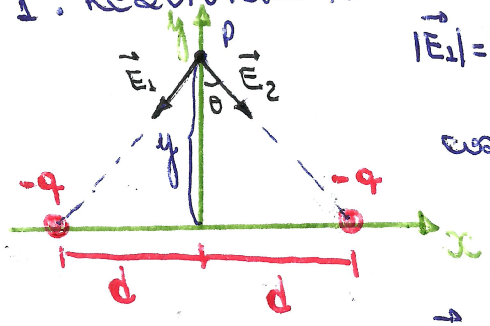
Prova 1 - Física III - Gabarito

Questão 1: Resolvida na aula de 11/03/2019



$$|\vec{E}_1| = |\vec{E}_2| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(y^2+d^2)}$$

$$\cos\theta = \frac{y}{(y^2+d^2)^{1/2}}$$

a) $\vec{E}_1 = -|\vec{E}_1|\sin\theta\,\hat{i} - |\vec{E}_1|\cos\theta\,\hat{j}$; $\vec{E}_2 = |\vec{E}_2|\sin\theta\,\hat{i} - |\vec{E}_2|\cos\theta\,\hat{j}$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = -(|\vec{E}_1| + |\vec{E}_2|)\cos\theta\,\hat{j} \Rightarrow \boxed{\vec{E} = -\frac{2q}{4\pi\varepsilon_0}\cdot\frac{y}{(y^2+d^2)^{3/2}}\hat{j}}$$

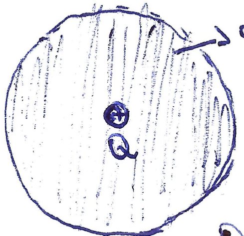
b) $$|\vec{E}| = E = \frac{2q}{4\pi\varepsilon_0}\cdot\frac{y}{(y^2+d^2)^{3/2}} \Rightarrow \boxed{E_{máx} = \frac{q}{3\sqrt{3}\pi\varepsilon_0 d^2}}$$

$E_{máx}$ qdo $\frac{dE}{dy} = 0 \Rightarrow \frac{2q}{4\pi\varepsilon_0}\frac{d}{dy}\left(\frac{y}{(y^2+d^2)^{3/2}}\right) = 0$

$$\frac{d}{dy}\left(\frac{y}{(y^2+d^2)^{3/2}}\right) = \frac{-2y^2+d^2}{(y^2+d^2)^{5/2}} = 0 \Rightarrow \boxed{y = \pm\frac{d}{\sqrt{2}}}$$

Questão 2: Prob. da Lista de exercícios (Prob. 43 Tipler 6ª Ed)
\+
carga no centro

a)



carga espalhada com $\rho = -\frac{\alpha}{r}$ → sim. esférica

Esfera tem carga nula:

Logo: $Q + \int_0^R \rho\, dv = 0 \Rightarrow Q + \int_0^R \left(-\frac{\alpha}{r}\right) 4\pi r^2 dr$

$$Q - 4\pi\alpha\int_0^R \frac{r^2}{r}dr = 0 \Rightarrow Q - 4\pi\alpha\frac{R^2}{2} = 0$$

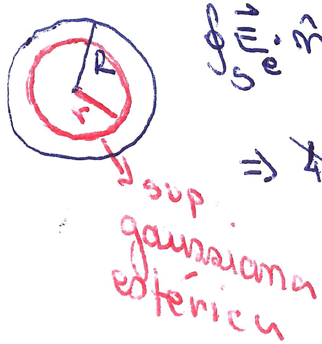
$$\boxed{\alpha = Q/2\pi R^2}$$

b) Campo Elétrico total é composto pelo campo da carga espalhada, $\vec{E}_e$, mais o campo da carga Q, $\vec{E}_Q$.

$$\vec{E}_Q = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad \text{para qualquer } r.$$

Cálculo de $\vec{E}_e$:

p/ $r \leq R$



$$\oint_S \vec{E}_e\cdot\hat{n}\,dA = \frac{q_{dentro}}{\varepsilon_0} \Rightarrow E_e 4\pi r^2 = \frac{\int\rho\, dv}{\varepsilon_0}$$

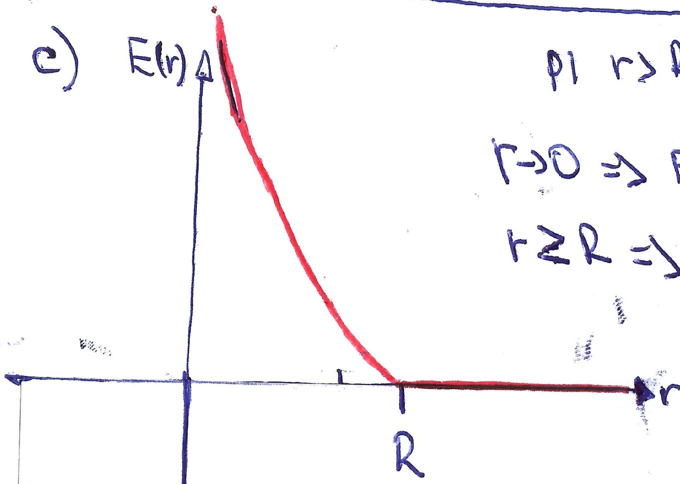
$$\Rightarrow \cancel{4\pi r^2} E_e = \frac{\int_0^r -\frac{\alpha}{r}4\pi r^2 dr}{\varepsilon_0} = -\frac{\cancel{4\pi}\alpha \cancel{r^2}}{2\varepsilon_0}$$

$$E_e = -\frac{\alpha}{2\varepsilon_0} = -\frac{Q}{4\pi\varepsilon_0 R^2}$$

$$\vec{E}_e = -\frac{Q}{4\pi\varepsilon_0 R^2}\hat{r}$$

$$\vec{E} = \vec{E}_Q + \vec{E}_e \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{1}{R^2}\right)\hat{r}}$$

c)



p/ $r > R$ $Q_{TOTAL} = 0$
↳ $\vec{E} = 0\hat{r}$

$r \to 0 \Rightarrow E \to \infty$

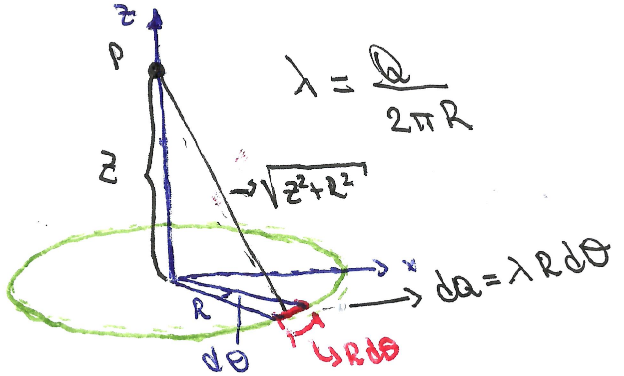
$r \geq R \Rightarrow E = 0$

↓

decresce com $\frac{1}{r^2}$

Questão 3 - Itens a) e b) resolvidos em sala
Item c) exemplo do Tipler 6ª Ed.

a)



Potencial devido a dQ no ponto P: $dV = \frac{dQ}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$

$$\Rightarrow dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda R\, d\theta}{(z^2+R^2)^{1/2}}$$

$$\Rightarrow V = \int dV = \frac{\lambda R}{4\pi\varepsilon_0 (z^2+R^2)^{1/2}}\int_0^{2\pi} d\theta \Rightarrow V(z) = \frac{\lambda R}{2\varepsilon_0 (z^2+R^2)^{1/2}}.$$

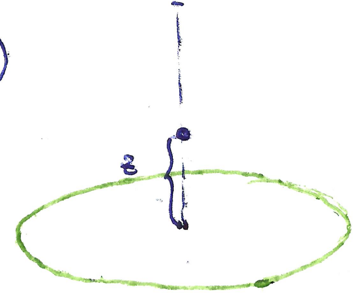
$$\Rightarrow \boxed{V(z) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{(z^2+R^2)^{1/2}}}$$

b) $E_z(z) = -\frac{dV}{dz} \Rightarrow E_z(z) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{z}{(z^2+R^2)^{3/2}}$

$E_x(z) = -\frac{dV}{dx} = 0$; $E_z(z) = -\frac{dV}{dy} = 0$.

$$\boxed{\vec{E}(z) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{z}{(z^2+R^2)^{3/2}}\hat{k}}$$

c)



Na posição z as energias potencial e cinética da partícula são:

$$U_1 = q\cdot V(z) = \frac{qQ}{4\pi\varepsilon_0 (z^2+R^2)^{1/2}}$$

$K_1 = 0$ (partícula liberada do repouso)

A uma distância muito grande do anel $z \to \infty$ essas energias ficam:

$$U_2 = qV(\infty) = 0 \quad ; \quad K_2 = \frac{1}{2} m v_f^2.$$

Pela cons. de energia $\Rightarrow U_1 + K_1 = U_2 + K_2$

$$\frac{qQ}{4\pi\varepsilon_0 (z^2+R^2)^{1/2}} = \frac{1}{2} m v_f^2 \Rightarrow \boxed{v_f = \sqrt{\frac{2qQ}{4\pi\varepsilon_0 m (z^2+R^2)^{1/2}}}}$$

Como $\vec{E} = E\hat{k} \Rightarrow \vec{F} = qE\hat{k} \Rightarrow \vec{v}_f = \sqrt{\frac{2qQ}{4\pi\varepsilon_0 (z^2+R^2)^{1/2}}}$