Covid-19 Case Fitting Curves

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Based on data from Covid-19 cases from China, a universal fitting curve is presented to describe a given Covid-19 case data, and its growth rates, accumulated over a number of days. Only four parameters are required to be determined by a non-linear optimization. Under the special condition of reliable mass testing, this curve can indicate the flattening of a given case data after its inflection point has been reached. Analyses of weekly new cases will be forecast on Mondays, including fitted case curves and growth rates.

CONTENTS

I. Model 3
I.1. Model function 3
I.2. Strategy 3
I.3. Parameters 4

II. Model cases 4
II.1. China 4
II.2. South Korea 5
II.3. New Zealand 5

III. Europe 5
III.1. Russia 5
III.2. Spain 6
III.3. UK 6
III.4. Italy 6
III.5. Germany 6
III.6. France 6
III.7. Portugal 6
III.8. Switzerland 6

IV. North America 7
IV.1. USA 7
IV.2. Mexico 7
IV.3. Canada 7
IV.4. Cuba 7

V. South America 7
V.1. Brazil 7
V.2. Peru 8
V.3. Colombia 8
V.4. Chile 8
V.5. Argentina 8
V.6. Bolivia 8
V.7. Venezuela 8

VI. Asia 8

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VI.1. India 8
VI.2. Saudi Arabia 9
VI.3. Japan 9
VI.4. Vietnam 9

VII. Africa 9
VII.1. South Africa 9
VII.2. Nigeria 9
VII.3. Ghana 9
VII.4. Cabo Verde 10
VII.5. Mozambique 10

VIII. Oceania 10
VIII.1. Australia 10

IX. Figures 11
IX.1. Russia 11
IX.2. Spain 13
IX.3. UK 15
IX.4. Italy 17
IX.5. Germany 19
IX.6. France 21
IX.7. Portugal 23
IX.8. Switzerland 25
IX.9. USA 27
IX.10. Mexico 29
IX.11. Canada 31
IX.12. Cuba 33
IX.13. Brazil 35
IX.14. Peru 37
IX.15. Colombia 39
IX.16. Chile 41
IX.17. Argentina 43
IX.18. Bolivia 45
IX.19. Venezuela 47
IX.20. India 49
IX.21. Saudi Arabia 51
IX.22. China 53
IX.23. Japan 55
IX.24. South Korea 57
IX.25. Vietnam 59
IX.26. South Africa 61
IX.27. Nigeria 63
IX.28. Ghana 65
IX.29. Cabo Verde 67
IX.30. Mozambique 69
IX.31. Australia 71
IX.32. New Zealand 73

References 75

I. MODEL

I.1. Model function

The hyperbolic tangent function,

$$Z_N(n) = a\tanh(bn - c) + d, \tag{1}$$

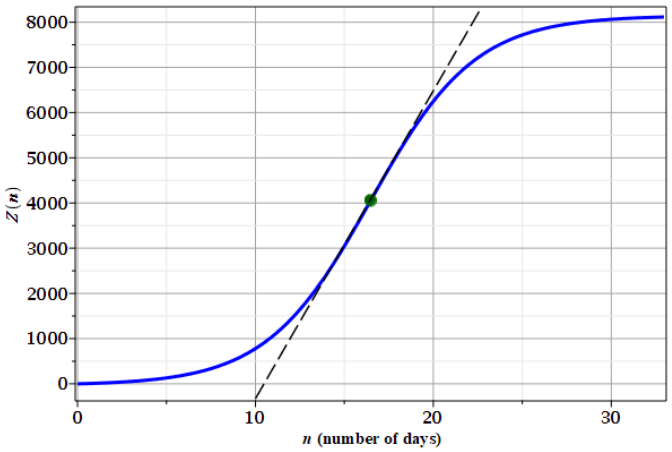
has two basic features of any Covid-19 case data over the number of days n: has flat endpoints and has one inflection point (at $n_i = c/b$). The final flattening indicates the stabilization of new infections. Parameter a, or $d = Z(n_i)$, gives the magnitude of the accumulated cases over N days. The amplitude is $2a$. The model function's steepness is tuned by the slope b of the straight line $bn - c$. Perhaps a q-deformed hyperbolic tangent could delivery a better response, using the deformation parameter to control its steepness as well.

Another important tools are the growth rates,

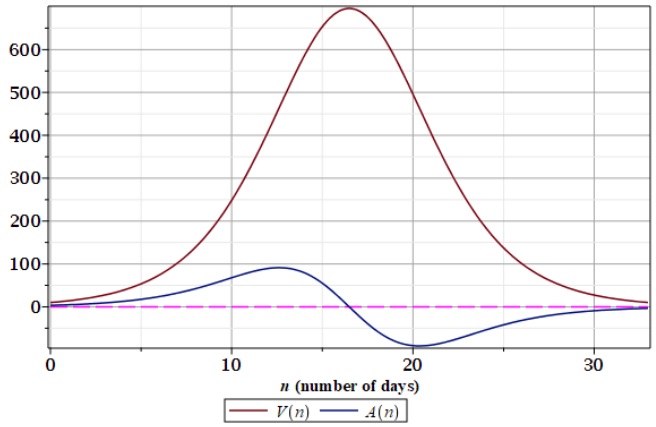
$$V(n) = \frac{dZ}{dn} = ab\,\mathrm{sech}(bn - c), \quad A(n) = \frac{dV}{dn} = -\frac{2b}{a}(Z(n) - d)V(n), \tag{2}$$

speed and acceleration, respectively. At the inflection point n_i, the speed is maximum ($V(n_i) = ab$) and the acceleration is null ($A(n_i) = 0$). Since the acceleration becomes negative after the inflection point, the speed diminishes and the stabilization is reached, the final flattening in the model function (1).

Figure 1a shows a typical case for the model function (1), where its magnitude is $a = 4086.9$ cases and its steepness is $0.170n - 2.807$ (dashed line). The green dot is the inflection point ($n_i = 16.48$). By the way, this model function represents the Covid-19 data case from New Zealand. Observe how the growth rates shown in Figure 1b are complete (as in no missing significant parts), indicating a real stabilization.



(a) Model function.



(b) Growth rates

FIG. 1: Model function $Z(n) = 4086.9\tanh(0.170n - 2.807) + 4057.2$, its inflection point (green dot at $n = 16.48$), its steepness (dashed line $0.170n - 2.807$) and its growth rates $V(n)$ and $A(n)$.

I.2. Strategy

The strategy is to learn possible universal features (conjectures) from analyses of Covid-19 case data of countries where the contamination process has been controlled. Subsequently, such conjectures will be applied to other cases in development.

Conjecture 1: monotonic convergence implies a possible stabilization (final flattening).

Conjecture 2: decreasing rms imply a possible stabilization.

Conjecture 3: complete growth rates imply a possible stabilization.

Conjecture 4: strong oscillations in the data require a multiple waves analysis.

Conjecture 5: stable inflection points (horizontal line) imply a possible stabilization.

Based on these conjectures, there are recommendations to extract information from the analyses given in the following sections. Important: the inflection point is a landmark in the middle of the road to stabilization.

How to read fitted curves:

1. Observe how close are the last fitted curve;
2. Observe the intensity and frequency of oscillations;
3. Observe how far is the final flattening;
4. Observe the root mean square (rms) deviations.

How to read growth rate curves:

1. Observe how intense is the speedy V;
2. Observe whether speedy V has reached its maximum or not;
3. Observe the sign of the acceleration A:
   (a) $A > 0$: speedy is trying to reach its maximum;
   (b) $A = 0$: speedy has reached its maximum (inflection point);
   (c) $A < 0$: speedy is decreasing and stability is foreseen.
4. Observe the behavior of the inflection point i_p:
   (a) horizontal line indicates a nearby stability;
   (b) non-horizontal line indicates new waves.

Nonetheless, it is important to say that despite the (weak) evidences presented below, any prediction here is not to be taken for granted. Also, this is a simple exercise of reconversion, since I am not an expert neither in Statistics Science nor Immunology. See Refs. [1]–[8] for more accurate fitting schemes.

I.3. Parameters

Parameters $\{a, b, c, d\}$ appearing in the model function (1) were obtained inside Maple and Mathematica by a (local) non-linear fitting minimizing the root mean square (rms) deviations. All case data points are equally weighted. Case data from every country are available at Worldometer.

Root mean square (rms) deviations shown by inlets in figures belong to the fitted curve Z_N, where N is the number of days in a given case data. Lower the rms, better the fitting. Residual is the difference between reported and predicted cases, using the last day fitted curve.

II. MODEL CASES

II.1. China

Figure 86 shows the case curves for the complete Covid-19 case data from China. The whole data seems to have a subseries described by the curve Z_{21}, obtained using the first 21 days, which indicates a (local) flattening of the cases over two weeks (around 53,700 cases). In spite of this curve Z_{21} being a false lead in searching for the case flattening, since we know the whole history, it has all features of a typical (complete) case curve. In fact, a close analysis in this subseries shows an inflection point around $n = 14$. Also, while the nearest case curves $Z_{N\leq 21}$ approach Z_{21} in an oscillatory way, all case curves $Z_{N\geq 23}$ approach Z_{74} monotonically. Curve Z_{74}, obtained using 74 days, is the global case curve, which indicates a flattening around 81,000 cases.

The (global) curve Z_{74} shown in Figure 86 is the (lower) limit for all curves $Z_{N\geq 22}$ and has its inflection point around $n = 21$. Since all curves $Z_{N\geq 29}$ converge monotonically and rapidly to Z_{74}, then we have a 10 days prediction for the (global) flattening occurring around day 50. **Conjecture 1: monotonic convergence implies a possible stabilization (final flattening).**

We have learned that the case curves Z_N obtained after the inflection point converge monotonically, in an increasing speed, to the global case curve. Of course, mass testing is presumed. Inset in Figure 86 shows the root mean square (rms) deviations of each case curve $Z_{N\geq 24}$, confirming the monotonic convergence.

Some questions. What did happen at day 22? Was it an increase in the total tests? Or is this jump in the number of cases from day 21 to the day 22 (see the solid green circles in Figure 86) a typical feature of the global case curve around its inflection point (where the rate of change is maximum)?

Figure 87 shows the case data accumulated over weeks and the last five fitted curves together with their root mean square (rms) deviations shown in the inset. Note that these last fitted curves are very close each other and the rms deviations are decreasing, indicating a stabilization. **Conjecture 2: decreasing rms imply a possible stabilization.**

Growth rates are shown in Figures 88 (daily) and 89 (weekly). They are complete curves, indicating a global stabilization starting around day 50 (week eight). The inflection point is around day 18 or around week three. **Conjecture 3: complete growth rates imply a possible stabilization.**

II.2. South Korea

South Korea has a case data with three waves, as shown in Figure 94. The second wave Z_b starts at day 31 and the last wave Z_c starts at day 81. These multiple waves represent new cases of infection, a very important lesson to be learned. Till day 80, stabilization seemed granted. Now a third wave is underway, growing slowly. Due these multiple waves, the global curve Z_{177} does not represent some portions of the observed data very well. The root mean square (rms) deviations from the fitted curves in the last wave are decreasing, as shown in the inset in Figure 94. Figure 95 shows the case data accumulated over weeks and the fitted curves for the three waves, $Z_a(n \leq 5)$, $Z_b(6 \leq n \leq 12)$ and $Z_c(n \leq 12)$. Daily and weekly growth rates for the third wave Z_c are shown in Figures 96 and 97, respectively. This last wave has an inflection point (i_p) around day 141 (week 21.6). The inset in Figure 96 shows the inflection point (i_p) for the last six fitted curves representing the last wave in the last six days. Note that the inflection point is changing towards the last days and that a fourth wave is coming. **Conjecture 4: strong oscillations in the data require a multiple waves analysis.**

II.3. New Zealand

New Zealand have reached a global flattening, givem by the global curve Z_{99}, as we can see in Figure 126. Figure 127 shows the case data accumulated over weeks and the last five fitted curves together with their root mean square (rms) deviations shown in the inset. Note that these last fitted curves are very close each other and the rms deviations are decreasing, indicating a stabilization given by the global curve Z_{15}. Growth rates are shown in Figures 128 (daily) and 129 (weekly). They are complete curves, indicating a global stabilization starting around day 70 (week 11). The inflection point is around day 33 (week 5.5). The insets show the inflection points (i_p) for the last curves. Note how horizontal they are. New Zealand has an almost perfect case curve. **Conjecture 5: stable inflection points (horizontal line) imply a possible stabilization.**

III. EUROPE

III.1. Russia

Daily and weekly accumulated case data from Russia are shown in Figures 2 and 3, respectively. The insets show the root mean square (rms) deviations. Note that all these last fitted curves are orderly approaching each other. Increasings in the rms in the last days are due to small oscillations in the data. Growth rates (2) from the last day fitted curves are shown in Figures 4 (daily) and 5 (weekly). They are incomplete curves and indicate the inflection point around day 99 (week 15.1). The insets show the inflection points (i_p) for the last curves.

III.2. Spain

Daily and weekly accumulated case data from Spain are shown in Figures 6 and 7, respectively. The insets show the root mean square (rms) deviations for the last wave Z_b, started at day 106 (week 10). Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 8 (daily) and 9 (weekly). They are incomplete curves and indicate that the inflection point reached at day 172 (week 25.2). The insets show the inflection points (i_p) for the last curves.

III.3. UK

Daily and weekly accumulated case data from UK are shown in Figures 10 and 11, respectively. The insets show the root mean square (rms) deviations. Note that all these last fitted curves are orderly approaching each other. Increasings in the rms in the last days are due to medium oscillations in the data. Growth rates (2) from the last day fitted curves are shown in Figures 12 (daily) and 13 (weekly). They are almost complete curves and indicate the inflection point around day 66 (week 10.3). The insets show the inflection points (i_p) for the last curves. There is a new wave growing slowly.

III.4. Italy

Daily and weekly accumulated case data from Italy are shown in Figures 14 and 15, respectively. The insets show the root mean square (rms) deviations. Increasings in the rms in the last days are due to small oscillations in the data. Growth rates (2) from the last day fitted curves are shown in Figures 16 (daily) and 17 (weekly). They are complete curves and indicate the inflection point around day 49 (week 7.8). The insets show the inflection points (i_p) for the last curves. There is a new wave growing slowly.

III.5. Germany

Daily and weekly accumulated case data from Germany are shown in Figures 18 and 19, respectively. The insets show the root mean square (rms) deviations for the last wave Z_b, started at day 126 (week 7). Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 20 (daily) and 21 (weekly). They are incomplete curves and indicate the inflection point was not reached yet.

III.6. France

Daily and weekly accumulated case data from France are shown in Figures 22 and 23, respectively. The insets show the root mean square (rms) deviations for the last wave Z_c. The second wave starts at day 75 (week 10) and the last wave at day 125 (week 15). Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 24 (daily) and 25 (weekly). They are incomplete curves and indicate that the inflection point was not reached yet. The last wave is growing strong.

III.7. Portugal

Daily and weekly accumulated case data from Portugal are shown in Figures 26 and 27, respectively. The insets show the root mean square (rms) deviations for the last wave. Increasings in the rms in the last days are due to strong oscillations in the data, which require at least to waves, Z_a and Z_b (starting at day 64, week 10). Growth rates (2) from the last day fitted curves for the last wave are shown in Figures 28 (daily) and 29 (weekly). They are incomplete curves and indicate the inflection point around day 109 (week 16.5). The insets show the inflection points (i_p) for the last curves. The last wave is closing.

III.8. Switzerland

Daily and weekly accumulated case data from Switzerland are shown in Figures 30 and 31, respectively. The insets show the root mean square (rms) deviations for the last wave. Increasings in the rms in the last days are due to strong oscillations in the

data, which require at least to waves, Z_a and Z_b (starting at day 90, week 13). Growth rates (2) from the last day fitted curves for the last wave are shown in Figures 32 (daily) and 33 (weekly). They are incomplete curves and indicate the inflection point around day 170 (week 25.4). The insets show the inflection points (i_p) for the last curves. The last wave is still growing and pushing the inflection points away.

IV. NORTH AMERICA

IV.1. USA

Daily and weekly accumulated case data from USA are shown in Figures 34 and 35, respectively. The insets show the root mean square (rms) deviations for the last wave Z_b. The last wave starts at day 105 (week 16). Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 36 (daily) and 37 (weekly). They are incomplete curves and indicate that the inflection point was reached at day 158 (week 23.5). The last wave is growing strong.

IV.2. Mexico

Daily and weekly accumulated case data from Mexico are shown in Figures 38 and 39, respectively. Their root mean square (rms) deviations are shown in the insets. So far there are no significant oscillations in the data. Growth rates (2) from the last day fitted curves are shown in Figures 40 (daily) and 41 (weekly). They are incomplete curves with a recent inflection point around day 142 (week 21.1). The insets show the inflection points (i_p) for the last curves.

IV.3. Canada

Daily and weekly accumulated case data from Canada are shown in Figures 42 and 43, respectively. The insets show the root mean square (rms) deviations for the last wave Z_b, which starts at day 135 (week). Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 44 (daily) and 45 (weekly). They are almost complete curves and indicate the inflection point around day 162 (week 24.1). The insets show the inflection points (i_p) for the last curves. The last wave is growing with a low intensity.

IV.4. Cuba

Daily and weekly accumulated case data from Cuba are shown in Figures 46 and 47, respectively. The insets show the root mean square (rms) deviations for the last wave Z_c. The second wave starts at day 71 (week 10) and the last wave at day 111 (week 15). Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 48 (daily) and 49 (weekly). They are incomplete curves and indicate the inflection point was not reached yet. The last wave can become strong.

V. SOUTH AMERICA

V.1. Brazil

Figure 50 shows the case curves for the last five days for the Brazilian case data (Ministery of Health). Figure 51 shows the case data accumulated over weeks and the last fitted curves. The insets show the increasing root mean square (rms) deviations. There are frequent small oscillations in the data. Growth rates (2) from the last day fitted curves are shown in Figures 52 (daily) and 53 (weekly). They are incomplete curves and indicate that the inflection point was reached recently, around day 141 (week 21.2). The insets show the inflection points (i_p) for the last curves.

V.2. Peru

Daily and weekly accumulated case data from Peru are shown in Figures 54 and 55, respectively. The insets show the root mean square (rms) deviations for the last wave Z_b. Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 56 (daily) and 57 (weekly). They are incomplete curves and indicate that the inflection point is absent.

V.3. Colombia

Daily and weekly accumulated case data from Colombia are shown in Figures 58 and 59, respectively. The insets show the increasing root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 60 (daily) and 61 (weekly). They are incomplete curves and the inflection point is absent.

V.4. Chile

Daily and weekly accumulated case data from Chile are shown in Figures 62 and 63, respectively. The insets show the increasing root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 64 (daily) and 65 (weekly). They are incomplete curves and indicate a recent inflection point around day 98 (week 15). The insets show the inflection points (i_p) for the last curves.

V.5. Argentina

Daily and weekly accumulated case data from Argentina are shown in Figures 66 and 67, respectively. The insets show the increasing root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 68 (daily) and 69 (weekly). They are incomplete curves and the inflection point is absent.

V.6. Bolivia

Daily and weekly accumulated case data from Bolivia are shown in Figures 70 and 71, respectively. The insets show the increasing root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 72 (daily) and 73 (weekly). They are incomplete curves and the inflection point was reached recently at day 134 (week 20.1).

V.7. Venezuela

Daily and weekly accumulated case data from Venezuela are shown in Figures 74 and 75, respectively. The insets show the increasing root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 76 (daily) and 77 (weekly). They are incomplete curves and the inflection point is absent.

VI. ASIA

VI.1. India

Daily and weekly accumulated case data from India are shown in Figures 78 and 79, respectively. The insets show the increasing root mean square (rms) deviations. Note that all these last fitted curves are still distant each other. Growth rates (2) from the last day fitted curves are shown in Figures 80 (daily) and 81 (weekly). They are incomplete curves and indicate that the inflection point is absent.

VI.2. Saudi Arabia

Daily and weekly accumulated case data from Saudi Arabia are shown in Figures 82 and 83, respectively. Their root mean square (rms) deviations are shown in the insets. There are medium oscillations in the data. Growth rates (2) from the last day fitted curves are shown in Figures 84 (daily) and 85 (weekly). They are incomplete curves with a recent inflection point around day 107 (week 16.2). The insets show the inflection points (i_p) for the last curves.

VI.3. Japan

Daily and weekly accumulated case data from Japan are shown in Figures 90 and 91, respectively. There is a second wave Z_b in progress, which started around day 92 (week 14), with a strong intensity. The insets show the root mean square (rms) deviations calculated from the last wave. Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 92 (daily) and 93 (weekly). They are incomplete curves and indicate that the inflection point was reached at day 179 (week 26.2). The insets show the inflection points (i_p) for the last curves.

VI.4. Vietnam

Daily and weekly accumulated case data from Vietnam are shown in Figures 98 and 99, respectively. The insets show the root mean square (rms) deviations calculated from the last wave. Although there are many waves, most of them of very small intensity, we chose only the three most relevant. The last wave Z_c started around day 112 (week 17) has the strongest intensity. Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 100 (daily) and 101 (weekly). They are incomplete curves and indicate that the inflection point was reached around day 158 (week 23.4). The insets show the inflection points (i_p) for the last curves.

VII. AFRICA

VII.1. South Africa

Daily and weekly accumulated case data from South Africa are shown in Figures 102 and 103, respectively. The insets show the root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 104 (daily) and 105 (weekly). They are incomplete curves and indicate that the inflection point was reached at day 133 (week 19.8). The insets show the inflection points (i_p) for the last curves.

VII.2. Nigeria

Daily and weekly accumulated case data from Nigeria are shown in Figures 106 and 107, respectively. The insets show the root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 108 (daily) and 109 (weekly). They are incomplete curves and indicate that the inflection point was reached at day 113 (week 17). The insets show the inflection points (i_p) for the last curves.

VII.3. Ghana

Daily and weekly accumulated case data from Ghana are shown in Figures 110 and 111, respectively. The insets show the root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 112 (daily) and 113 (weekly). They are incomplete curves and indicate that the inflection point was reached at day 142 (week 20.4). The insets show the inflection points (i_p) for the last curves. There are strong oscillations in the data.

VII.4. Cabo Verde

Daily and weekly accumulated case data from Cabo Verde are shown in Figures 114 and 115, respectively. The insets show the root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 116 (daily) and 117 (weekly). They are incomplete curves and indicate that the inflection point was reached at day 106 (week 16.3). The insets show the inflection points (i_p) for the last curves. There are strong oscillations in the data.

VII.5. Mozambique

Daily and weekly accumulated case data from Mozambique are shown in Figures 118 and 119, respectively. The insets show the root mean square (rms) deviations. Growth rates (2) from the last day fitted curves are shown in Figures 120 (daily) and 121 (weekly). They are incomplete curves and indicate that the inflection point has been pushed away. The insets show the inflection points (i_p) for the last curves. There are strong oscillations in the data.

VIII. OCEANIA

VIII.1. Australia

Daily and weekly accumulated case data from Australia are shown in Figures 122 and 123, respectively. There is a second wave Z_b in progress, which started around day 81 (week 13), with a strong intensity. The insets show the root mean square (rms) deviations calculated from the last wave. Growth rates (2) from the last day fitted curves in the last wave are shown in Figures 124 (daily) and 125 (weekly). They are incomplete curves and indicate that the inflection point was reached at day 168 (week 24.9).

IX. FIGURES

IX.1. Russia

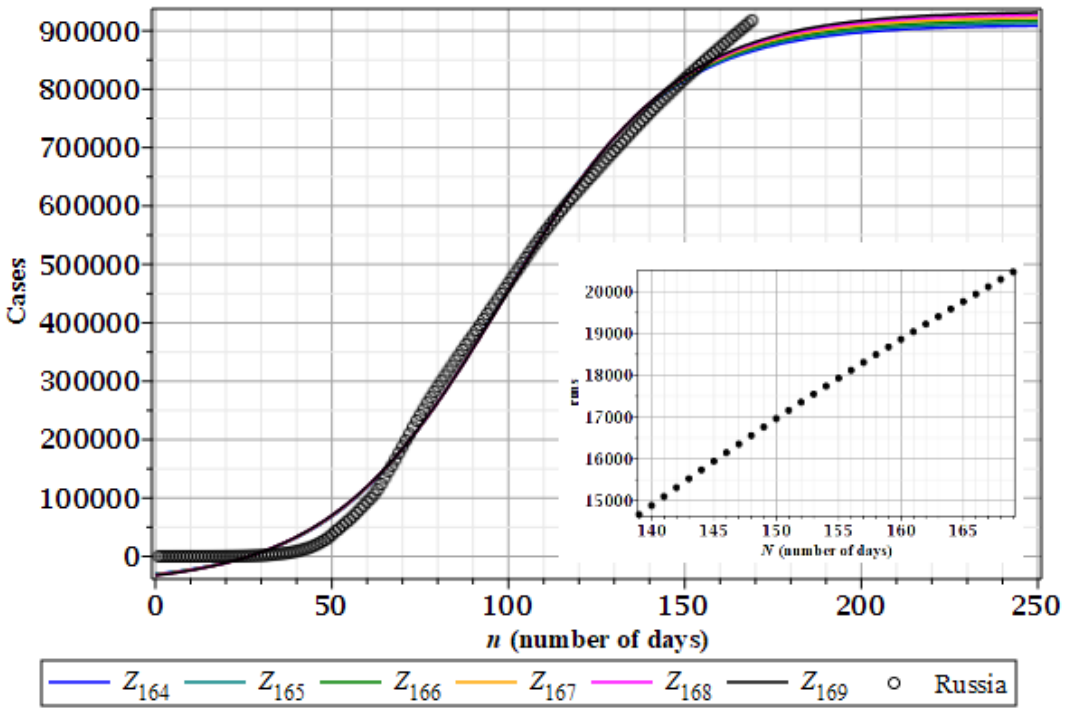


FIG. 2: Daily Covid-19 case curves and rms for the data from Russia.

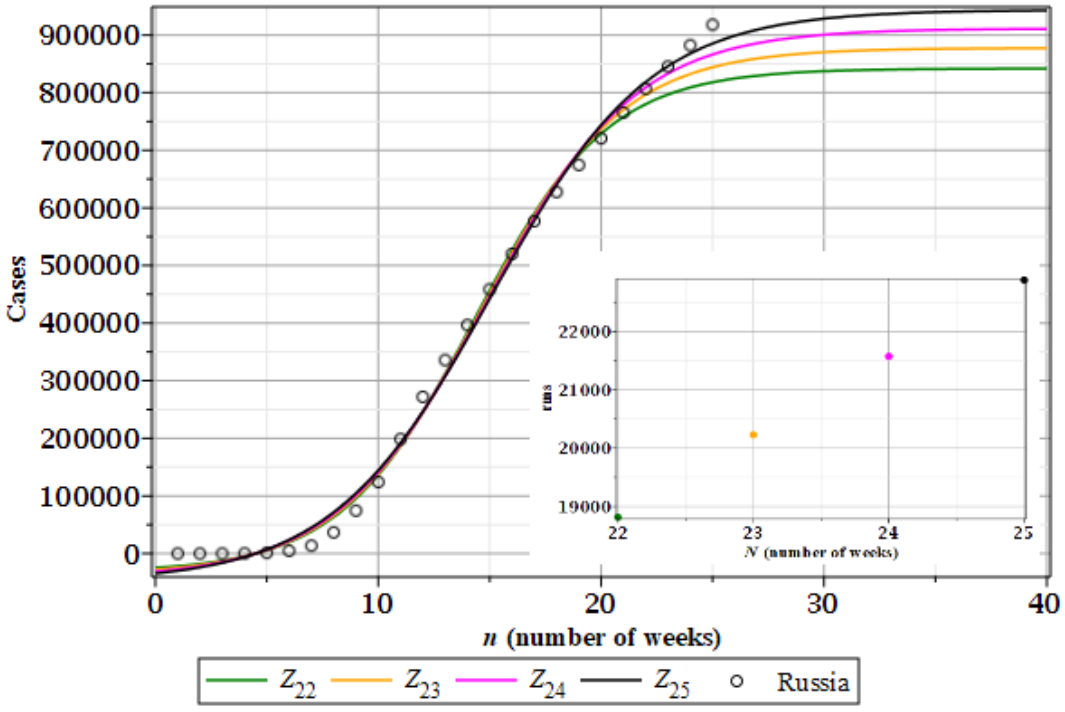


FIG. 3: Weekly Covid-19 case curves and rms for the data from Russia.

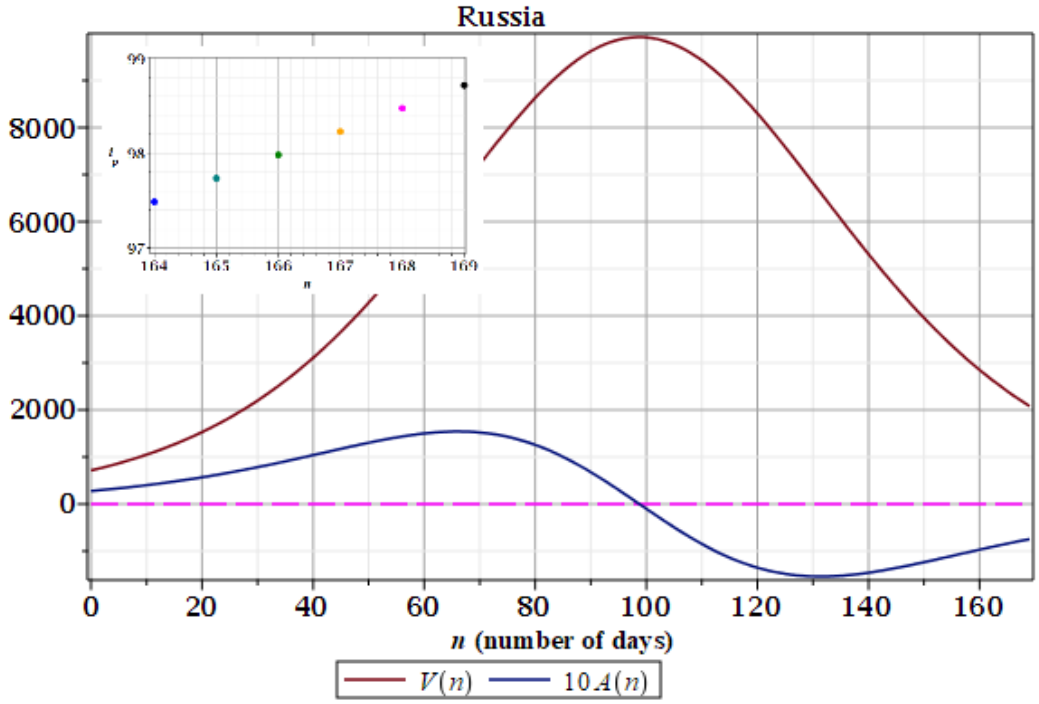


FIG. 4: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{169} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 99$.

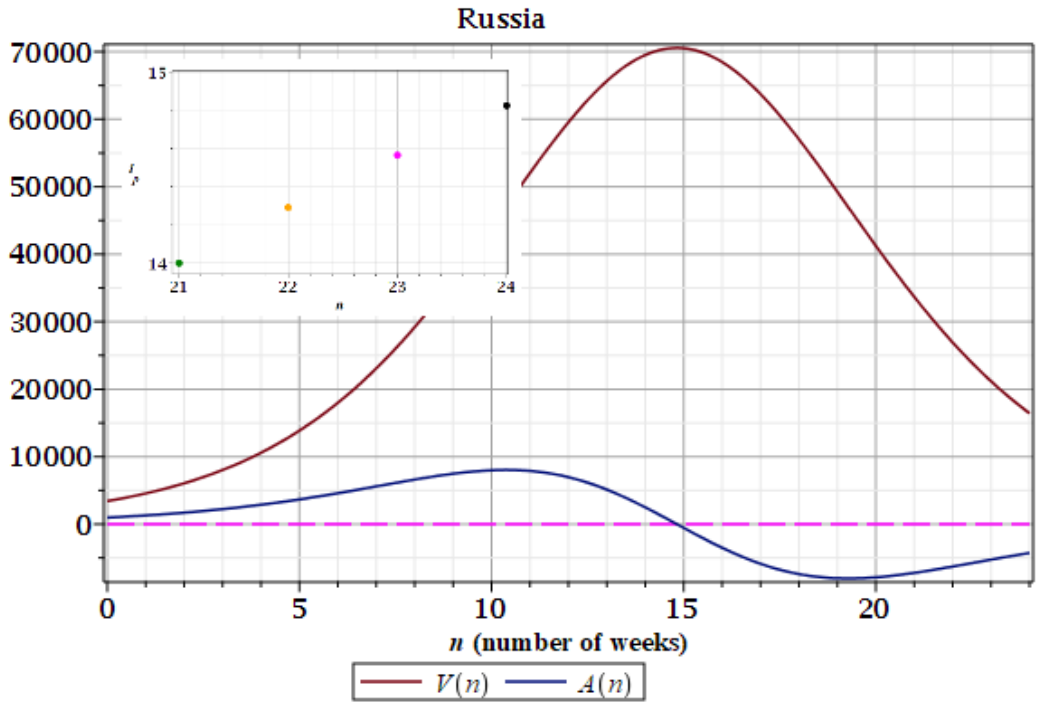


FIG. 5: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{25} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 15.1$.

IX.2. Spain

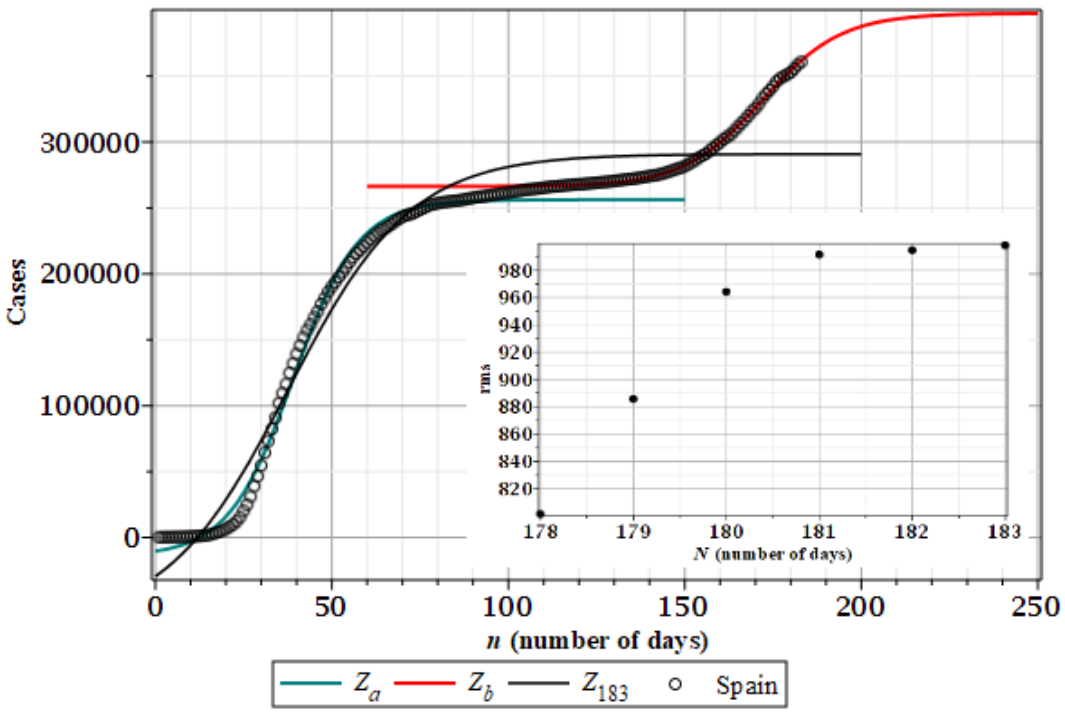


FIG. 6: Daily Covid-19 case curves and rms (last wave) for the data from Spain.

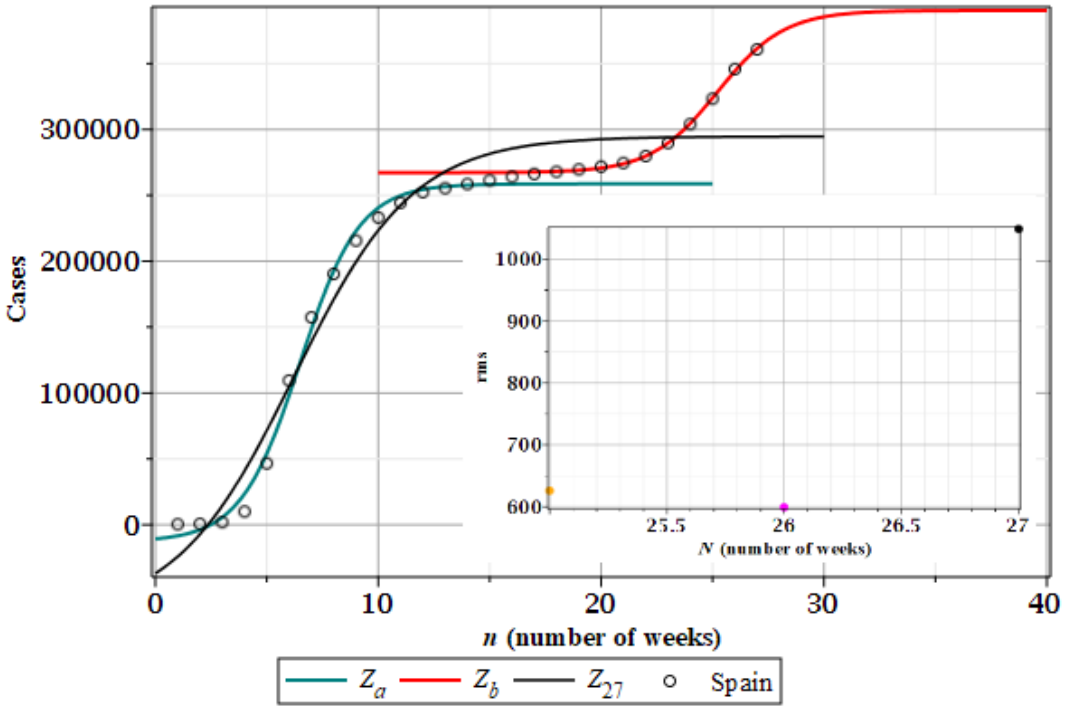


FIG. 7: Weekly Covid-19 case curves and rms (last wave) for the data from Spain.

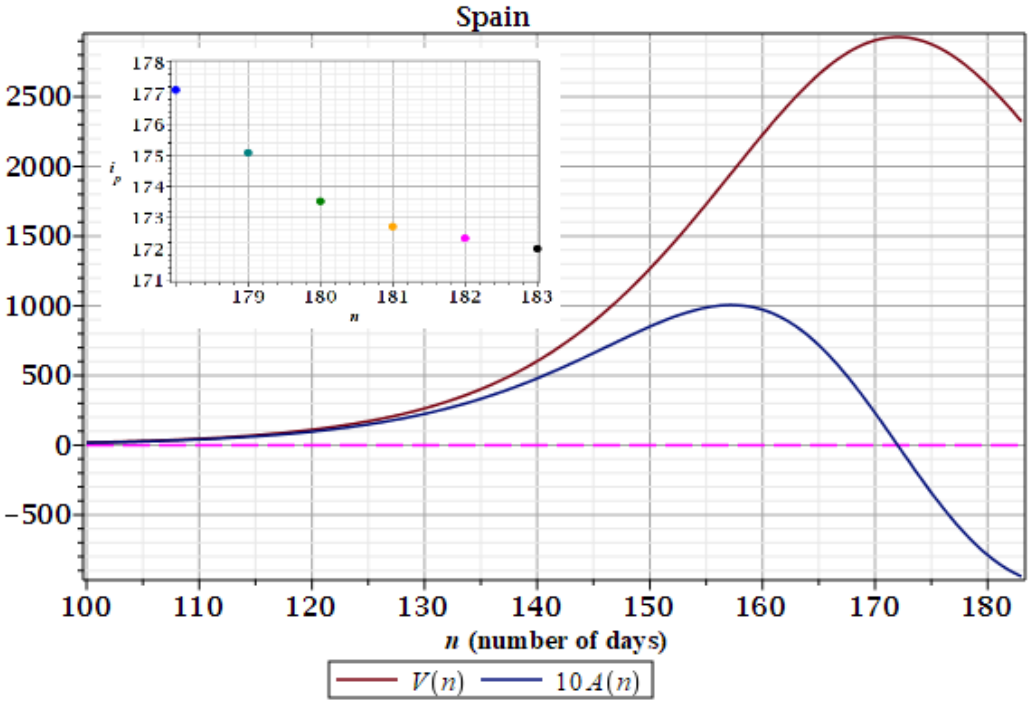


FIG. 8: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b (the last wave) and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 172$.

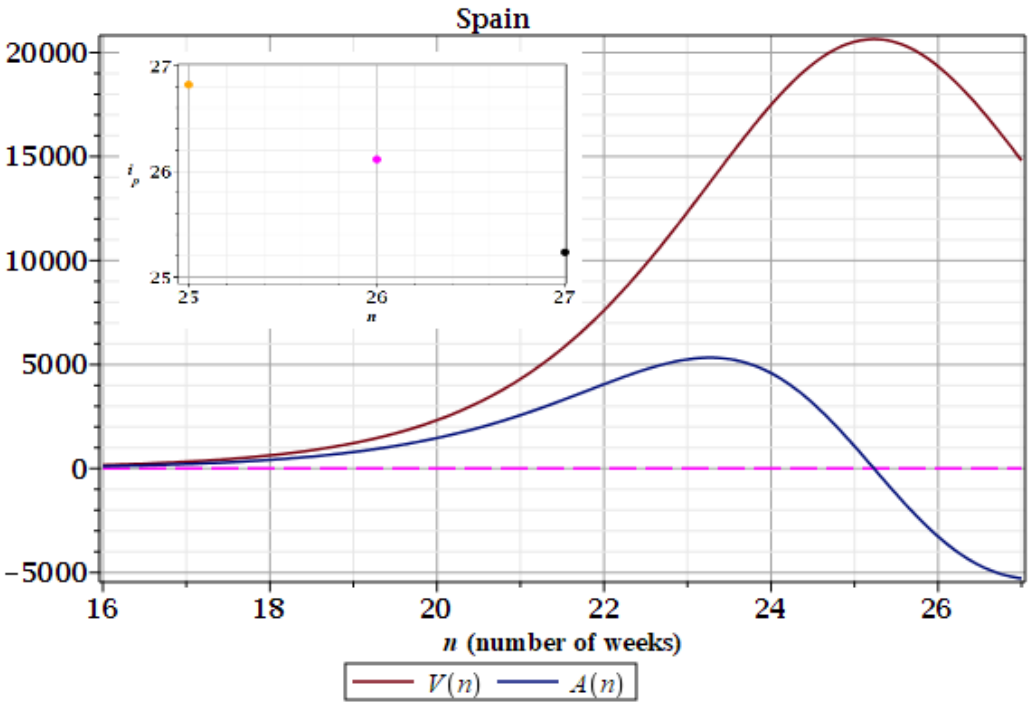


FIG. 9: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b (the last wave) and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 25.2$.

IX.3. UK

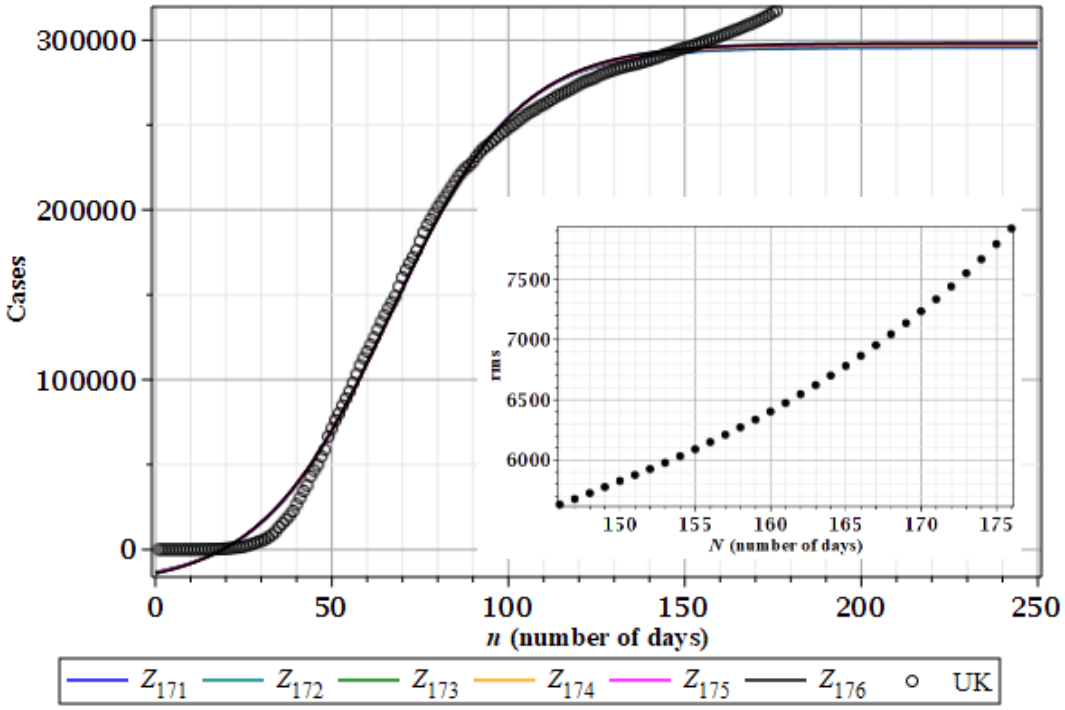


FIG. 10: Daily Covid-19 case curves and rms for the data from UK.

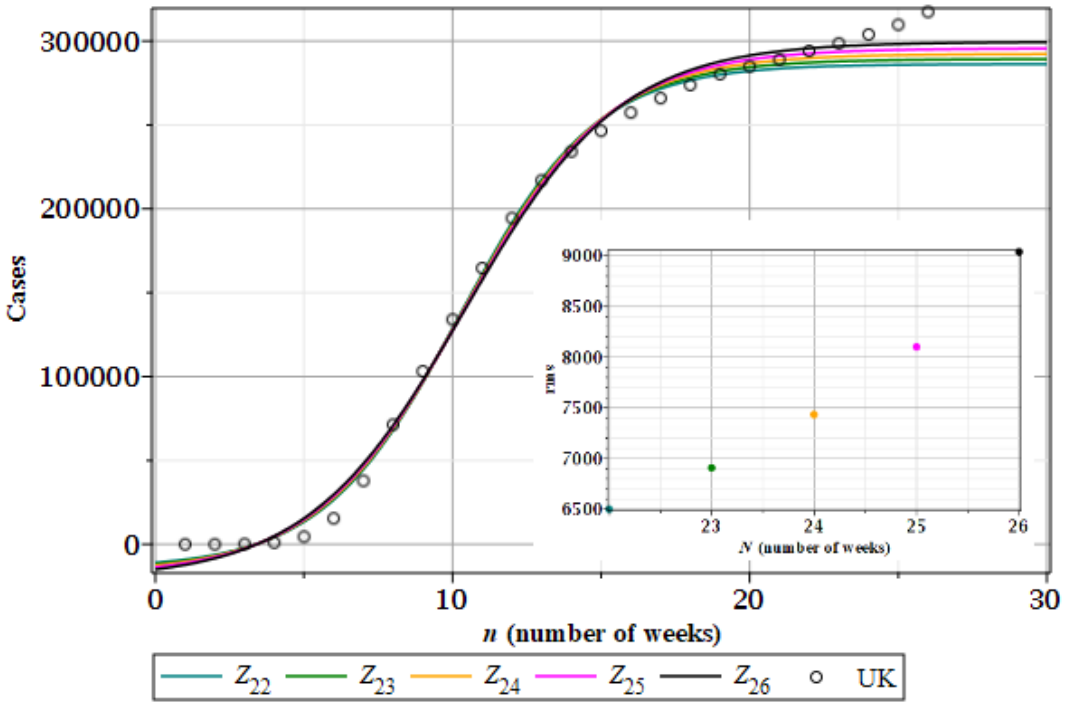


FIG. 11: Weekly Covid-19 case curves and rms for the data from UK.

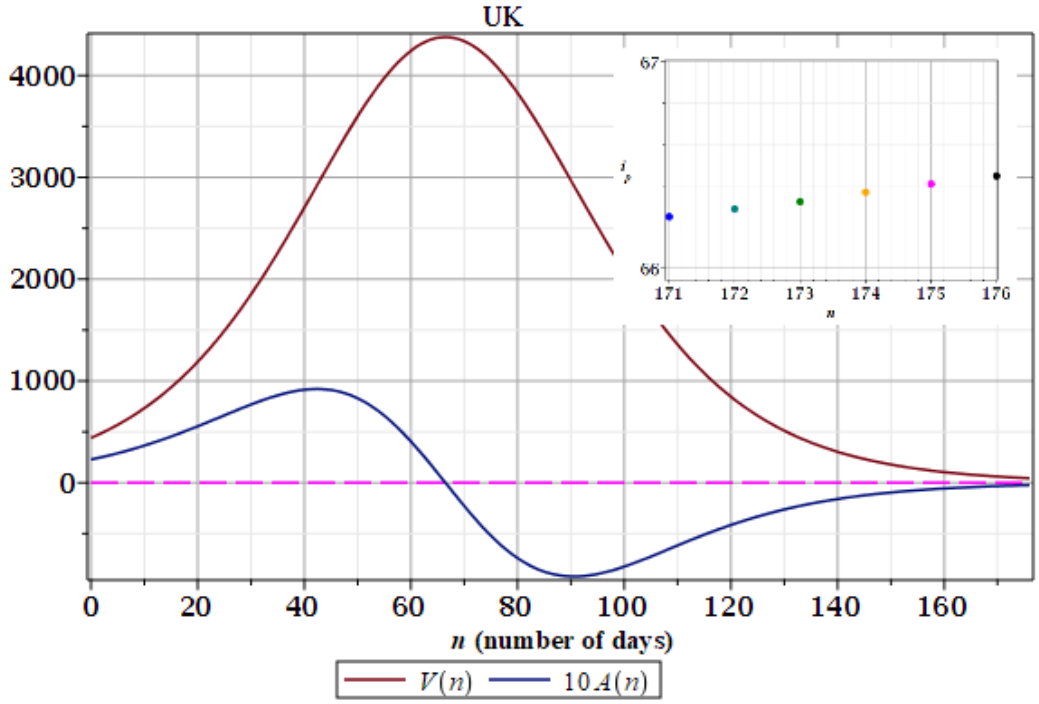


FIG. 12: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{176} and inflection points for the last curves. Almost complete curves. Inflection point: $n \approx 66$.

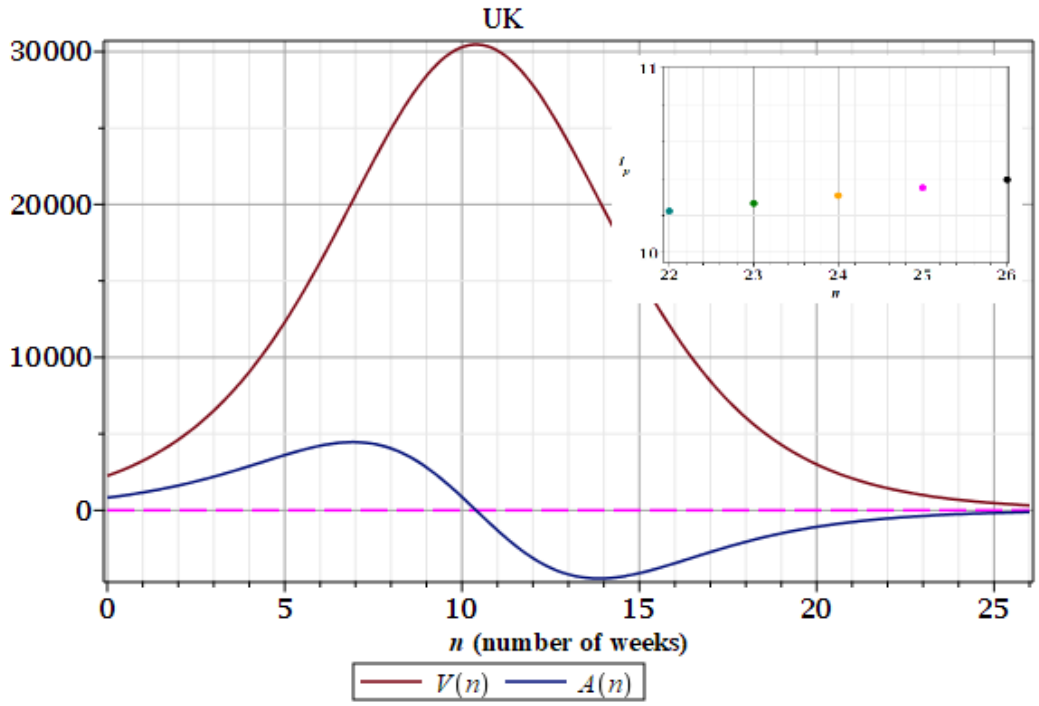


FIG. 13: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{26} and inflection points for the last curves. Almost complete curves. Inflection point: $n \approx 10.4$.

IX.4. Italy

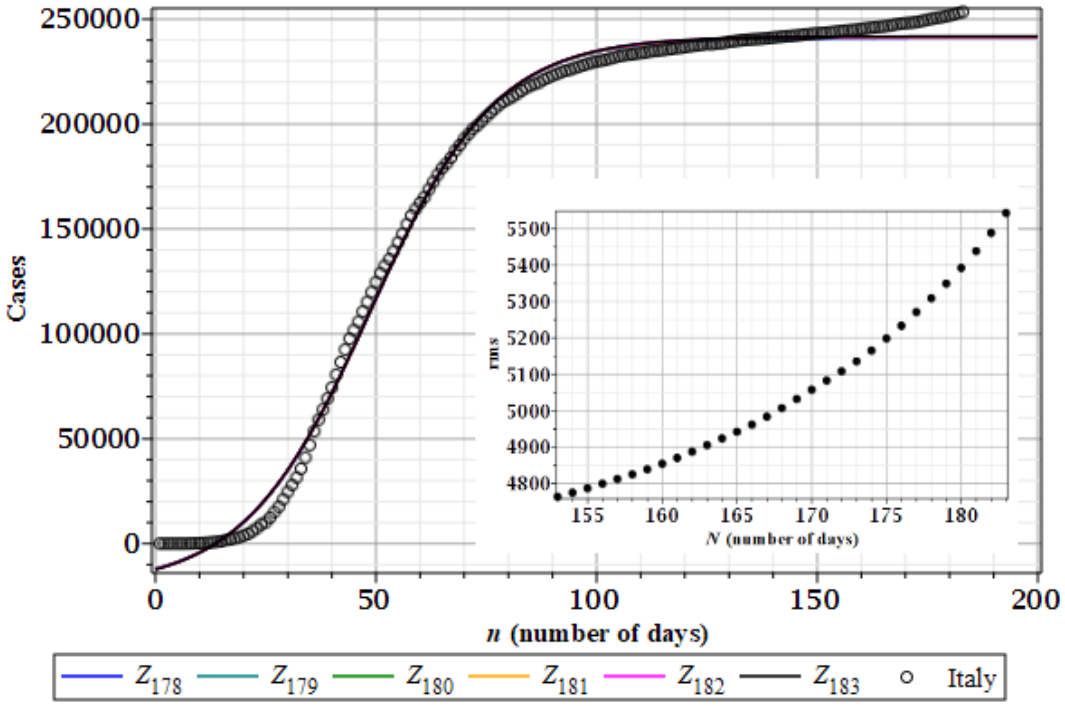


FIG. 14: Daily Covid-19 case curves and rms for the data from Italy.

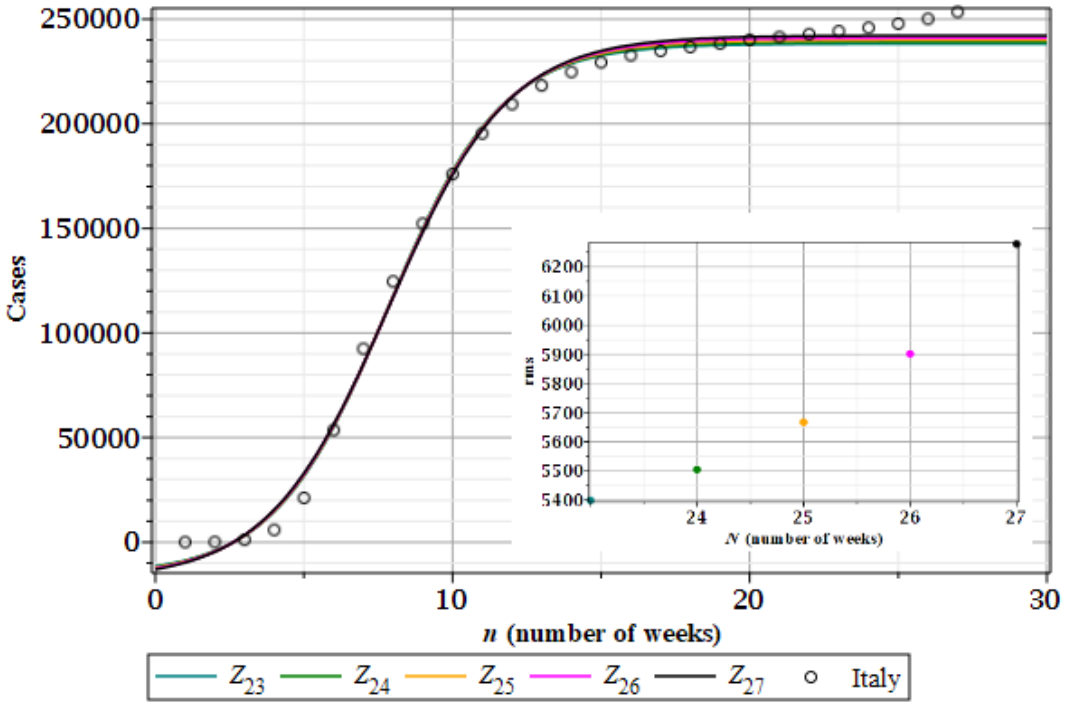


FIG. 15: Weekly Covid-19 case curves and rms for the data from Italy.

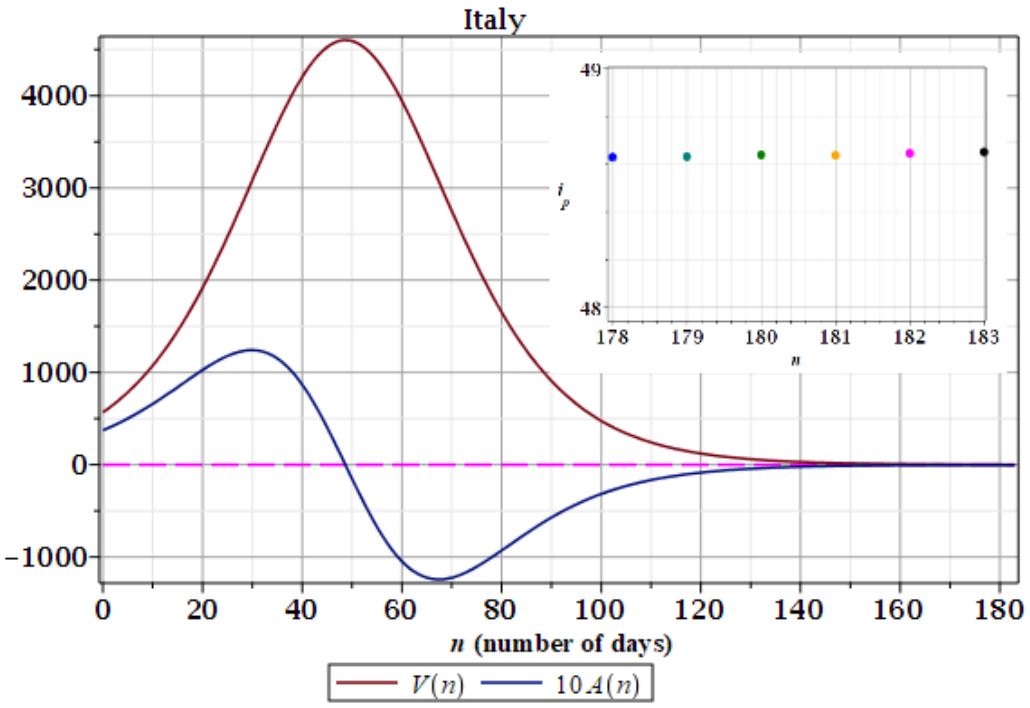


FIG. 16: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{184} and inflection points for the last curves. Complete curves. Inflection point: $n \approx 49$.

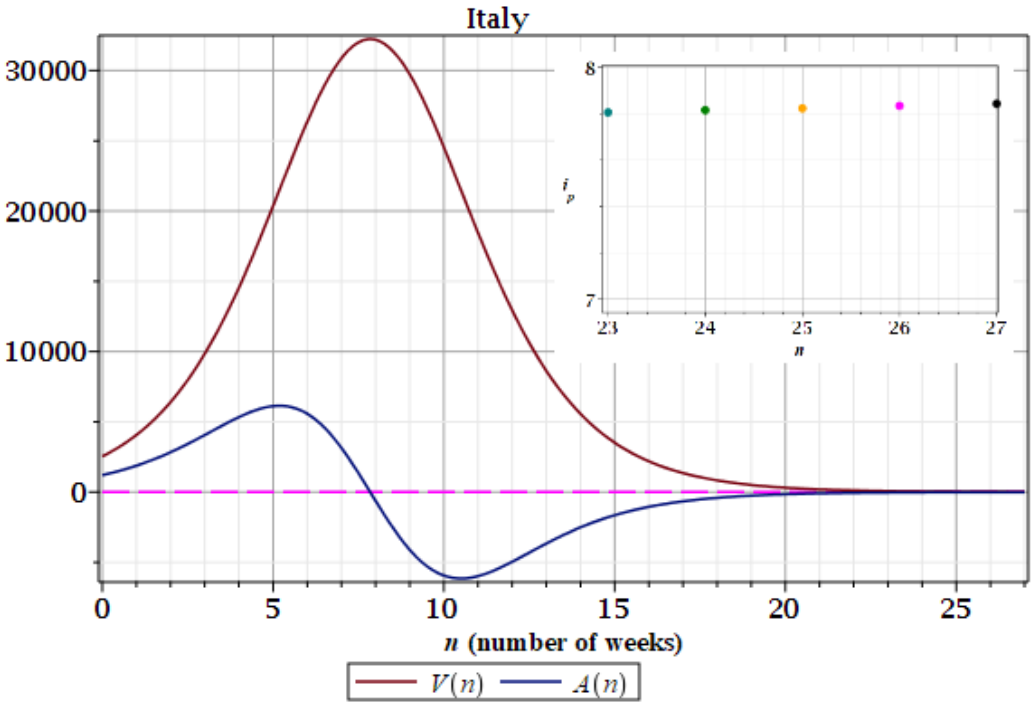


FIG. 17: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{27} and inflection points for the last curves. Complete curves. Inflection point: $n \approx 7.8$.

IX.5. Germany

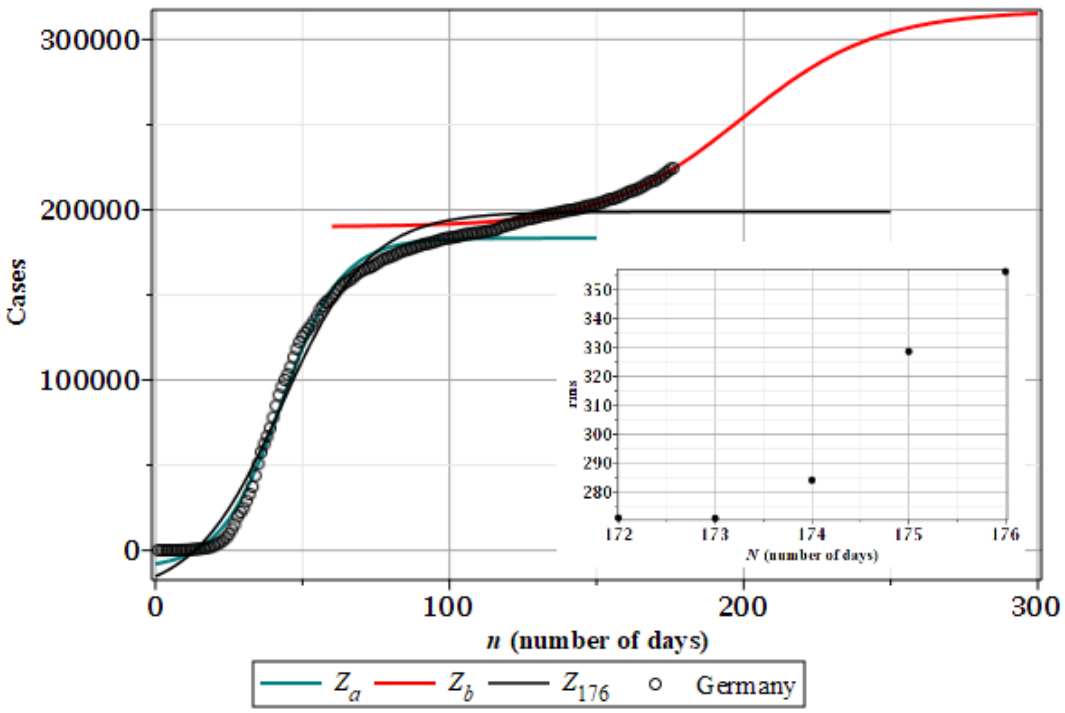


FIG. 18: Daily Covid-19 case curves and rms (last wave) for the data from Germany.

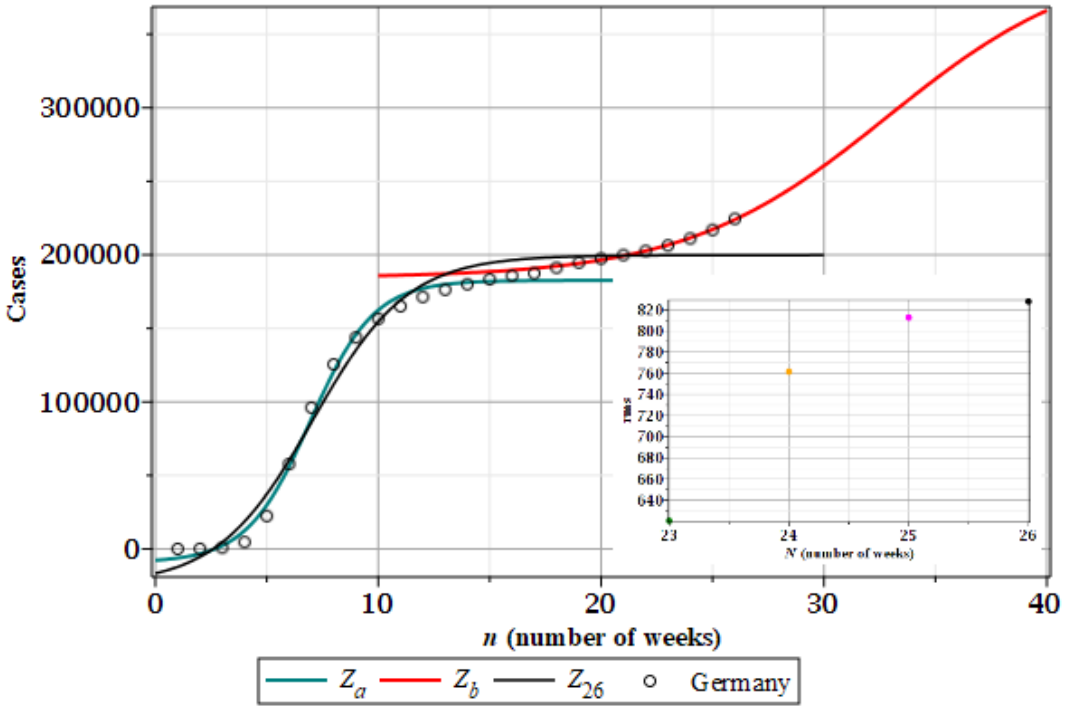


FIG. 19: Weekly Covid-19 case curves and rms (last wave) for the data from Germany.

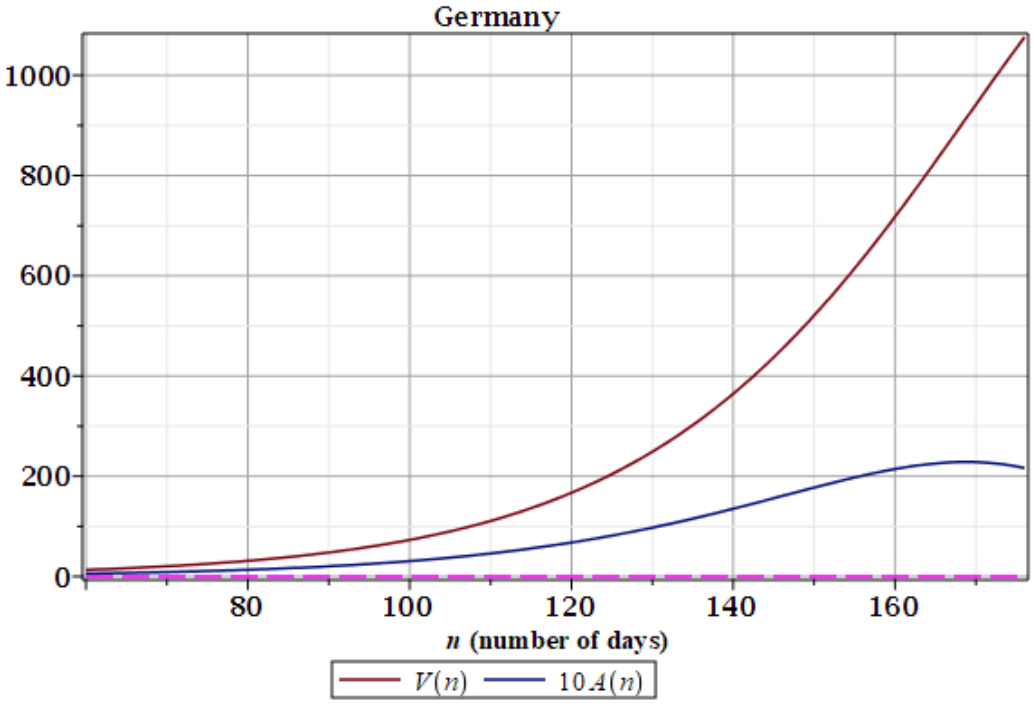


FIG. 20: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b. Incomplete curves. Inflection point: absent.

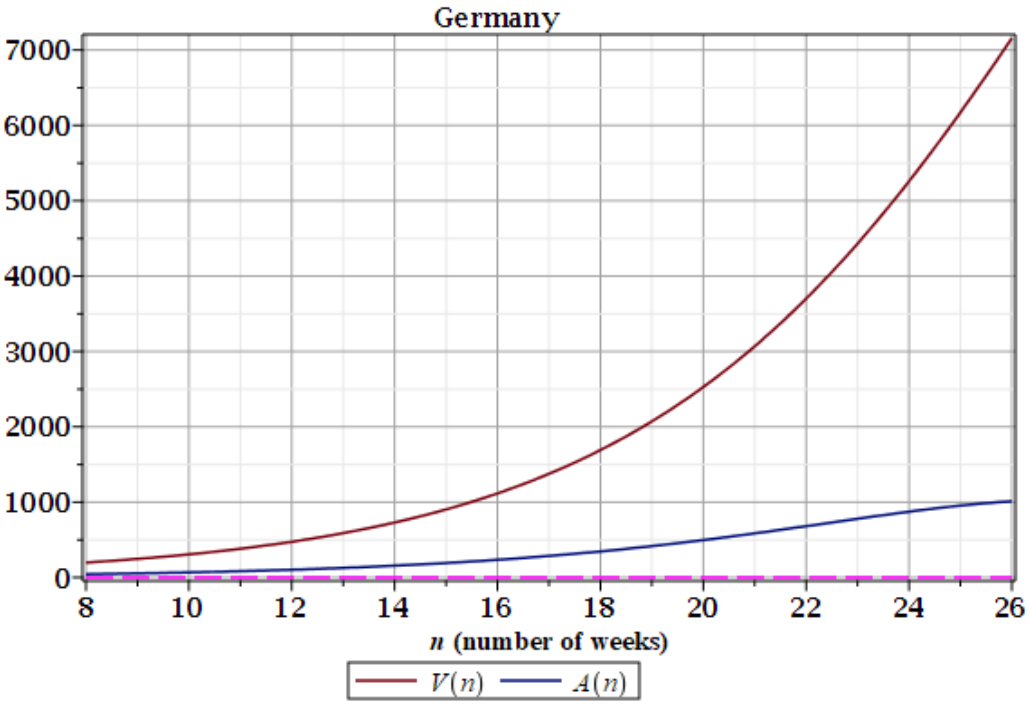


FIG. 21: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b. Incomplete curves. Inflection point: absent.

France

IX.6. France

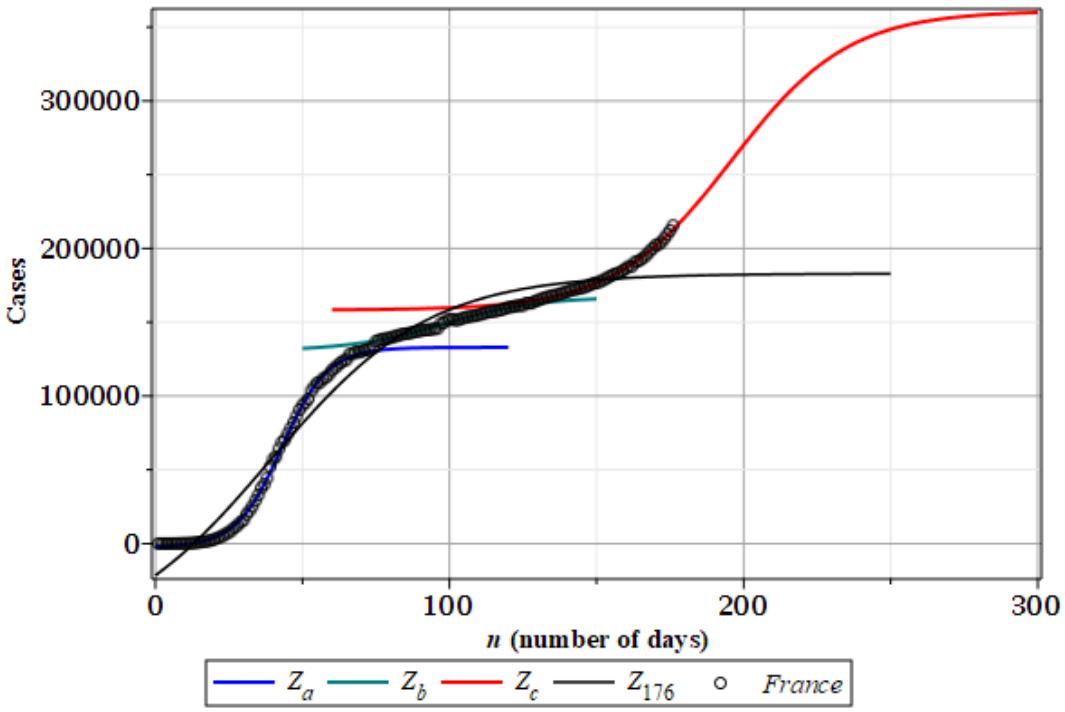


FIG. 22: Covid-19 case curves and rms (last wave) for the data from France.

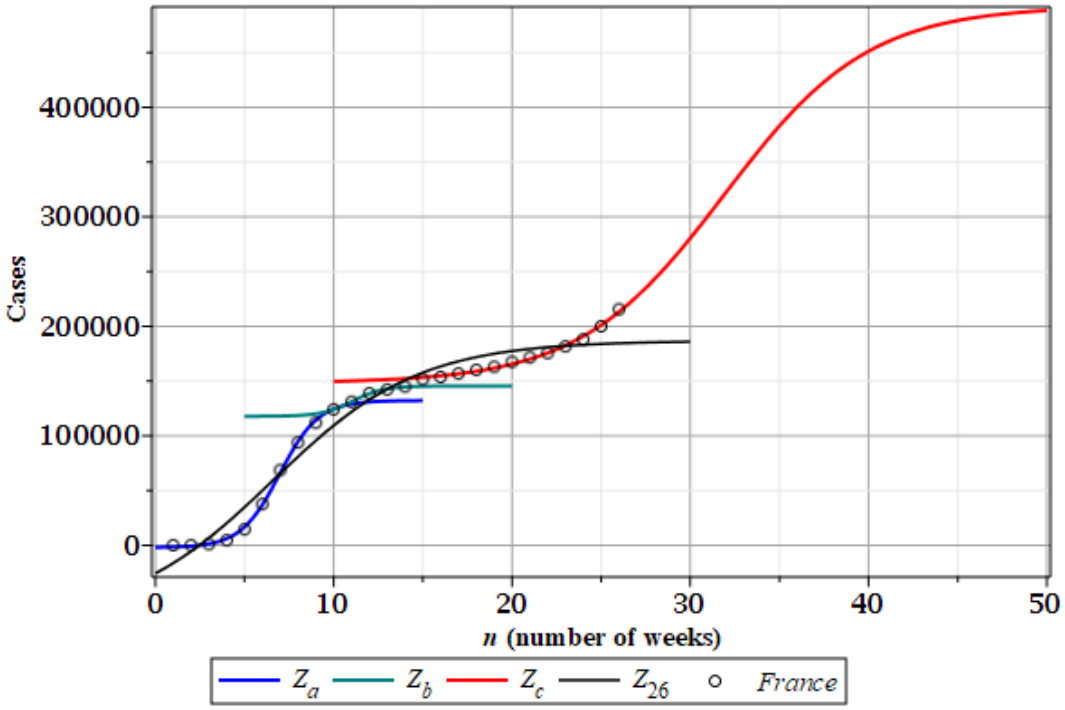


FIG. 23: Weekly Covid-19 case curves and rms (last wave) for the data from France.

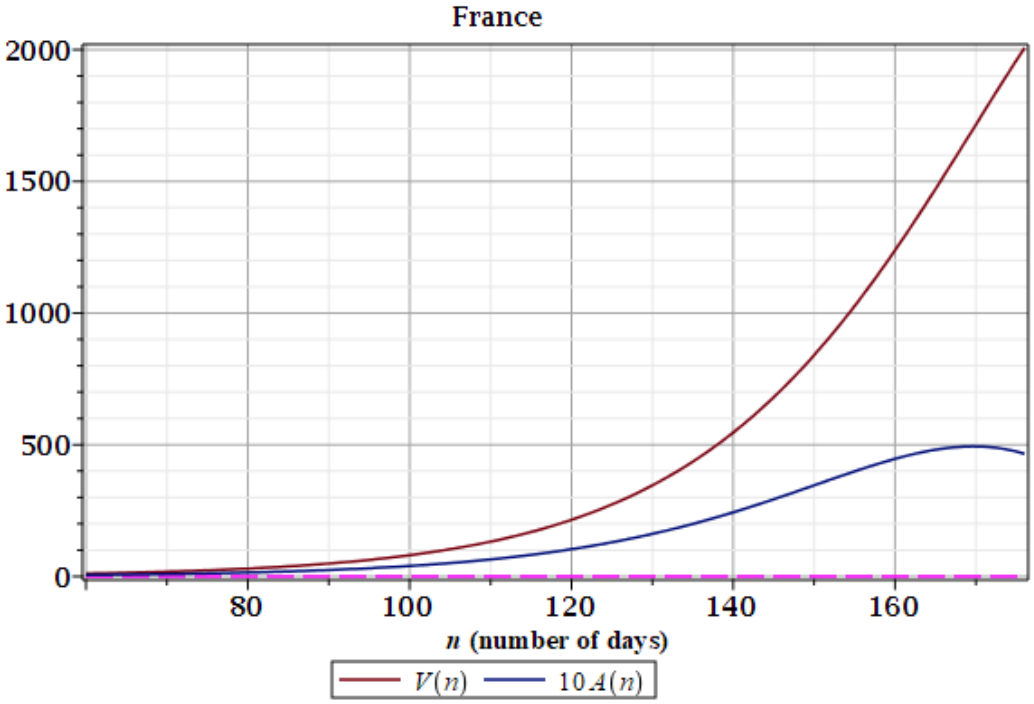


FIG. 24: Daily growth rates (speed V and acceleration A) from the fitted curve Z_c (the last wave) and inflection points for the last curves. Incomplete curves. Inflection point: absent.

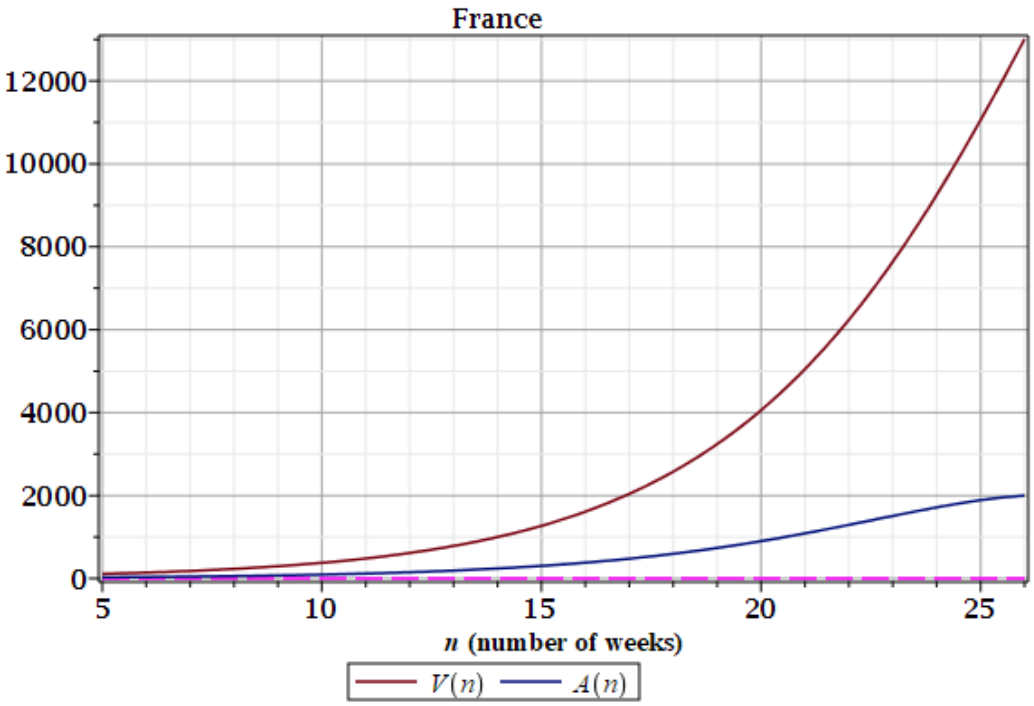


FIG. 25: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_c (the last wave) and inflection points for the last curves. Incomplete curves. Inflection point: absent.

IX.7. Portugal

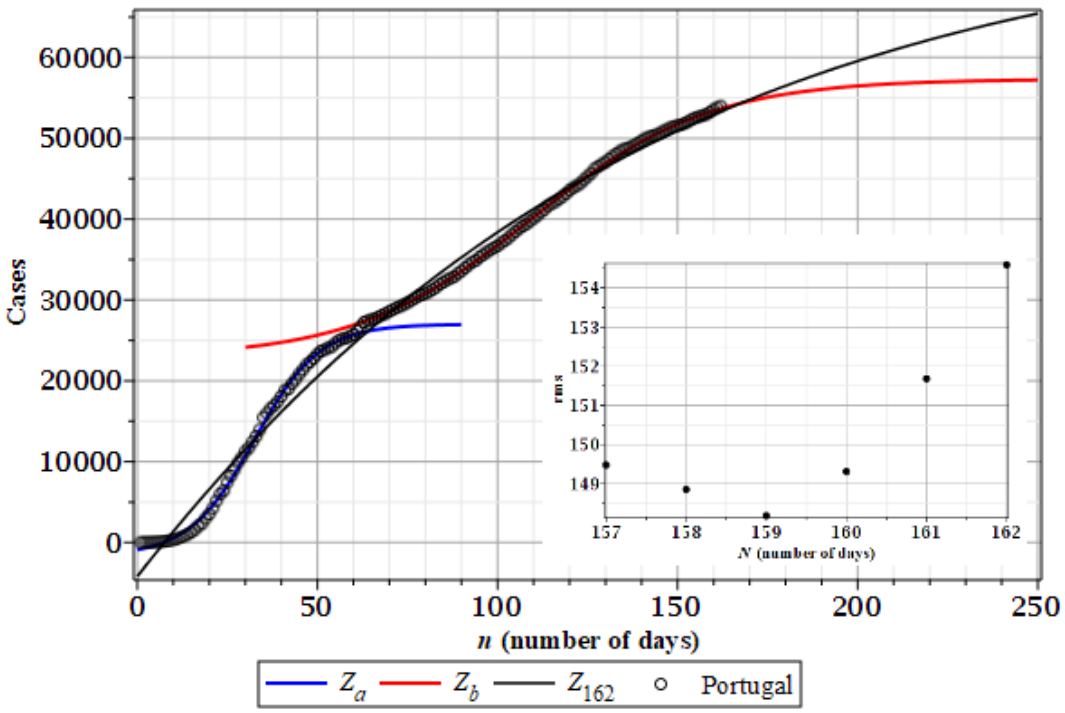


FIG. 26: Covid-19 case curves and rms (last wave) for the data from Portugal.

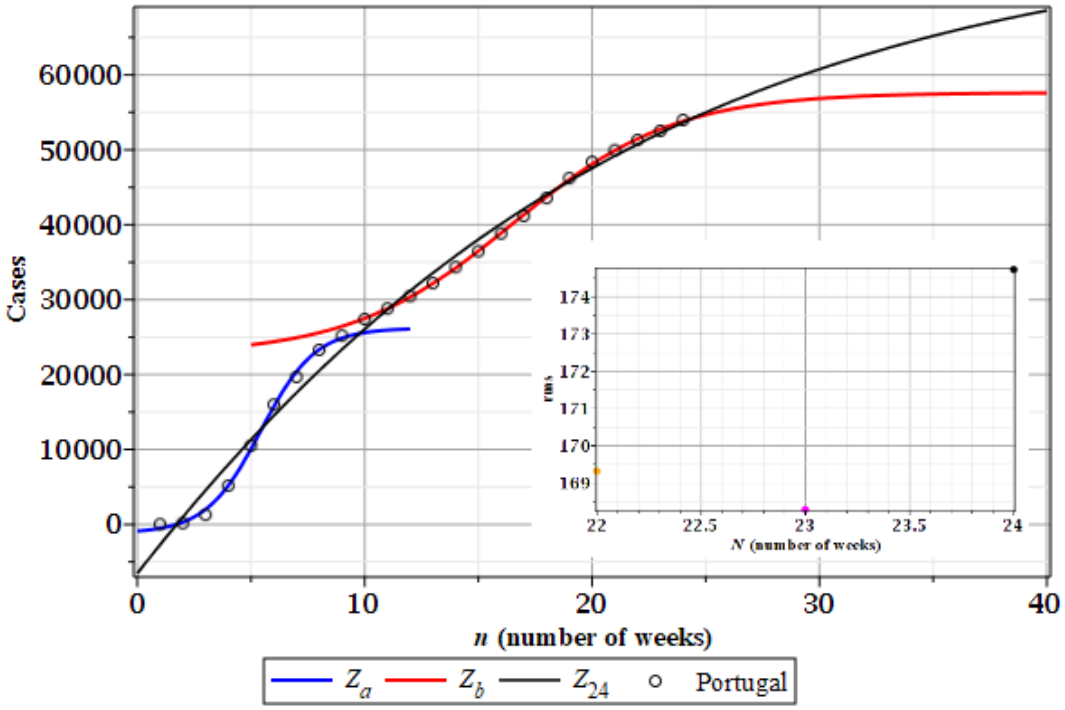


FIG. 27: Weekly Covid-19 case curves and rms (last wave) for the data from Portugal.

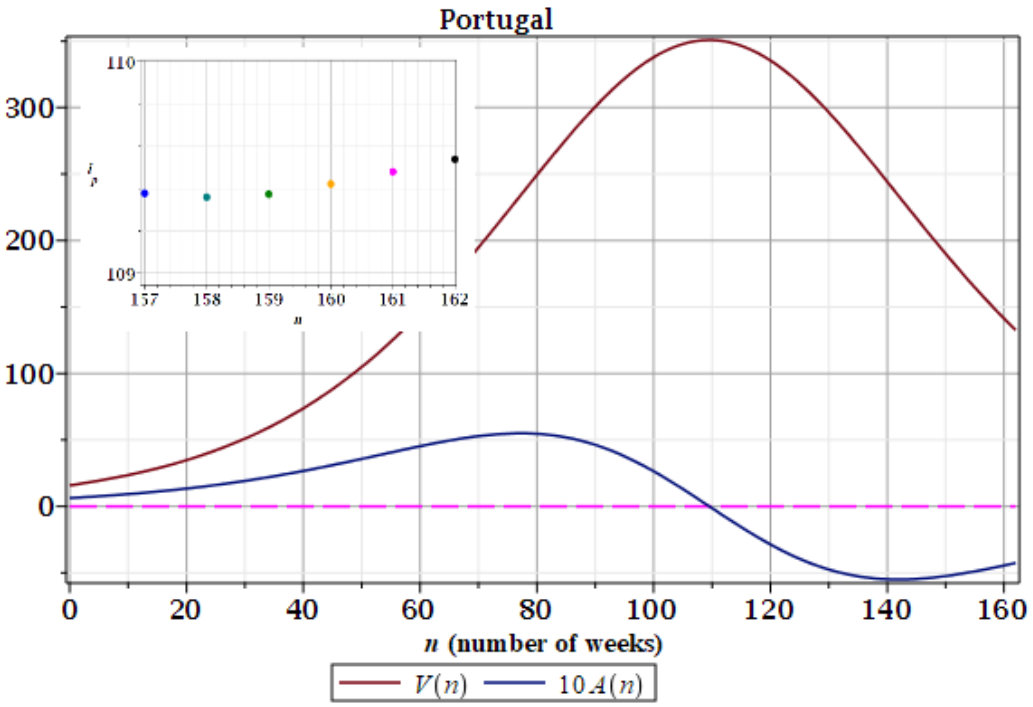


FIG. 28: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 109$.

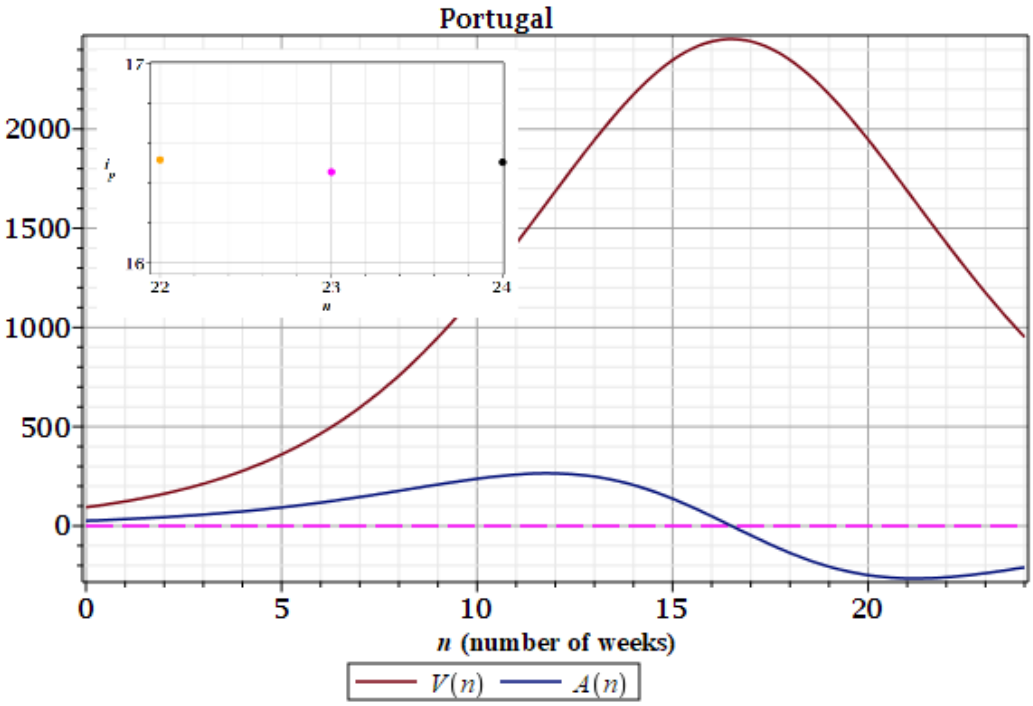


FIG. 29: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 16.5$.

IX.8. Switzerland

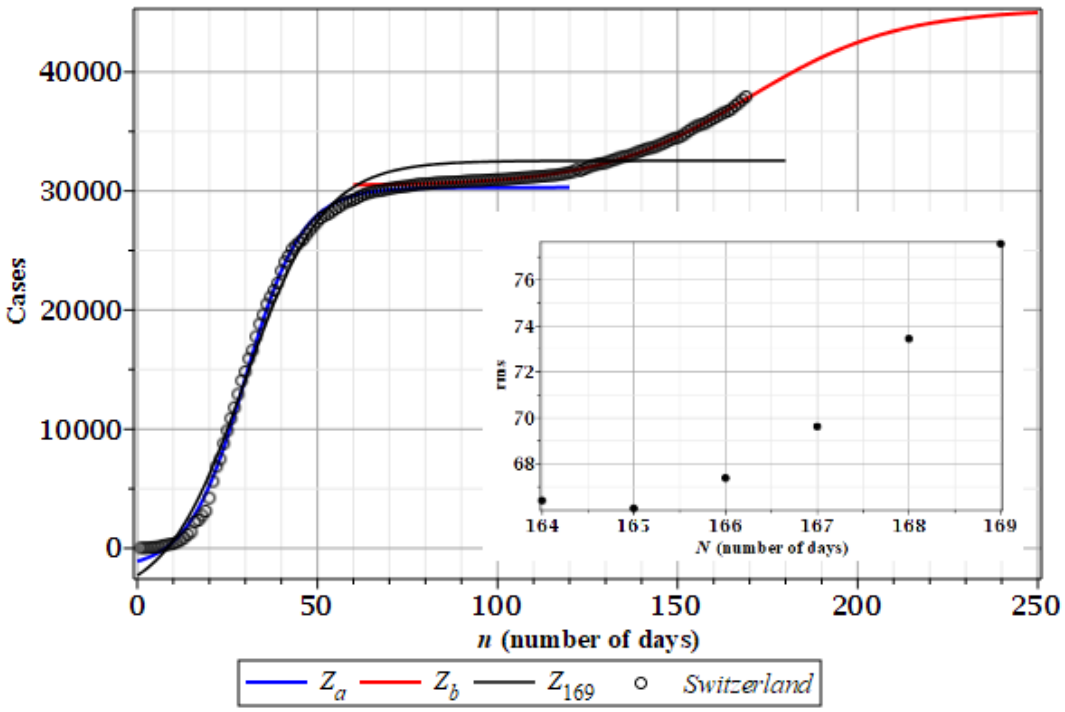


FIG. 30: Covid-19 case curves and rms (last wave) for the data from Switzerland.

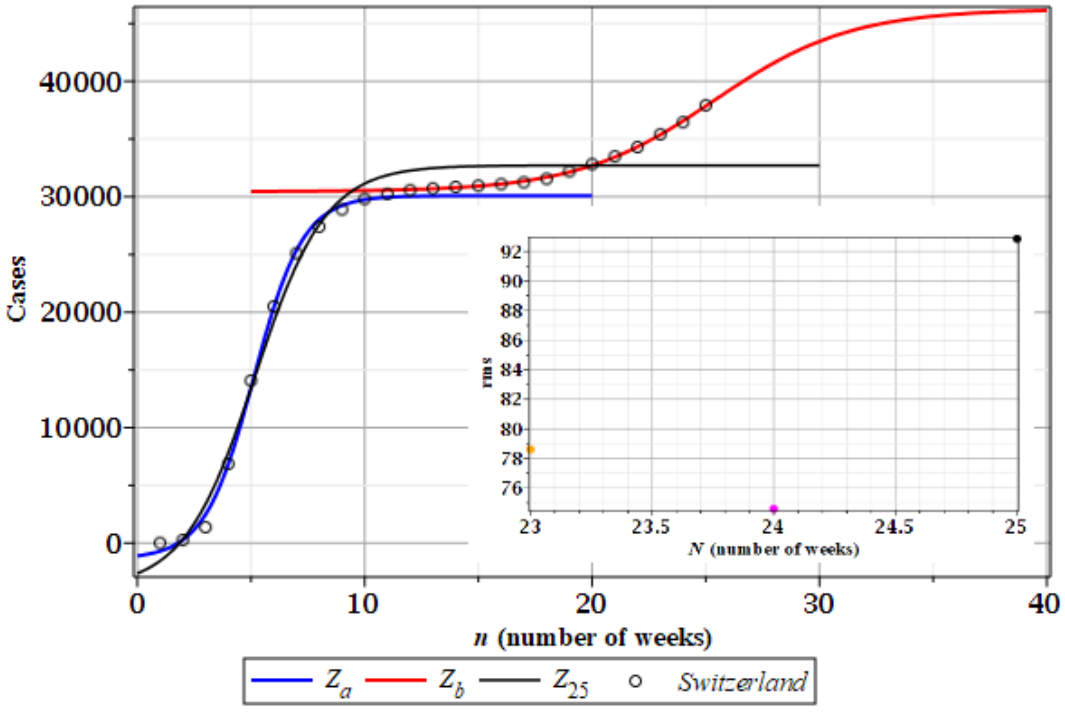


FIG. 31: Weekly Covid-19 case curves and rms (last wave) for the data from Switzerland.

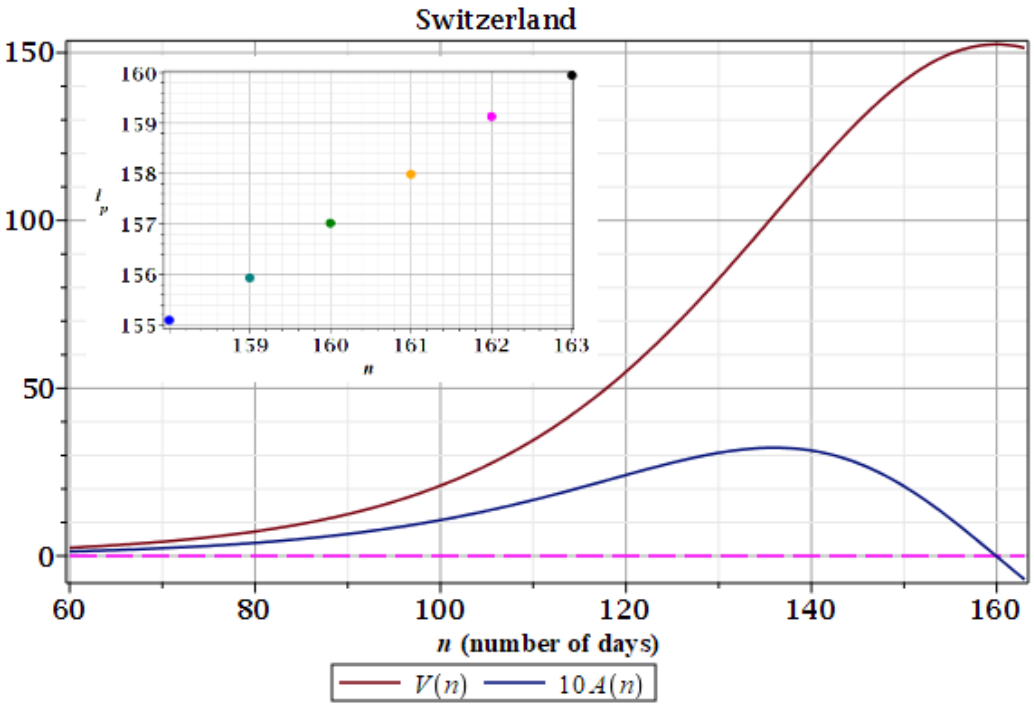


FIG. 32: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 170$.

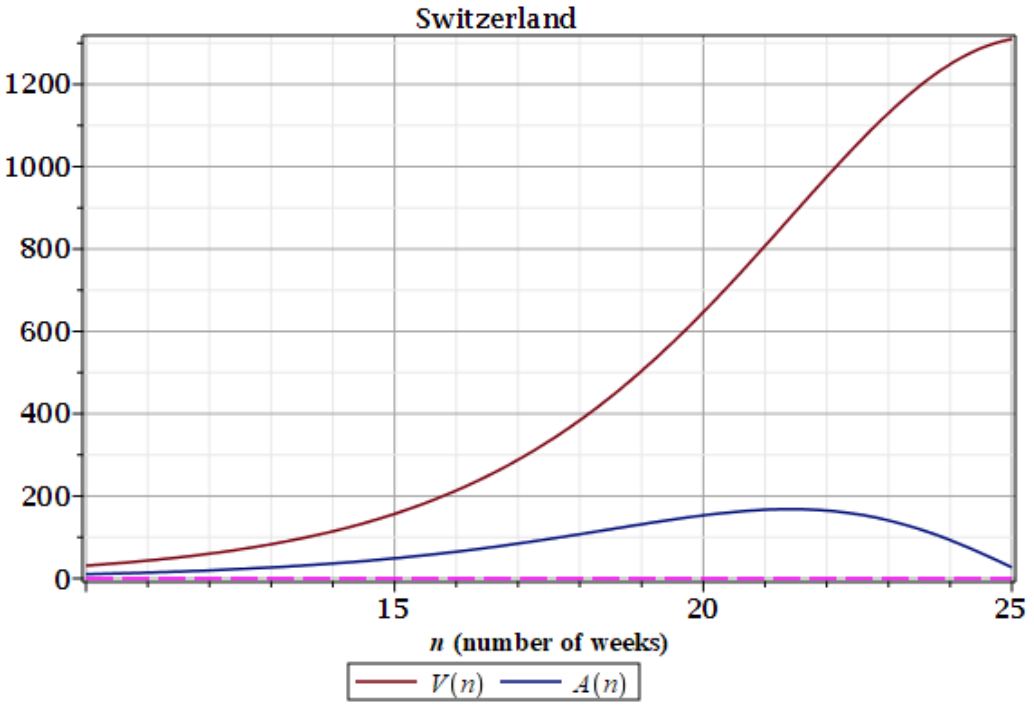


FIG. 33: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 25.4$.

IX.9. USA

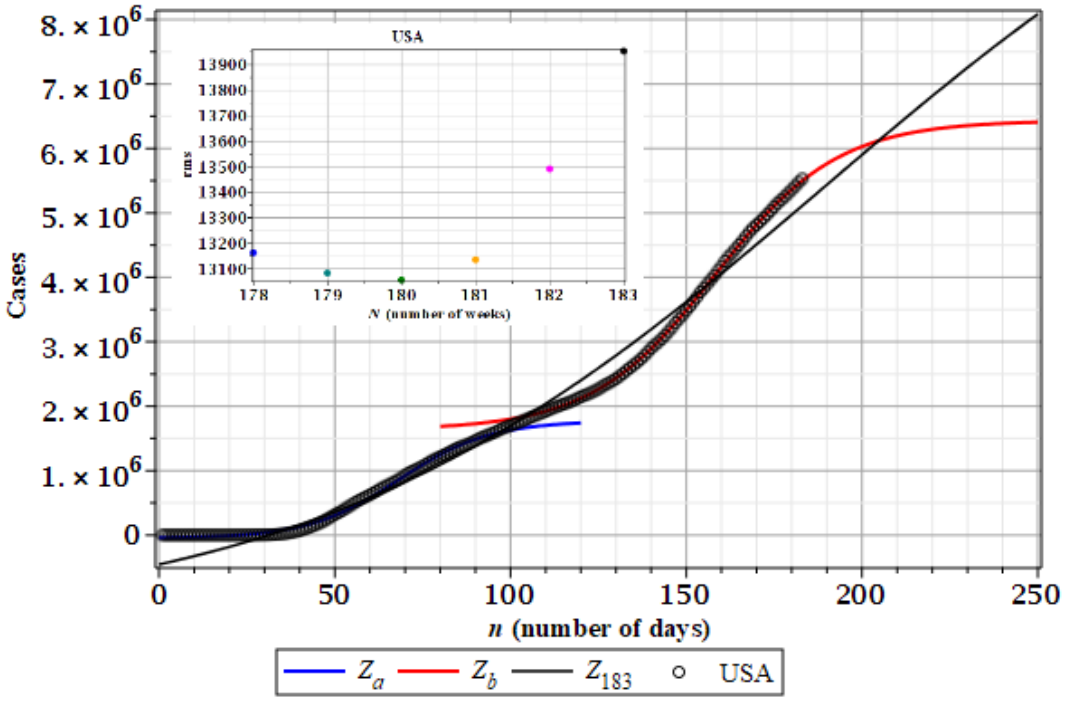


FIG. 34: Daily Covid-19 case curves and rms (last wave) for the data from USA.

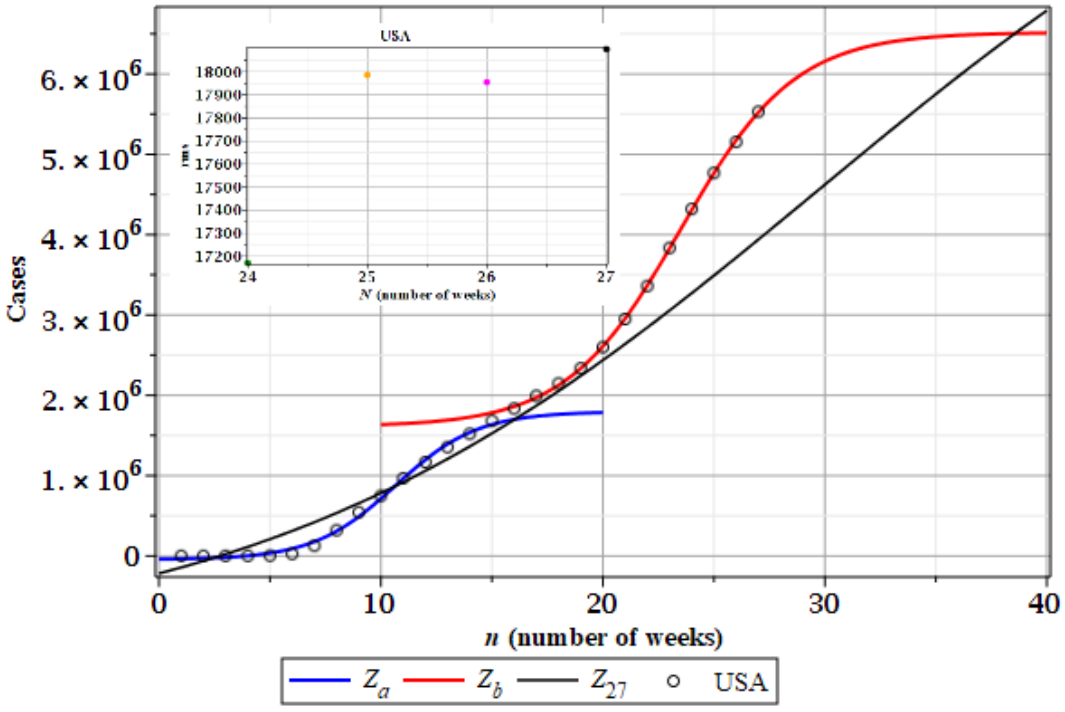


FIG. 35: Weekly Covid-19 case curves and rms (last wave) for the data from USA.

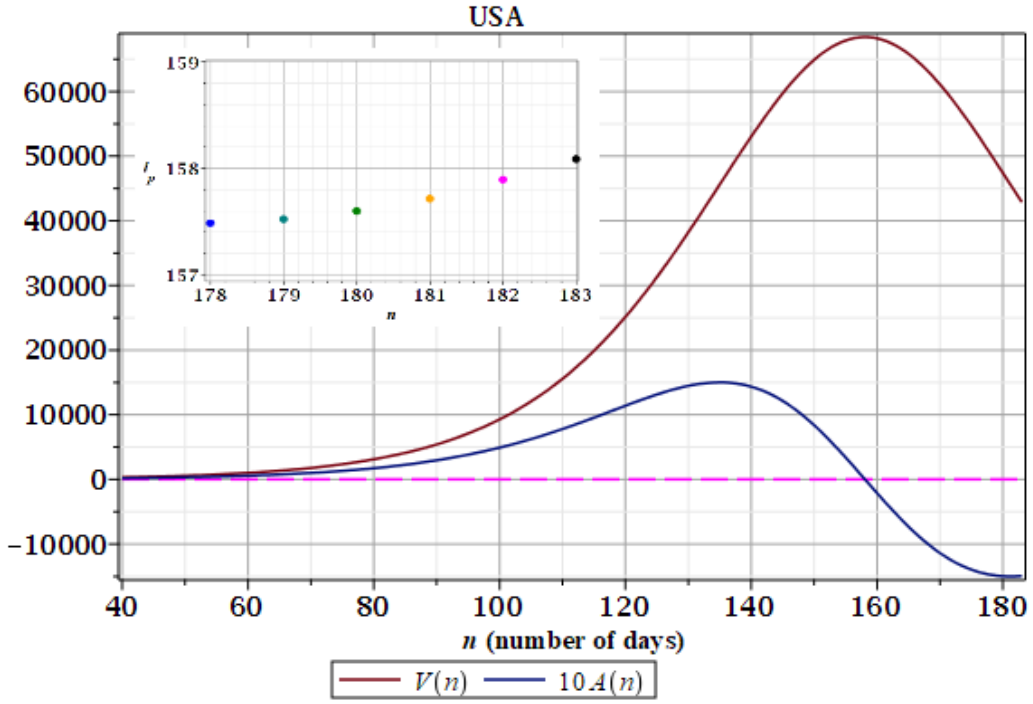


FIG. 36: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 158$.

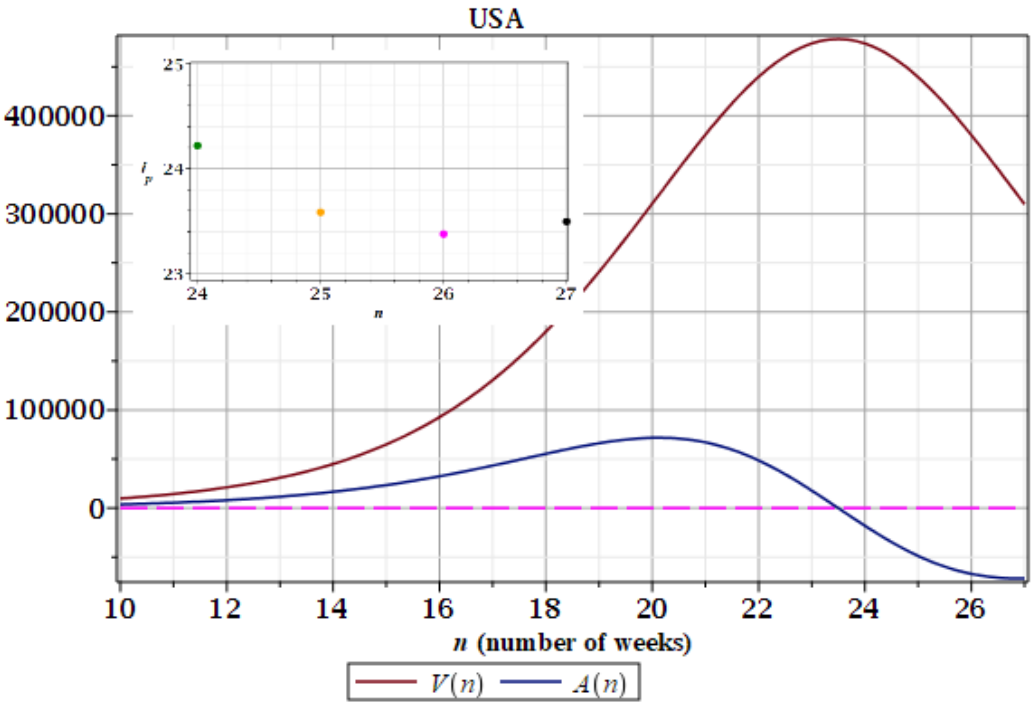


FIG. 37: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 23.5$.

IX.10. Mexico

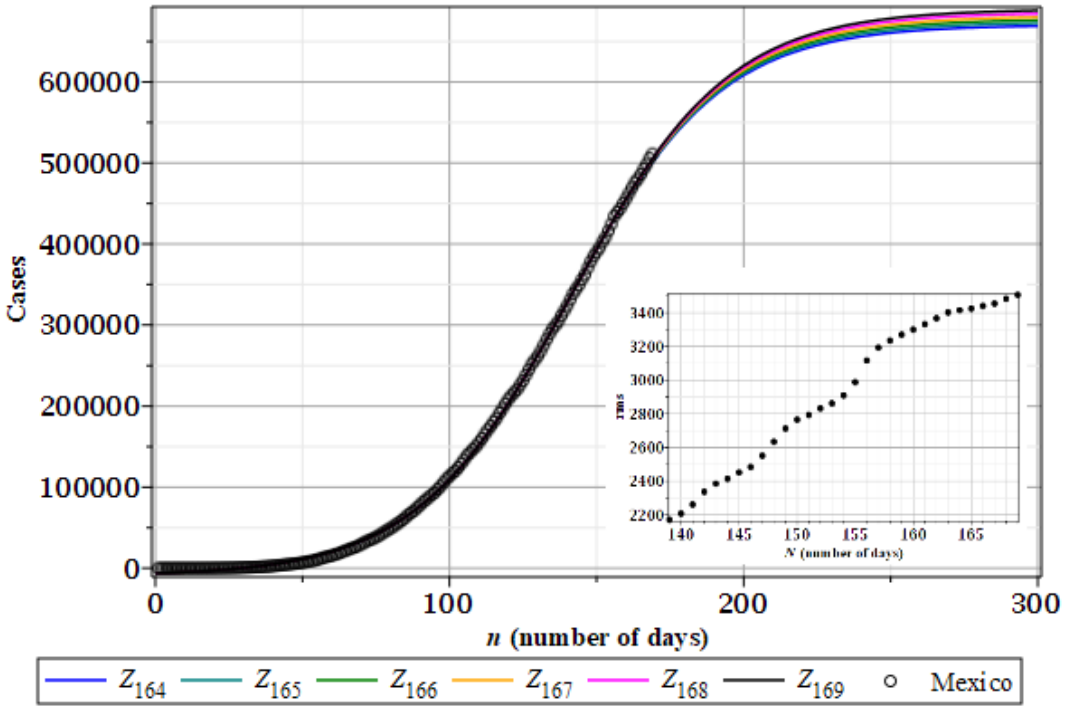


FIG. 38: Daily Covid-19 case curves and rms for the data from Mexico

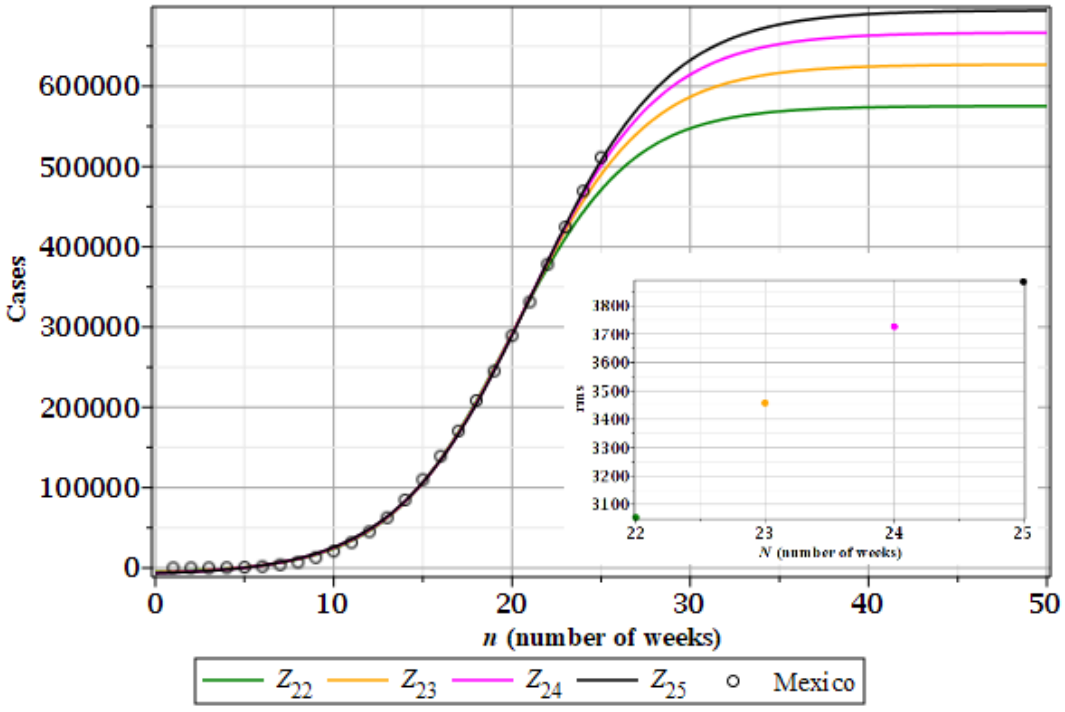


FIG. 39: Weekly Covid-19 case curves and rms for the data from Mexico.

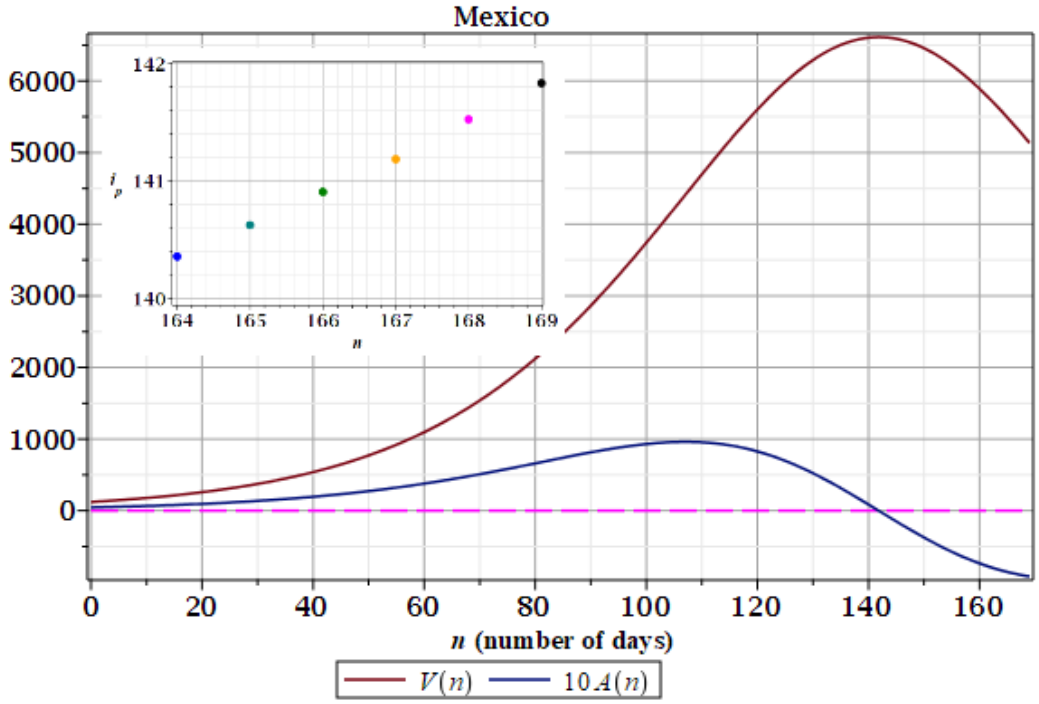


FIG. 40: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{169} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 142$.

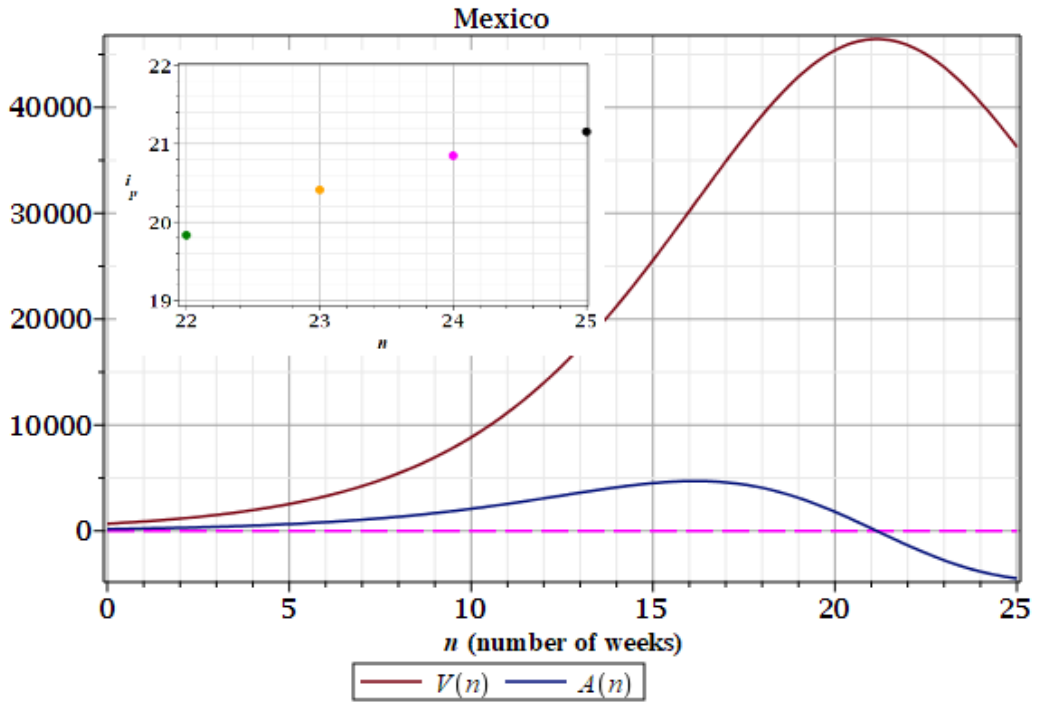


FIG. 41: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{25} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 21.1$.

IX.11. Canada

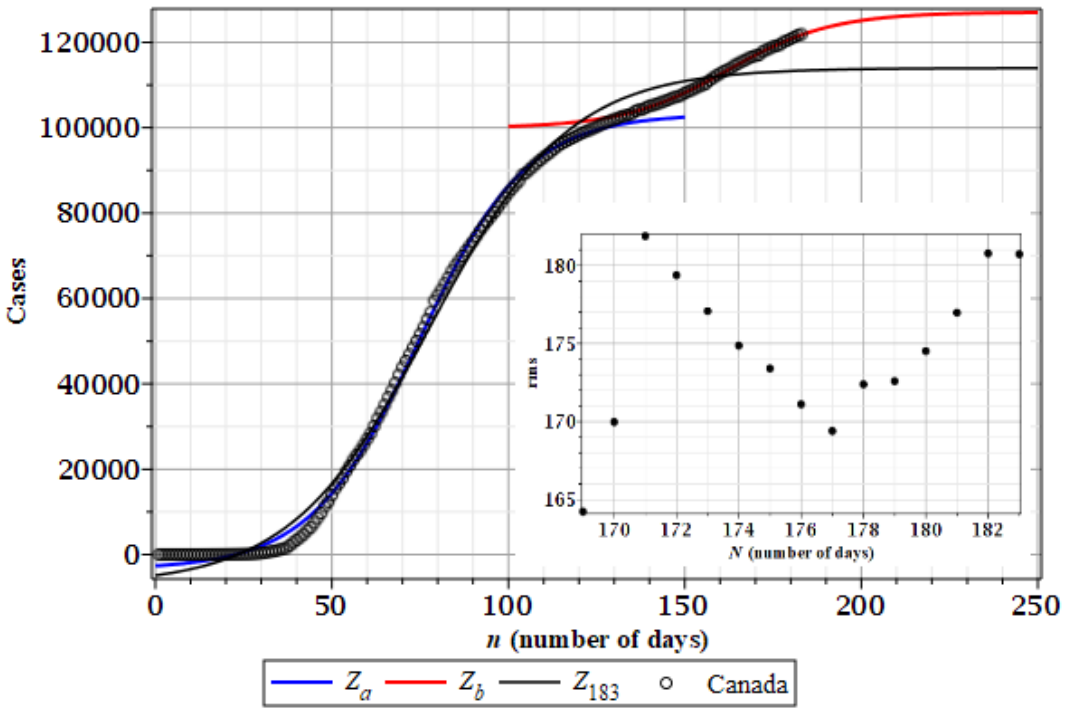


FIG. 42: Daily Covid-19 case curves and rms (last wave) for the data from Canada.

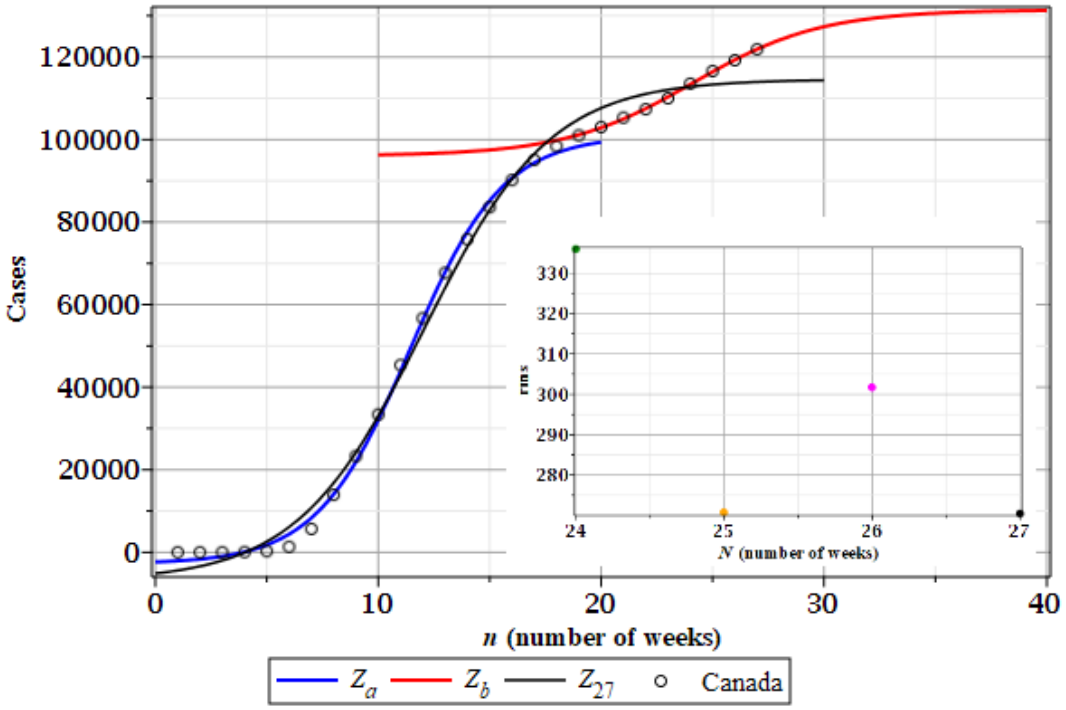


FIG. 43: Weekly Covid-19 case curves and rms (last wave) for the data from Canada.

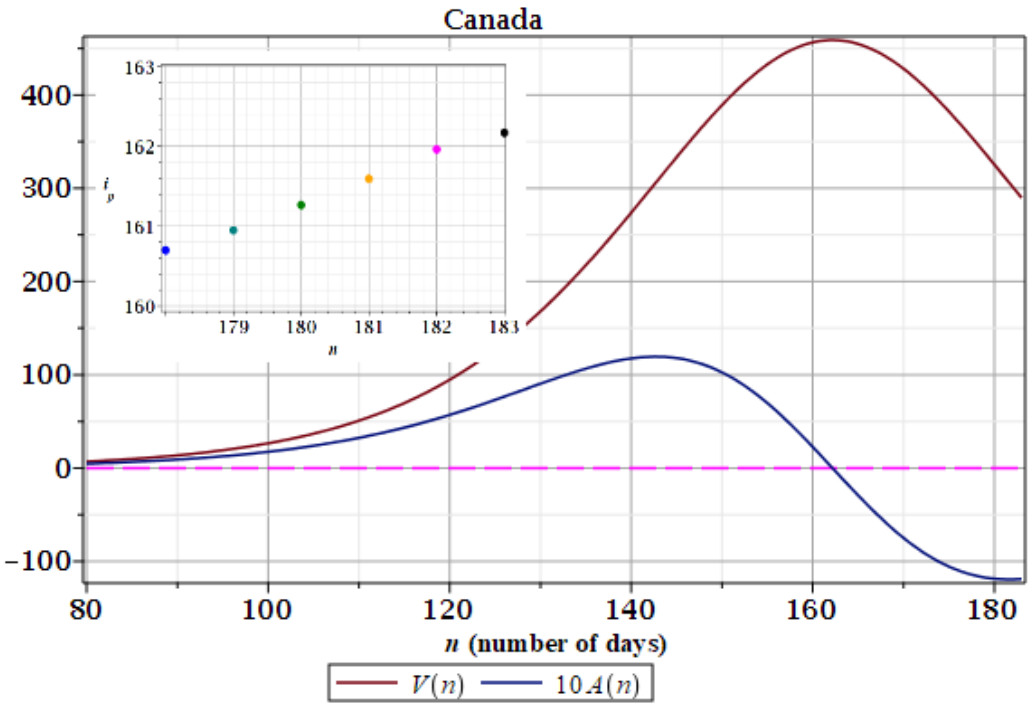


FIG. 44: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b (last wave) and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 162$.

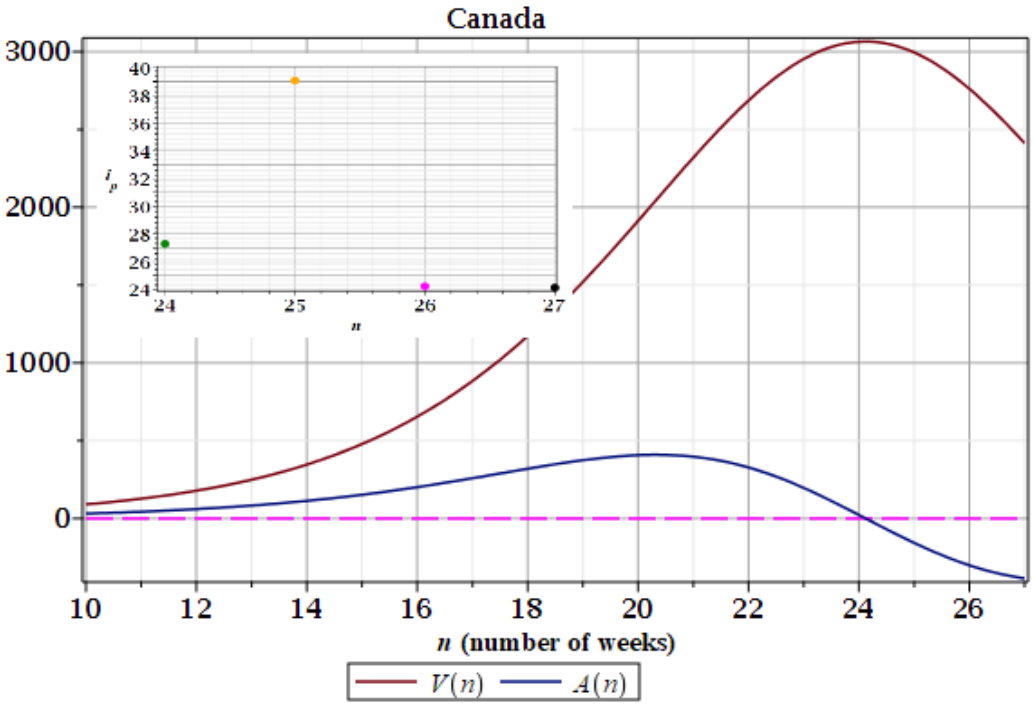


FIG. 45: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b (last wave) and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 24.1$.

IX.12. Cuba

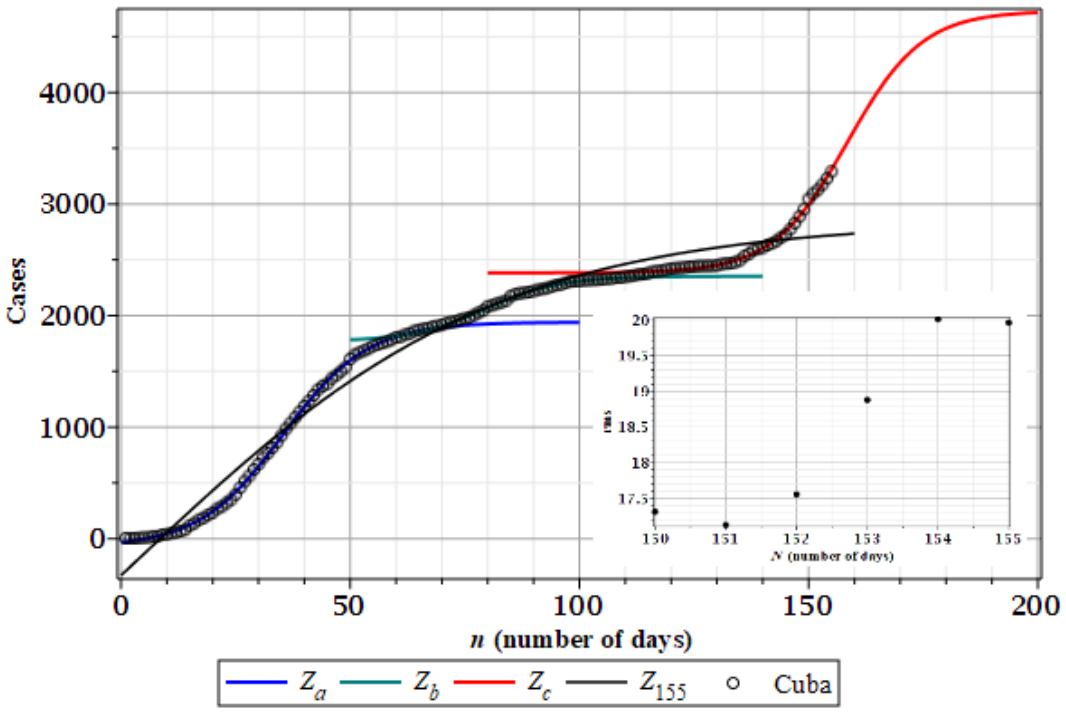


FIG. 46: Daily Covid-19 case curves and rms (last wave) for the data from Cuba.

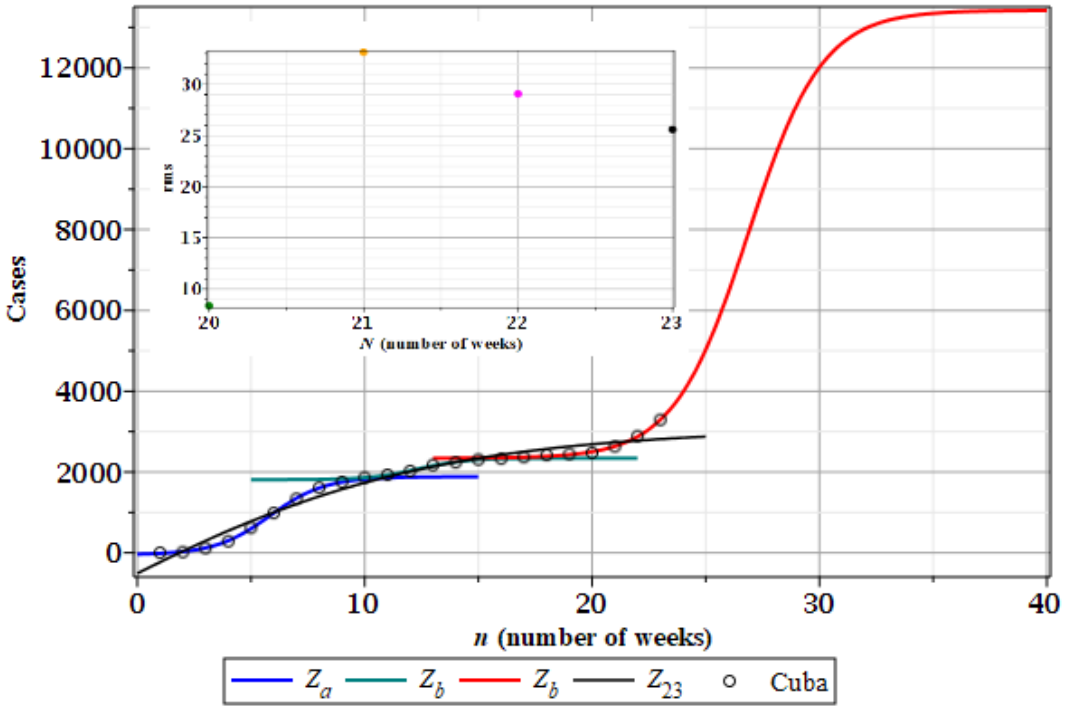


FIG. 47: Weekly Covid-19 case curves and rms (last wave) for the data from Cuba.

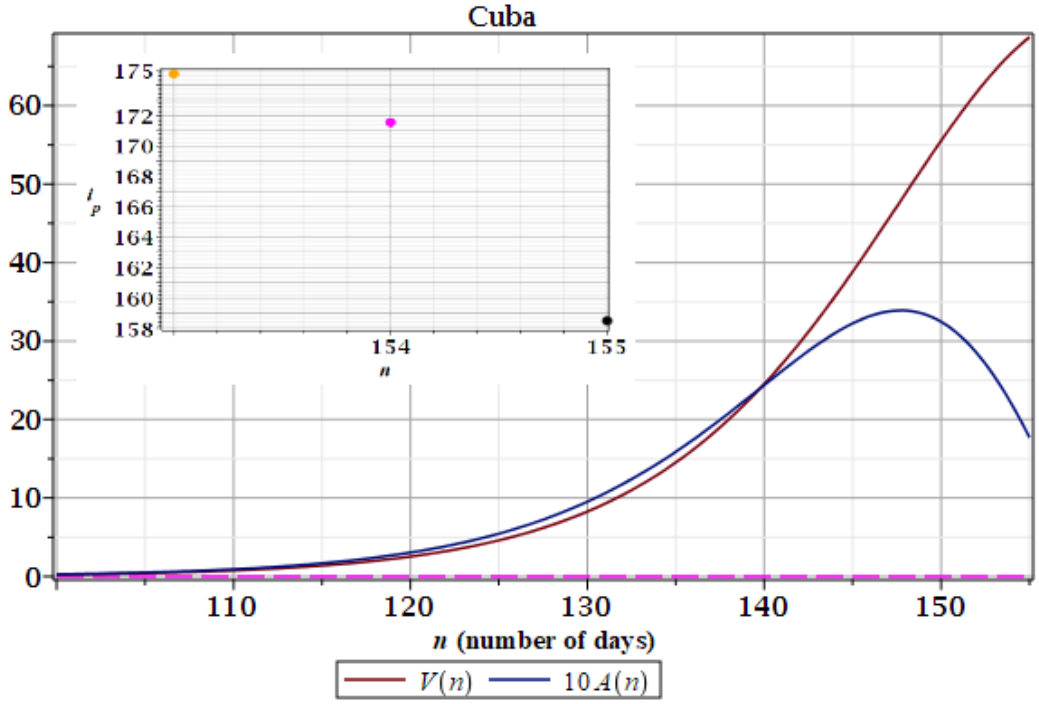


FIG. 48: Daily growth rates (speed V and acceleration A) from the fitted curve Z_c (last wave) and inflection points for the last curves. Incomplete curves. Inflection point: absent.

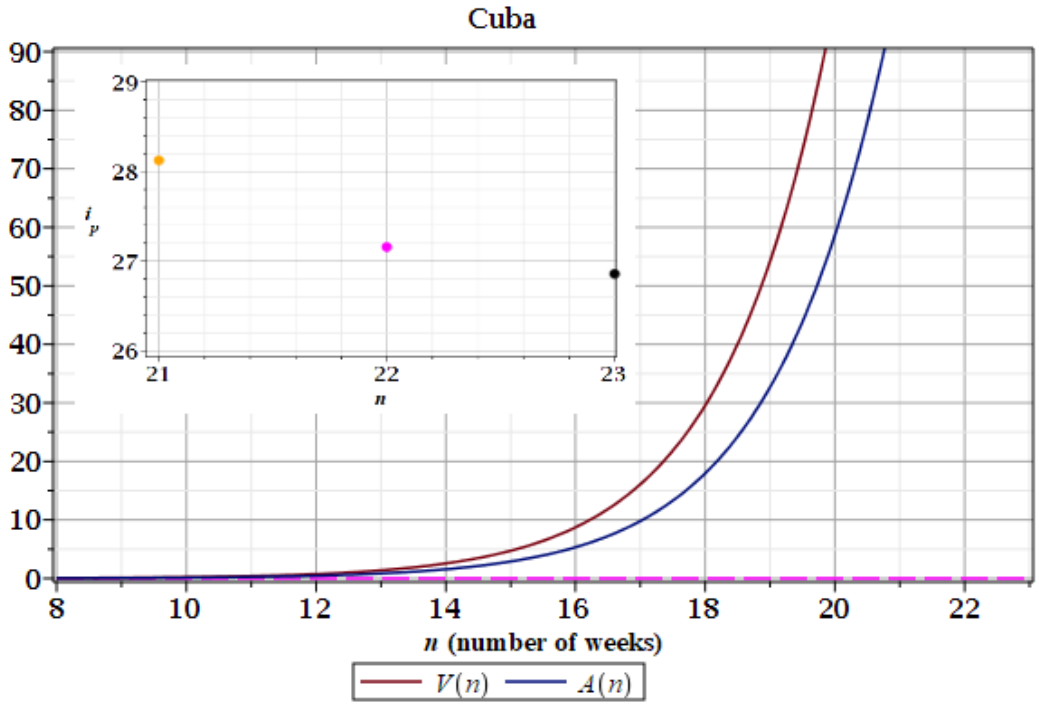


FIG. 49: Weekly growth rates (speed V and acceleration A) from the last wave Z_c (last wave) and inflection points for the last curves. Incomplete curves. Inflection point: absent.

IX.13. Brazil

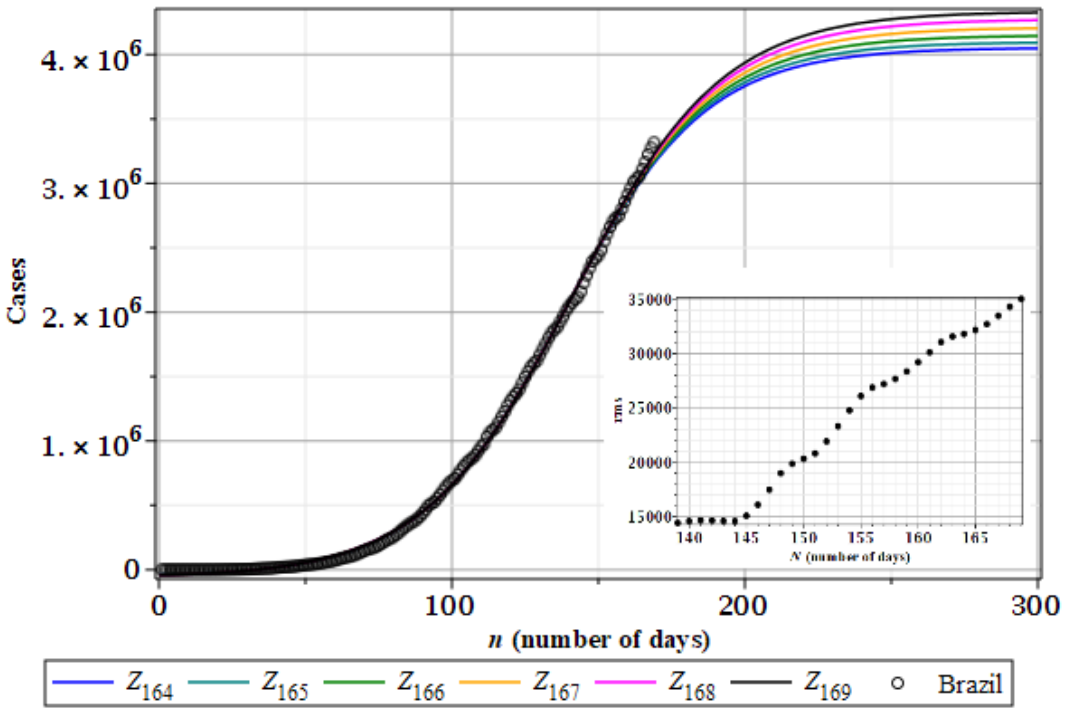


FIG. 50: Daily Covid-19 case curves and rms for the data from Brazil.

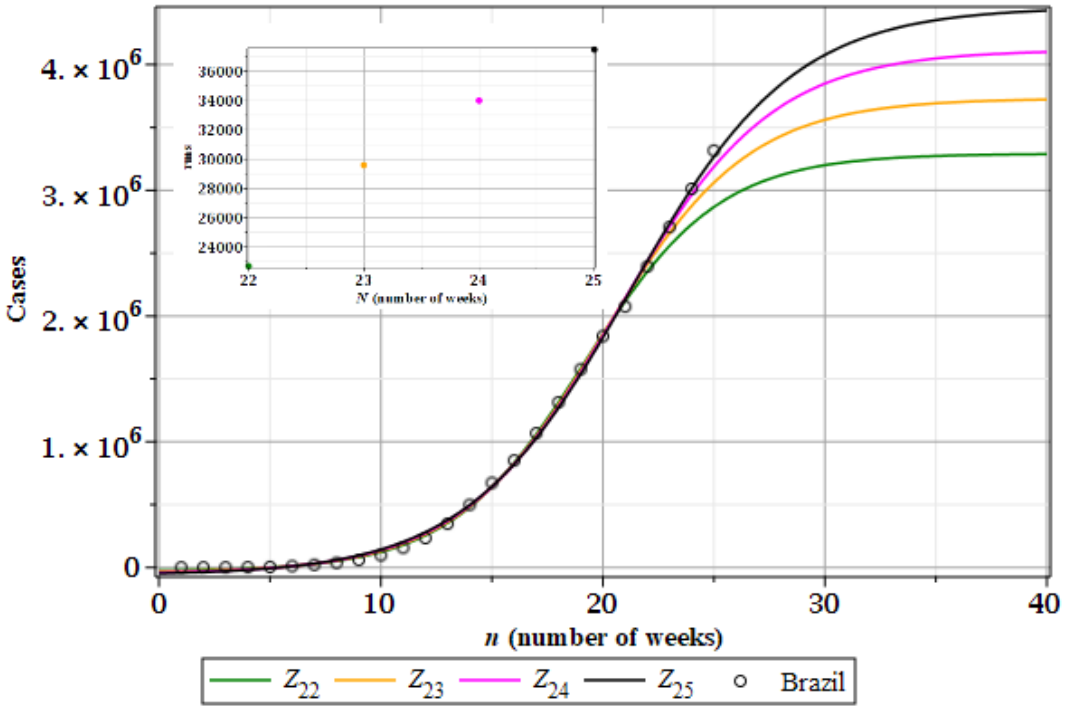


FIG. 51: Weekly Covid-19 case curves and rms for the data from Brazil.

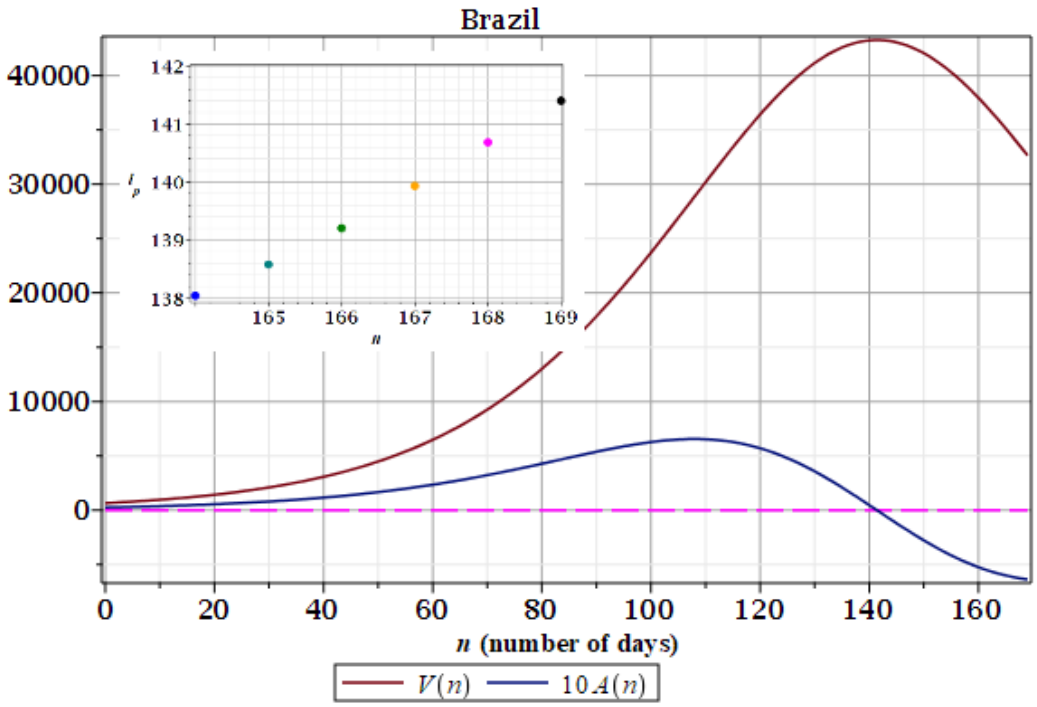


FIG. 52: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{169} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 141$.

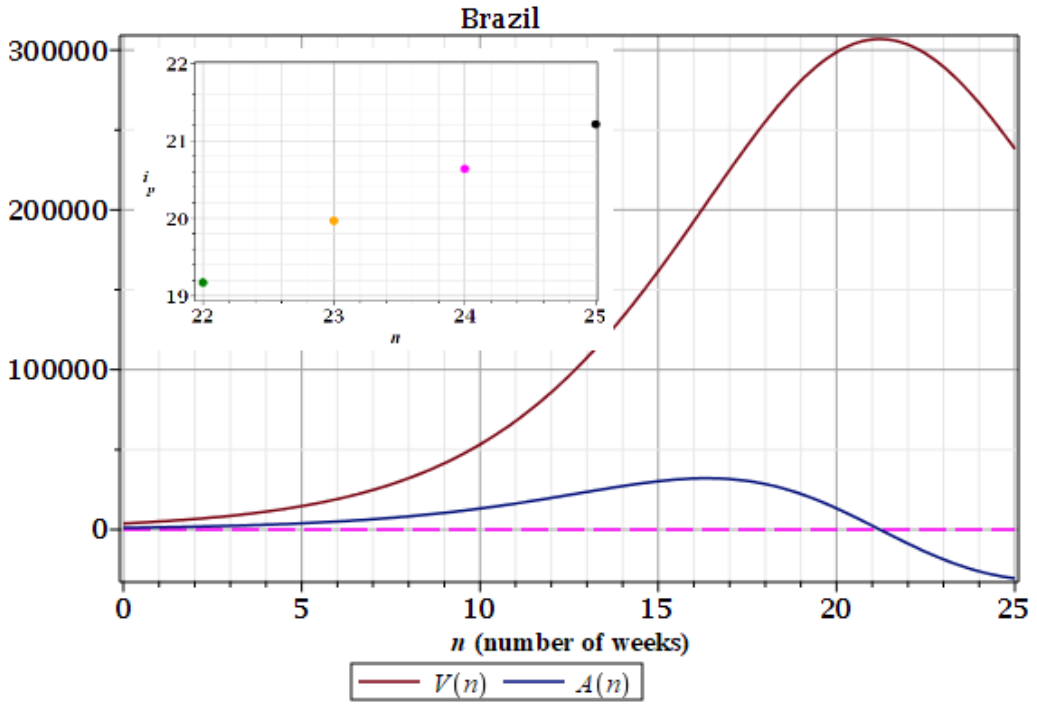


FIG. 53: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{25} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 21.2$.

IX.14. Peru

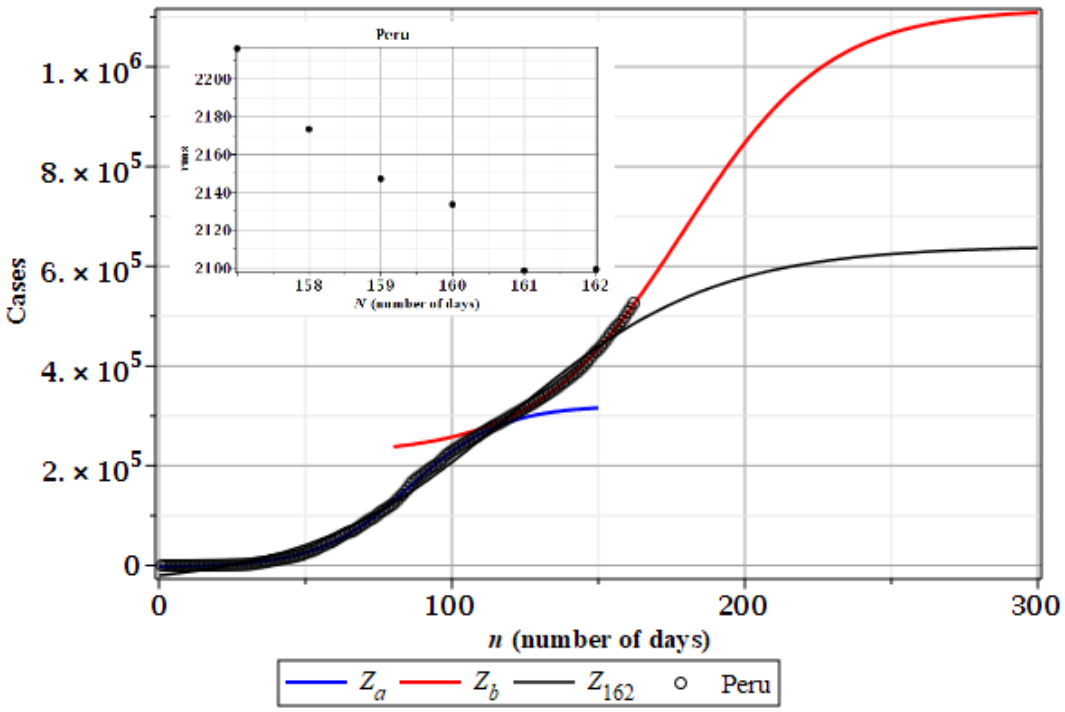


FIG. 54: Daily Covid-19 case curves and rms (last wave) for the data from Peru.

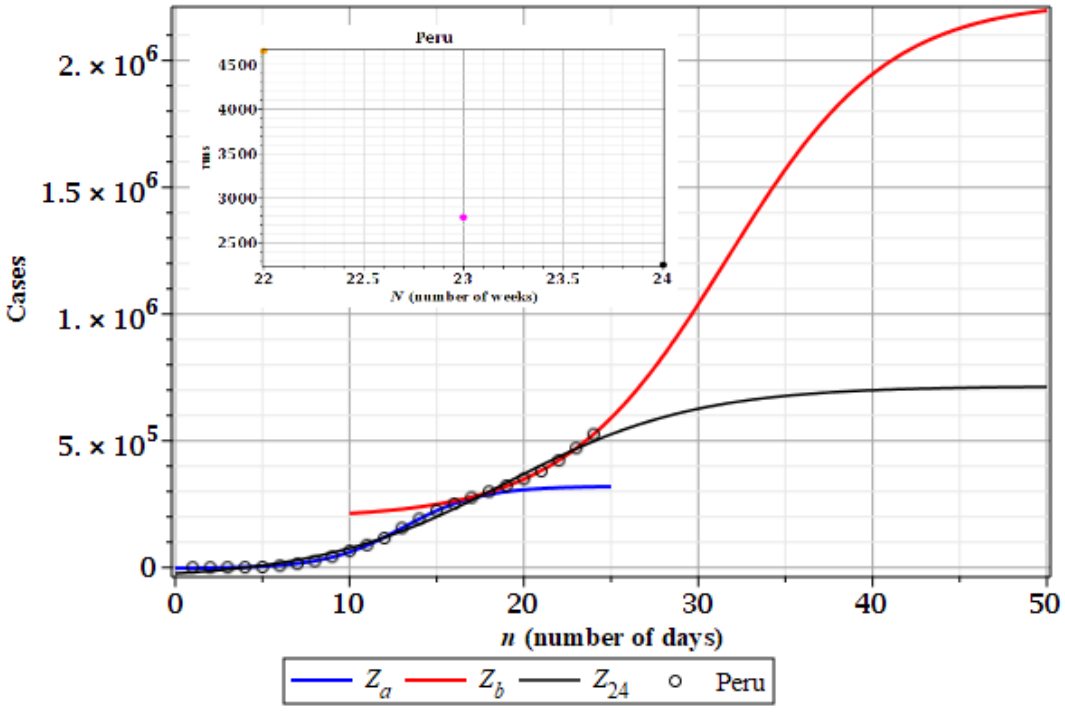


FIG. 55: Weekly Covid-19 case curves and rms (last wave) for the data from Peru.

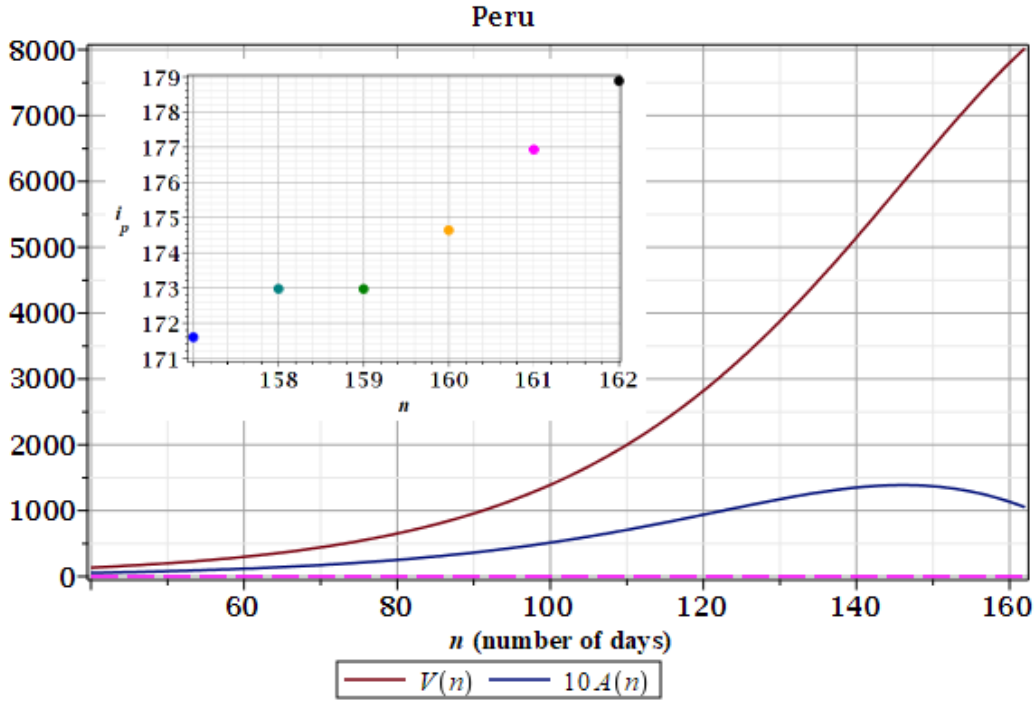


FIG. 56: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{162} and inflection points for the last curves in the last wave. Incomplete curves. Inflection point: absent.

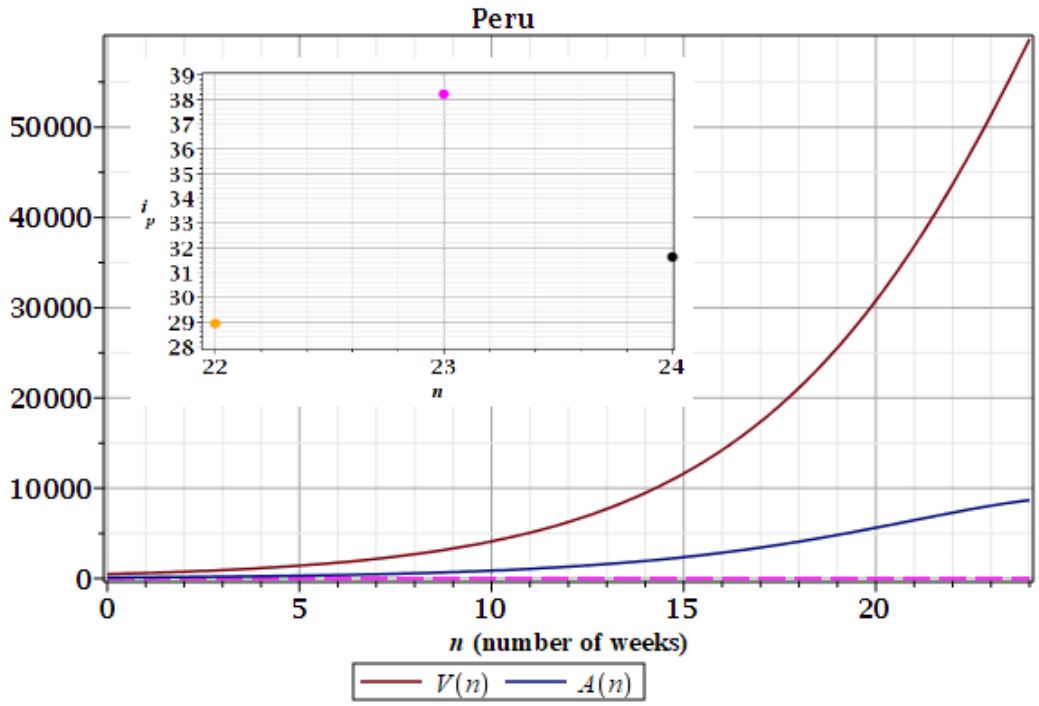


FIG. 57: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{24} and inflection points for the last curves in the last wave. Incomplete curves. Inflection point: absent.

IX.15. Colombia

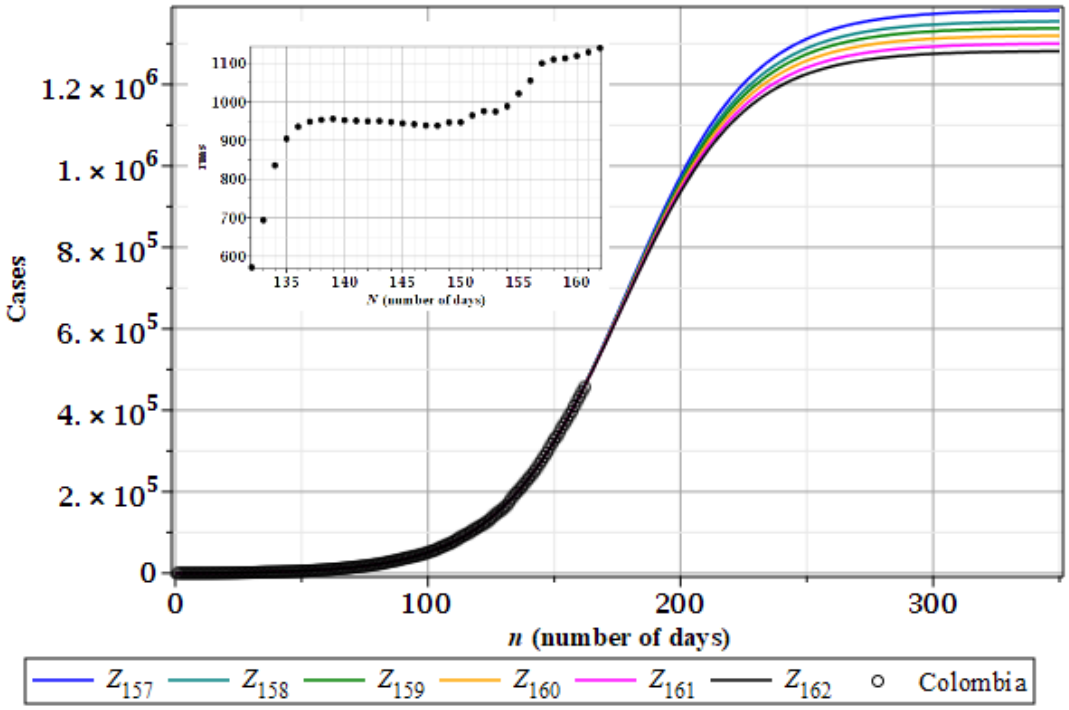


FIG. 58: Daily Covid-19 case curves and rms for the data from Colombia.

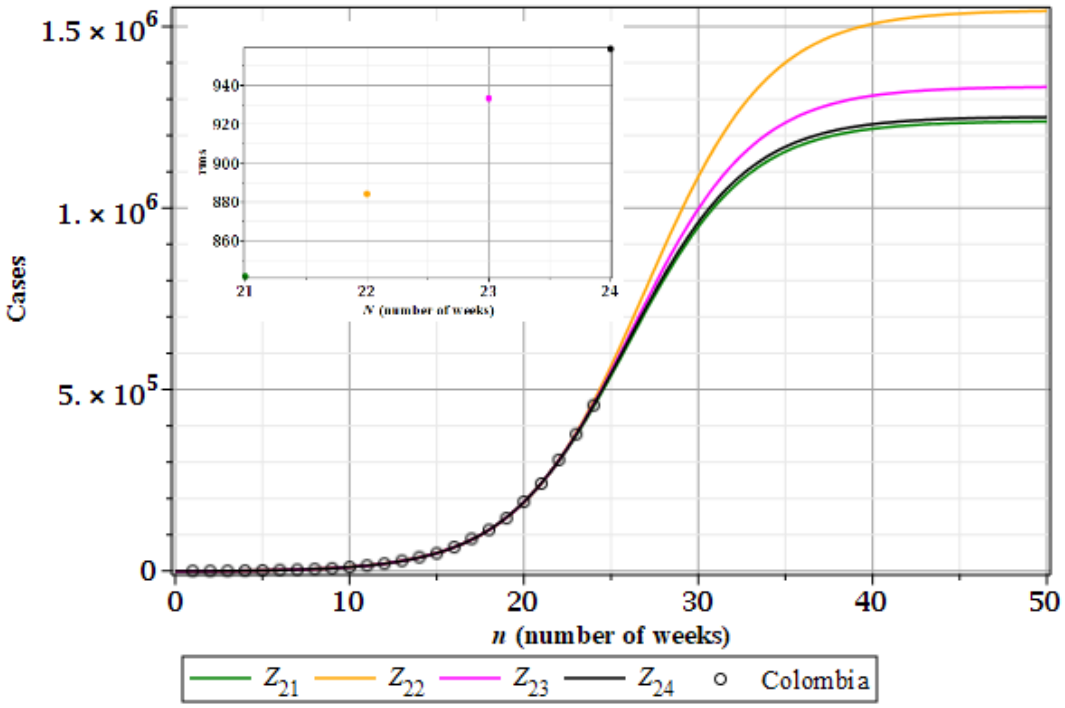


FIG. 59: Weekly Covid-19 case curves and rms for the data from Colombia.

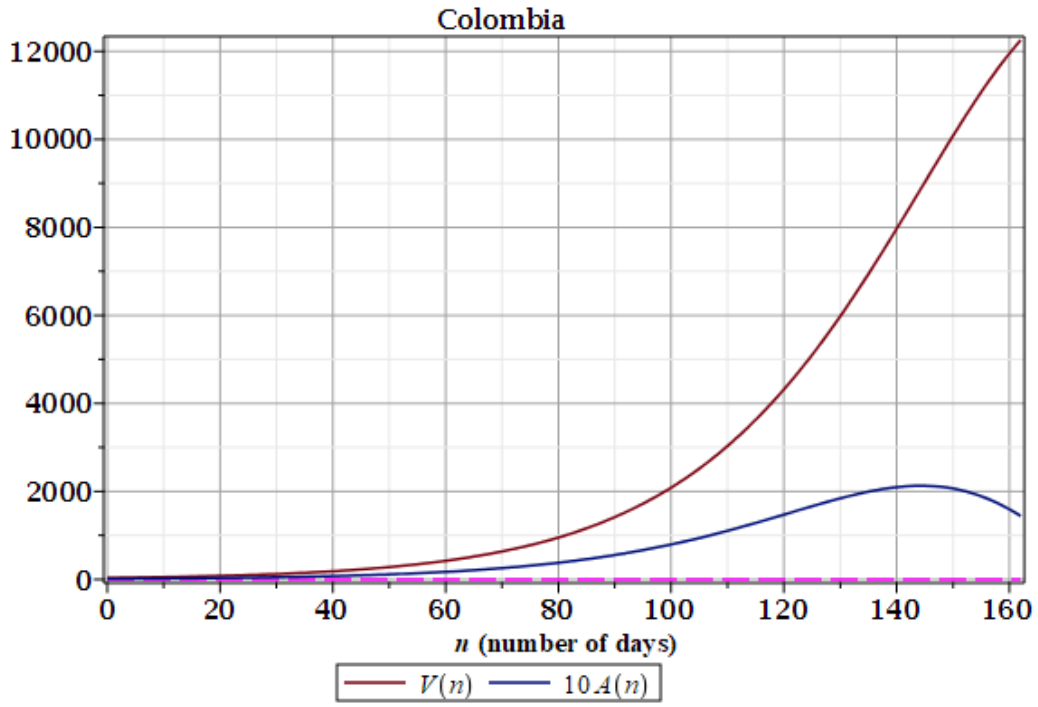


FIG. 60: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{162} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

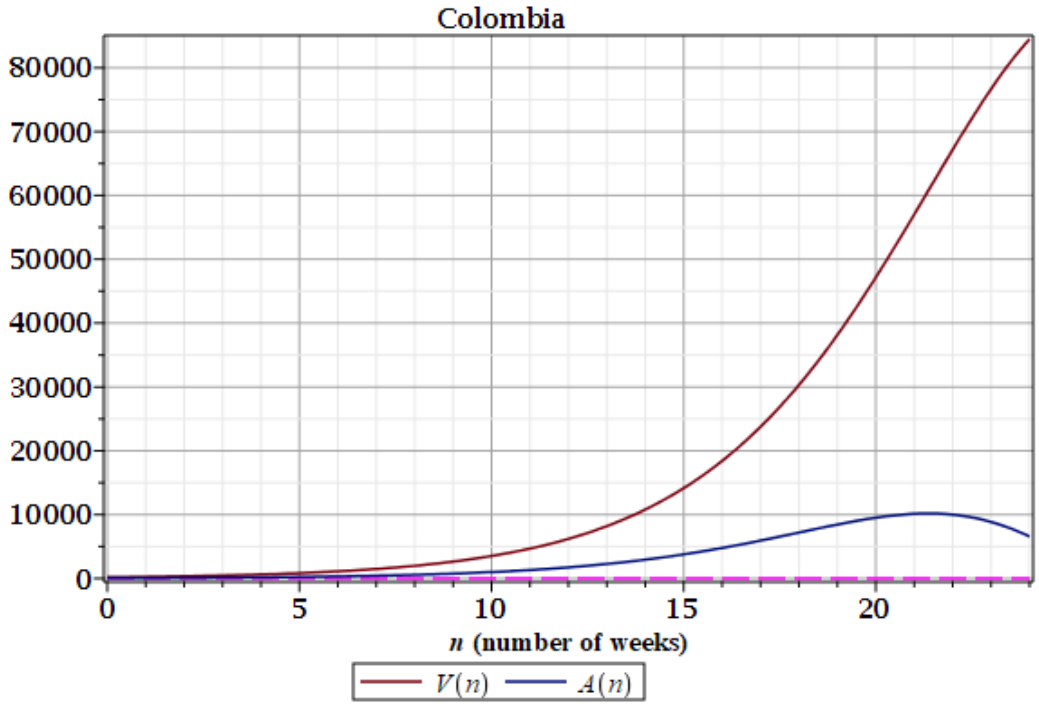


FIG. 61: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{24} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

IX.16. Chile

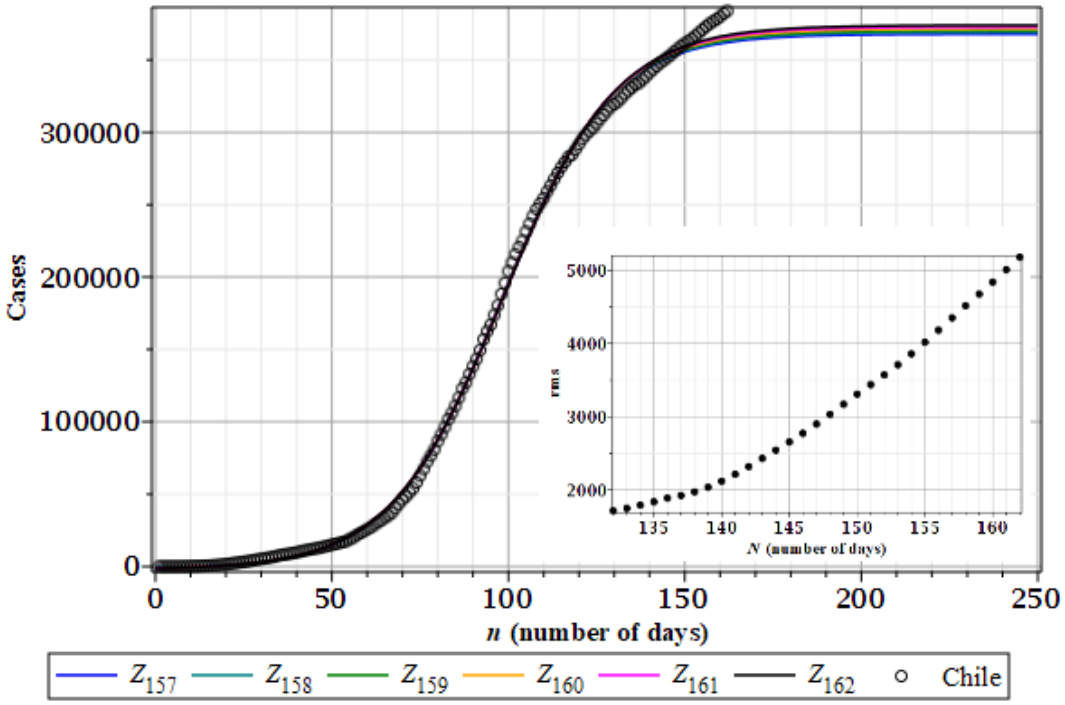


FIG. 62: Daily Covid-19 case curves and rms for the data from Chile.

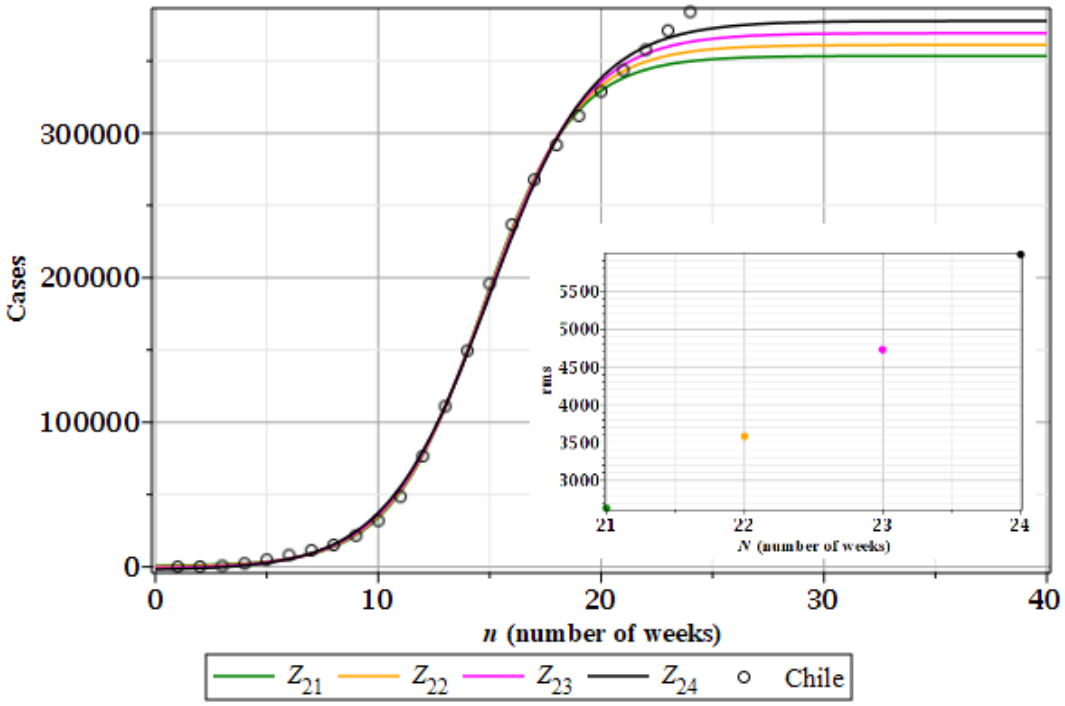


FIG. 63: Weekly Covid-19 case curves and rms for the data from Chile.

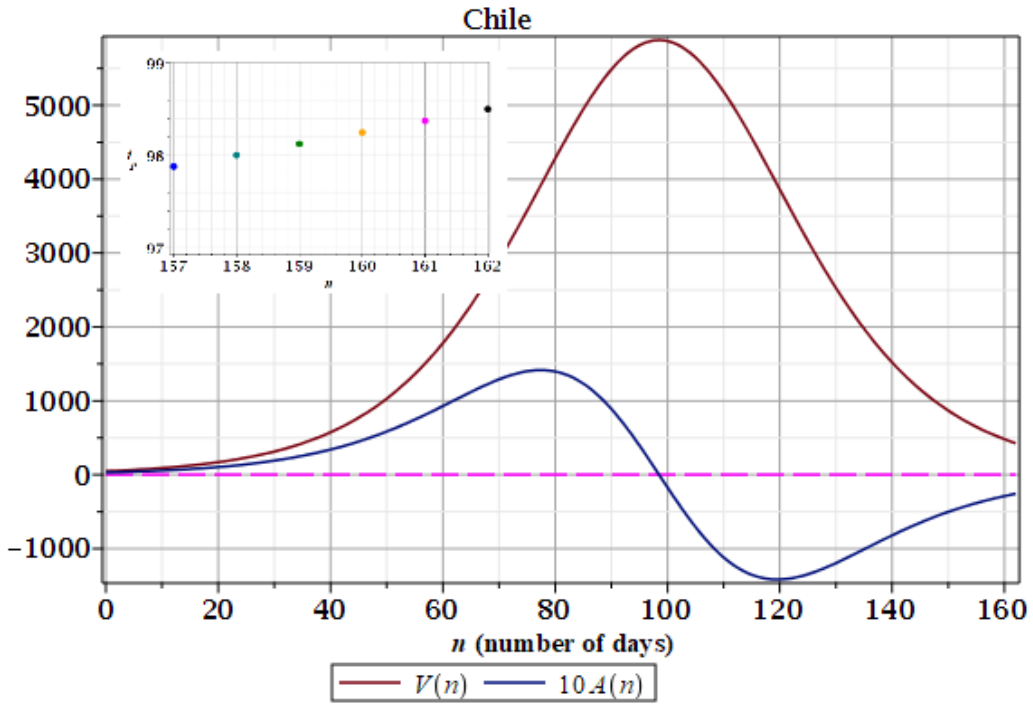


FIG. 64: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{162} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 98$.

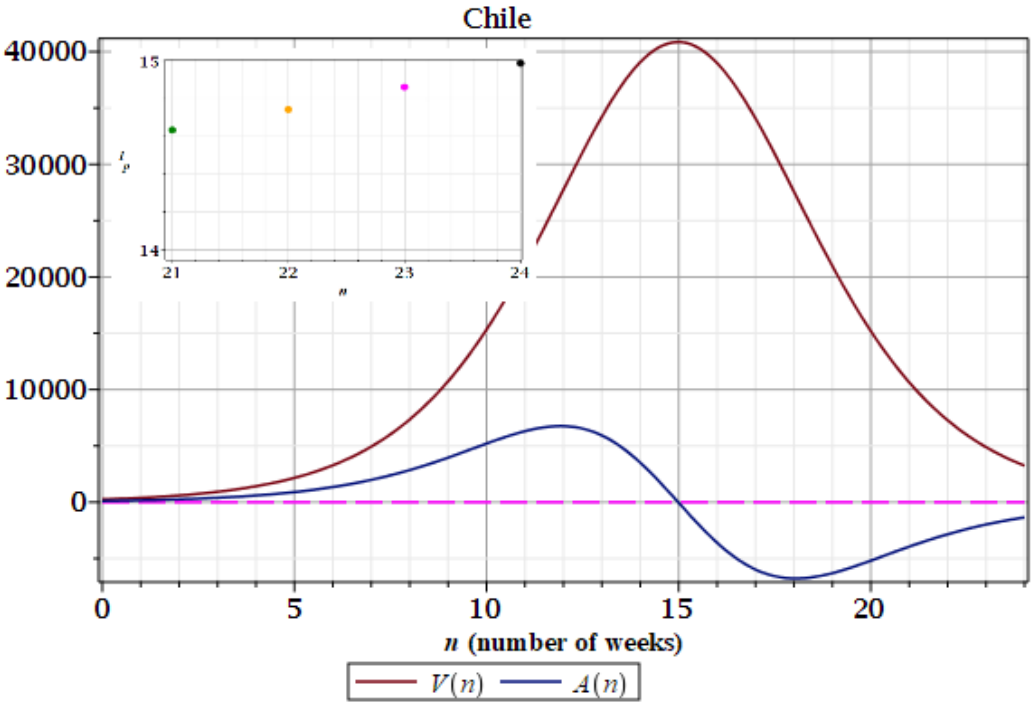


FIG. 65: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{24} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 15$.

IX.17. Argentina

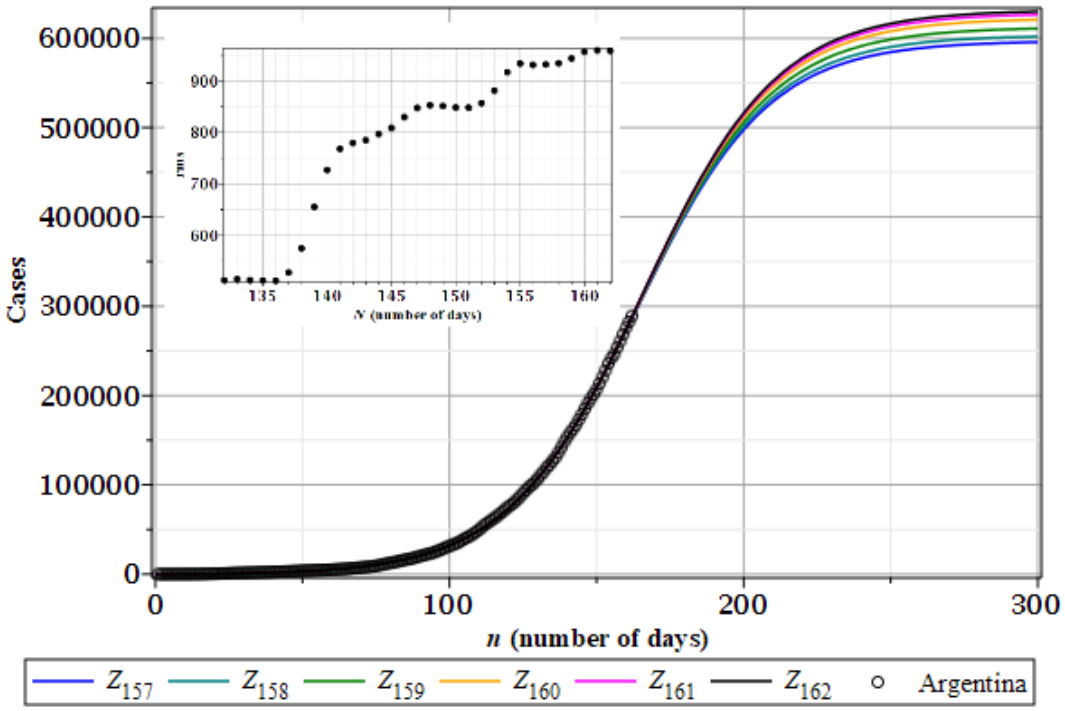


FIG. 66: Daily Covid-19 case curves and rms for the data from Argentina.

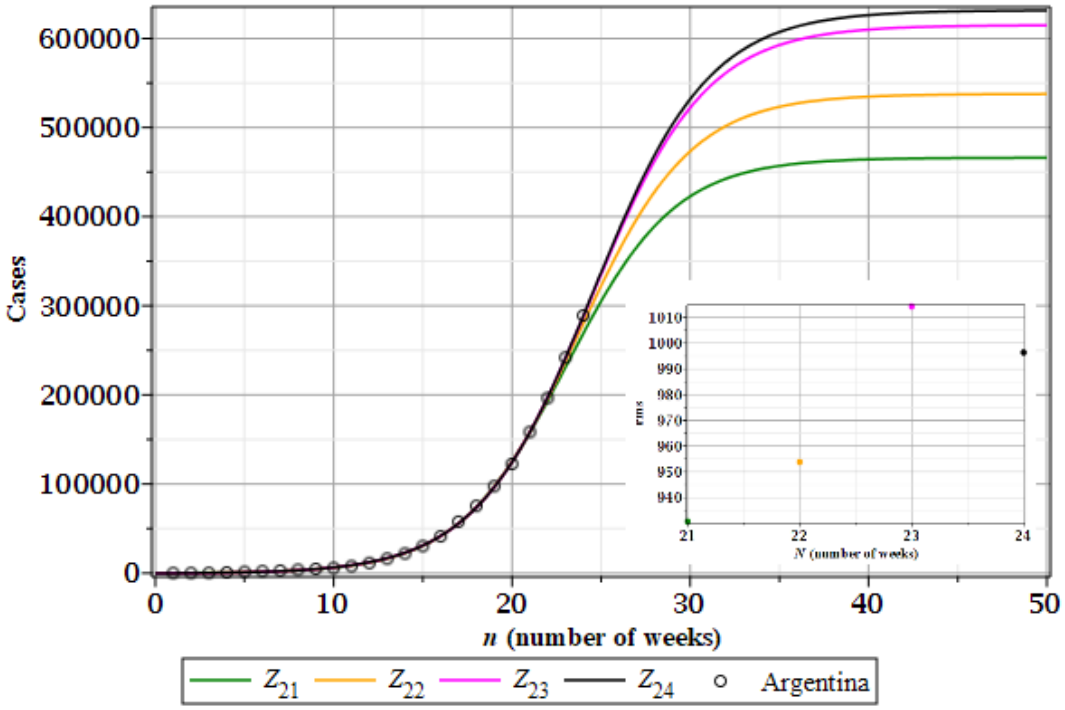


FIG. 67: Weekly Covid-19 case curves and rms for the data from Argentina.

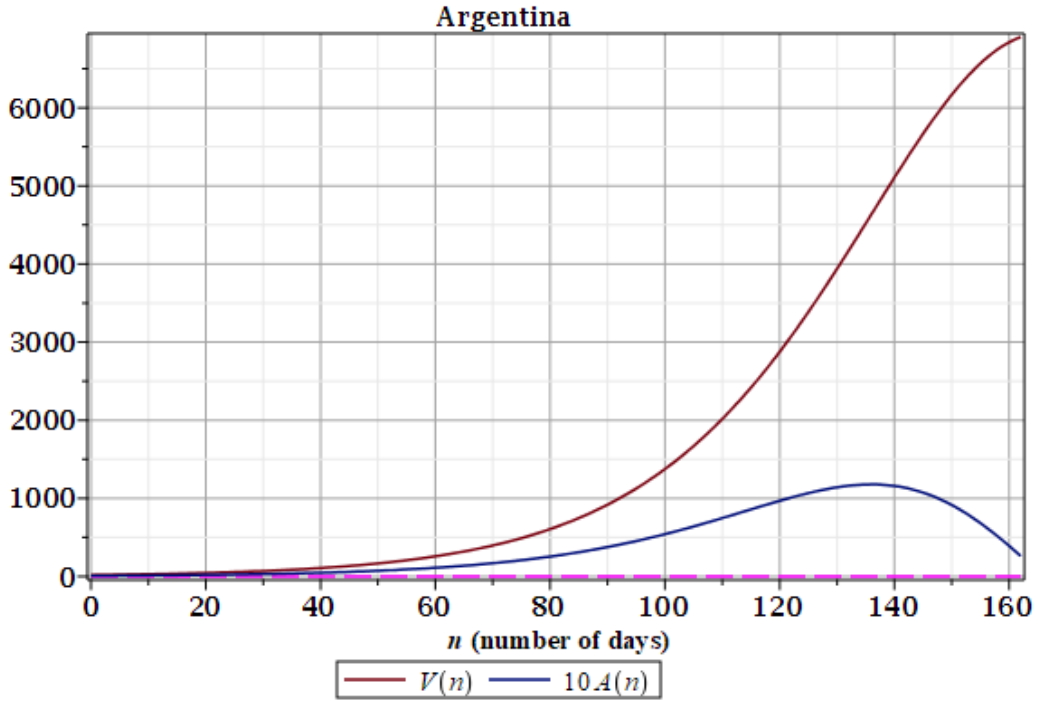


FIG. 68: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{162} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

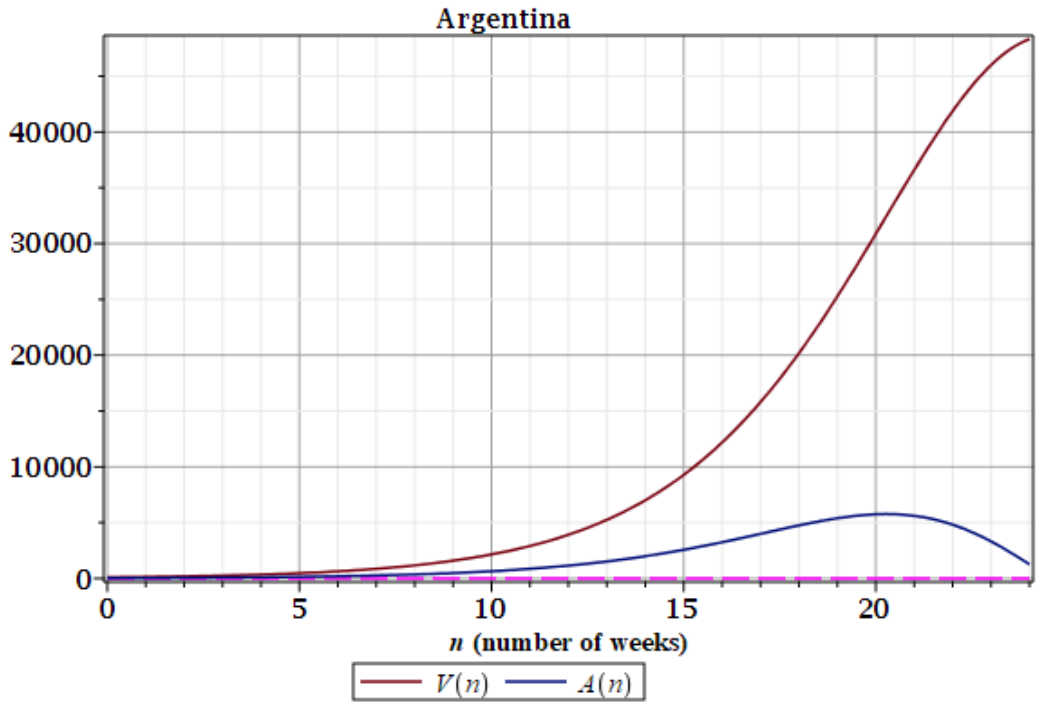


FIG. 69: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{24} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

IX.18. Bolivia

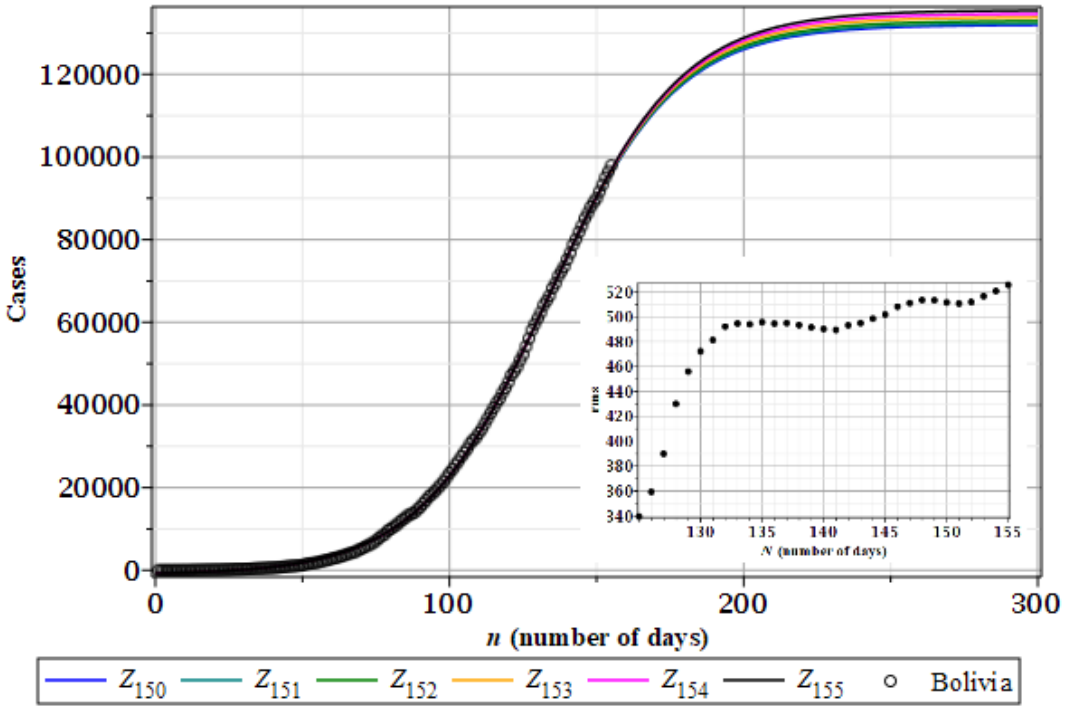


FIG. 70: Daily Covid-19 case curves and rms for the data from Bolivia.

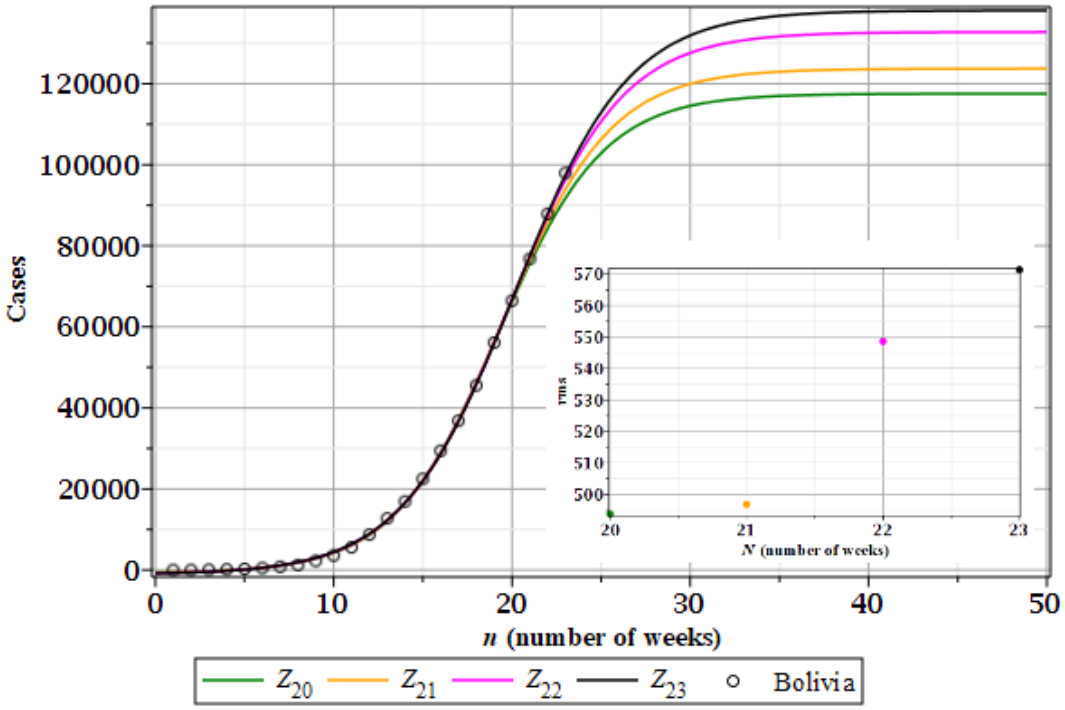


FIG. 71: Weekly Covid-19 case curves and rms for the data from Bolivia.

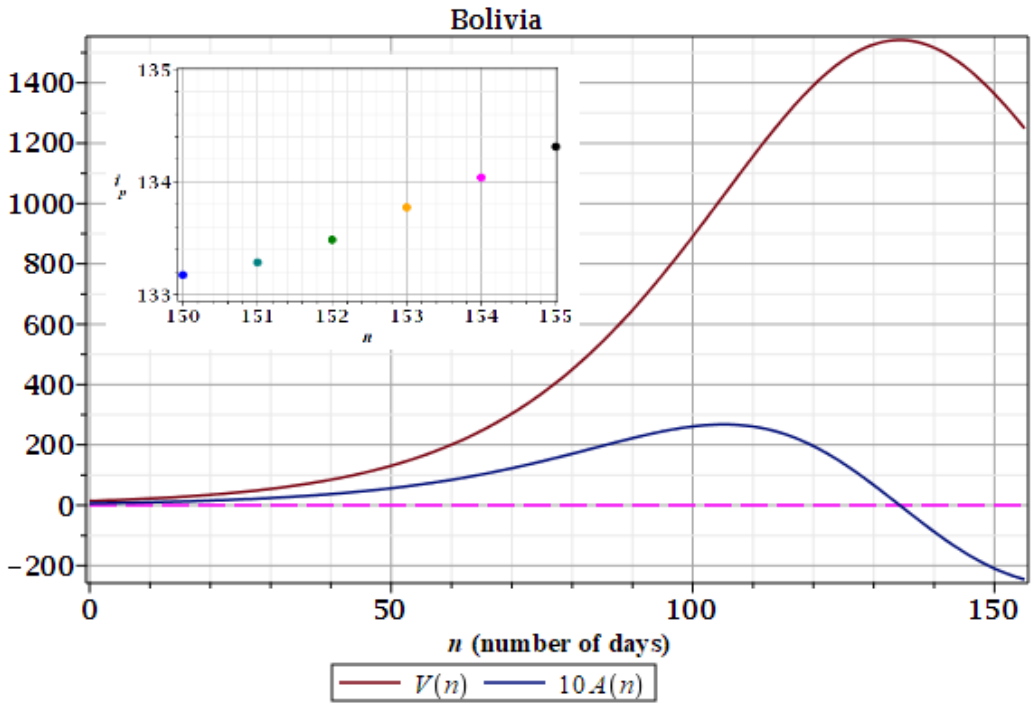


FIG. 72: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{155} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 134$.

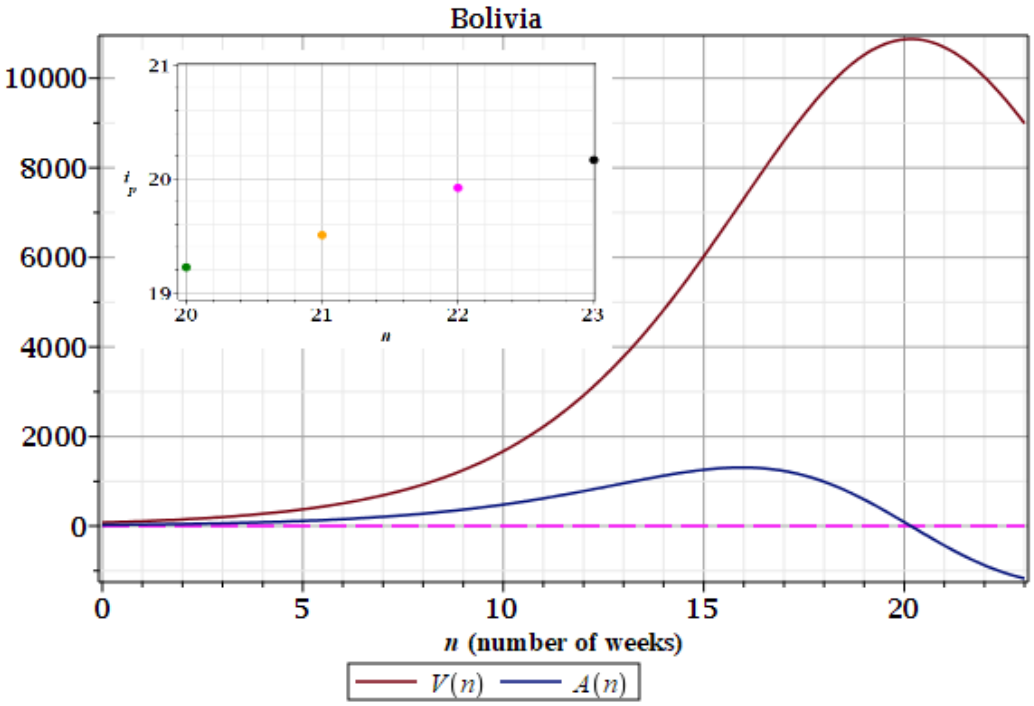


FIG. 73: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{23} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 20.1$.

IX.19. Venezuela

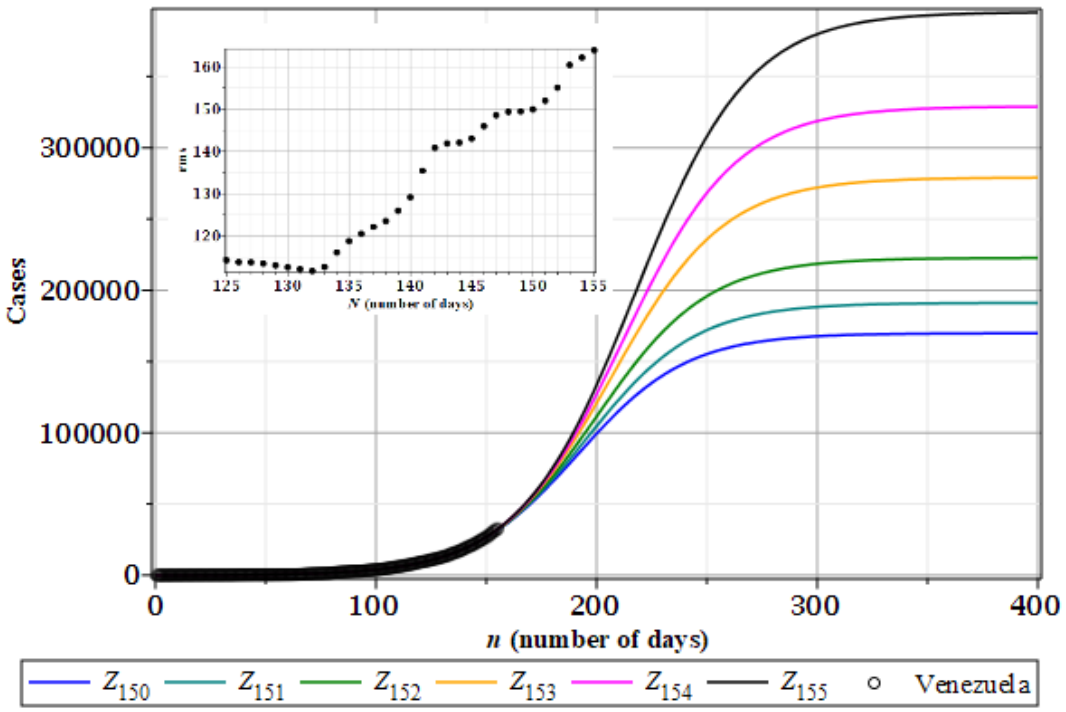


FIG. 74: Daily Covid-19 case curves and rms for the data from Venezuela.

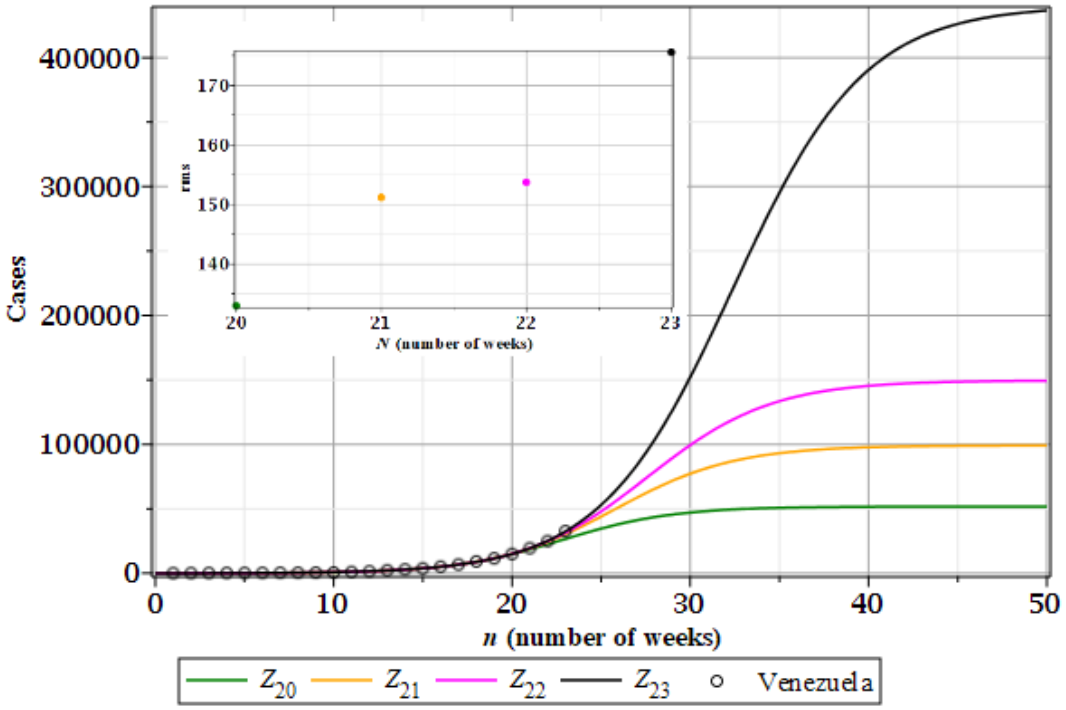


FIG. 75: Weekly Covid-19 case curves and rms for the data from Venezuela.

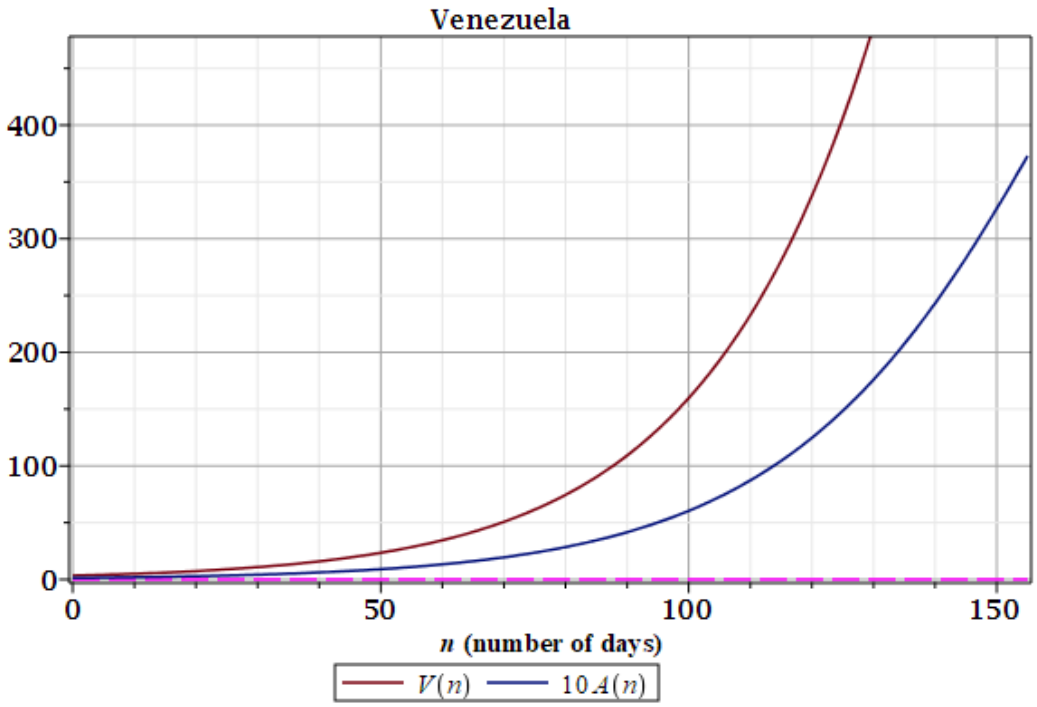


FIG. 76: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{155} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

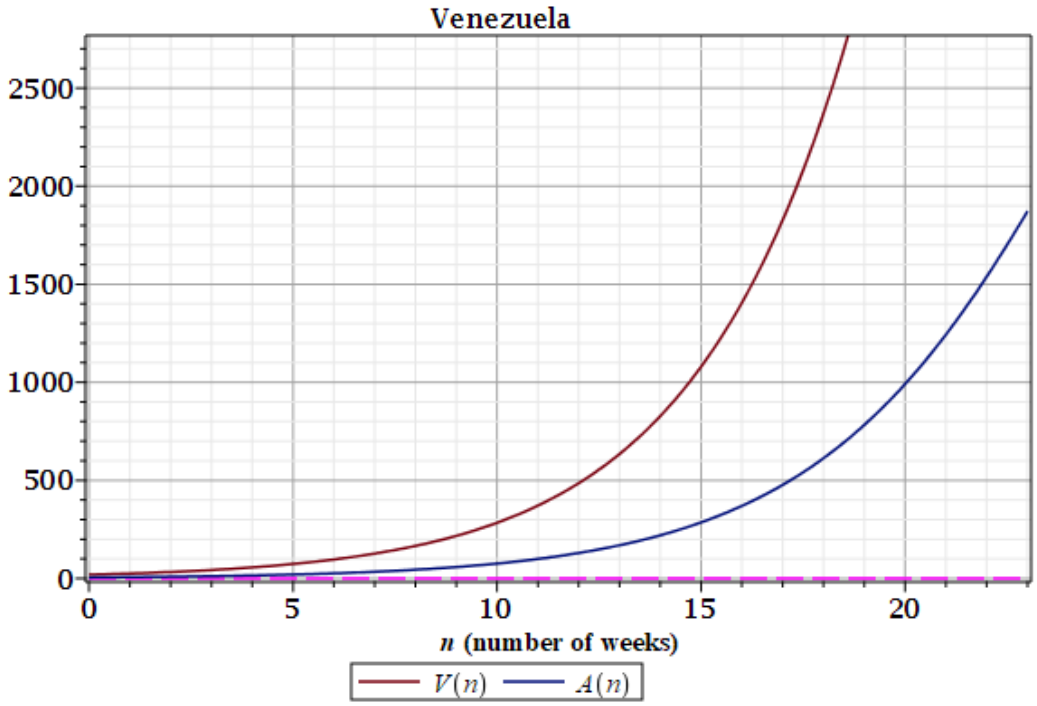


FIG. 77: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{23} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

IX.20. India

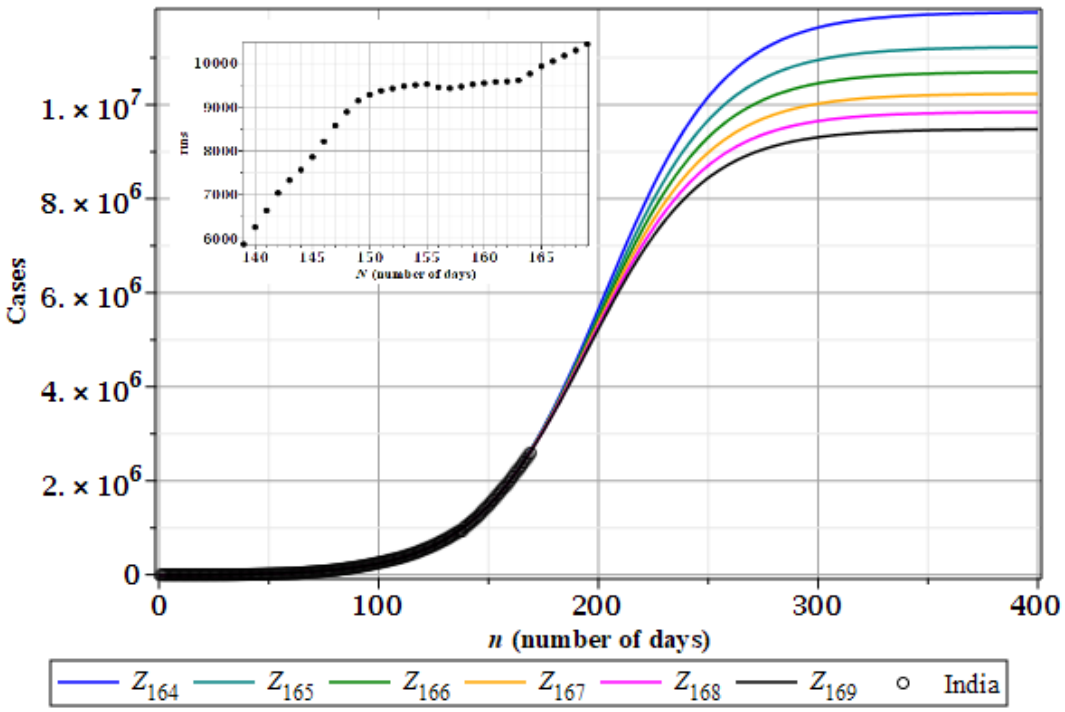


FIG. 78: Daily Covid-19 case curves and rms for the data from India.

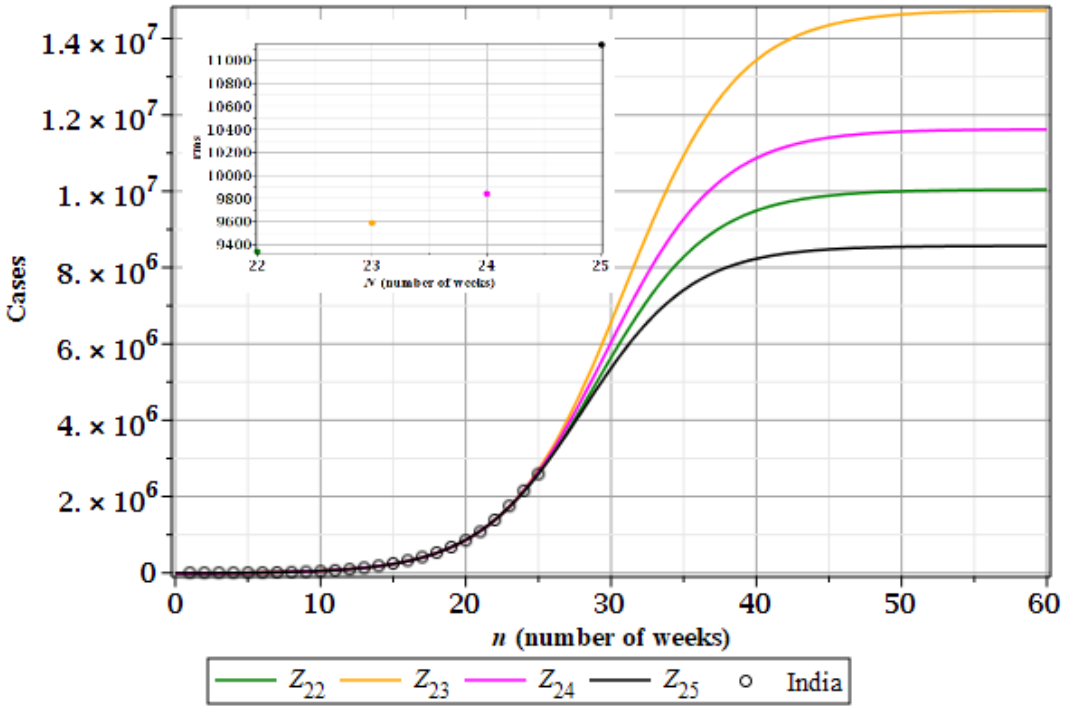


FIG. 79: Weekly Covid-19 case curves and rms for the data from India.

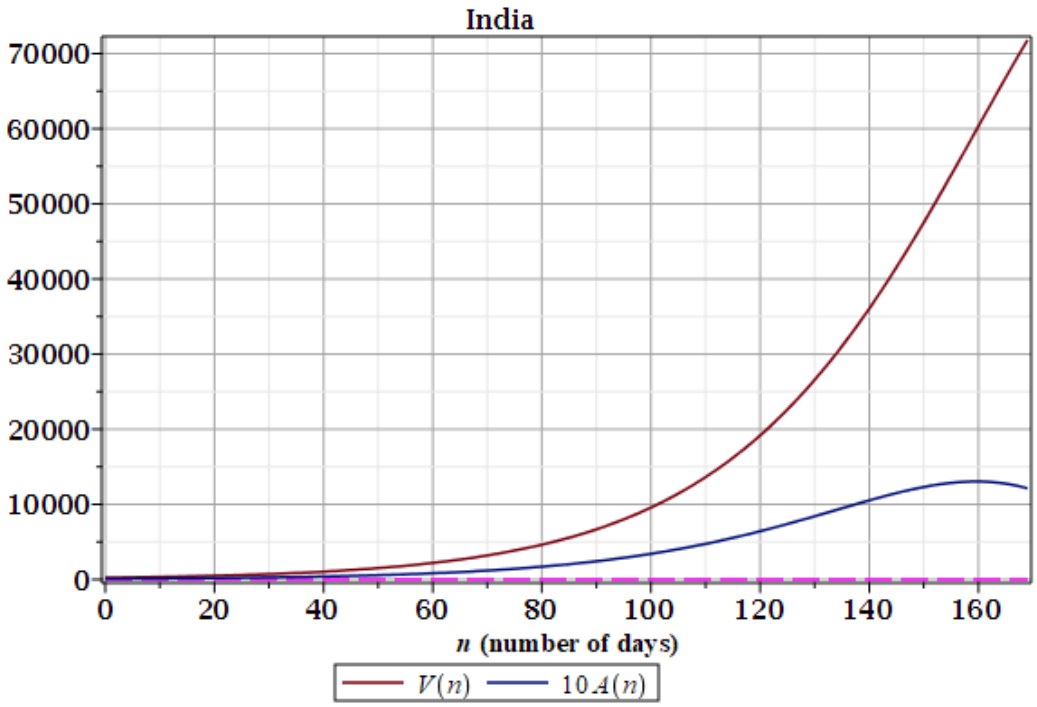


FIG. 80: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{169}. Incomplete curves. Inflection point: absent.

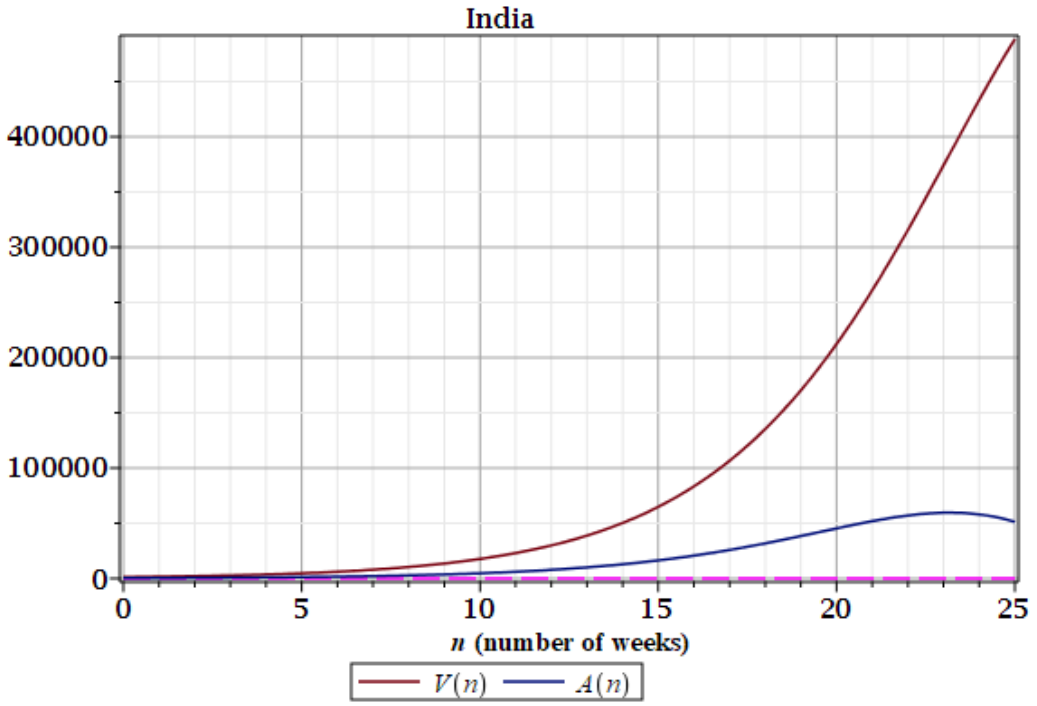


FIG. 81: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{25}. Incomplete curves. Inflection point: absent.

IX.21. Saudi Arabia

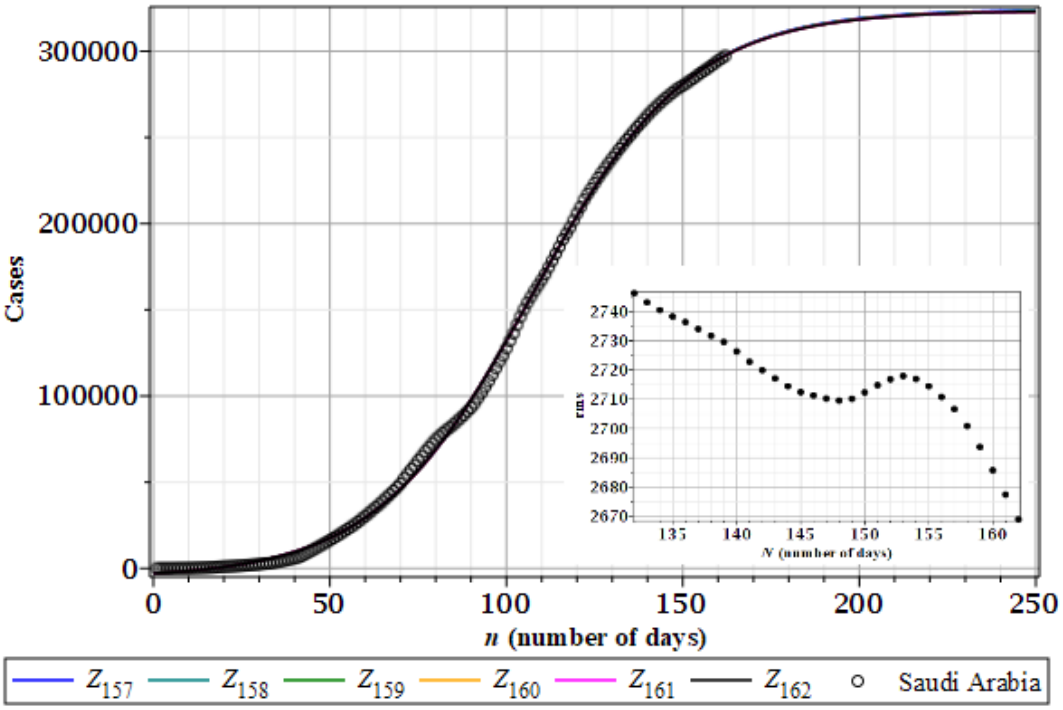


FIG. 82: Daily Covid-19 case curves and rms for the data from Saudi Arabia.

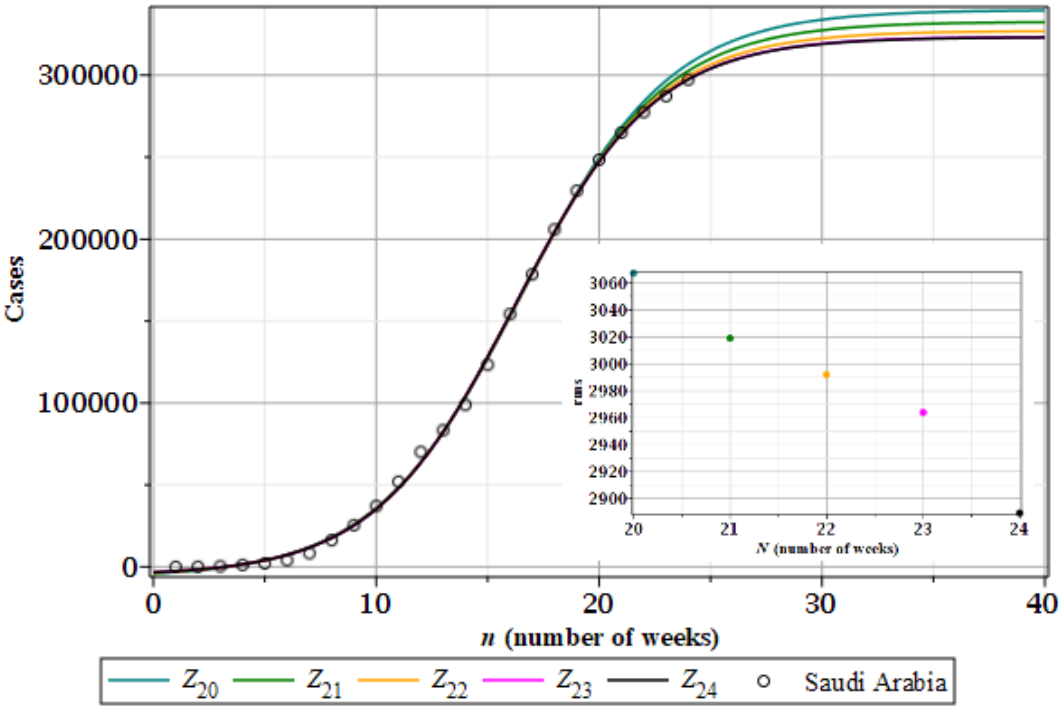


FIG. 83: Weekly Covid-19 case curves and rms for the data from Saudi Arabia.

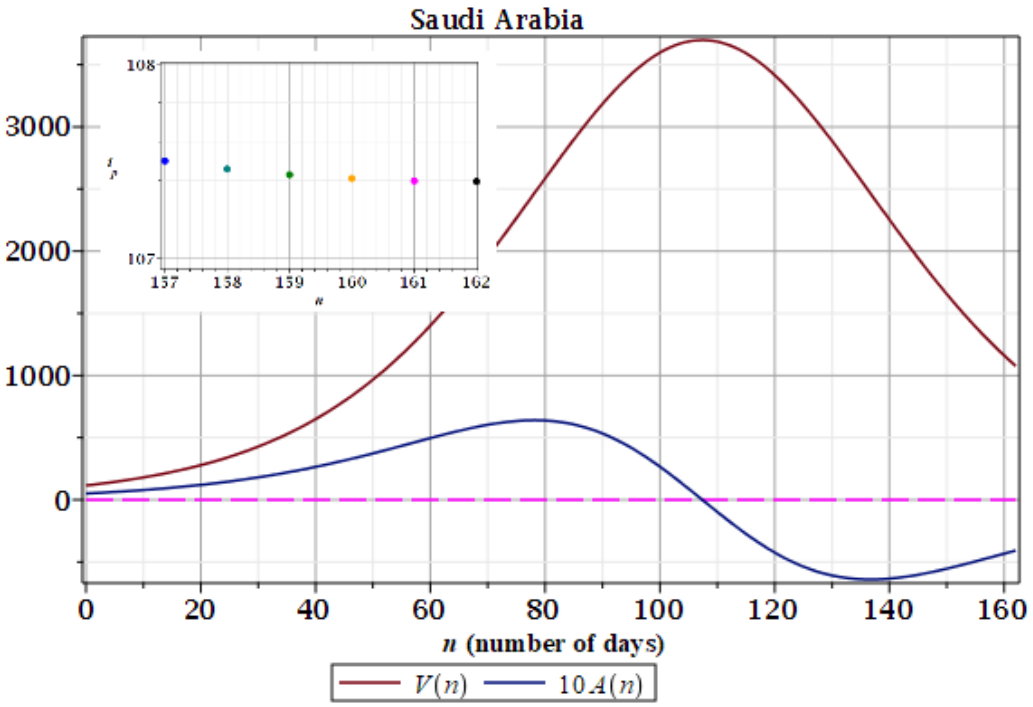


FIG. 84: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{162} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 107$.

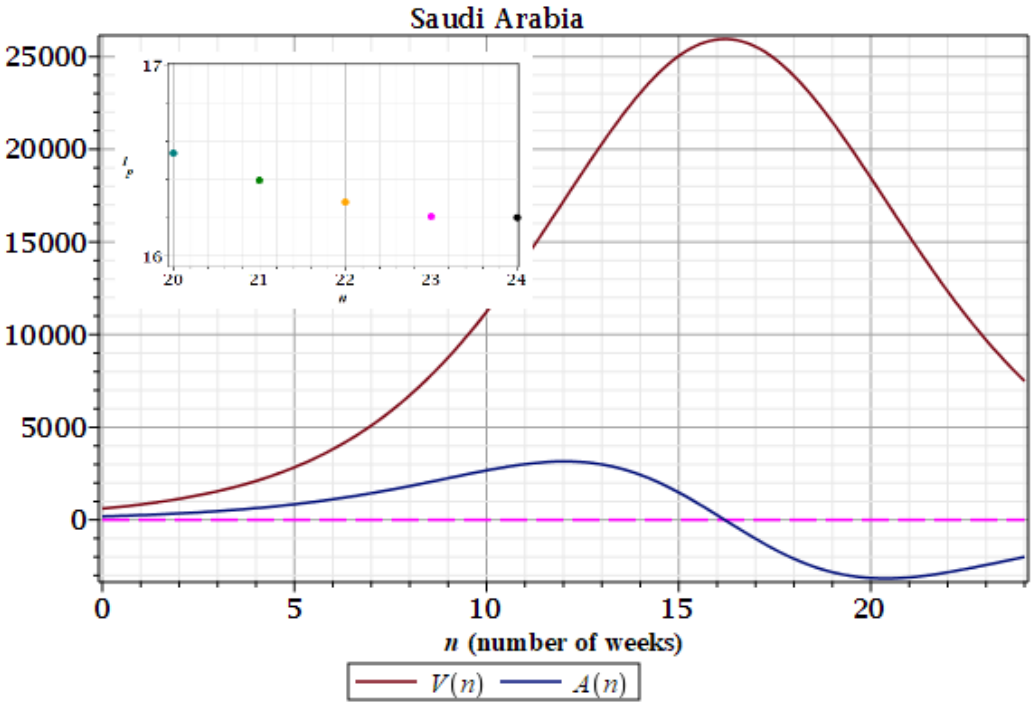


FIG. 85: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{24} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 16.2$.

IX.22. China

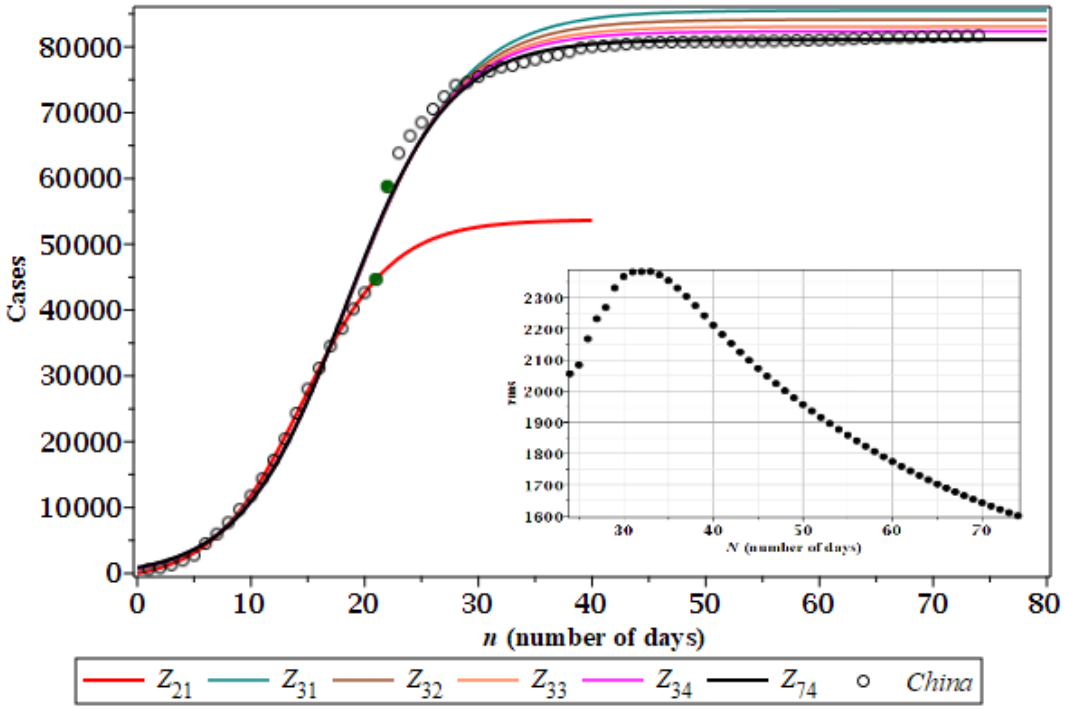


FIG. 86: Daily Covid-19 case curves and rms for the data from China. $Z_{74} = 40823\tanh(0.110n - 2.005) + 40264$.

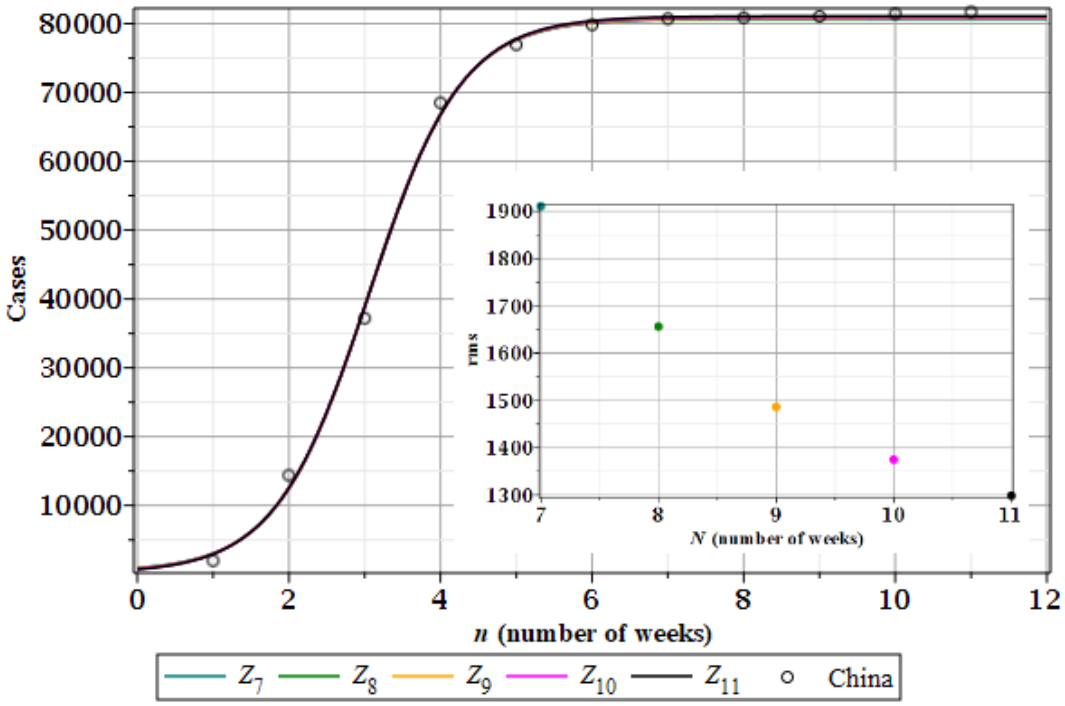


FIG. 87: Weekly Covid-19 case curves and rms for the data from China. $Z_{11} = 40466\tanh(0.811n - 2.479) + 40611$.

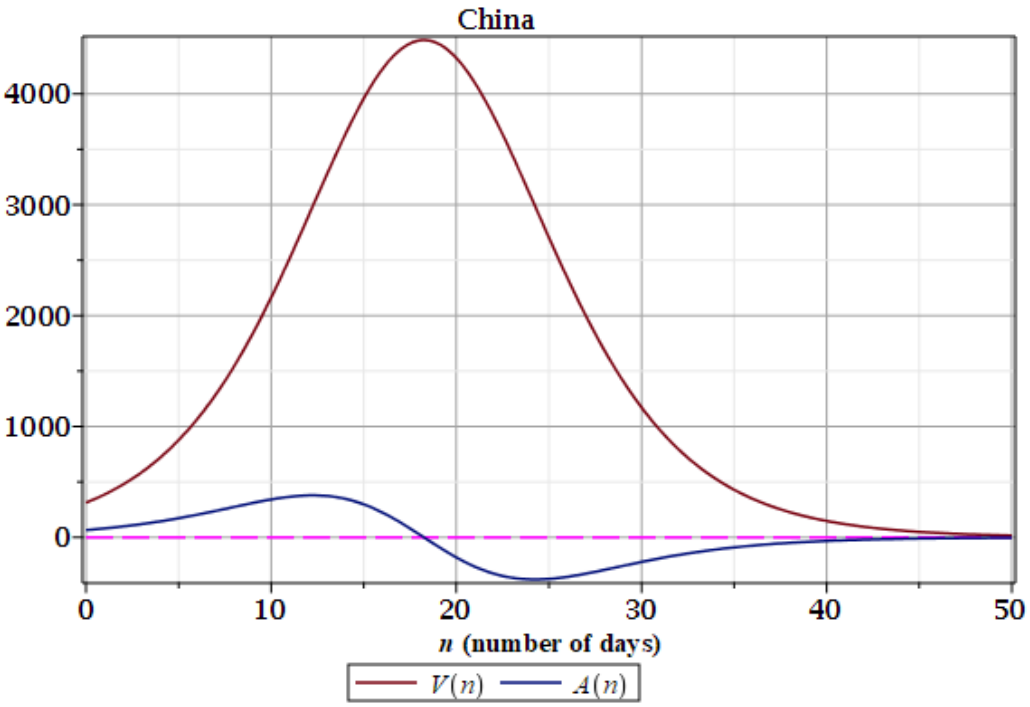


FIG. 88: Daily growth rates from $Z_{74} = 40823 \tanh(0.110n - 2.005) + 40264$. Complete curves. Inflection point: $n \approx 18$.

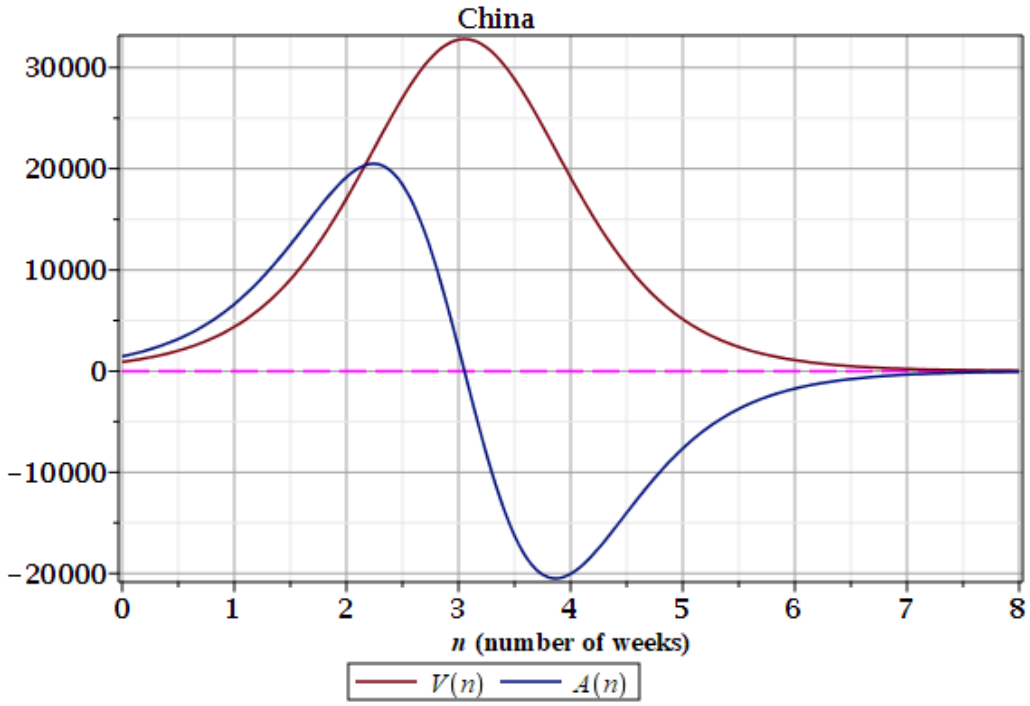


FIG. 89: Weekly growth rates from $Z_{11} = 40466 \tanh(0.811n - 2.479) + 40611$. Complete curves. Inflection point: $n \approx 3$.

IX.23. Japan

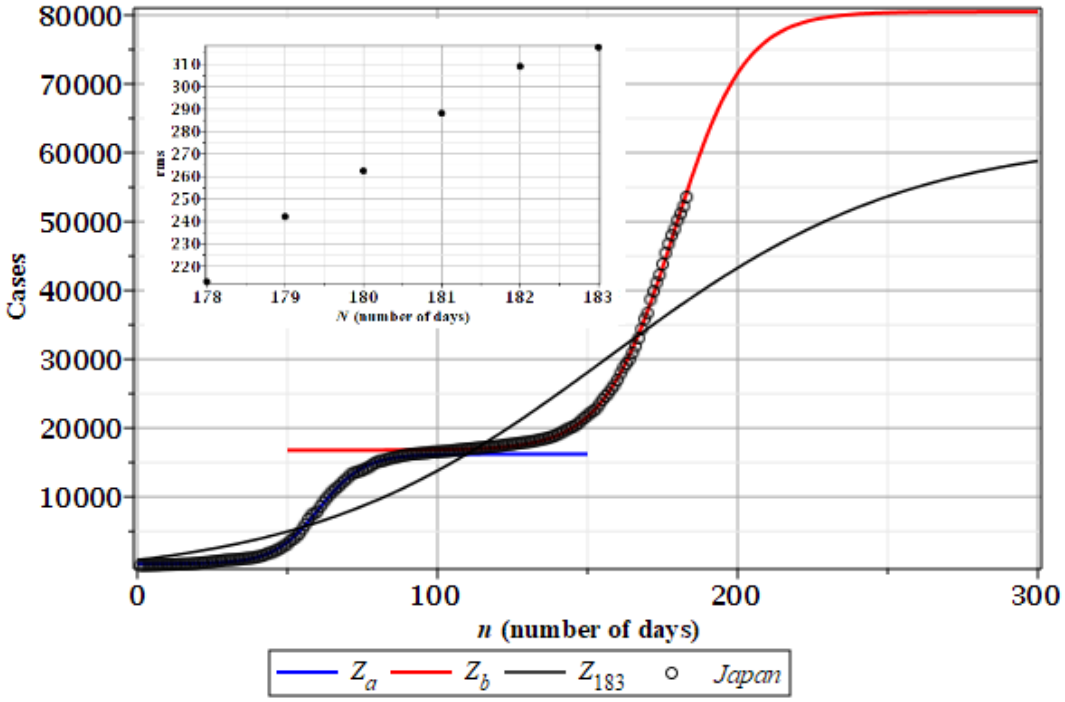


FIG. 90: Daily Covid-19 case curves and rms (from the last wave) for the data from Japan.

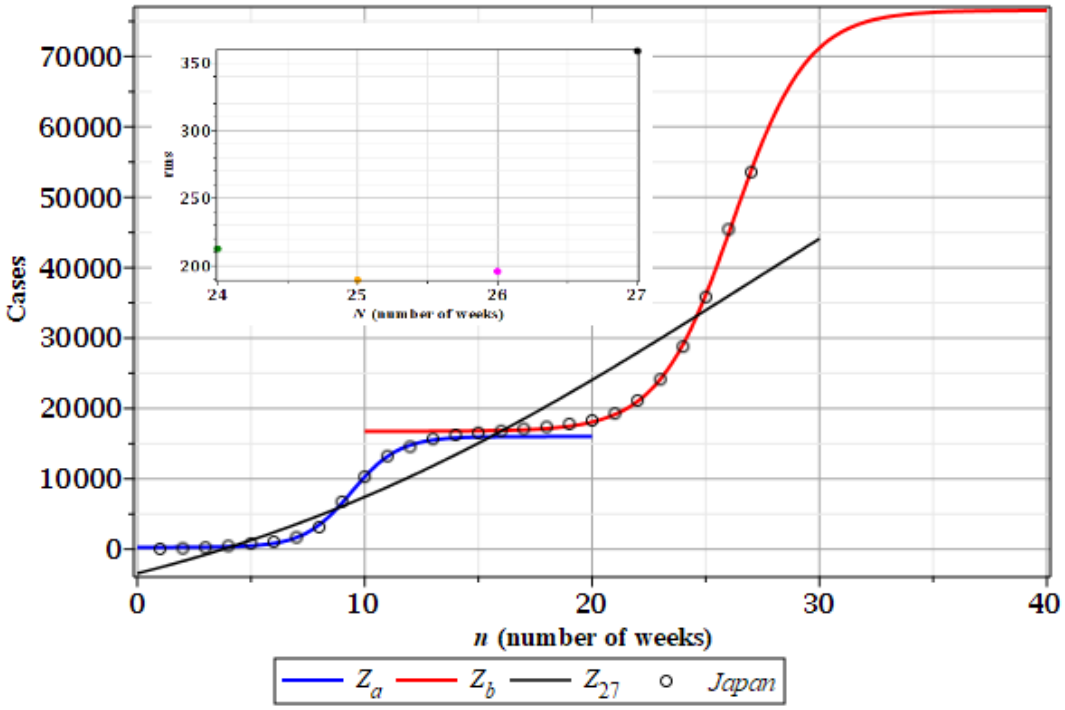


FIG. 91: Weekly Covid-19 case curves and rms (from the last wave) for the data from Japan.

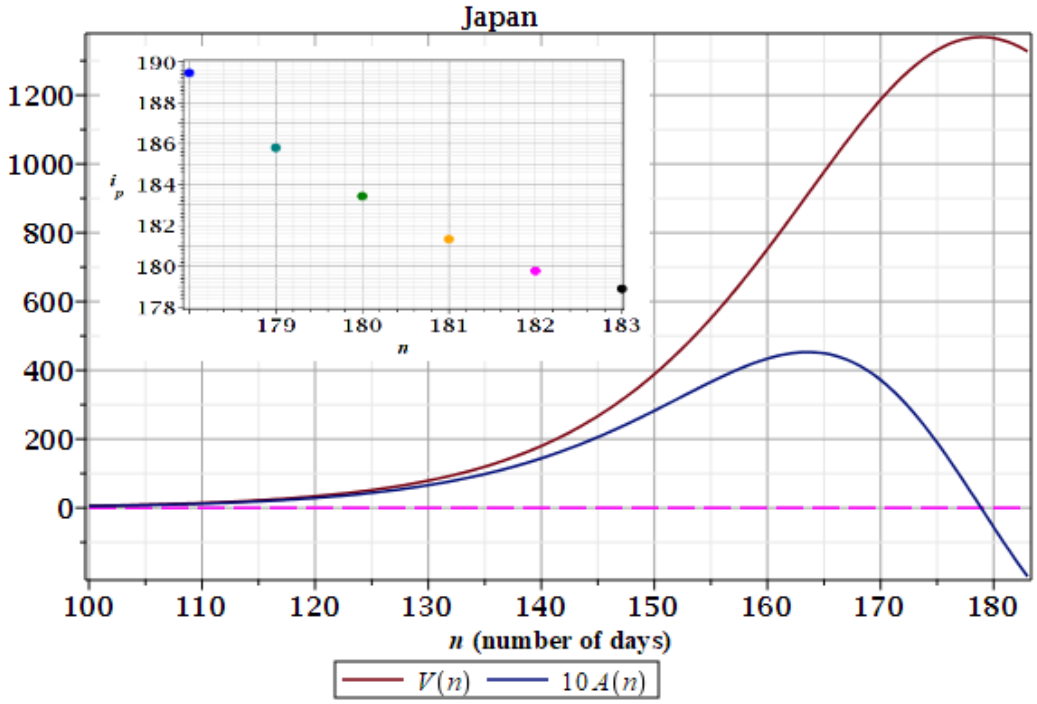


FIG. 92: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b (the last wave). Incomplete curves. Inflection point: $n \approx 179$.

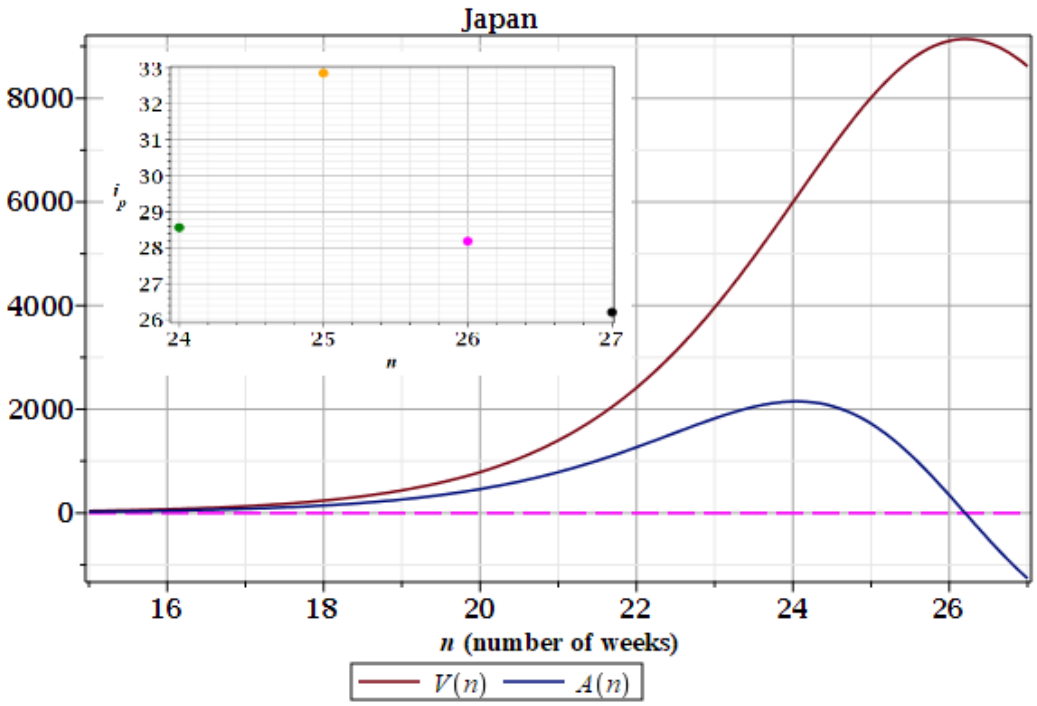


FIG. 93: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b (the last wave). Incomplete curves. Inflection point: $n \approx 26.2$.

IX.24. South Korea

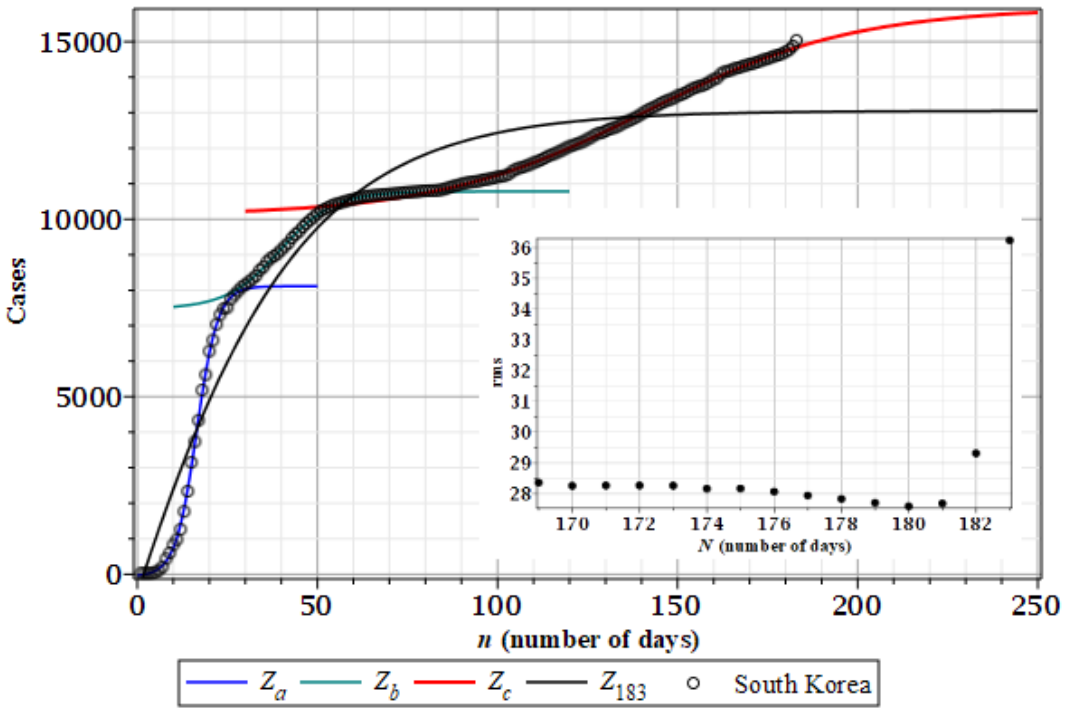


FIG. 94: Daily Covid-19 case curves for the data from South Korea. Inset: rms for the last wave Z_c.

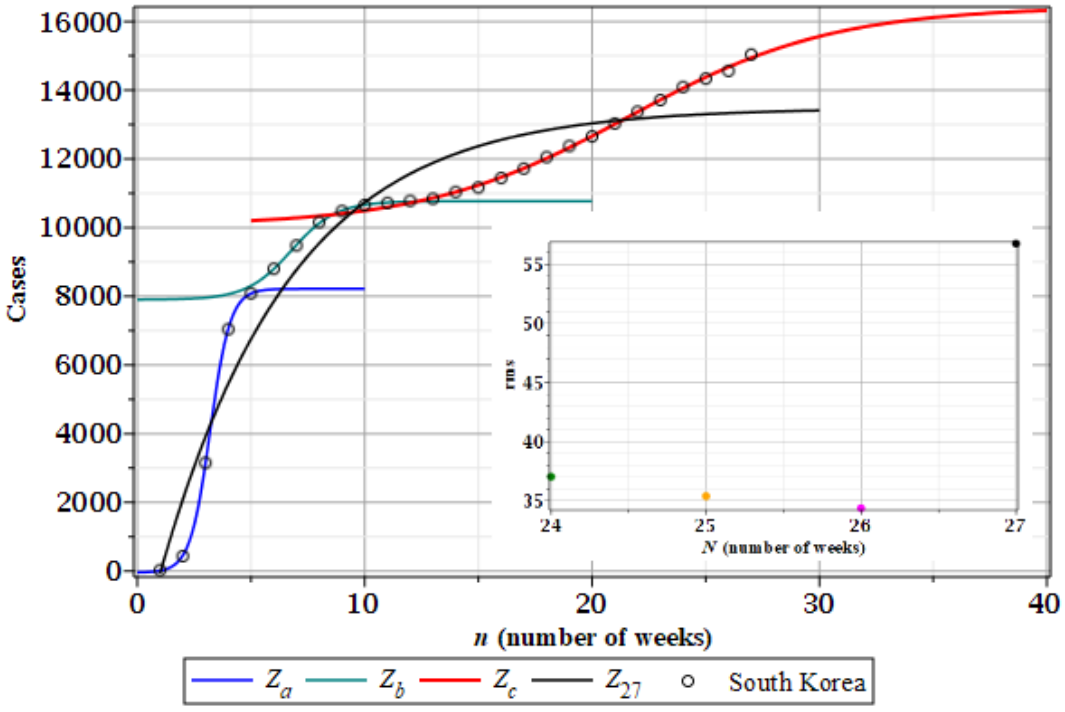


FIG. 95: Weekly Covid-19 case curves for the data from South Korea. Inset: rms for the last wave Z_c.

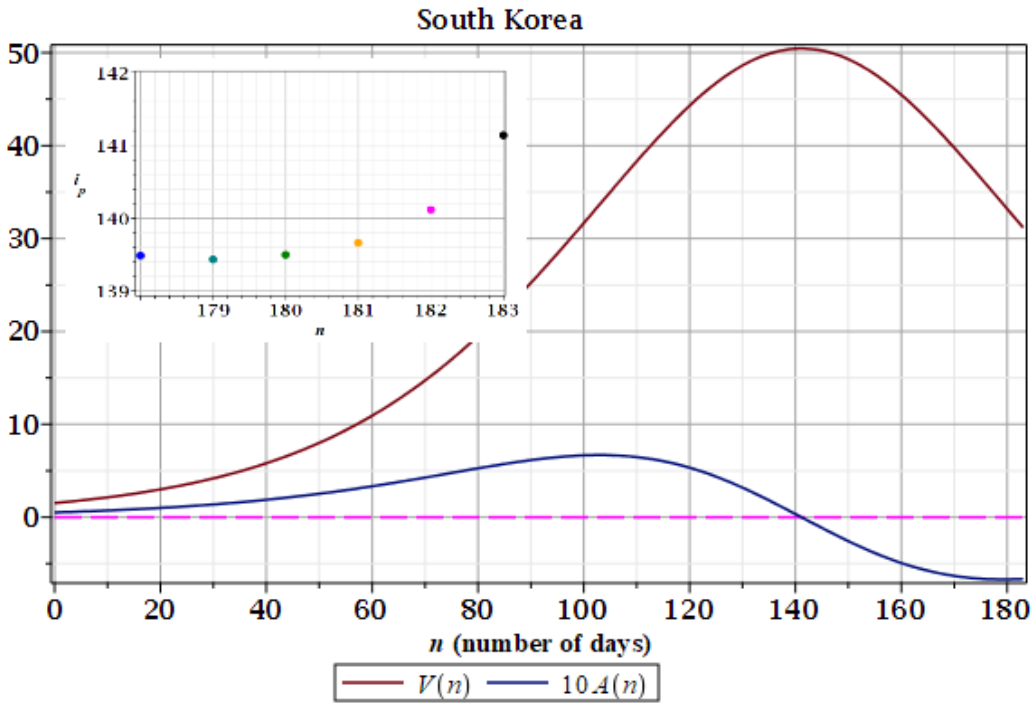


FIG. 96: Daily growth rates and inflection points from the last wave Z_c. Incomplete curves. Inflection point: $n \approx 141$.

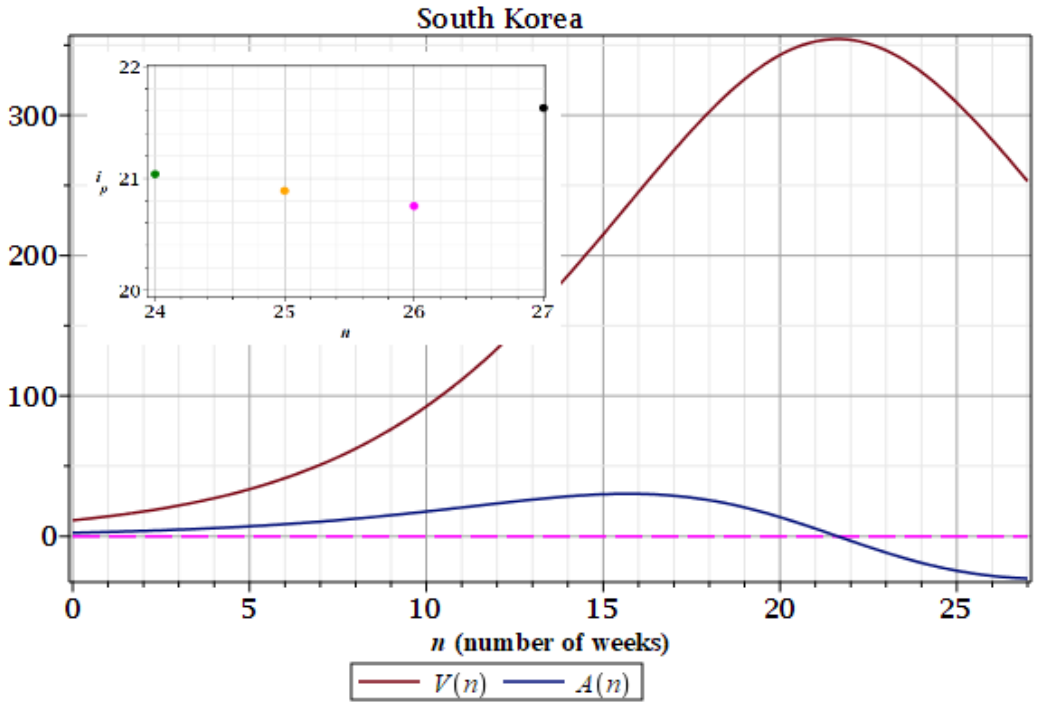


FIG. 97: Weekly growth rates and inflection points from the last wave Z_c. Incomplete curves. Inflection point: $n \approx 21.6$.

IX.25. Vietnam

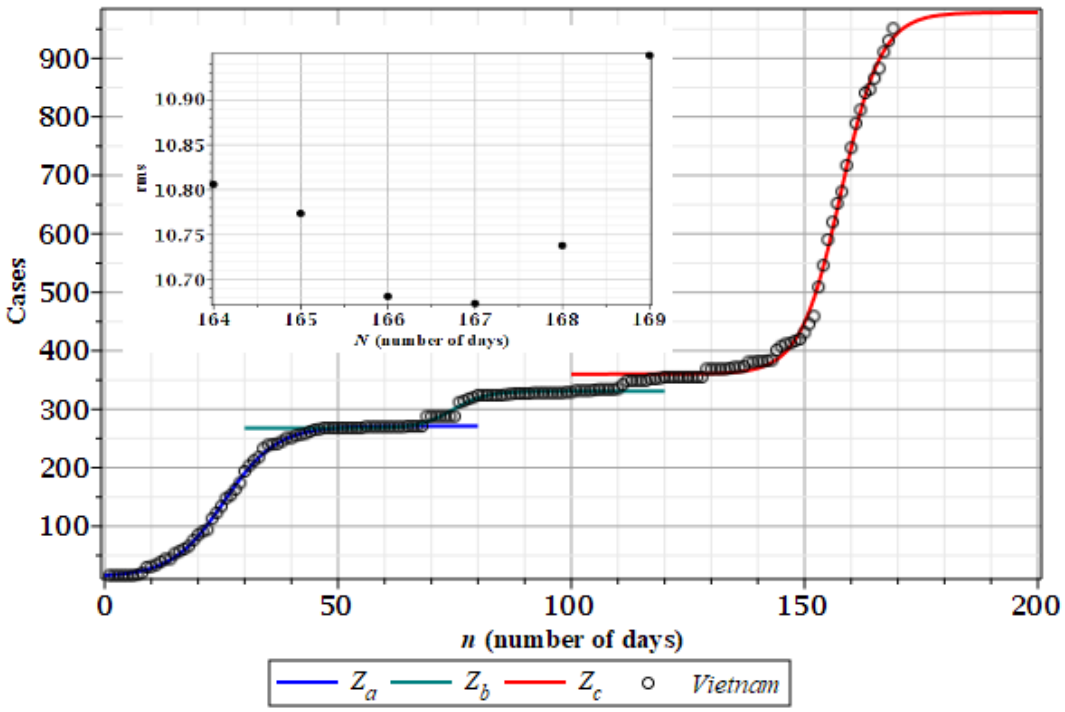


FIG. 98: Daily Covid-19 case curves for the data from Vietnam. Inset: rms for the last wave Z_c.

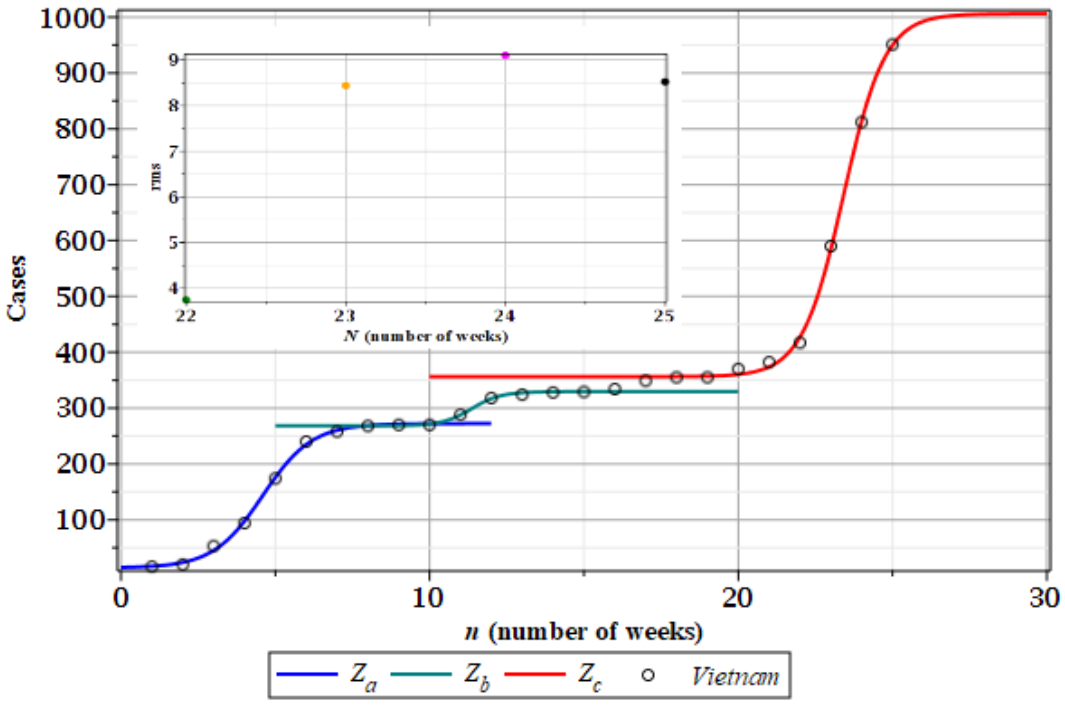


FIG. 99: Weekly Covid-19 case curves for the data from Vietnam. Inset: rms for the last wave Z_c.

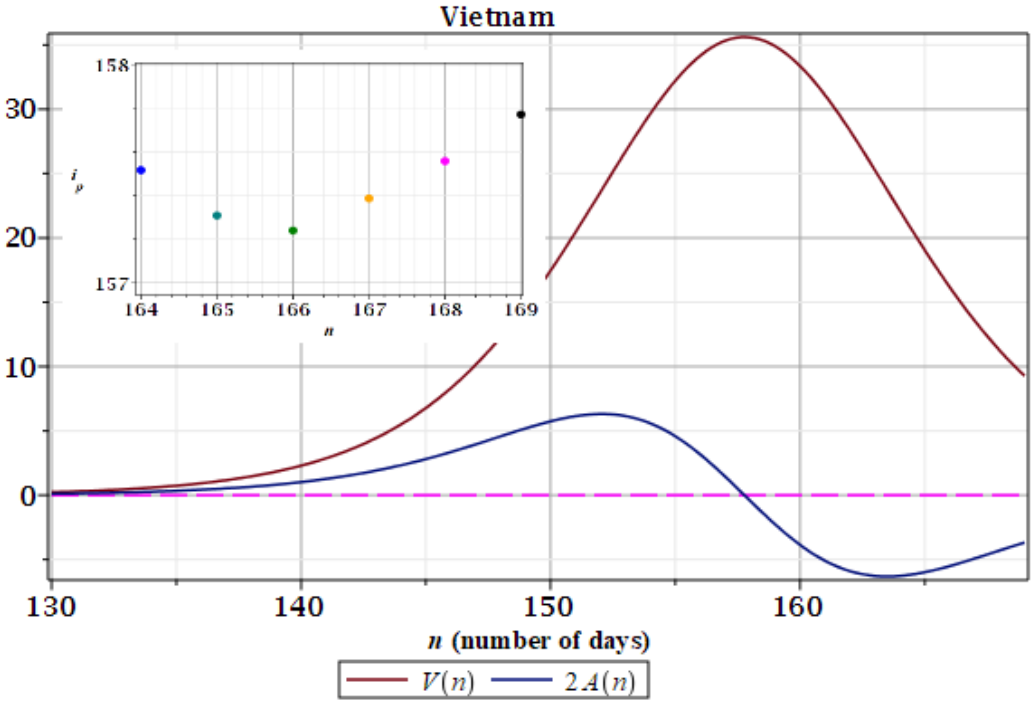


FIG. 100: Daily growth rates and inflection points from the last wave Z_c. Incomplete curves. Inflection point: $n \approx 158$.

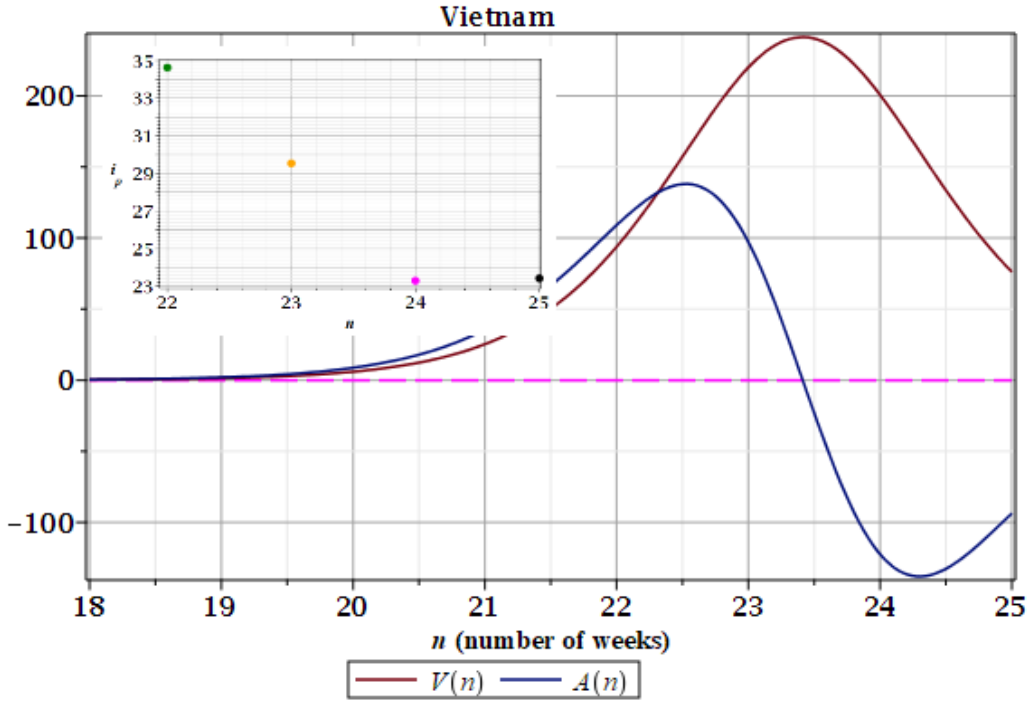


FIG. 101: Weekly growth rates and inflection points from the last wave Z_c. Incomplete curves. Inflection point: $n \approx 23.4$.

IX.26. South Africa

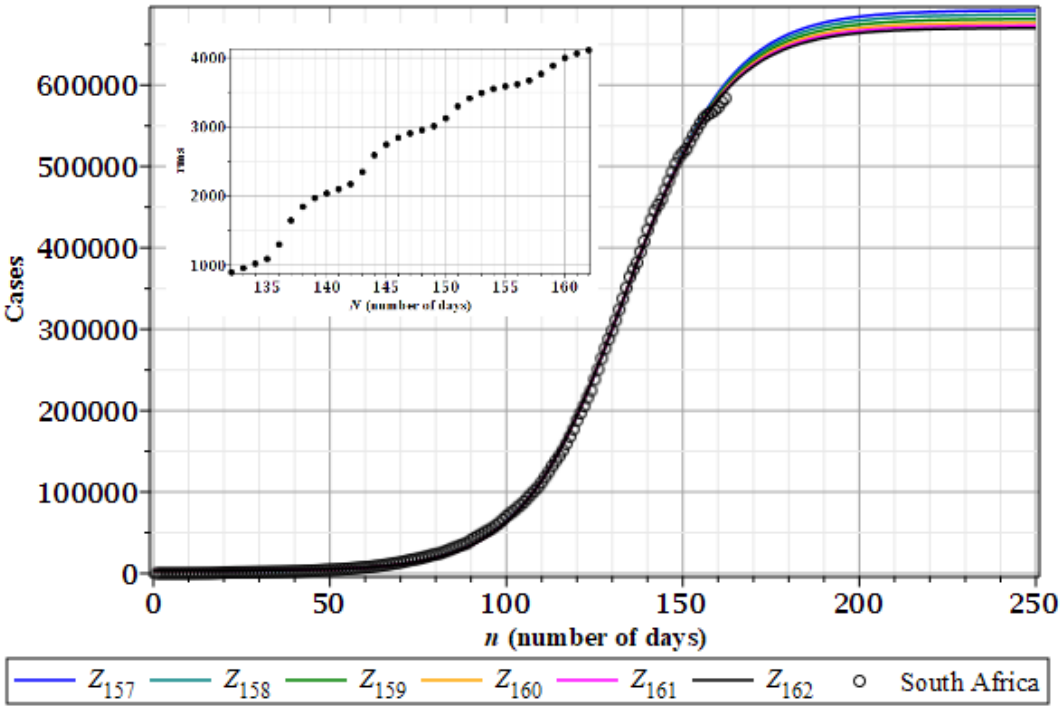


FIG. 102: Daily Covid-19 case curves and rms for the data from South Africa.

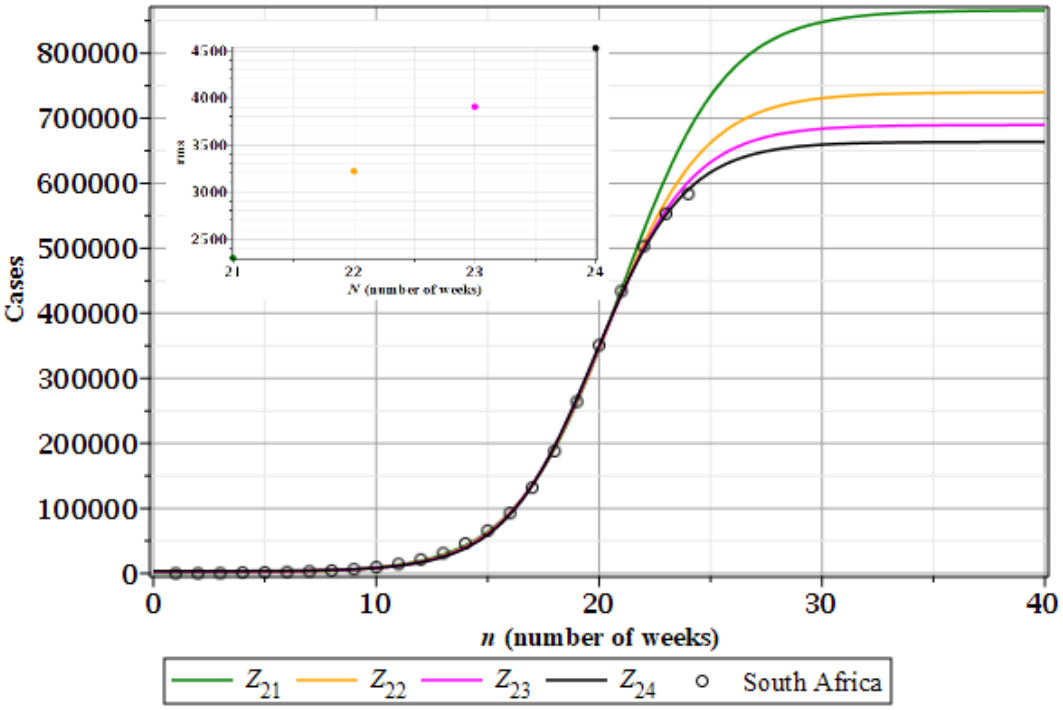


FIG. 103: Weekly Covid-19 case curves and rms for the data from South Africa.

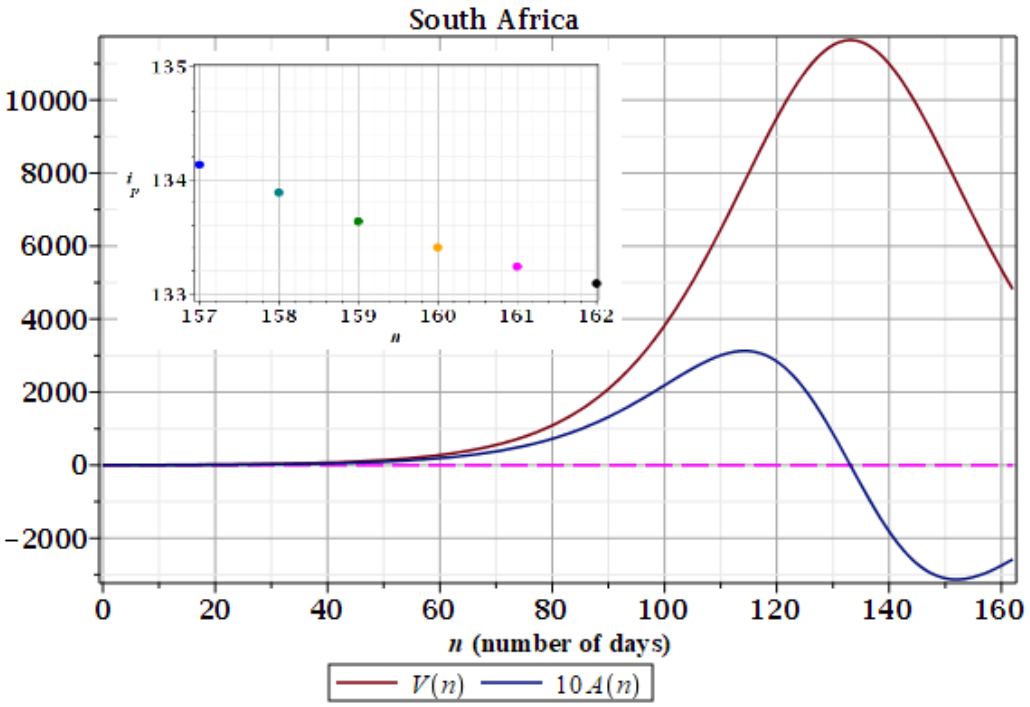


FIG. 104: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{162}. Incomplete curves. Inflection point: $n \approx 133$.

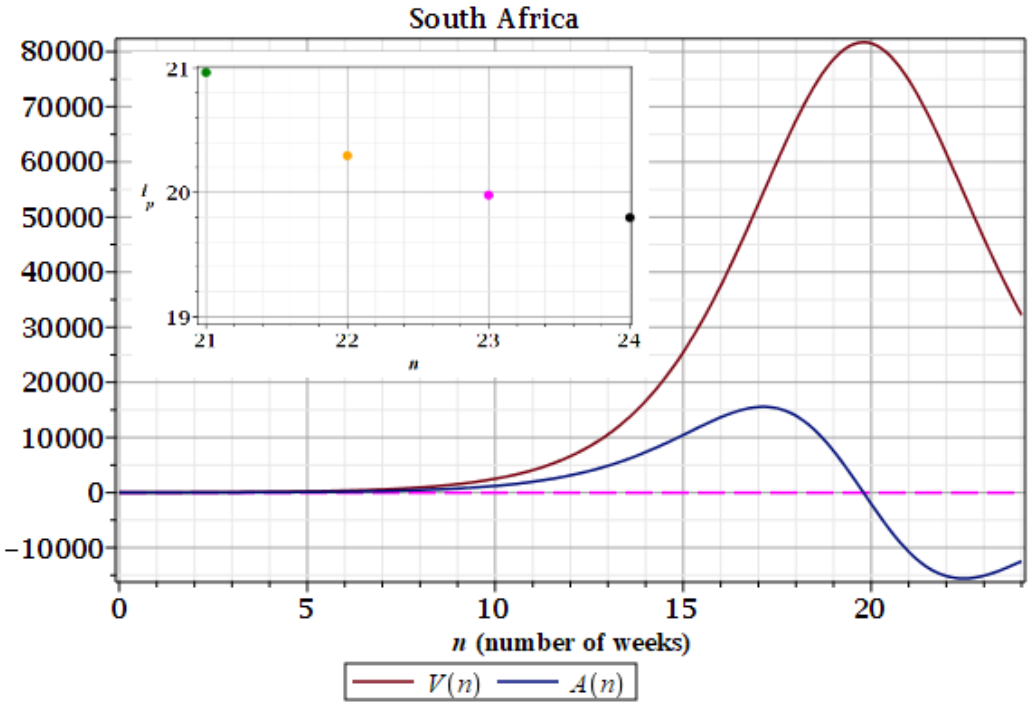


FIG. 105: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{24}. Incomplete curves. Inflection point: $n \approx 19.8$.

IX.27. Nigeria

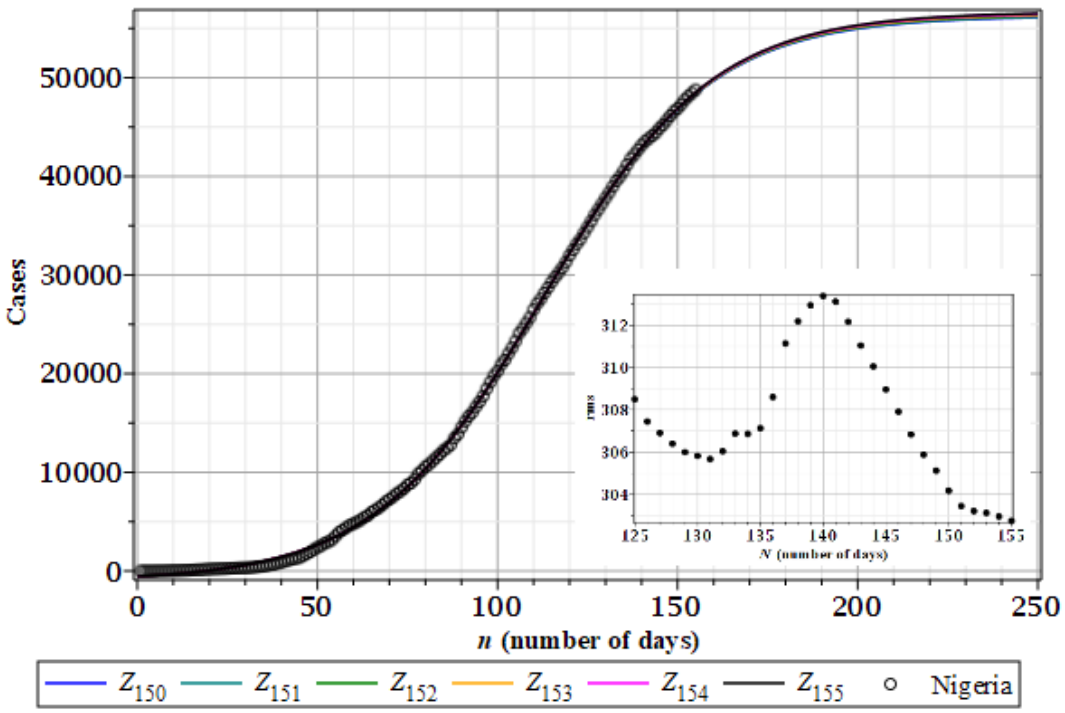


FIG. 106: Daily Covid-19 case curves and rms for the data from Nigeria.

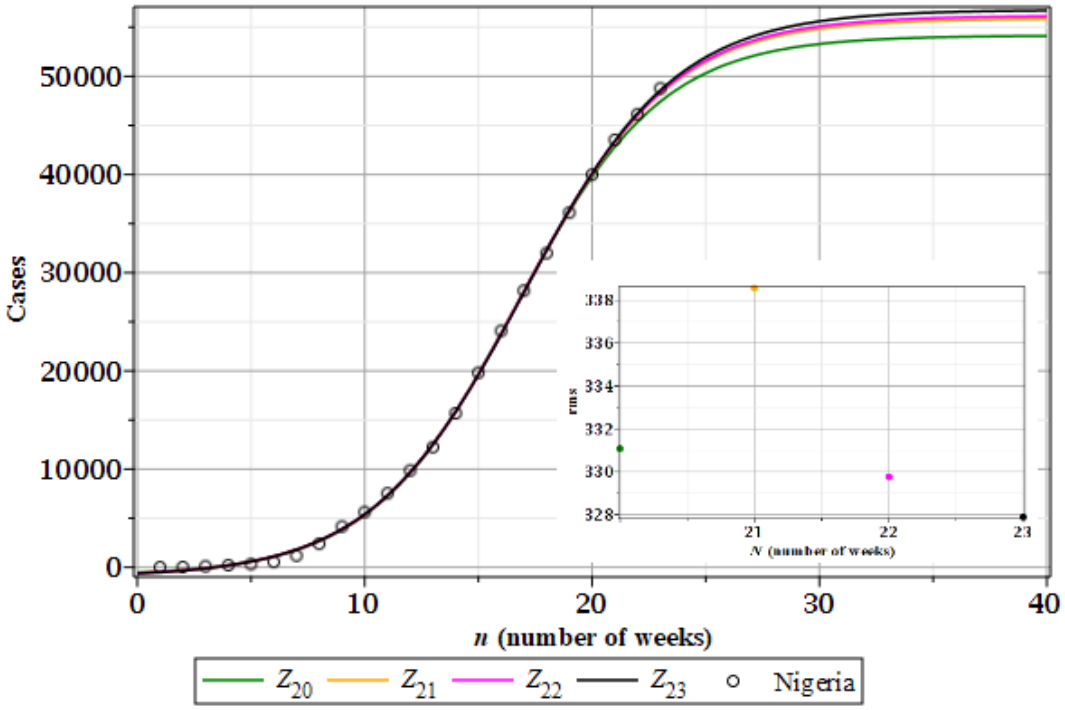


FIG. 107: Weekly Covid-19 case curves and rms for the data from Nigeria.

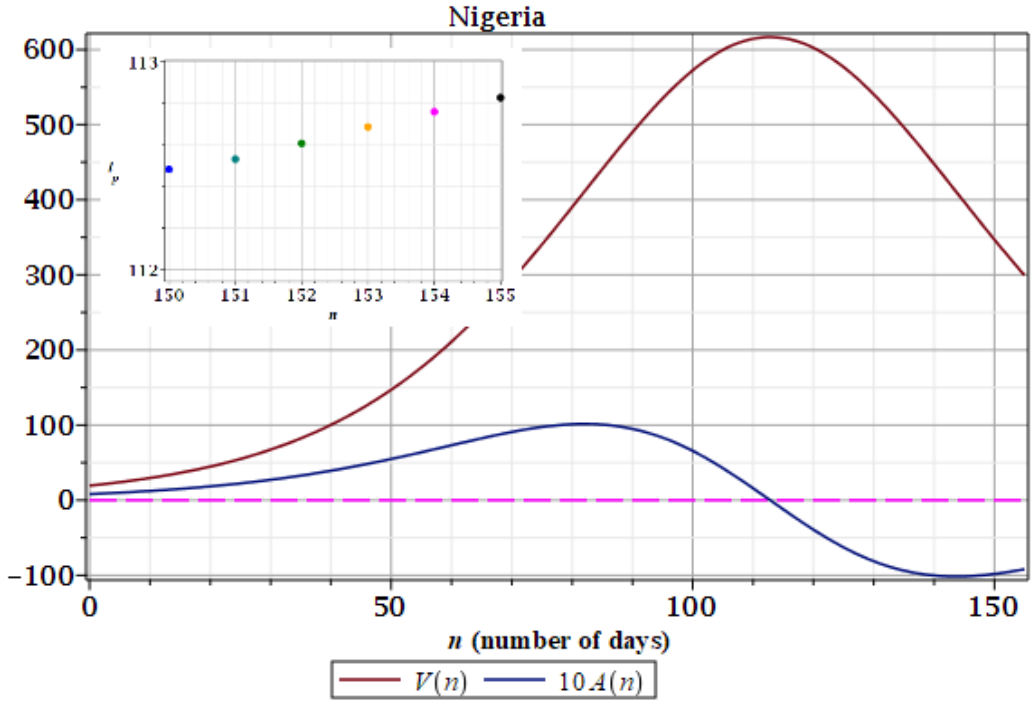


FIG. 108: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{155} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 113$.

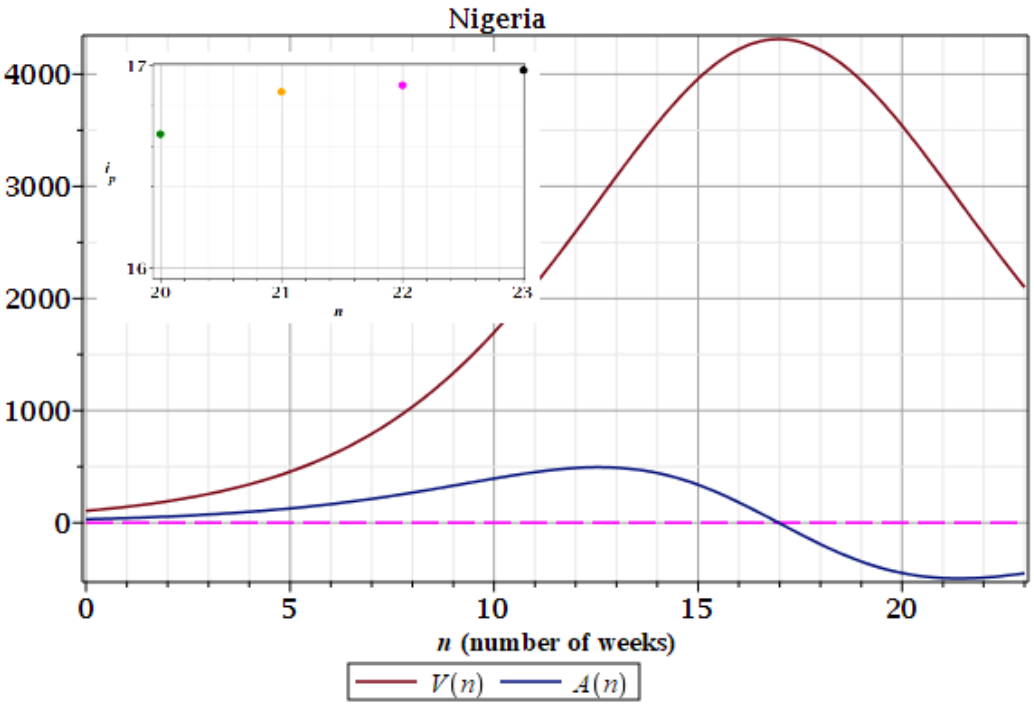


FIG. 109: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{23} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 17$.

IX.28. Ghana

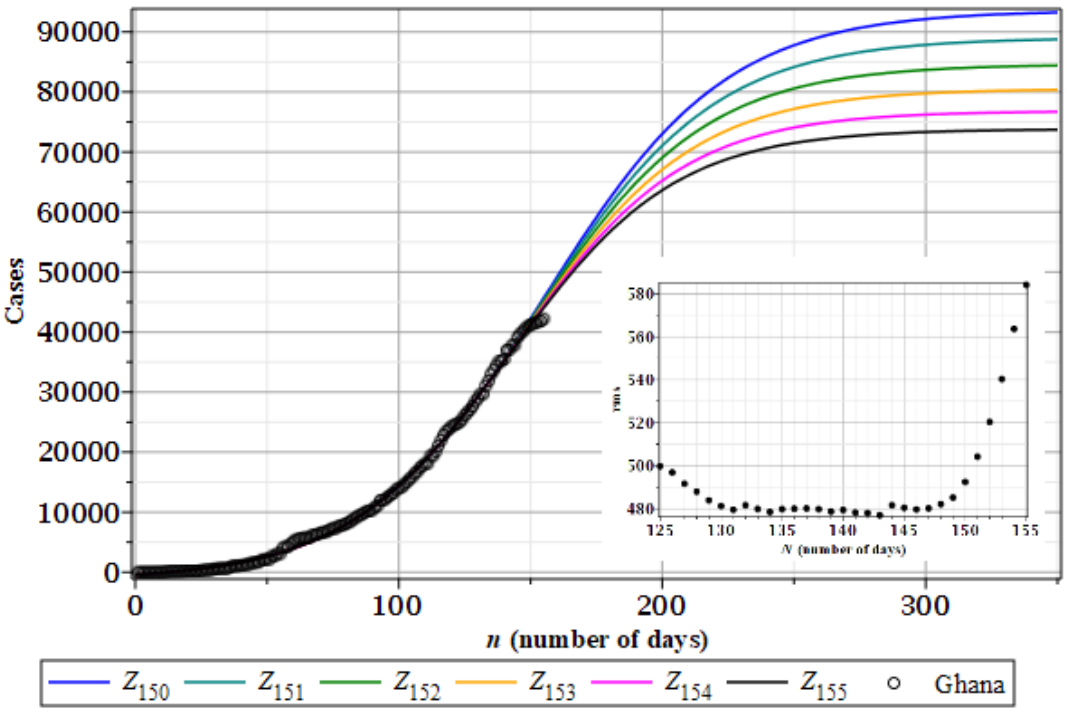


FIG. 110: Daily Covid-19 case curves and rms for the data from Ghana.

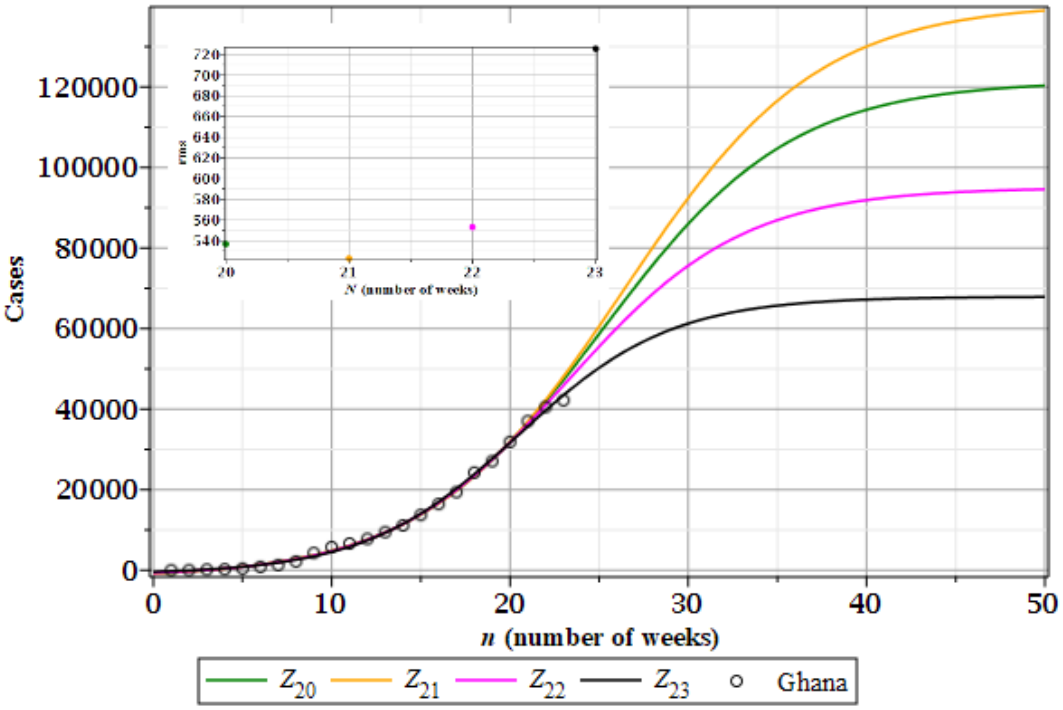


FIG. 111: Weekly Covid-19 case curves and rms for the data from Ghana.

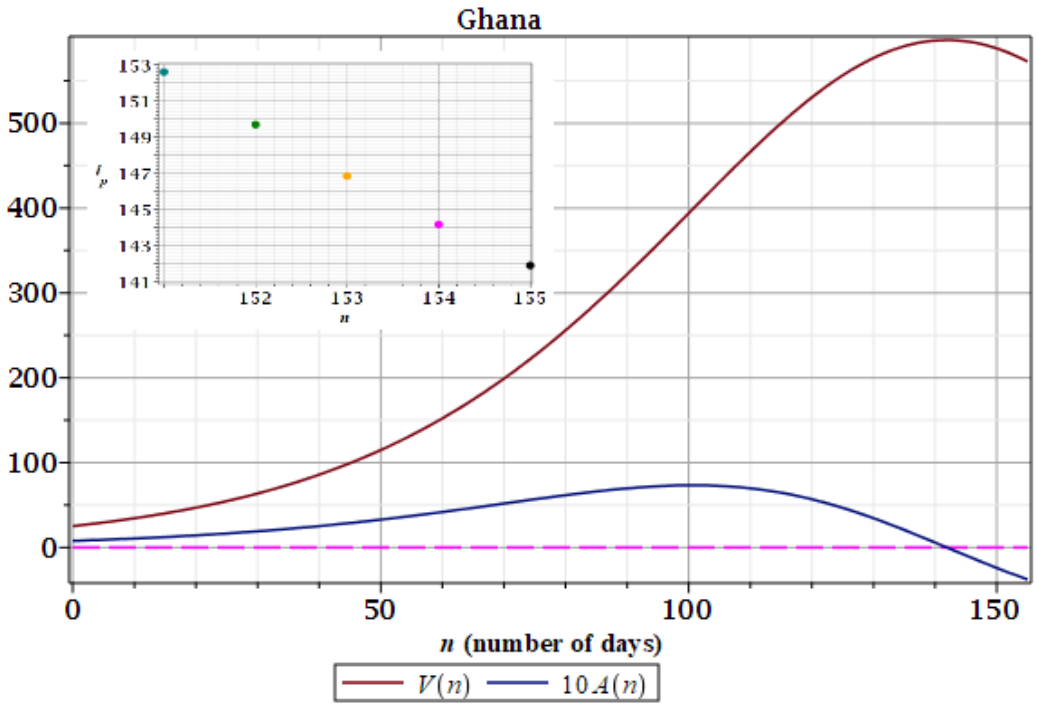


FIG. 112: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{155}. Incomplete curves. Inflection point: $n \approx 142$.

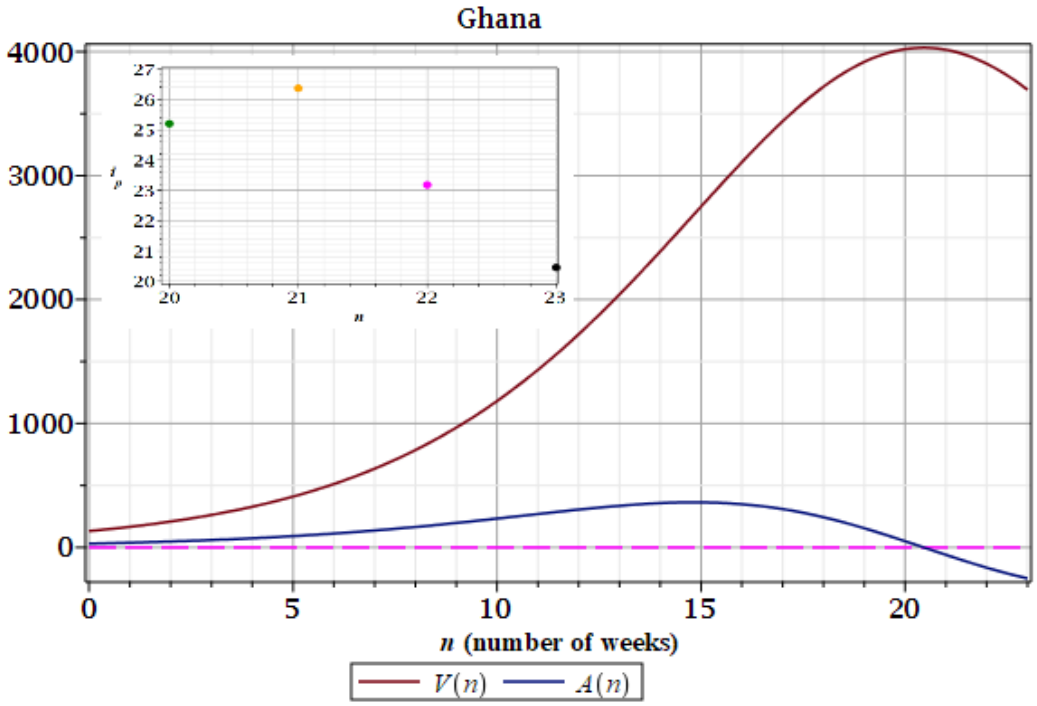


FIG. 113: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{23}. Incomplete curves. Inflection point: $n \approx 20.4$.

IX.29. Cabo Verde

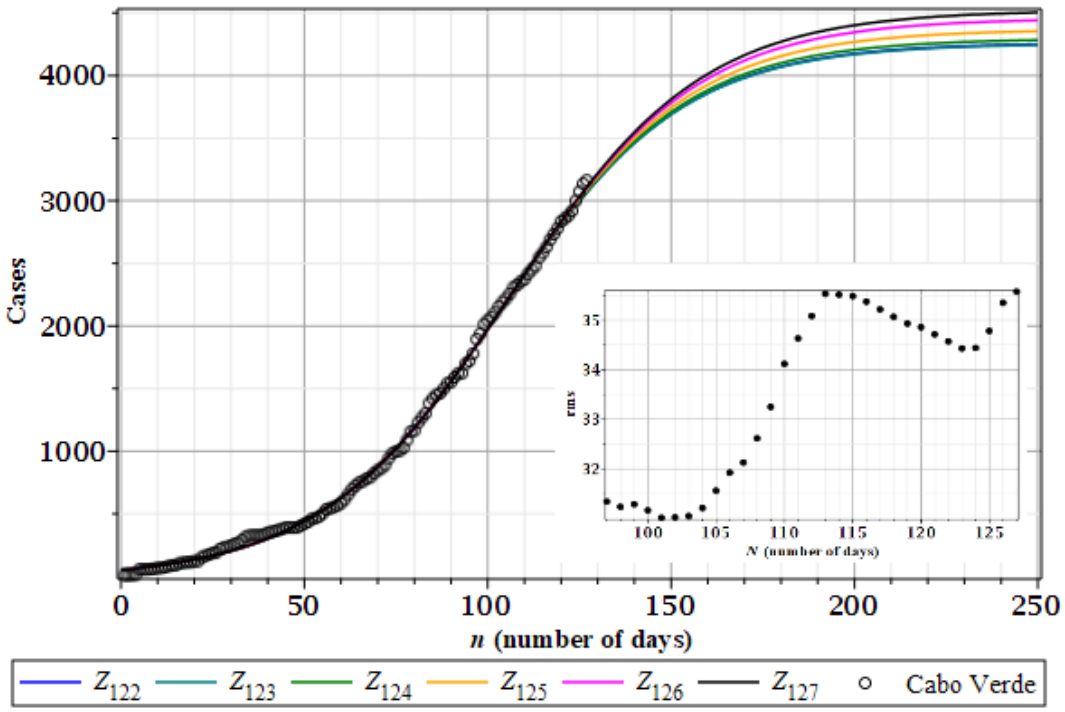


FIG. 114: Daily Covid-19 case curves and rms for the data from Cabo Verde.

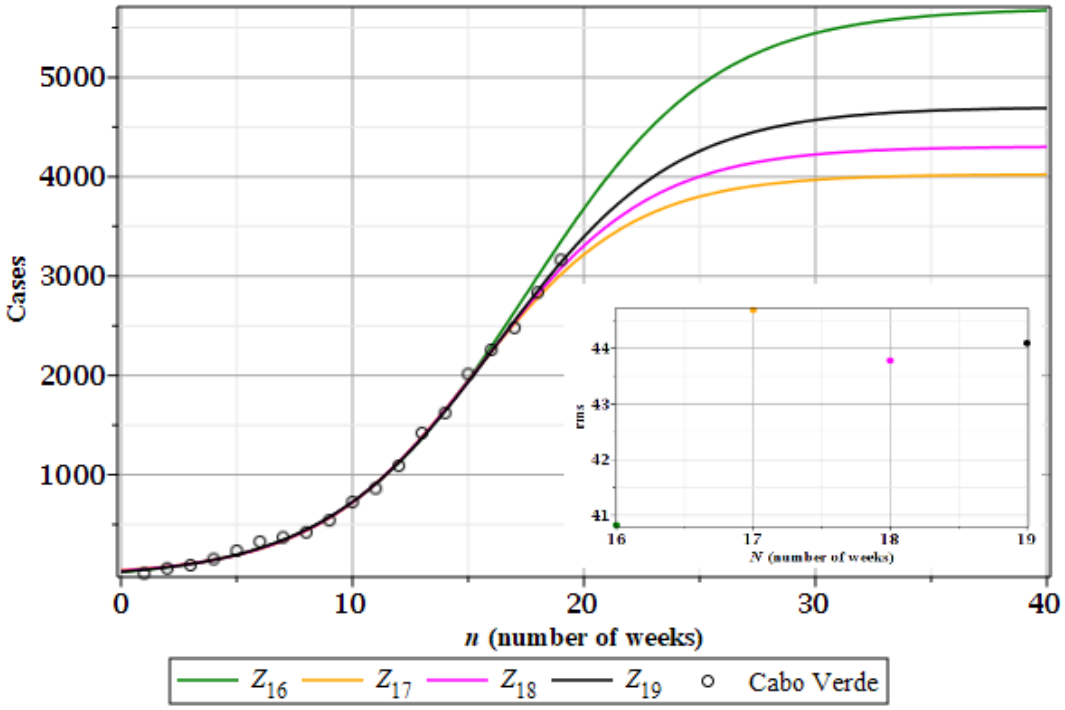


FIG. 115: Weekly Covid-19 case curves and rms for the data from Cabo Verde.

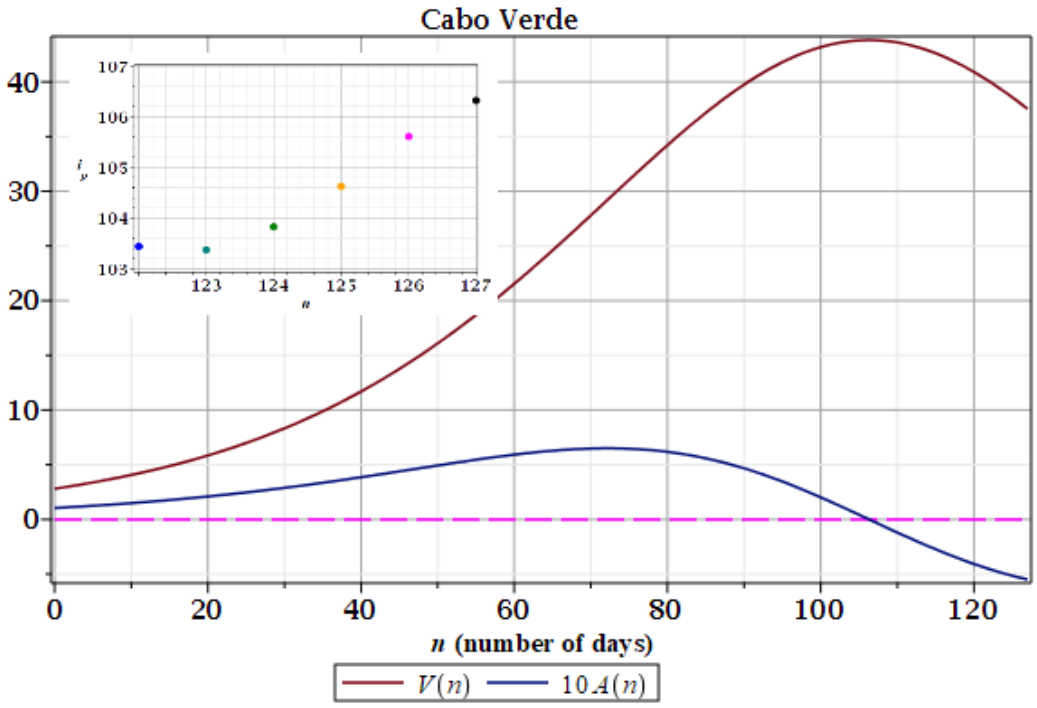


FIG. 116: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{127} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 106$.

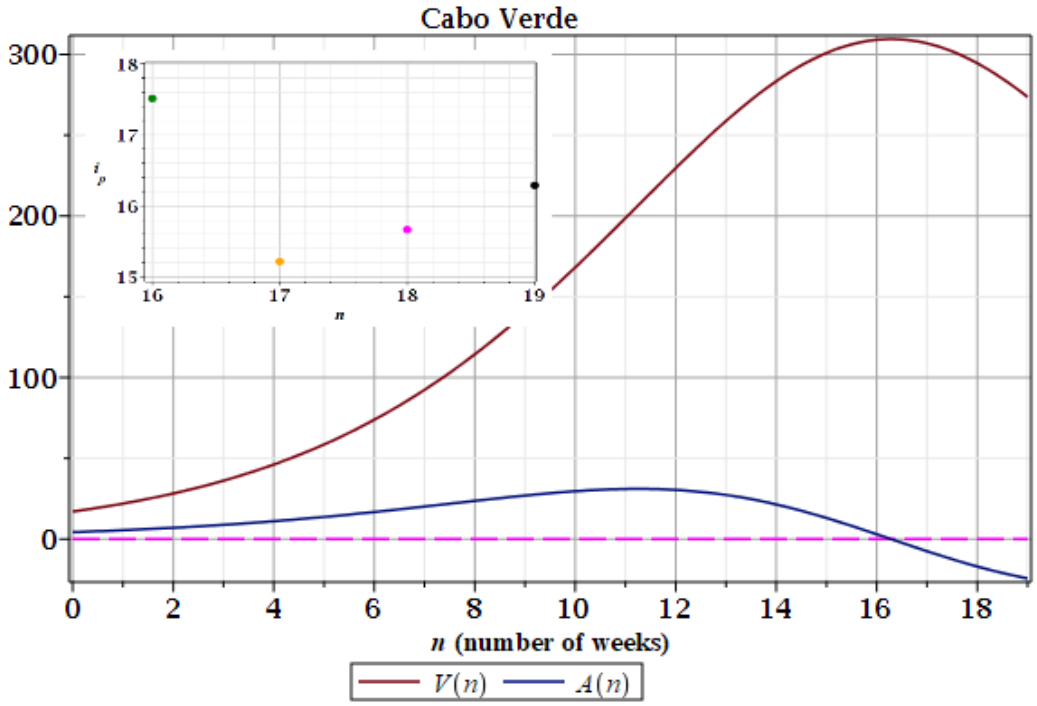


FIG. 117: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{19} and inflection points for the last curves. Incomplete curves. Inflection point: $n \approx 16.3$.

IX.30. Mozambique

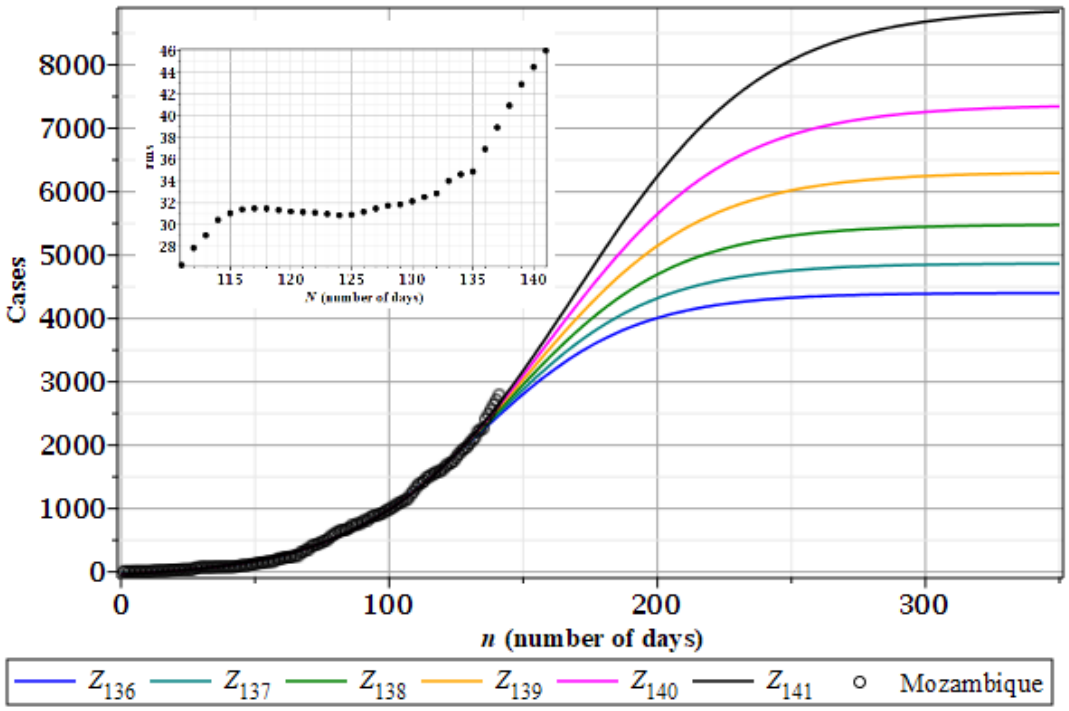


FIG. 118: Daily Covid-19 case curves and rms for the data from Mozambique.

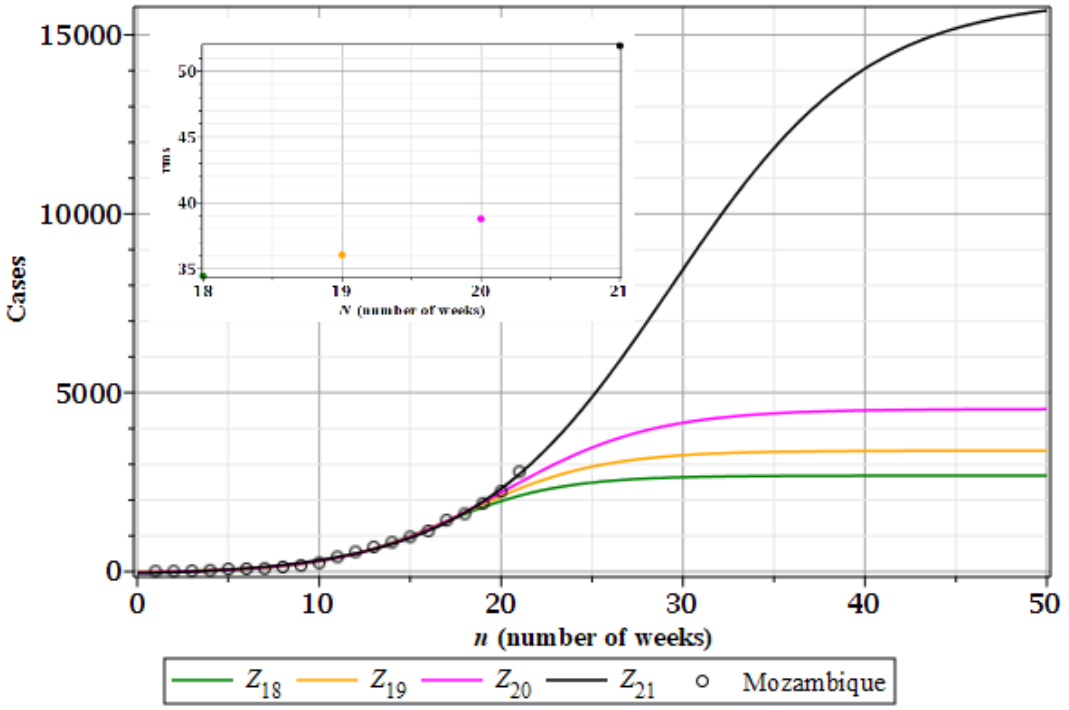


FIG. 119: Weekly Covid-19 case curves and rms for the data from Mozambique.

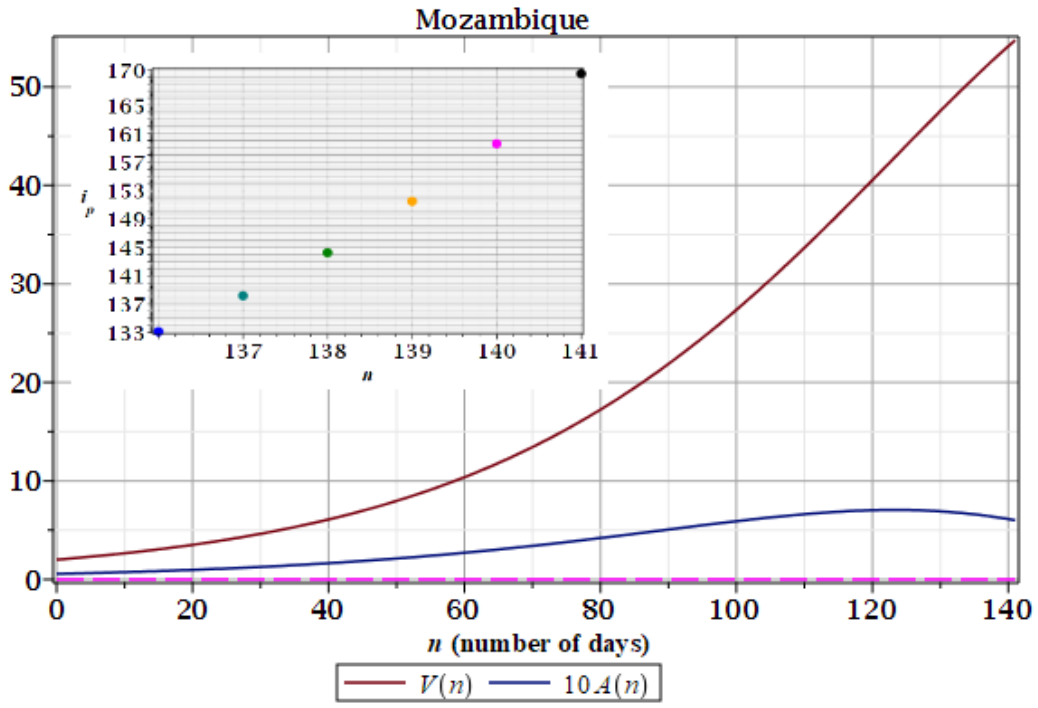


FIG. 120: Daily growth rates (speed V and acceleration A) from the fitted curve Z_{141} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

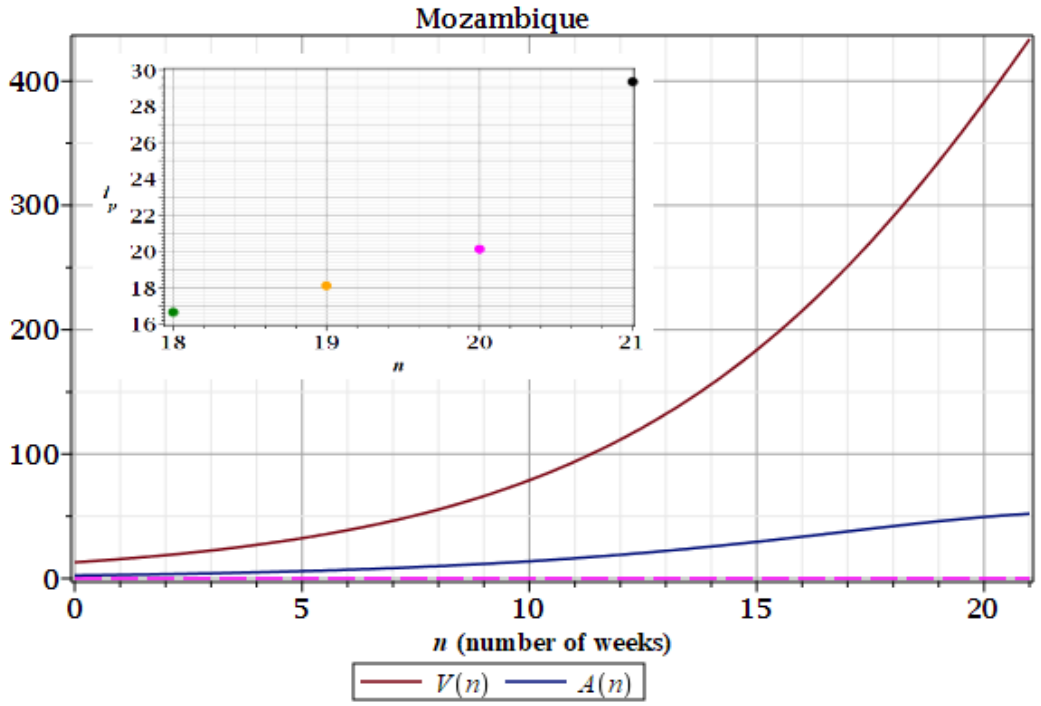


FIG. 121: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_{21} and inflection points for the last curves. Incomplete curves. Inflection point: absent.

IX.31. Australia

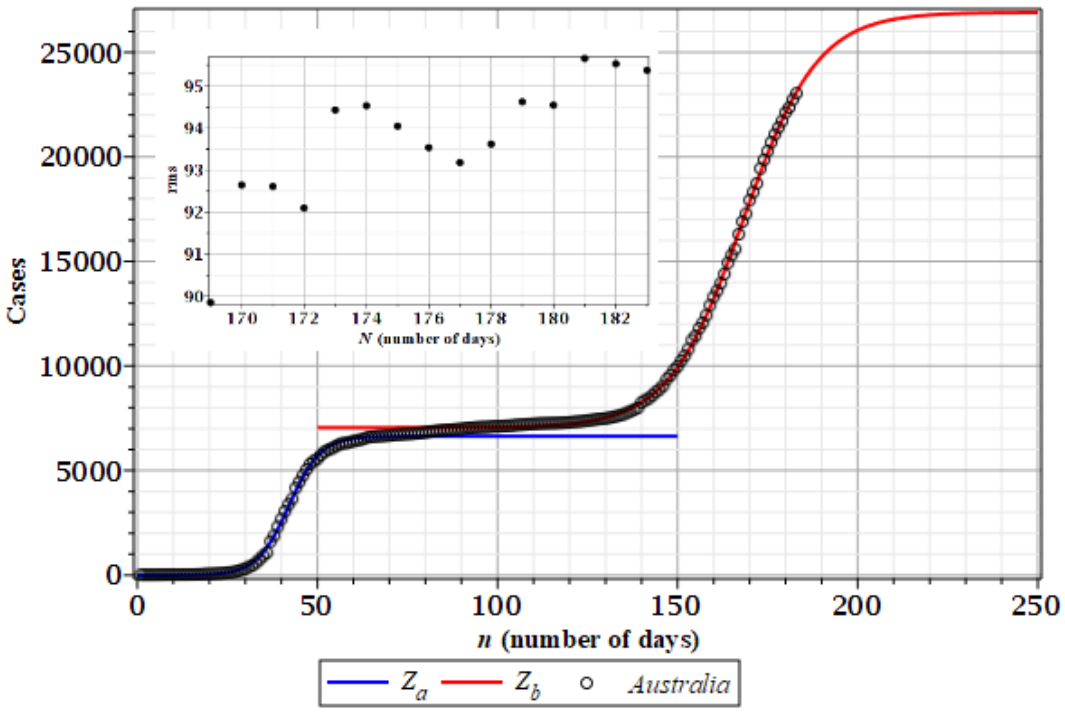


FIG. 122: Daily Covid-19 case curves and rms (from the last wave) for the data from Australia.

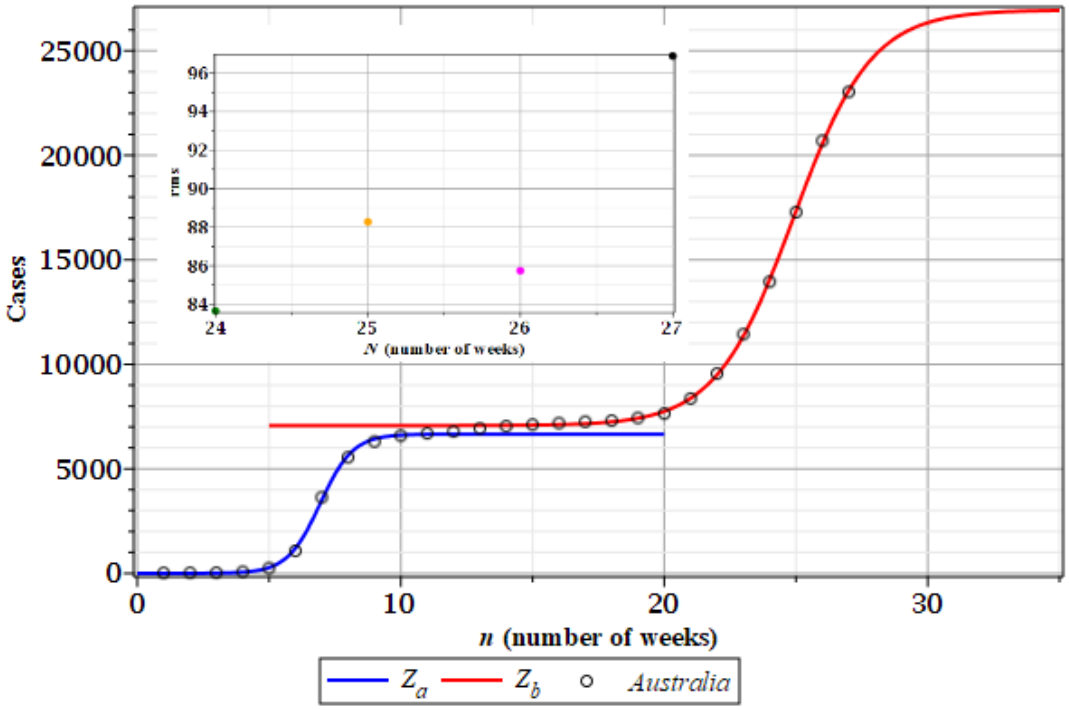


FIG. 123: Weekly Covid-19 case curves and rms (from the last wave) for the data from Australia.

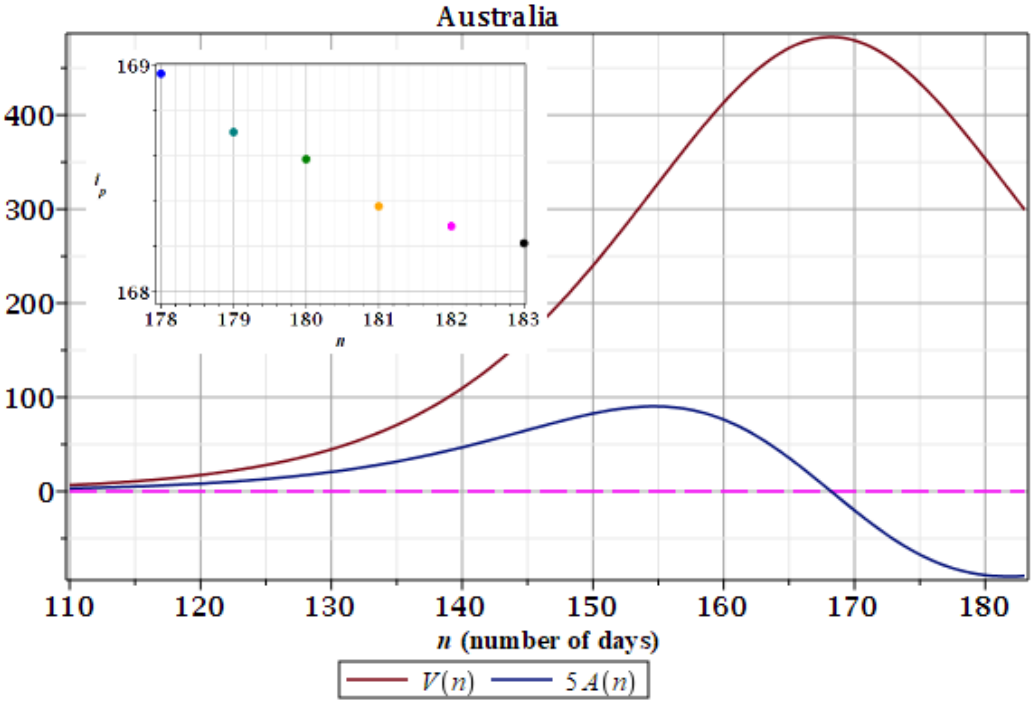


FIG. 124: Daily growth rates (speed V and acceleration A) from the fitted curve Z_b (the last wave). Inflection point: $n \approx 168$.

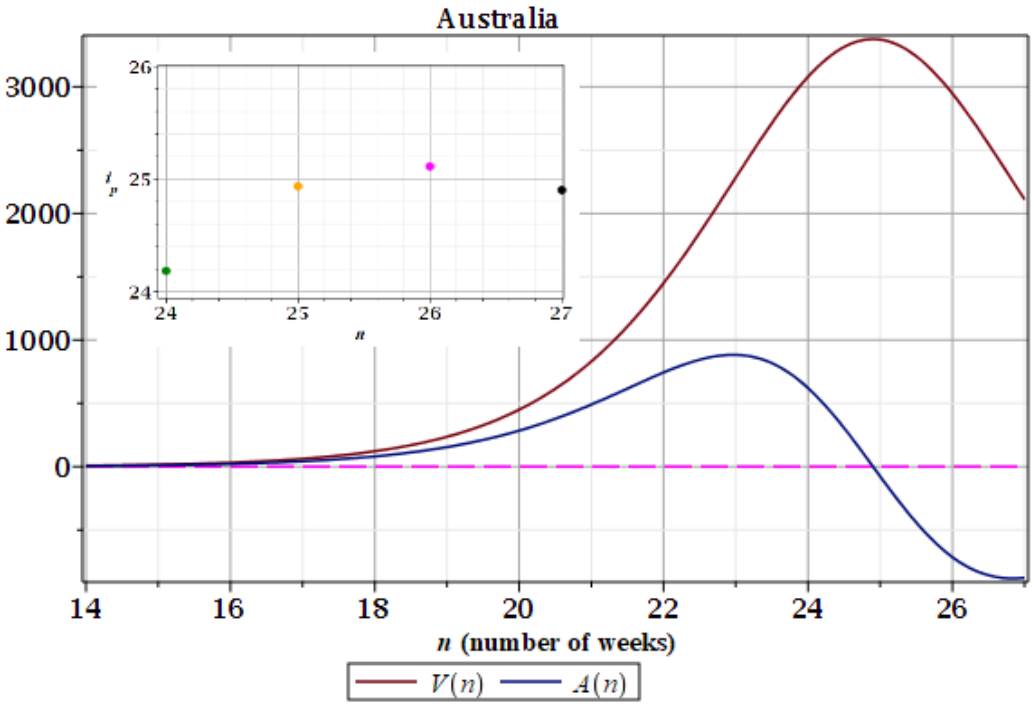


FIG. 125: Weekly growth rates (speed V and acceleration A) from the fitted curve Z_b (the last wave). Inflection point: $n \approx 24.9$.

IX.32. New Zealand

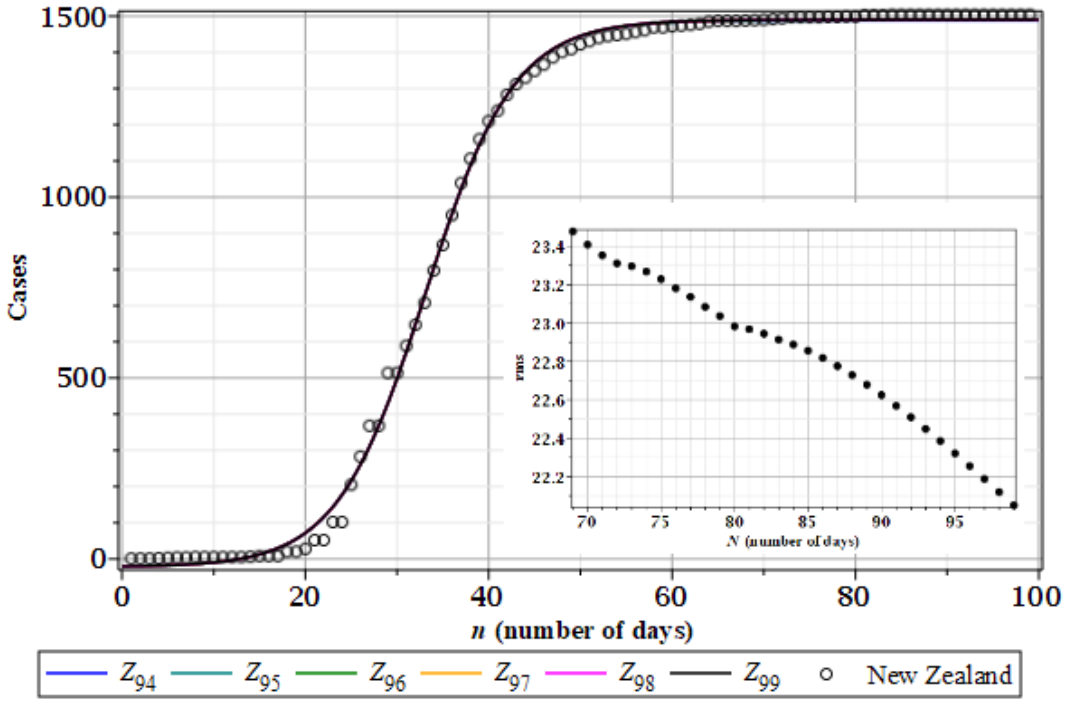


FIG. 126: Daily Covid-19 case curves amd rms for the data from New Zealand. $Z_{99} = 756\tanh(0.103n - 3.424) + 734$.

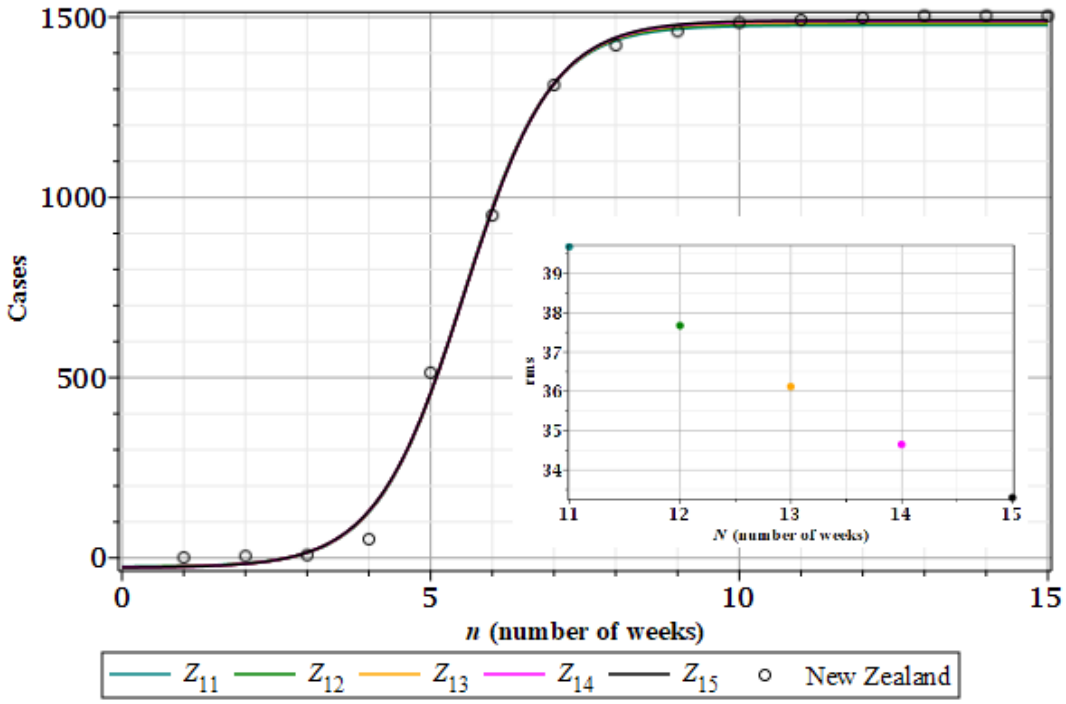


FIG. 127: Weekly Covid-19 case curves amd rms for the data from New Zealand. $Z_{15} = 759\tanh(0.698n - 3.871) + 732$.

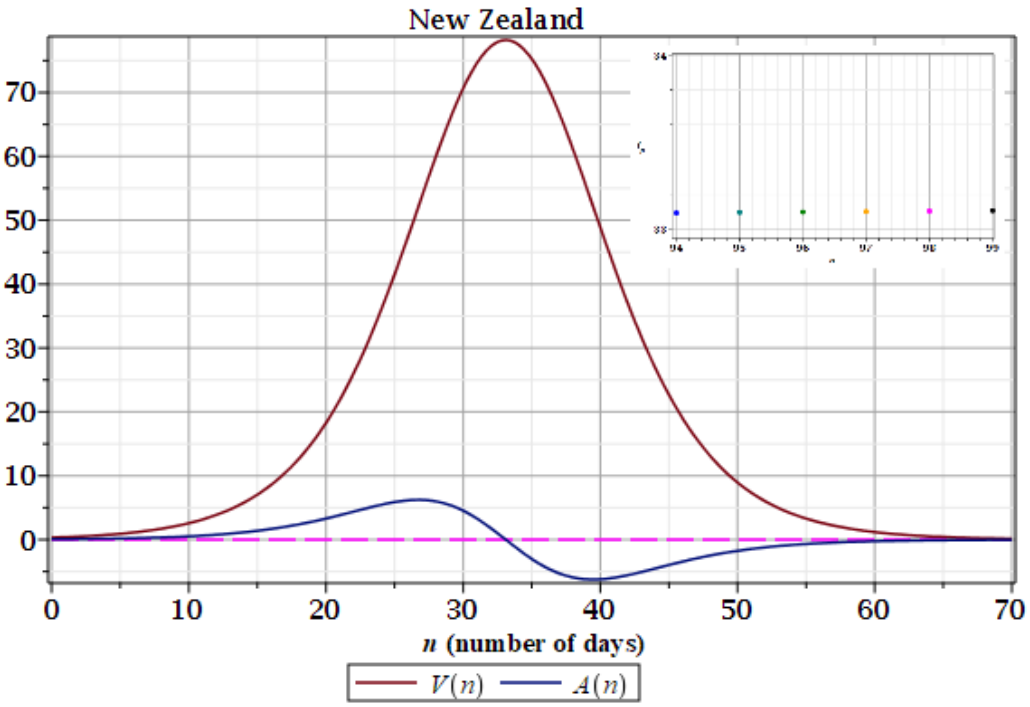


FIG. 128: Daily growth rates from Z_{99} and inflection points for the last curves. Complete curves. Inflection point: $n \approx 33$.

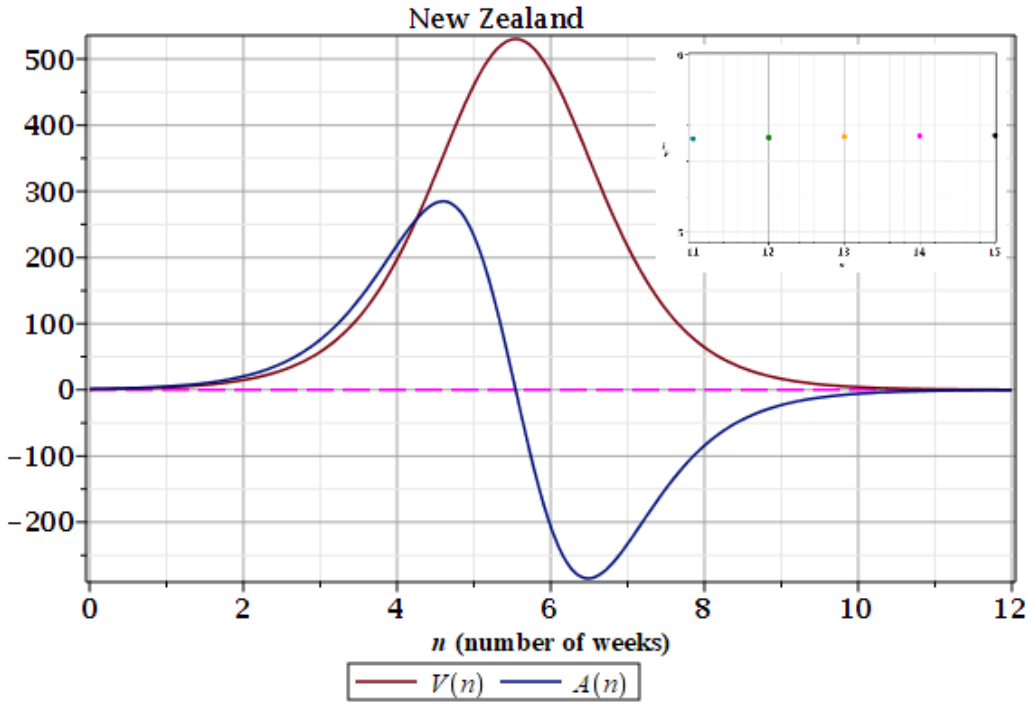


FIG. 129: Daily growth rates from Z_{15} and inflection points for the last curves. Complete curves. Inflection point: $n \approx 5.5$.

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