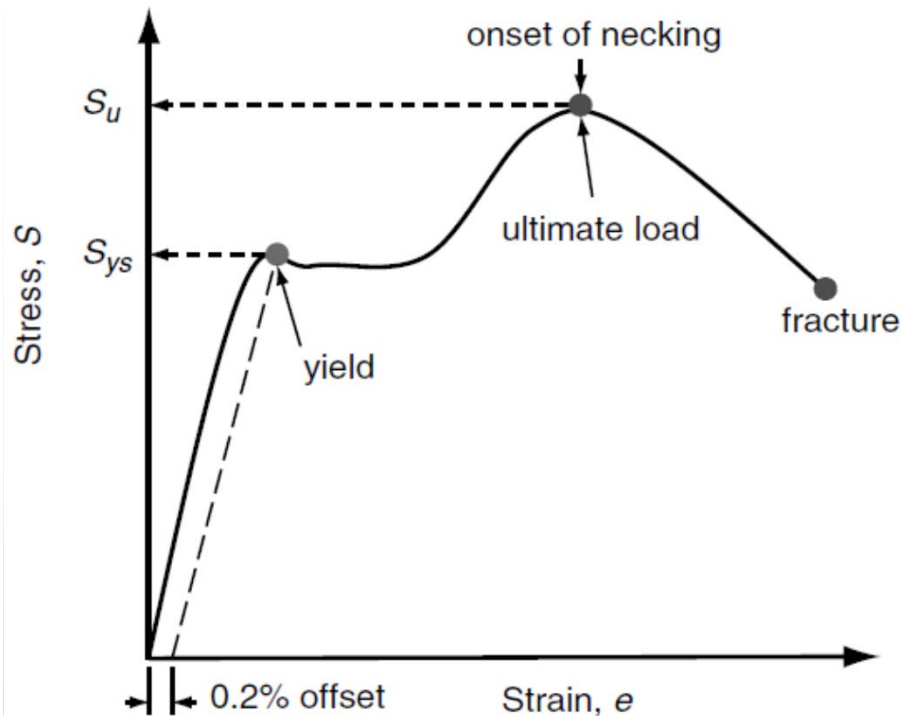


Four parameters can be measured directly from test:  $S_{ys}$ ,  $S_u$ , %EI, %RA

# Monotonic Stress-Strain Behavior

ASTM E8M

Engineering stress-strain curve



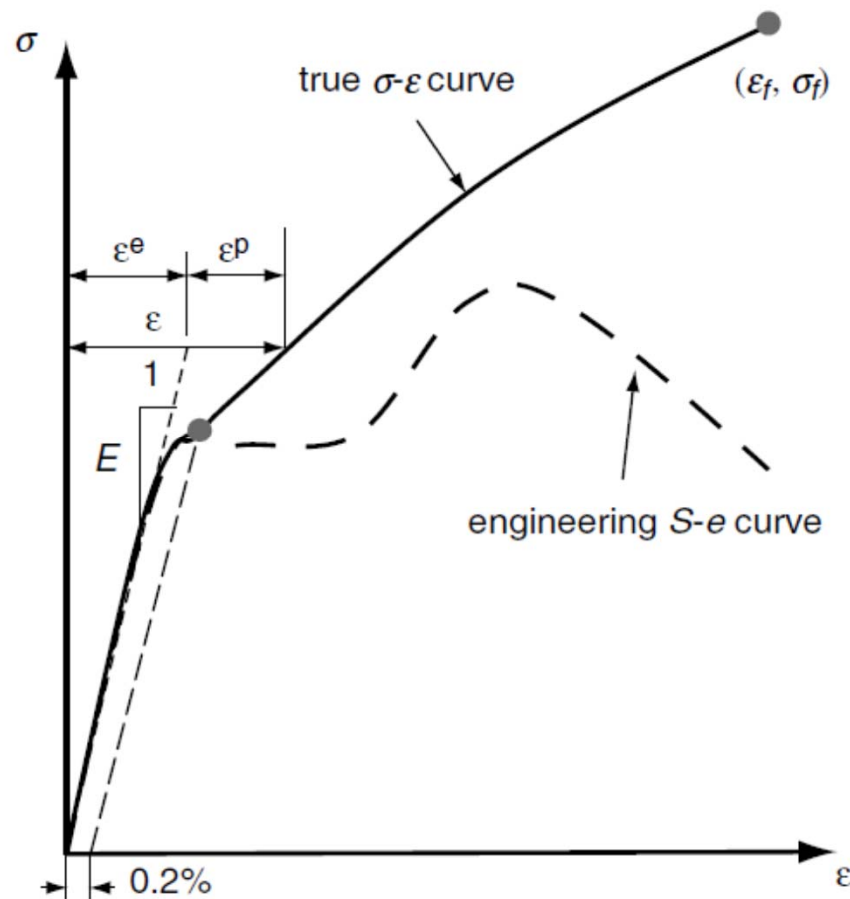
$$S_u = \frac{P_{\max}}{A_o}$$

$$\%EI = 100 \times \frac{l_f - l_o}{l_o}$$

$$\%RA = 100 \times \frac{A_f - A_o}{A_o}$$

Ductility

# Monotonic Stress-Strain Behavior



True stress- true strain curve



Use of instantaneous area (A) and length (l)

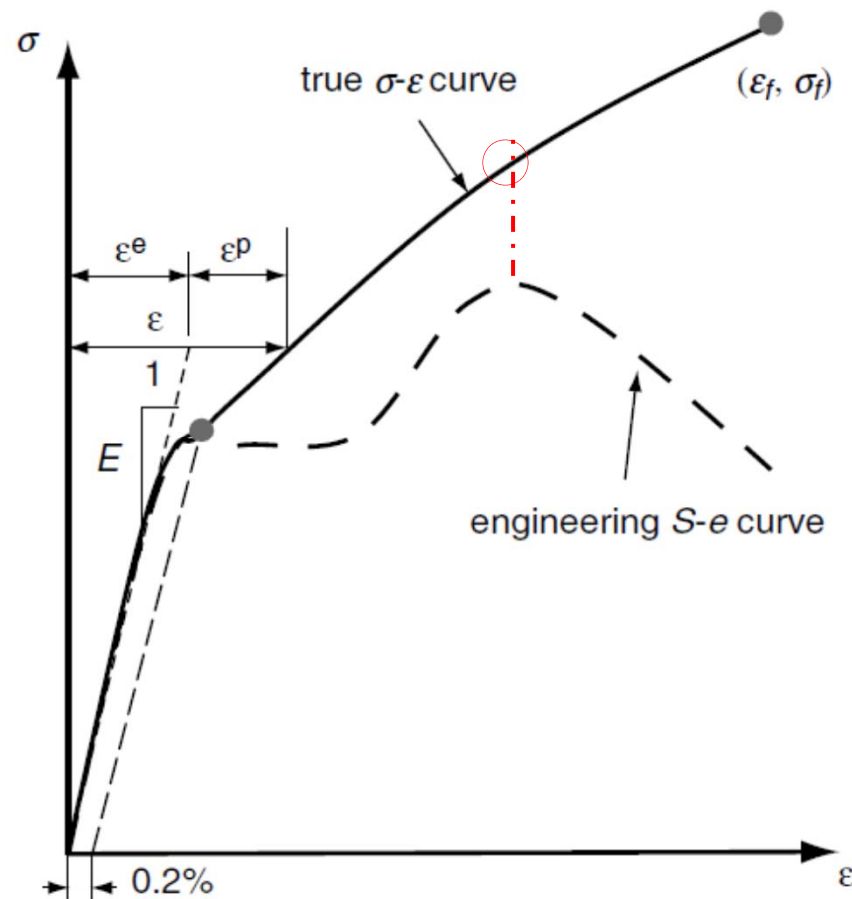


$$\sigma = \frac{P}{A}$$

$$\epsilon = \int_{l_0}^l \frac{dl}{l} = \ln \frac{l}{l_0}$$



# Monotonic Stress-Strain Behavior



Engineering stress-strain curve



True stress- true strain curve

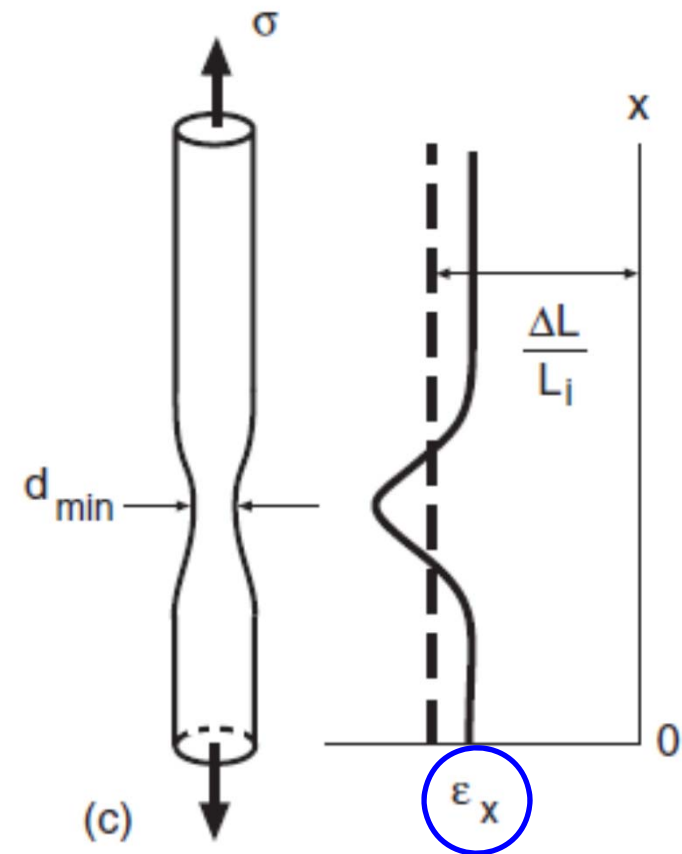
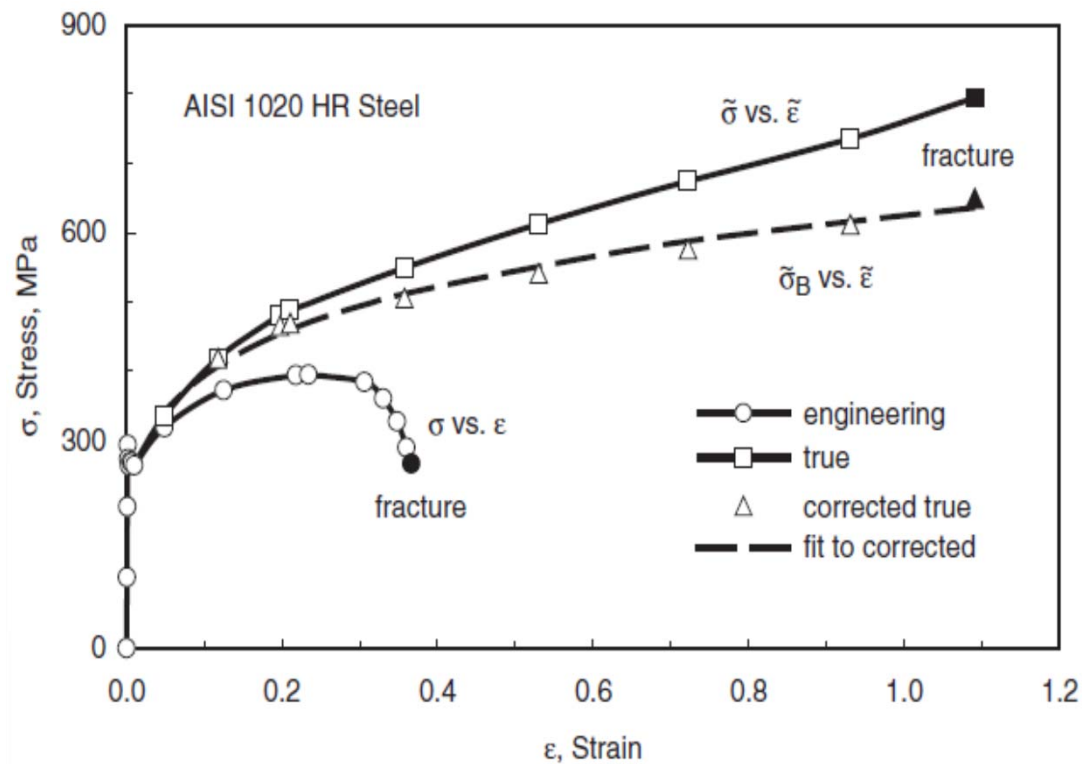
$$\sigma = S(1 + e)$$

$$\epsilon = \ln(1 + e)$$

- Assumption of constant volume
- Valid up to the onset of necking

# Monotonic Stress-Strain Behavior

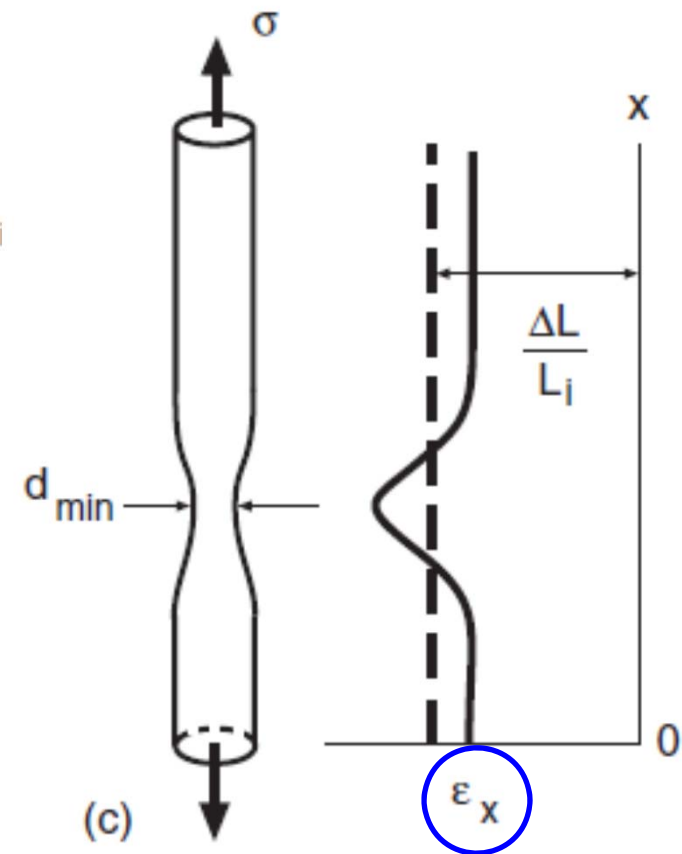
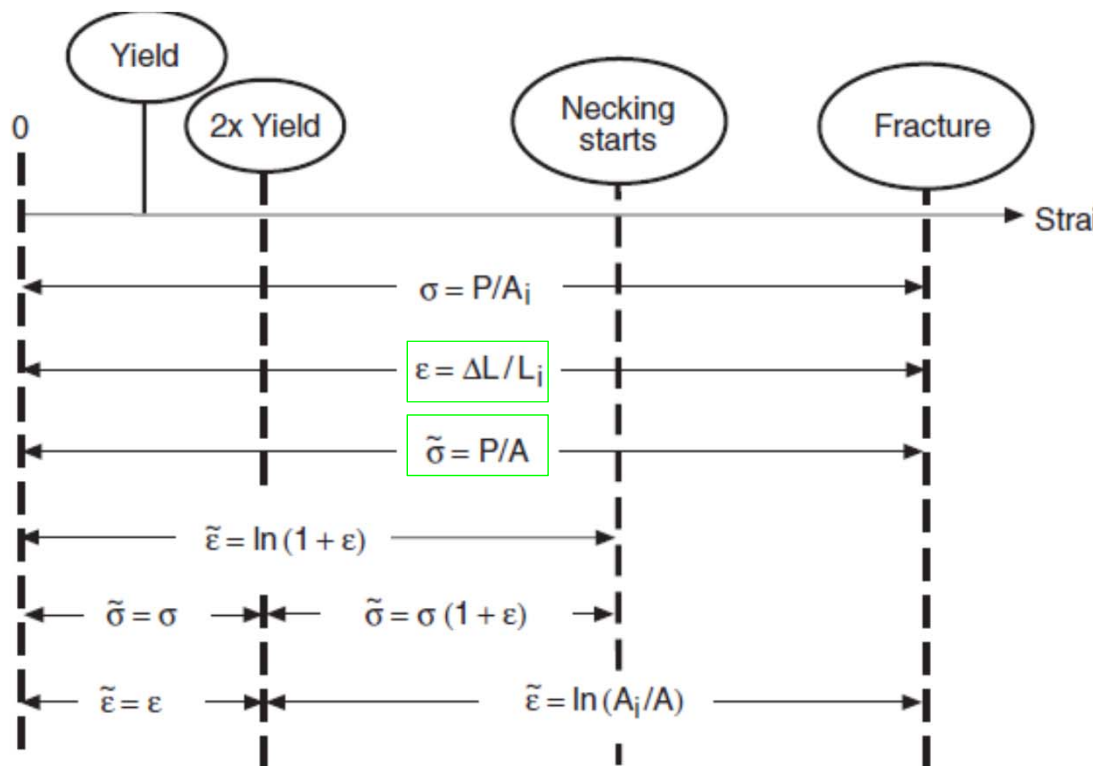
## Empescoamento (Necking)



Decrease in force is a consequence of cross-sectional area rapidly decrease

# Monotonic Stress-Strain Behavior

## Limitations on True Stress – Strain Equations

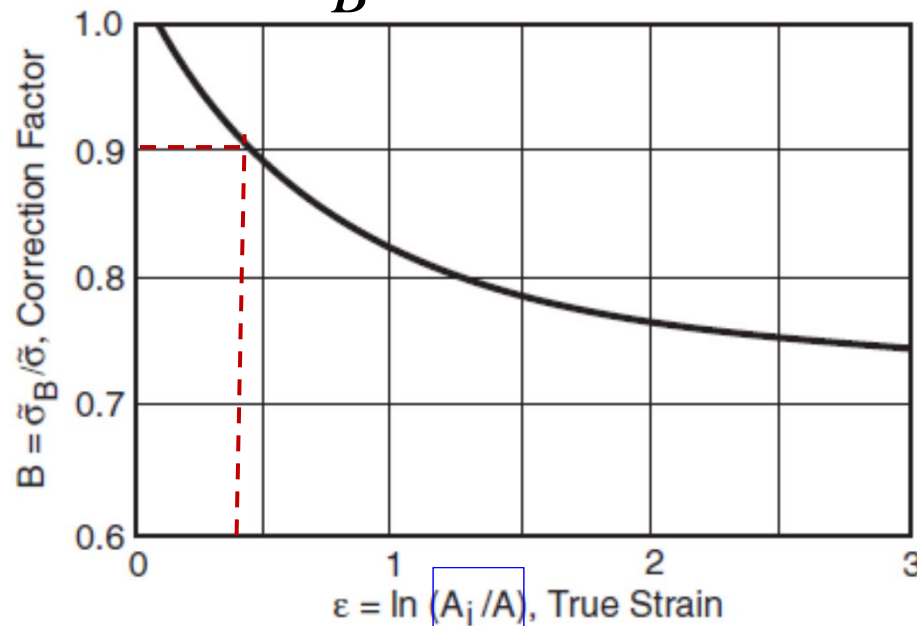




# Monotonic Stress-Strain Behavior

## Bridman Correction (Empirical)

Steels :  $\tilde{\sigma}_B = B\tilde{\sigma}$

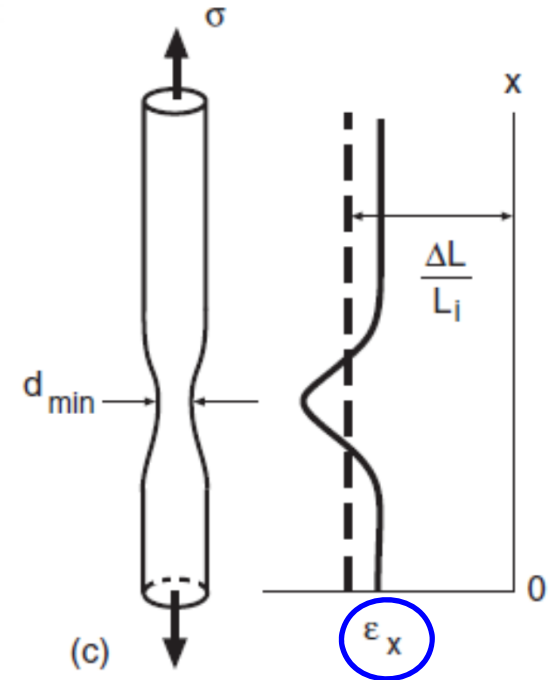


$$\tilde{\epsilon} = \ln \frac{L}{L_i}$$

$$\tilde{\epsilon} \approx 0.44$$

$$\downarrow$$

$$\frac{L}{L_i} = 1.55$$

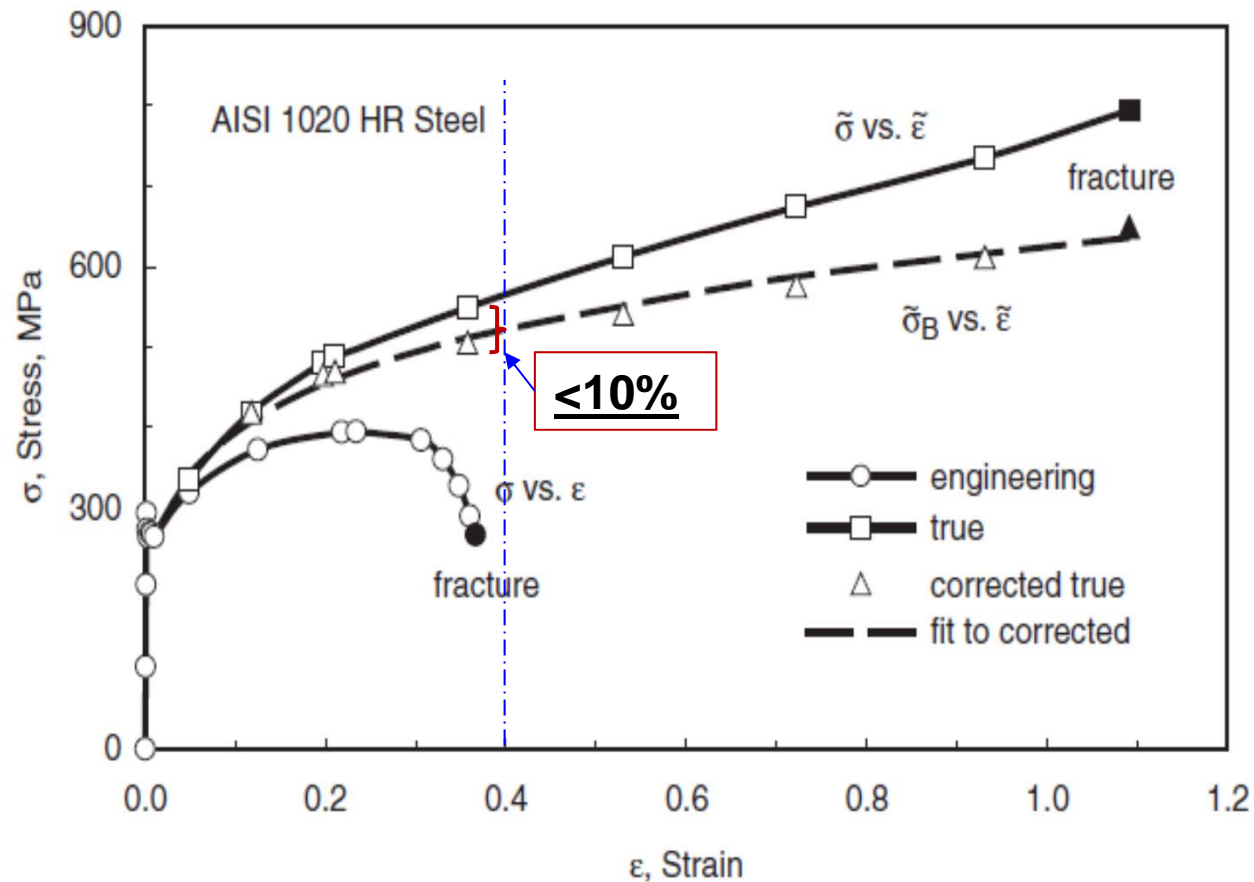


$$B = 0.0684(\log \tilde{\epsilon})^3 + 0.0461(\log \tilde{\epsilon})^2 - 0.205(\log \tilde{\epsilon}) + 0.825$$

The correction is not need for true strain  $(A_i/A) < 0.12$

# Monotonic Stress-Strain Behavior

## Limitations on True Stress – Strain Equations

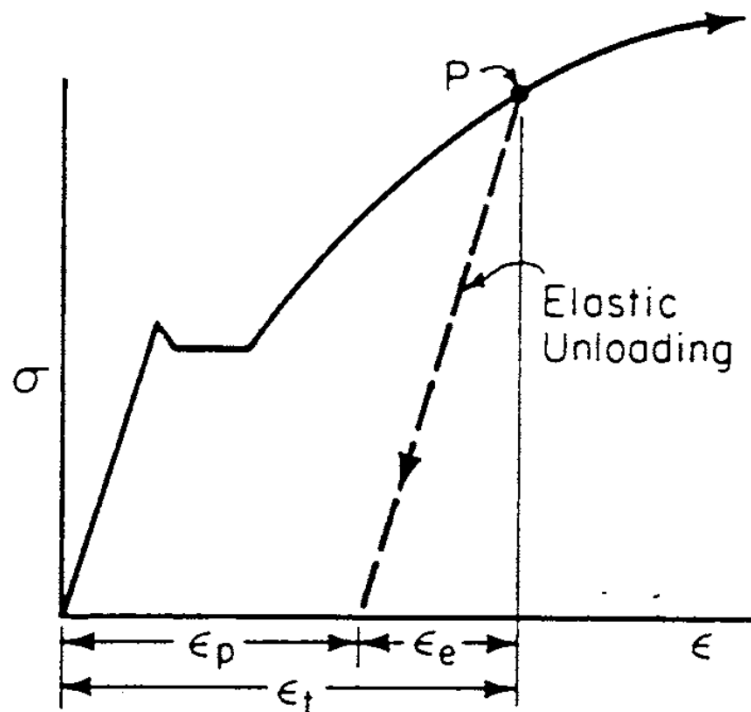


$$\tilde{\sigma}_B = B\tilde{\sigma}$$

$$B = 0.0684(\log \tilde{\epsilon})^3 + 0.0461(\log \tilde{\epsilon})^2 - 0.205(\log \tilde{\epsilon}) + 0.825$$

# Stress-Strain Relationships

Total deformation can be separated in two parts:



$$\epsilon = \epsilon_e + \epsilon_p$$

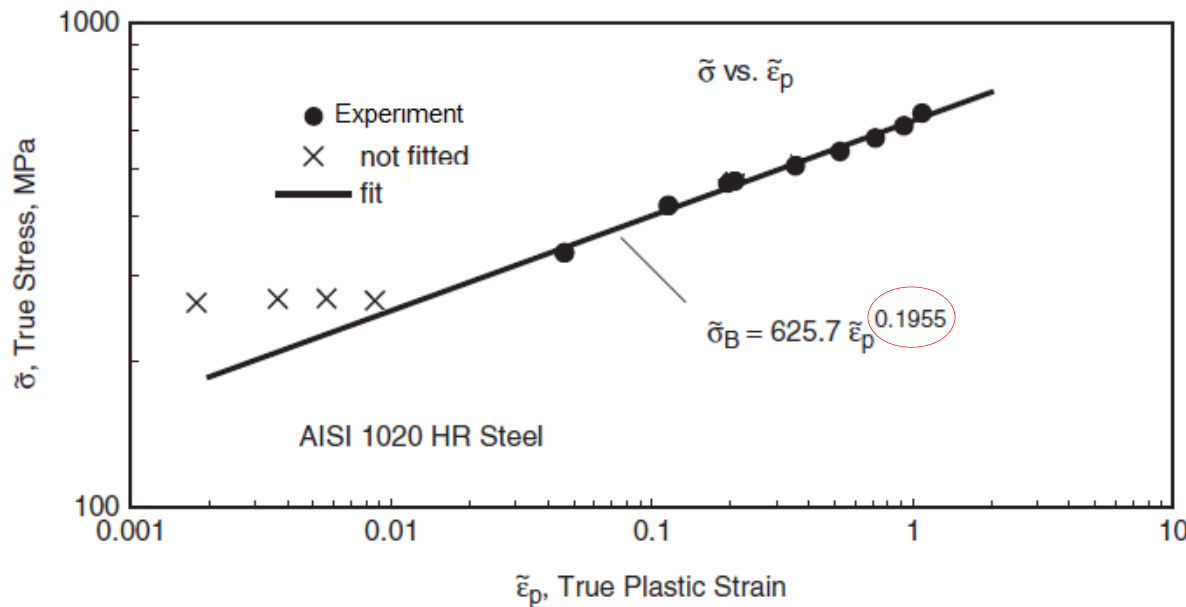
Recovered upon unloading

Can not be recovered upon unloading



# Stress-Strain Relationships

For metals a log-log plot of experimental true stress versus true plastic strain often fits a straight line.



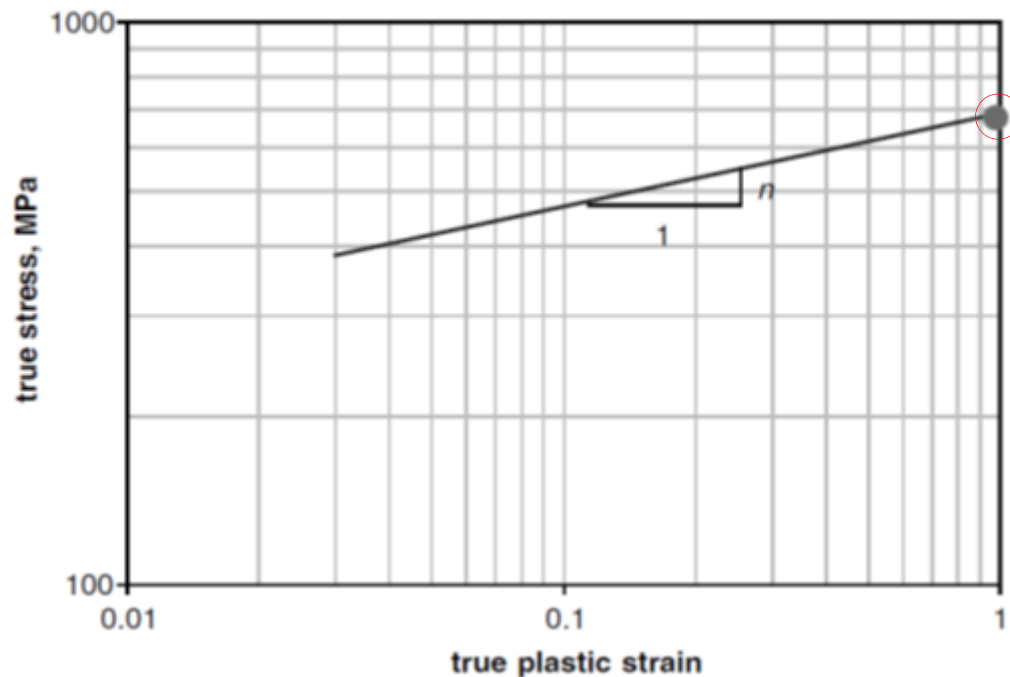
$$\tilde{\sigma} = H \tilde{\epsilon}_p^n$$

$n$ : strain hardening exponent ( $0 < n < 1$ )

$H$ : strength coefficient (MPa)

# Stress-Strain Relationships

For metals a log-log plot of experimental true stress versus true plastic strain often fits a straight line.



$$\tilde{\sigma} = H \tilde{\varepsilon}_p^n$$

$$\tilde{\varepsilon}_p = 1$$



$$\tilde{\sigma} = H$$

**Table 4.6 True Stress–Strain Tensile Properties for Some Engineering Metals, and Also Hardness**

Material	True Fracture		Strength Coefficient $H$	Strain Hardening Exponent $n$	Brinell Hardness <sup>1</sup> $HB$
	Strength $\tilde{\sigma}_{fB}$	Strain $\tilde{\epsilon}_f$			
	MPa (ksi)		MPa (ksi)		
Ductile cast iron A536 (65-45-12)	524 (76)	0.222	456 (66.1)	0.0455	167
AISI 1020 steel as rolled	713 (103)	0.96	737 (107)	0.19	107
ASTM A514, T1 structural steel	1213 (176)	1.08	1103 (160)	0.088	256
AISI 4142 steel as quenched	2580 (375)	0.060	—	0.136	670
AISI 4142 steel 205°C temper	2650 (385)	0.310	—	0.091	560
AISI 4142 steel 370°C temper	1998 (290)	0.540	—	0.043	450
AISI 4142 steel 450°C temper	1826 (265)	0.660	—	0.051	380
18 Ni maraging steel (250)	2136 (310)	0.82	—	0.02	460
SAE 308 cast aluminum	232 (33.6)	0.009	567 (82.2)	0.196	80
2024-T4 aluminum	631 (91.5)	0.43	806 (117)	0.20	120
7075-T6 aluminum	744 (108)	0.41	827 (120)	0.113	150
AZ91C-T6 cast magnesium	137 (20)	0.004	653 (94.7)	0.282	61

Note: <sup>1</sup>Load 3000 kg for irons and steels, 500 kg otherwise; typical values from [Boyer 85] are listed in some cases.

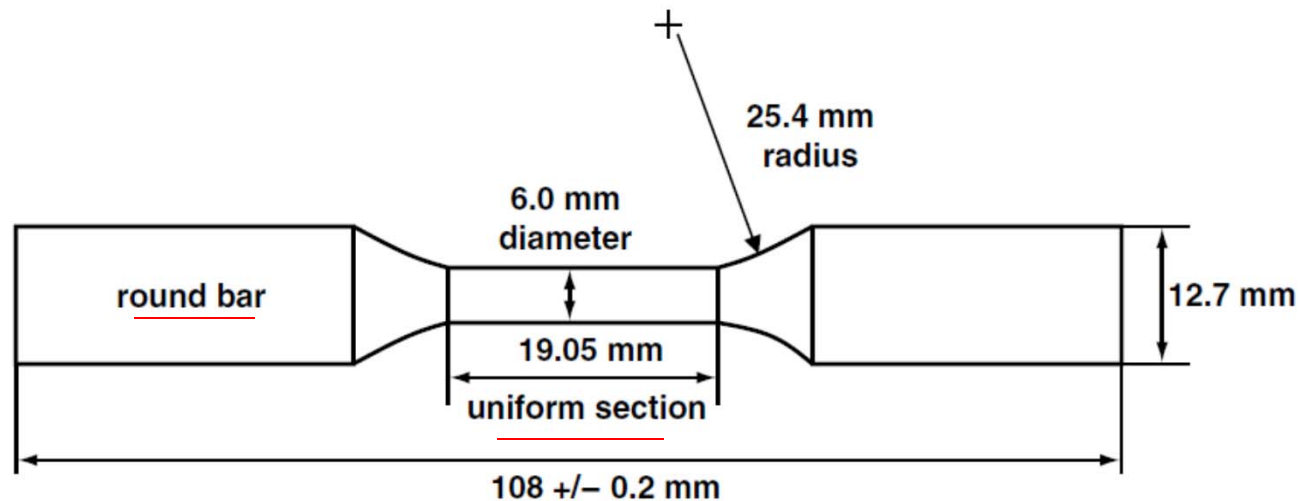
Sources: Data in [Conle 84] and [SAE 89].



# Cyclic Stress-Strain Behavior

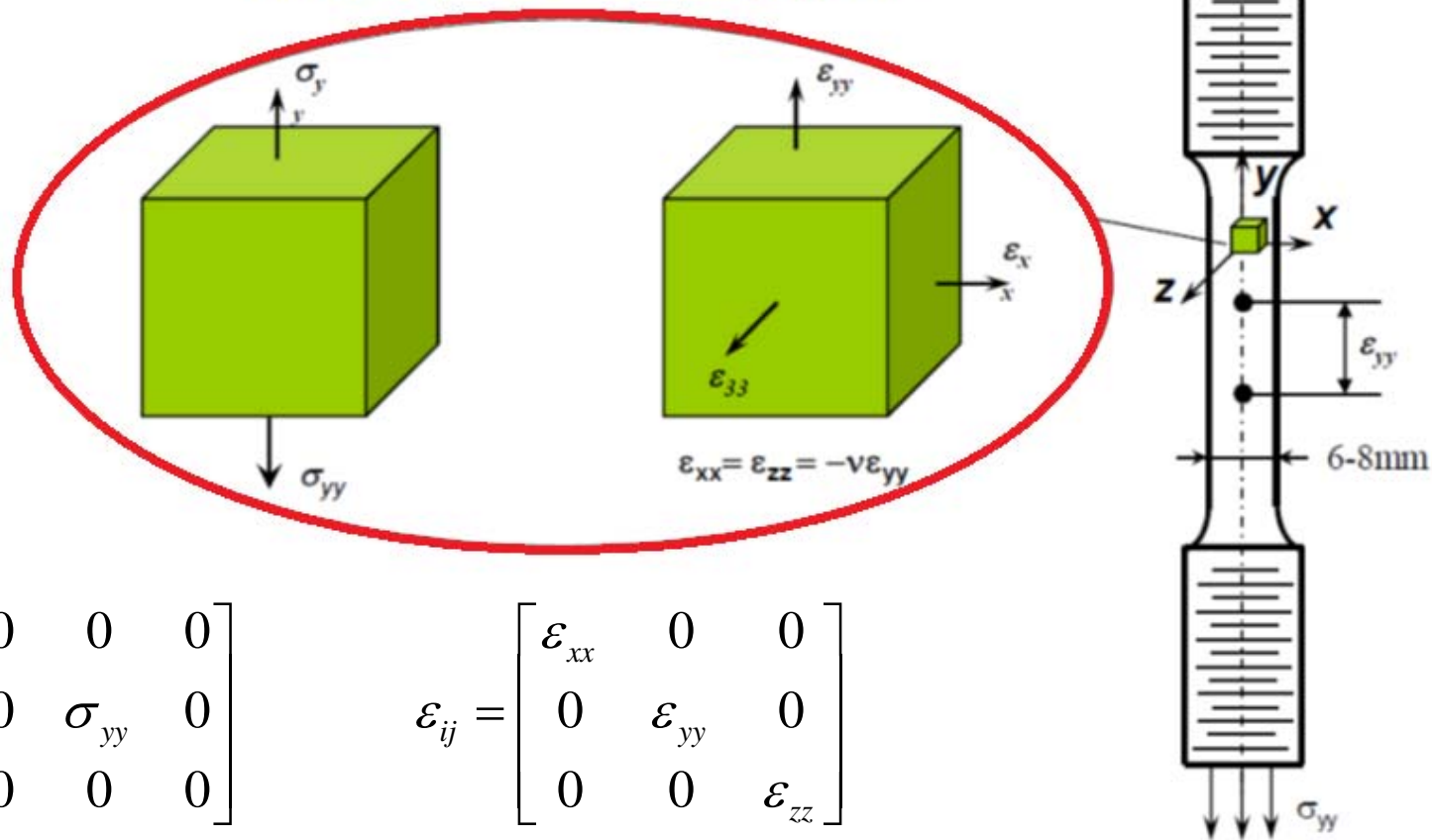
ASTM E466

- Cylindrical specimens  $\phi \sim 6 \text{ mm} - 12 \text{ mm}$



# Smooth Specimen for $\sigma$ - $\varepsilon$ determination under monotonic and cyclic loading

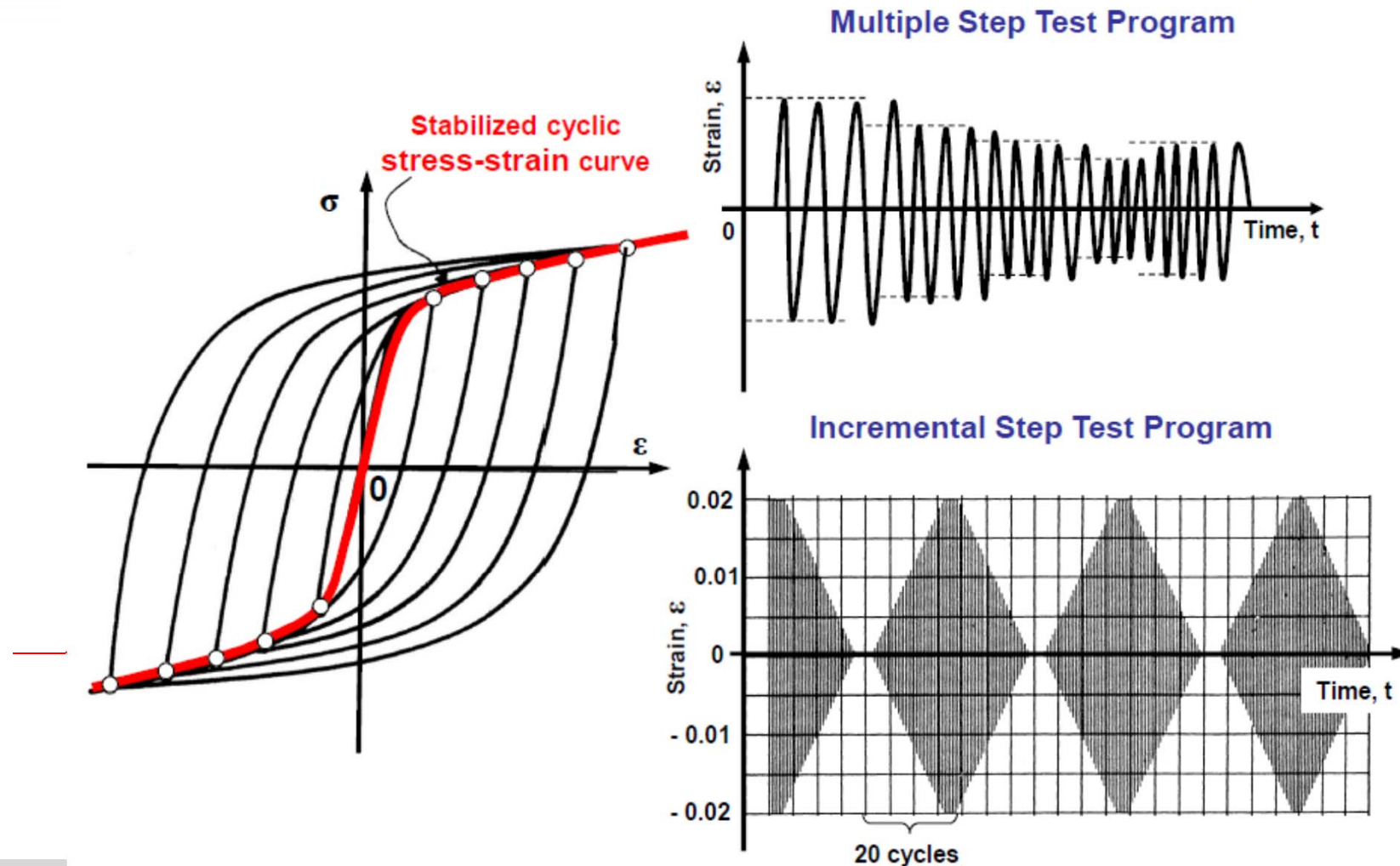
Stress and strain state in specimens used for determination of material properties



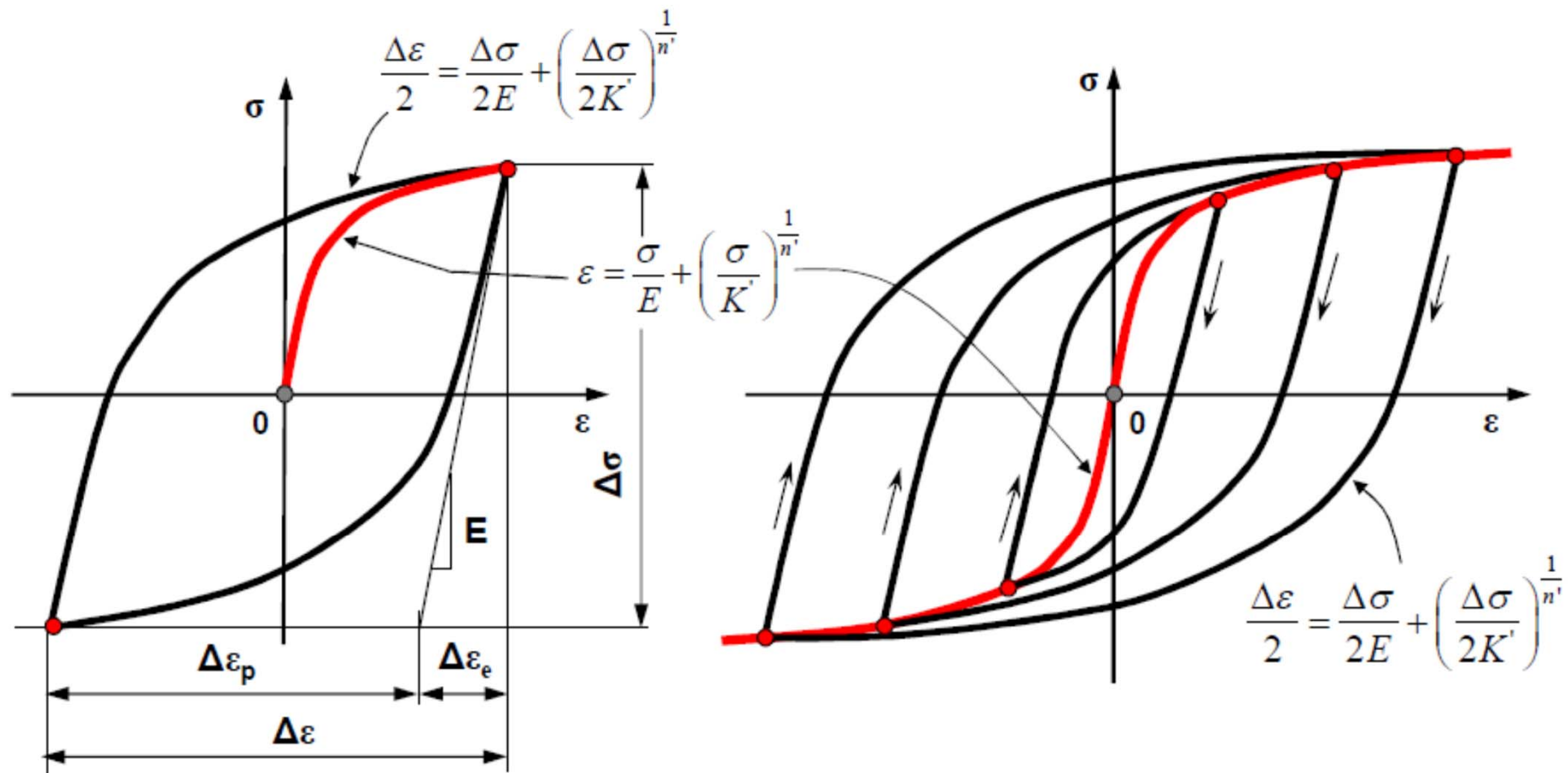
$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

# Cyclic Stress-Strain Behavior

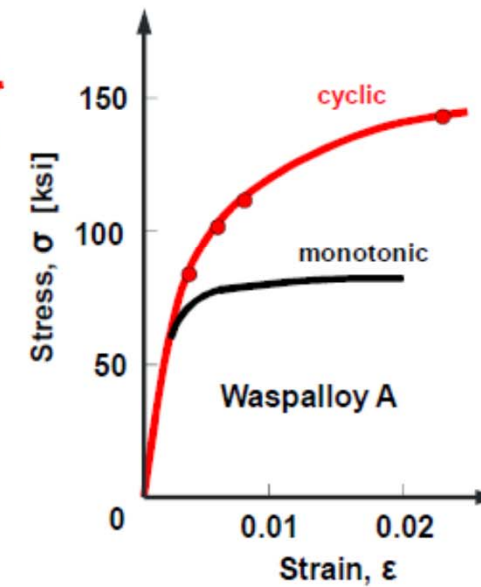
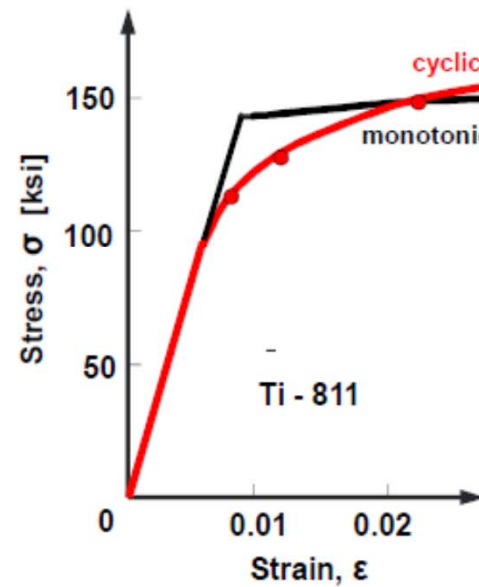
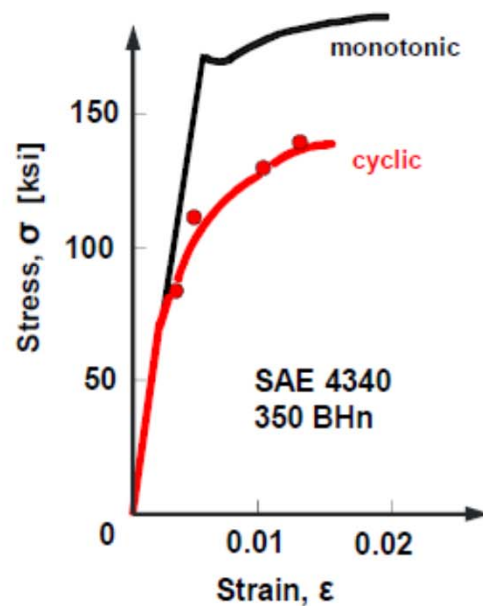
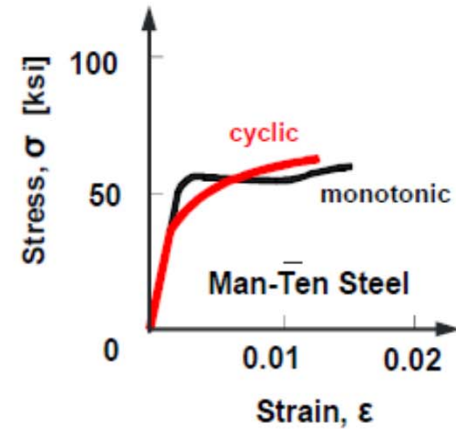
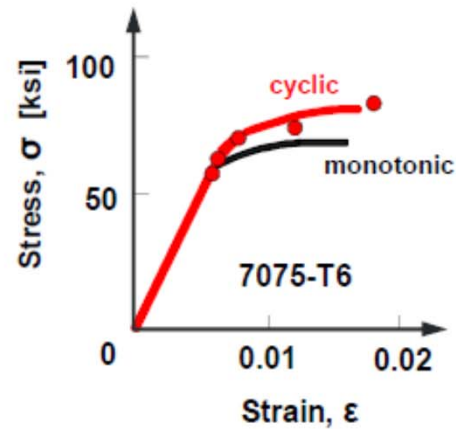
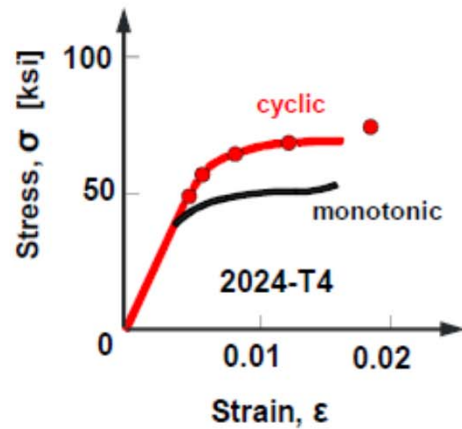


# Cyclic Stress-Strain Behavior

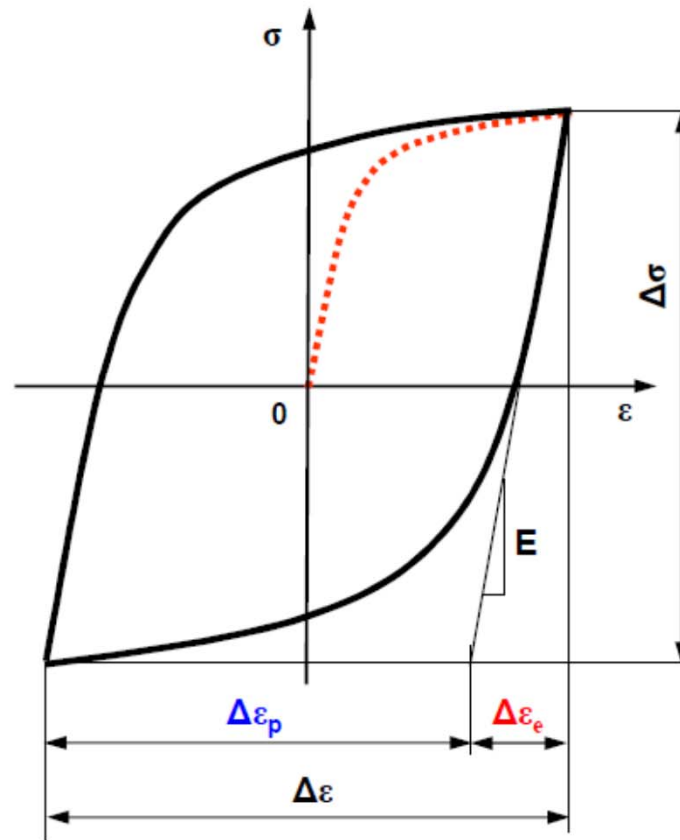
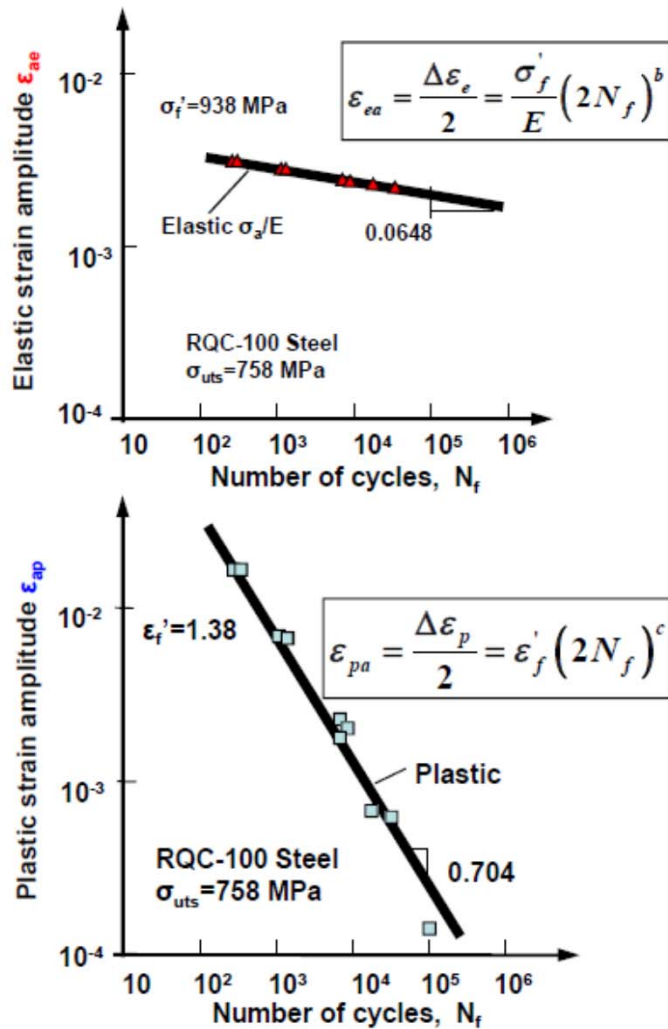




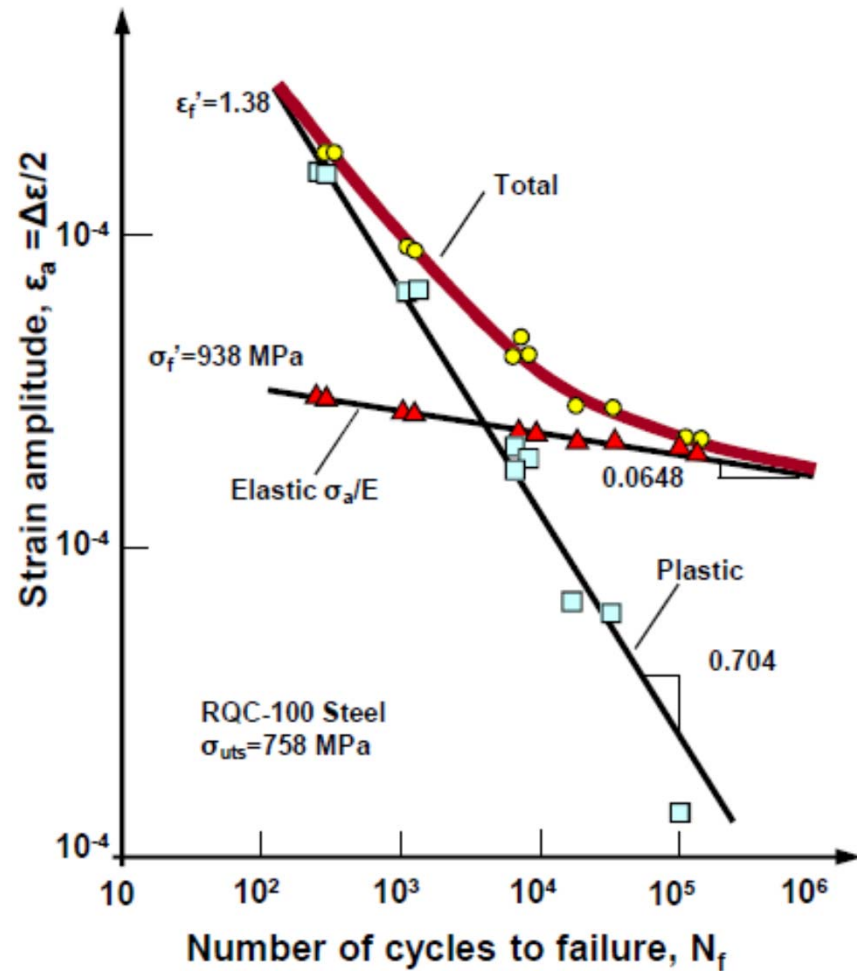
# Cyclic vs. Monotonic



# Fatigue Strain-Life Curve



# Fatigue Strain-Life Curve



- Coffin & Manson Equation

$$\frac{\Delta\epsilon}{2} = \epsilon_a = \frac{\sigma'_f}{E} (2N_f)^b + \epsilon'_f (2N_f)^c$$

# Fatigue Strain-Life Properties

Table 14.1 Cyclic Stress–Strain and Strain–Life Constants for Selected Engineering Metals.<sup>1</sup>

Material	Source	Tensile Properties				Cyclic $\sigma$ - $\epsilon$ Curve			Strain–Life Curve			
		$\sigma_o$	$\sigma_u$	$\bar{\sigma}_{fB}$	% RA	$E$	$H'$	$n'$	$\sigma'_f$	$b$	$\epsilon'_f$	$c$
<i>(a) Steels</i>												
SAE 1015 (normalized)	(8)	228 (33.0)	415 (60.2)	726 (105)	68	207,000 (30,000)	1349 (196)	0.282	1020 (148)	-0.138	0.439	-0.513
Man-Ten <sup>2</sup> (hot rolled)	(7)	322 (46.7)	557 (80.8)	990 (144)	67	203,000 (29,500)	1096 (159)	0.187	1089 (158)	-0.115	0.912	-0.606
RQC-100 (roller Q & T)	(2)	683 (99.0)	758 (110)	1186 (172)	64	200,000 (29,000)	903 (131)	0.0905	938 (136)	-0.0648	1.38	-0.704
SAE 1045 (HR & norm.)	(6)	382 (55.4)	621 (90.1)	985 (143)	51	202,000 (29,400)	1258 (182)	0.208	948 (137)	-0.092	0.260	-0.445
SAE 4142 (As Q, 670 HB)	(1)	1619 (235)	2450 (355)	2580 (375)	6	200,000 (29,000)	2810 (407)	0.040	2550 (370)	-0.0778	0.0032	-0.436
SAE 4142 (Q & T, 560 HB)	(1)	1688 (245)	2240 (325)	2650 (385)	27	207,000 (30,000)	4140 (600)	0.126	3410 (494)	-0.121	0.0732	-0.805
SAE 4142 (Q & T, 450 HB)	(1)	1584 (230)	1757 (255)	1998 (290)	42	207,000 (30,000)	2080 (302)	0.093	1937 (281)	-0.0762	0.706	-0.869
SAE 4142 (Q & T, 380 HB)	(1)	1378 (200)	1413 (205)	1826 (265)	48	207,000 (30,000)	2210 (321)	0.133	2140 (311)	-0.0944	0.637	-0.761
AISI 4340 <sup>2</sup> (Aircraft Qual.)	(3)	1103 (160)	1172 (170)	1634 (237)	56	207,000 (30,000)	1655 (240)	0.131	1758 (255)	-0.0977	2.12	-0.774
AISI 4340 (409 HB)	(1)	1371 (199)	1468 (213)	1557 (226)	38	200,000 (29,000)	1910 (277)	0.123	1879 (273)	-0.0859	0.640	-0.636
Ausformed H-11 (660 HB)	(1)	2030 (295)	2580 (375)	3170 (460)	33	207,000 (30,000)	3475 (504)	0.059	3810 (553)	-0.0928	0.0743	-0.7144



# Fatigue Strain-Life Properties

**Table 14.1** Cyclic Stress–Strain and Strain–Life Constants for Selected Engineering Metals.<sup>1</sup>

Material	Source	Tensile Properties				Cyclic $\sigma$ - $\epsilon$ Curve			Strain–Life Curve			
		$\sigma_o$	$\sigma_u$	$\bar{\sigma}_{fB}$	% RA	$E$	$H'$	$n'$	$\sigma'_f$	$b$	$\epsilon'_f$	$c$
AISI 4340 <sup>2</sup> (Aircraft Qual.)	(3)	1103 (160)	1172 (170)	1634 (237)	56	207,000 (30,000)	1655 (240)	0.131	1758 (255)	−0.0977	2.12	−0.774
AISI 4340 (409 HB)	(1)	1371 (199)	1468 (213)	1557 (226)	38	200,000 (29,000)	1910 (277)	0.123	1879 (273)	−0.0859	0.640	−0.636
Ausformed H-11 (660 HB)	(1)	2030 (295)	2580 (375)	3170 (460)	33	207,000 (30,000)	3475 (504)	0.059	3810 (553)	−0.0928	0.0743	−0.7144
<i>(b) Other Metals</i>												
2024-T351 Al	(1)	379 (55.0)	469 (68.0)	558 (81.0)	25	73,100 (10,600)	662 (96.0)	0.070	927 (134)	−0.113	0.409	−0.713
2024-T4 Al <sup>3</sup> (Prestrained)	(4)	303 (44.0)	476 (69.0)	631 (91.5)	35	73,100 (10,600)	738 (107)	0.080	1294 (188)	−0.142	0.327	−0.645
7075-T6 Al	(5)	469 (68.0)	578 (84)	744 (108)	33	71,000 (10,300)	977 (142)	0.106	1466 (213)	−0.143	0.262	−0.619
Ti-6Al-4V (soln. tr. & age)	(1)	1185 (172)	1233 (179)	1717 (249)	41	117,000 (17,000)	1772 (257)	0.106	2030 (295)	−0.104	0.841	−0.688
Inconel X (Ni base, annl.)	(1)	703 (102)	1213 (176)	1309 (190)	20	214,000 (31,000)	1855 (269)	0.120	2255 (327)	−0.117	1.16	−0.749

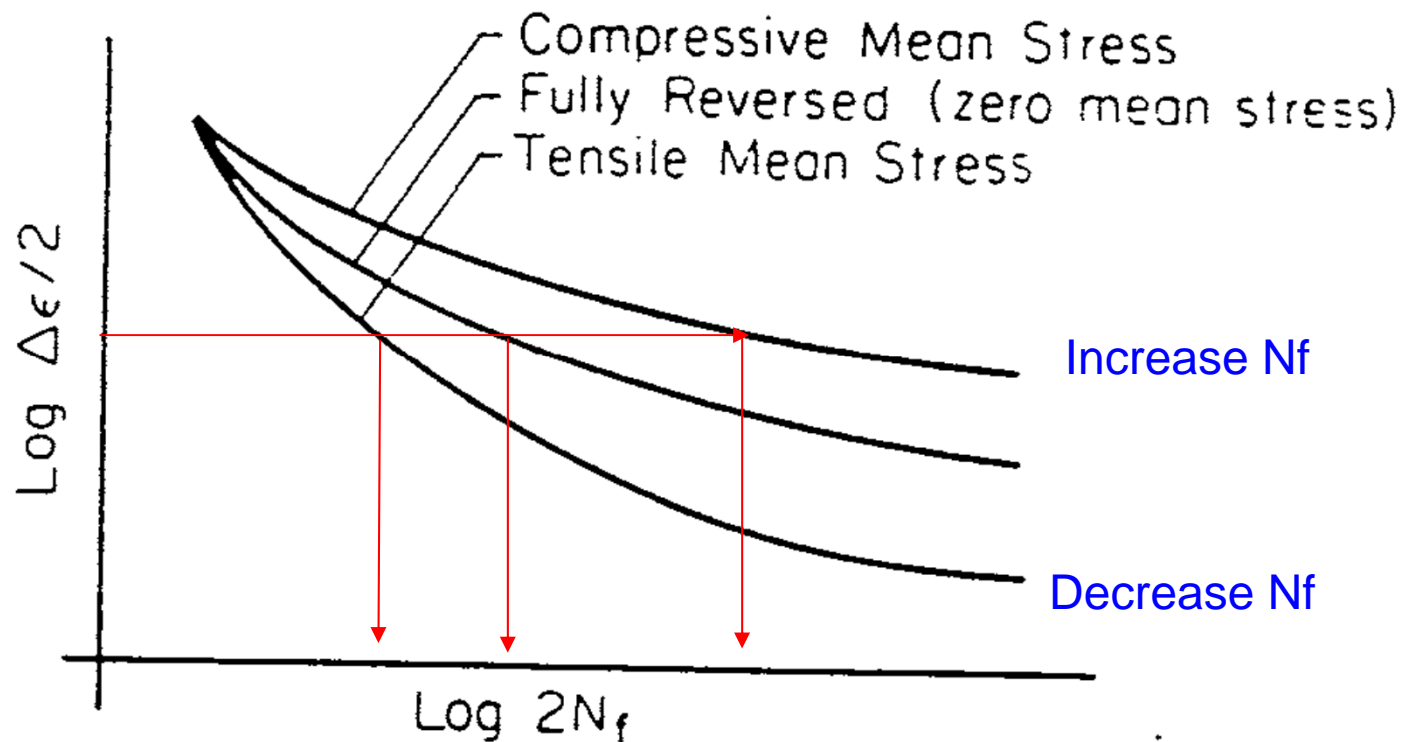
Notes: <sup>1</sup>The tabulated values either have units of MPa (ksi), or they are dimensionless. <sup>2</sup>Test specimens prestrained, except at short lives, also periodically overstrained at long lives. <sup>3</sup>For nonprestrained tests, use same constants, except  $\sigma'_f = 900(131)$  and  $b = -0.102$ .

Sources: Data in (1) [Conle 84]; (2) author's data on the ASTM Committee E9 material; (3) [Dowling 73]; (4) [Dowling 89] and [Topper 70]; (5) [Endo 69] and [Raske 72]; (6) [Leese 85]; (7) [Wetzel 77] pp. 41 and 66; (8) [Keshavan 67] and [Smith 70].

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# Mean Stress Effect



Mean stress effects are predominantly at longer lives

# Stress Relaxation

At high stress amplitude

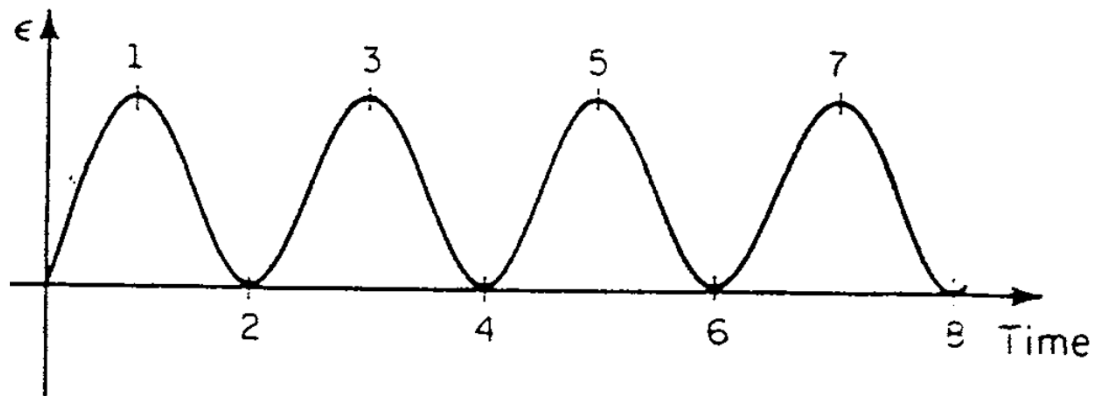
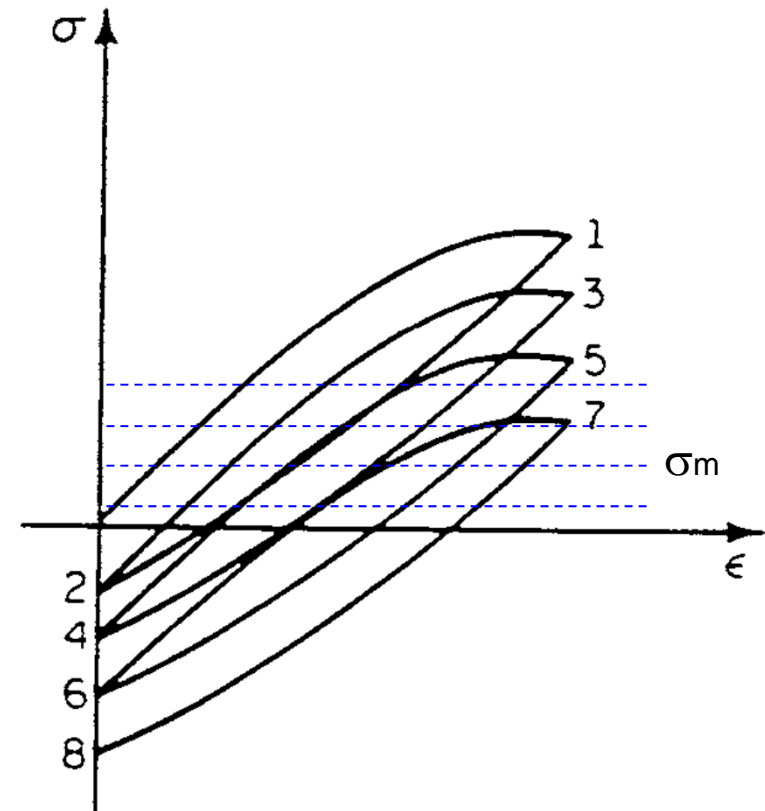


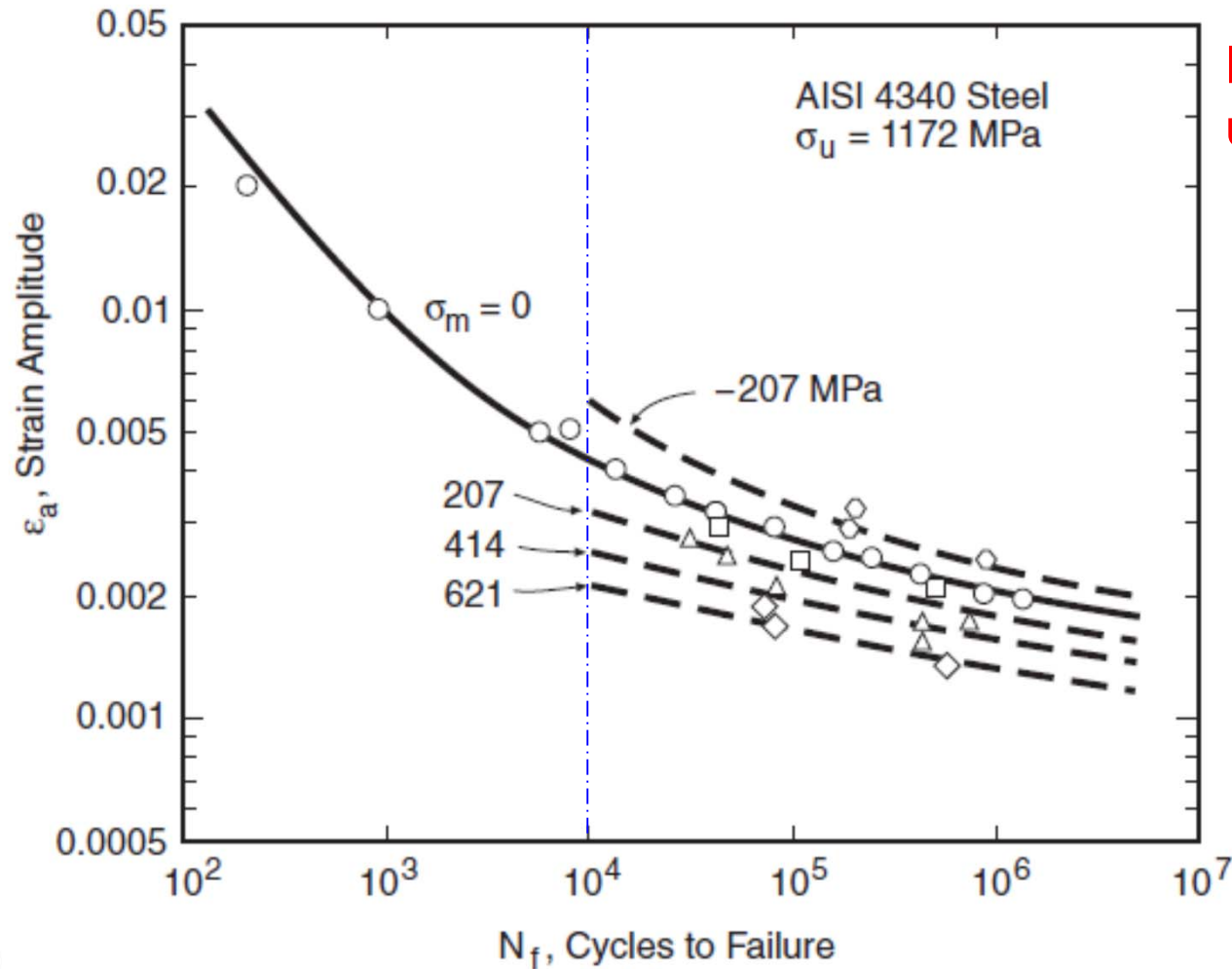
Figure 2.22 Mean stress relaxation.



Mean stress effects can be neglected at short lives



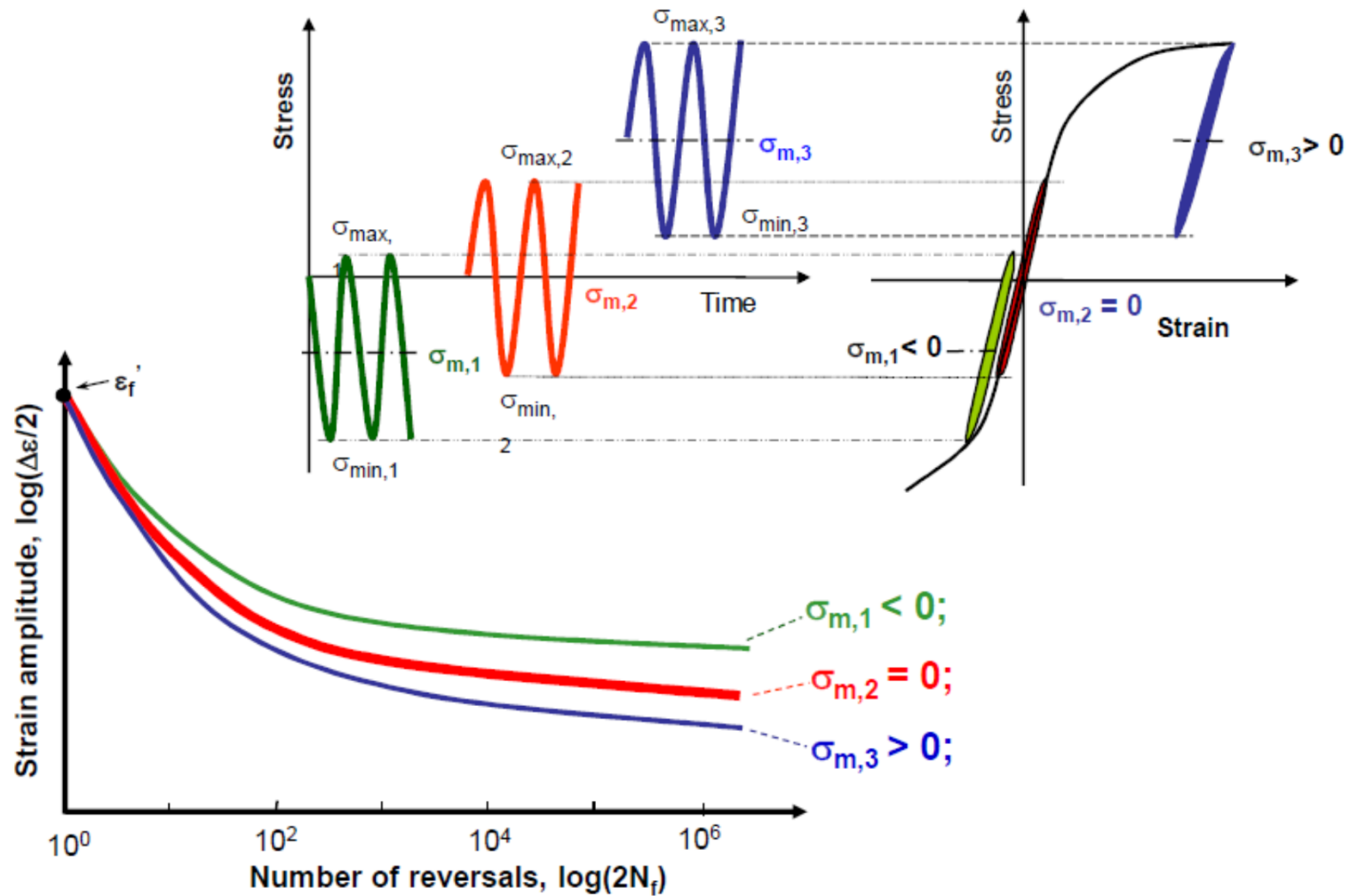
# Mean Stress Effect



Dashed curves obtained using Morrow's equation

[Dowling 73].)

# Mean Stress Effect

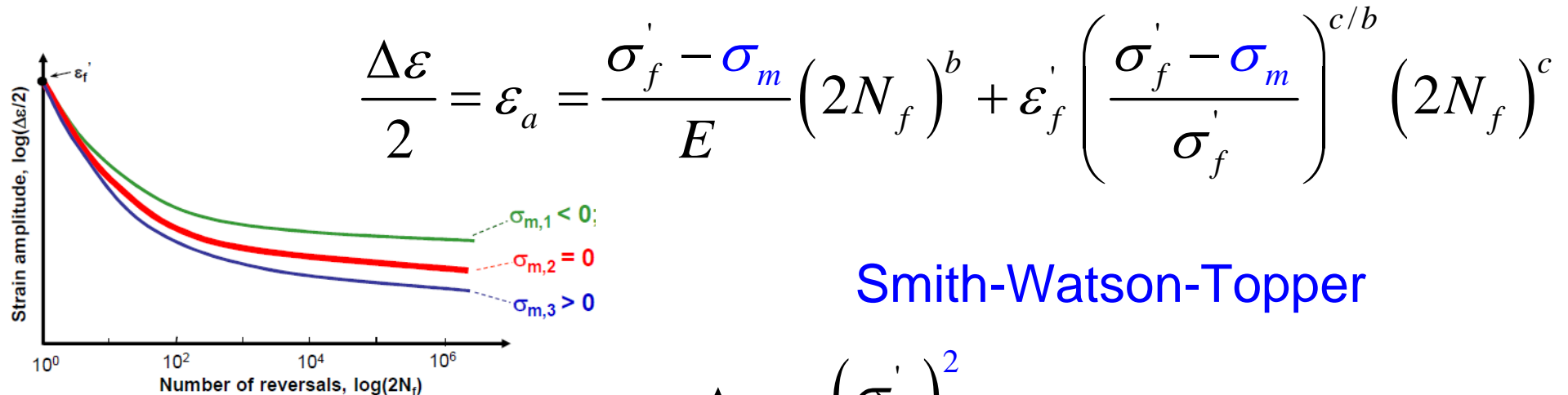


# Mean Stress Effect

## Morrow

$$\frac{\Delta \varepsilon}{2} = \varepsilon_a = \frac{\sigma'_f - \sigma_m}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$

## Manson-Halford

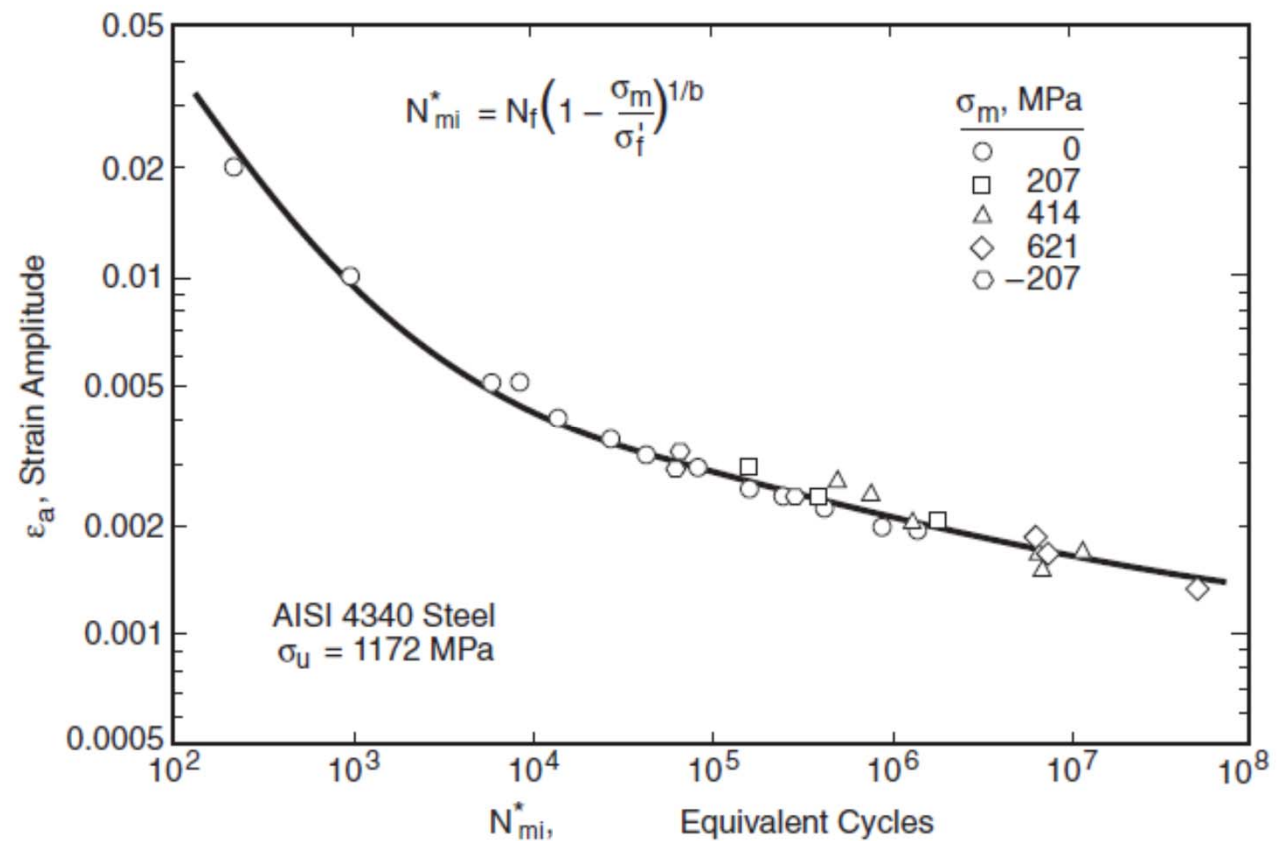


## Smith-Watson-Topper

$$\sigma_{\max} \frac{\Delta \varepsilon}{2} = \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \varepsilon'_f (2N_f)^{c+b}$$

# Mean Stress Effect

$$\frac{\Delta \varepsilon}{2} = \varepsilon_a = \frac{\sigma_f' - \sigma_m}{E} (2N_f)^b + \varepsilon_f' \left( \frac{\sigma_f' - \sigma_m}{\sigma_f'} \right)^{c/b} (2N_f)^c$$

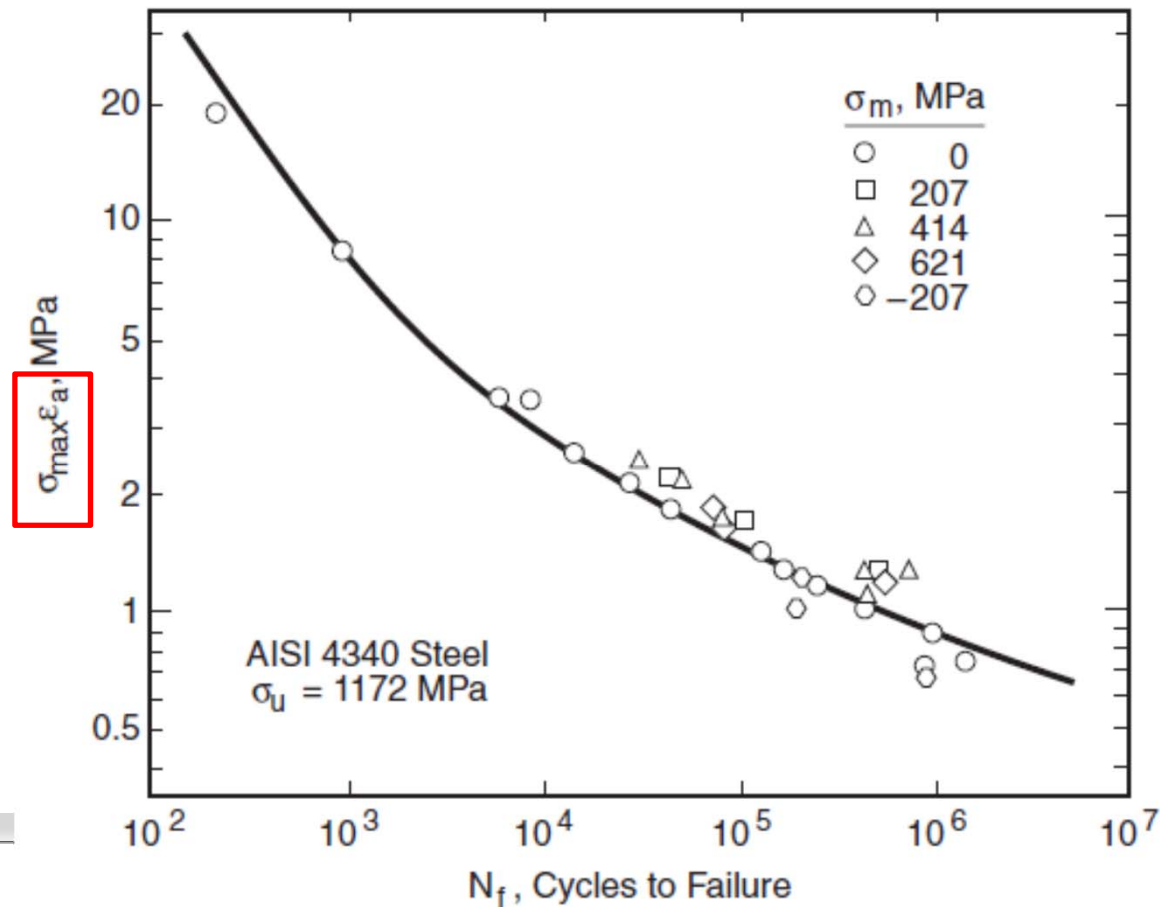


[Dowling 73].)



# Mean Stress Effect

$$\sigma_{\max} \frac{\Delta \varepsilon}{2} = \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \varepsilon'_f (2N_f)^{c+b}$$



[Dowling 73].)

# Deformation Plasticity Theory

## Stress-Strain relations

$$\varepsilon_x = \varepsilon_{ex} + \varepsilon_{px},$$

$$\varepsilon_y = \varepsilon_{ey} + \varepsilon_{py},$$

$$\varepsilon_z = \varepsilon_{ez} + \varepsilon_{pz}$$

$$\gamma_{xy} = \gamma_{exy} + \gamma_{pxy},$$

$$\gamma_{yz} = \gamma_{eyz} + \gamma_{pyz},$$

$$\gamma_{zx} = \gamma_{ezx} + \gamma_{pzx}$$

## Elastic strains

$$\varepsilon_{ex} = \frac{1}{E} [\sigma_x - \nu (\sigma_y + \sigma_z)]$$

$$\varepsilon_{ey} = \frac{1}{E} [\sigma_y - \nu (\sigma_x + \sigma_z)]$$

$$\varepsilon_{ez} = \frac{1}{E} [\sigma_z - \nu (\sigma_x + \sigma_y)]$$

$$\gamma_{exy} = \frac{\tau_{xy}}{G}, \quad \gamma_{eyz} = \frac{\tau_{yz}}{G}, \quad \gamma_{ezx} = \frac{\tau_{zx}}{G}$$

## Plastic Strains

$$\varepsilon_{px} = \frac{1}{E_p} [\sigma_x - 0.5(\sigma_y + \sigma_z)]$$

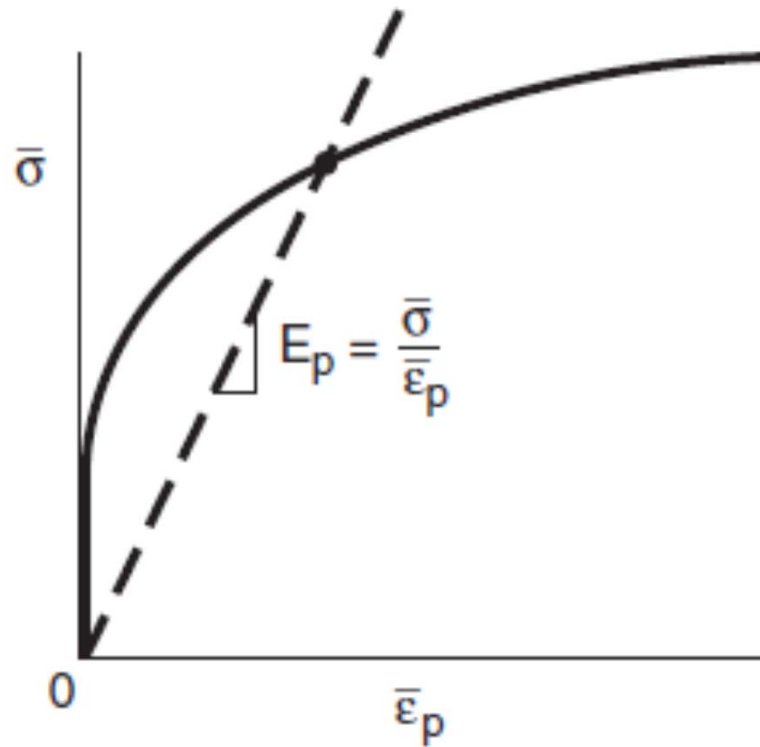
$$\varepsilon_{py} = \frac{1}{E_p} [\sigma_y - 0.5(\sigma_x + \sigma_z)]$$

$$\varepsilon_{pz} = \frac{1}{E_p} [\sigma_z - 0.5(\sigma_x + \sigma_y)]$$

$$\gamma_{pxy} = \frac{3}{E_p} \tau_{xy}, \quad \gamma_{pyz} = \frac{3}{E_p} \tau_{yz}, \quad \gamma_{pzx} = \frac{3}{E_p} \tau_{zx}$$

# Deformation Plasticity Theory

## Plastic Modulus



Effective total strain

$$\bar{\epsilon} = \frac{\bar{\sigma}}{E} + \bar{\epsilon}_p$$

# Deformation Plasticity Theory

Effective Stress

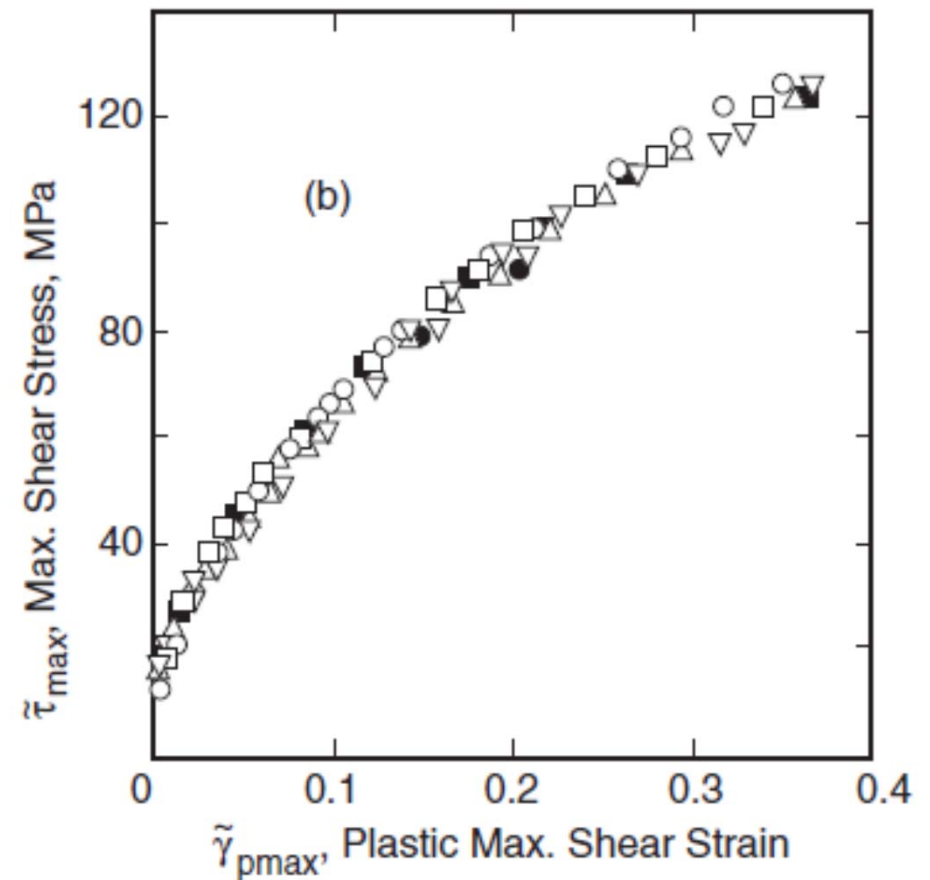
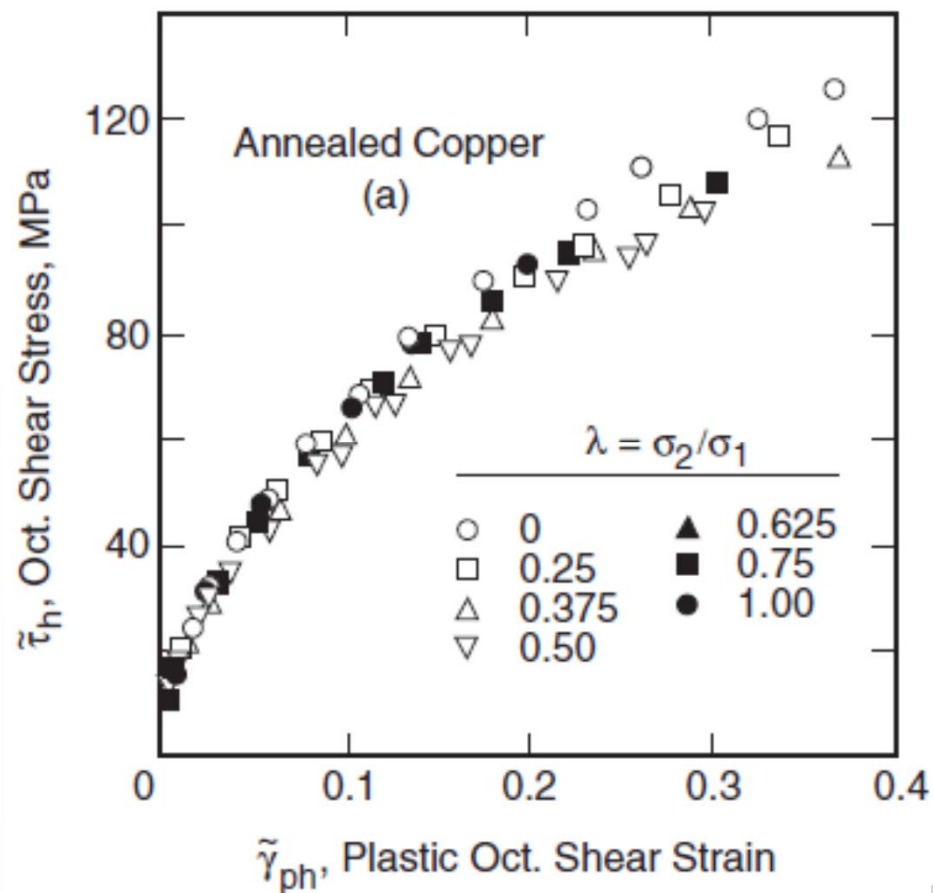
$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

Effective Plastic Strain

$$\bar{\varepsilon}_p = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_{p1} - \varepsilon_{p2})^2 + (\varepsilon_{p2} - \varepsilon_{p3})^2 + (\varepsilon_{p3} - \varepsilon_{p1})^2}$$



# Deformation Plasticity Theory



(Adapted from [Davis .

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# Modifying Factors

- Hostile chemical environments
- Temperature
- At short lives e-N curves are not highly sensitive to:
  - Surface finish
  - Residual stress
  - Small crack growth is important
- Size Effects ( experimental data limited)

