Issei Sasaki was born in Japan in 1947. He graduated from Mororan Institute of Technology, Hokkaido, Japan, in 1970.

From 1970 to 1978 he was a Research Associate at Hokkaido University and since 1978 he has been engaged in research on optical fibers at the University of Southampton, Hampshire, England.

Mr. Sasaki is a member of the Institute of Electronics and Communication Engineers of Japan.

M. J. Adams graduated from Imperial College, London University, London, England, in 1966 and received the M.Sc. and Ph.D. degrees from the University of Wales in 1967 and 1970, respectively.

After three years as a Research Fellow at University College, Cardiff, Wales, he worked from 1973 to 1975 at Plessey Radar, Cowes, Isle of Wight. Since 1975 he has been employed at the University of Southampton, Hampshire, England, where he is now a Lecturer in the Department of Electronics. His research interests include theoretical aspects of optical fibers and semiconductor optical components. He is the author or coauthor of over 50 papers and has recently published a book on optical waveguides.

Propagation in Doubly Clad Single-Mode Fibers

MICHEL MONERIE

Abstract—**General propagation properties and universal curves are given for doubly clad single-mode fibers with inner cladding index higher or lower than outer cladding index, using the two parameters: inner cladding/core radii ratio and inner cladding/core index differences ratio. LP01, LP11, and LP02 cutoff conditions are examined. It is shown that dispersion properties largely differ from the singly clad single-mode fiber case, leading to large new possibilities for low-loss dispersion-free fibers at any wavelength between 1.3 and 1.7 μm.**

I. Introduction

THIS paper results from calculations made on doubly clad fibers. We observed experimentally that the cutoff properties of LP11 and LP02 modes did not always match the values predicted by the weakly guiding singly-clad fiber theory and calculated with data issued from other experiments (refracted near-field pattern and preform measurements). Theoretical results on doubly clad fibers with depressed inner cladding have been previously published [1]–[3], but we experimentally study low-index inner cladding and high-index inner cladding as well. We then wanted to extend the theory to all types of doubly clad fibers. Some developments of our calculations led us to pay more attention to the dispersion properties of such structures. Kawakami and Nishida [1] already reported some features of the anomalous dispersion of W-type fibers, principally the theoretical possibility to cancel the glass dispersion at a wavelength of 1 μm, but with fiber specifications hardly very obtainable in practice. However, we show here that another part of the dispersion curves of W-type fibers allow us to obtain free dispersion operation with very low doping levels, contrary to singly clad fibers which require high doping levels. Now it has been demonstrated that particular attention should be paid to minimize the amount of germanium dopant required for any fiber design, in order to reduce loss [4]. This leads us to propose a new low-loss fiber structure with zero total dispersion at any wavelength between 1.3 and 1.7 μm.

Manuscript received August 19, 1981.

The author is with the Centre National d'Etudes des Télécommunications, Lannion, France.

The purpose of this paper is then to study the inner cladding effect on the propagation properties of the first guided modes of doubly clad fibers: cutoff, normalized propagation parameter, dispersion properties. We solve numerically the Maxwell equations without trying to find analytical formulas approaching the exact solution. However, as far as possible we give the physical meaning of some unusual results, especially when they differ from those of the singly clad fiber case.

Section II of this paper is devoted to the mathematical formulation of the problem: field solutions, dispersion equation, and its resolution.

Section III deals with results concerning the cutoff conditions for LP01, LP11, and LP02 for some practical cases. We show that a very simple general formula gives the condition for a nonzero LP01 cutoff in the case of depressed inner cladding.

Universal curves giving the normalized propagation parameter B are shown in Section IV for various cases of doubly clad fibers. We deduce from these data the LP01 modal dispersion properties of these fibers and review the possibility to obtain low-loss dispersion-free fibers in the range 1.3 to 1.7 μm.

We study the structure shown in Fig. 1: a weakly guiding fiber has a core radius a and a core refractive index n_1. The

0018-9480/82/0400-0381$00.75 © 1982 IEEE

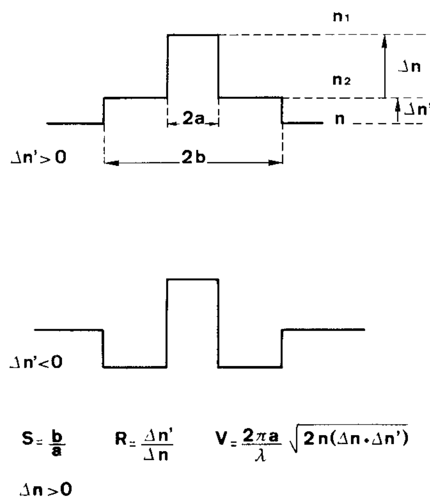


Fig. 1. Refractive index variation along a cross section diameter of a doubly clad weakly guiding fiber.

inner cladding has a radius b and a refractive index n_2. The refractive index of the surrounding medium (outer cladding) is n.

II. Mathematical Resolution

We note (see Fig. 1) that

$$\Delta n = n_1 - n_2$$

$$\Delta n' = n_2 - n$$

$$V = V_1 = k_o a(n_1^2 - n^2)^{1/2} = k_o a[2n(\Delta n + \Delta n')]^{1/2}$$

$$V_2 = k_o b(|n_2^2 - n^2|)^{1/2} = k_o b[2n|\Delta n'|]^{1/2}$$

where k_o is the vacuum wave number.

We also introduce the two main parameters $R = \Delta n'/\Delta n$ and $S = b/a$. It will be shown further that propagation characteristics depend only on R, S, and V. Note that $R > -1$ for mathematical guiding (propagation constant $\beta > k_o n$). In this work we limit ourselves to $\Delta n > 0$; $\Delta n < 0$ means a dip in the inner cladding but this does not correspond to a practical fiber. It rather describes a single-mode fiber with a dip in the core, the inner cladding (with $\Delta n' > 0$) becoming the core.

A. Eigenfields

β being the mode propagation constant, we define the usual normalized propagation parameter B

$$B = \frac{\beta^2 - k_o^2 n^2}{k_o^2(n_1^2 - n^2)} = \frac{\beta - k_o n}{k_o(\Delta n + \Delta n')} \text{ (weakly guiding case).}$$

Our purpose is to complete $B(V)$ and the mode dispersion parameters $d(VB)/dV$ and $V(d^2(VB)/dV^2)$ for various values of $R = \Delta n'/\Delta n$ and $S = b/a$.

We define mode parameters

$$u = a(k_o^2 n_1^2 - \beta^2)^{1/2}$$

$$u' = b(k_o^2 n_2^2 - \beta^2)^{1/2}$$

$$v' = b(\beta^2 - k_o^2 n_2^2)^{1/2}$$

$$v = b(\beta^2 - k_o^2 n^2)^{1/2}.$$

For the azimuthal order m, the radial dependance $\psi(r)$ of the axial fields components are expressed as [5]

$$\psi = A_o J_m\left(u\frac{r}{a}\right) \qquad \text{for } r \leqslant a$$

$$\psi = A_1 J_m\left(u'\frac{r}{b}\right) + A_2 Y_m\left(u'\frac{r}{b}\right) \qquad \text{for } a \leqslant r \leqslant b \text{ if } \beta < k_o n_2$$

$$\psi = A_3 K_m\left(v\frac{r}{b}\right) \qquad \text{for } r \geqslant b \tag{1}$$

and as

$$\psi = A'_o J_m\left(u\frac{r}{a}\right) \qquad \text{for } r \leqslant a$$

$$\psi = A'_1 I_m\left(v'\frac{r}{b}\right) + A'_2 K_m\left(v'\frac{r}{b}\right) \qquad \text{for } a \leqslant r \leqslant b \text{ if } \beta > k_o n_2$$

$$\psi = A'_3 K_m\left(v\frac{r}{b}\right) \qquad \text{for } r \geqslant b \tag{2}$$

where J_m, Y_m, I_m, and K_m are the usual Bessel and modified Bessel functions. A_o and A'_o are normalization coefficients. Analytical expressions for A_1, A_2, A_3 and A'_1, A'_2, A'_3 are given in Appendix I. The field components are derived as usual from ψ [5].

B. The Dispersion Equation

We find the dispersion equation by matching ψ and $\delta\psi/\delta r$ at interfaces between core, inner cladding, and outer cladding. In the limit of a weakly guiding fiber (all relative index differences $\ll 1$), this continuity is equivalent to the continuity of the transverse components [6]. This leads to a (4×4) matrix which determinant must be equal to zero to ensure a nontrivial solution.

For $\beta < k_o n_2$, we have

$$\frac{[\hat{J}_m(u) - \hat{Y}_m(u'c)]\;[\hat{K}_m(v) - \hat{J}_m(u')]}{[\hat{J}_m(u) - \hat{J}_m(u'c)]\;[\hat{K}_m(v) - \hat{Y}_m(u')]} = \frac{J_{m+1}(u'c)\,Y_{m+1}(u')}{J_{m+1}(u')\,Y_{m+1}(u'c)}. \tag{3}$$

For $\beta > k_o n_2$, we have

$$\frac{[\hat{J}_m(u) - \hat{K}_m(v'c)]\;[\hat{K}_m(v) + \hat{I}_m(v')]}{[\hat{J}_m(u) + \hat{I}_m(v'c)]\;[\hat{K}_m(v) - \hat{K}_m(v')]} = \frac{I_{m+1}(v'c)\,K_{m+1}(v')}{I_{m+1}(v')\,K_{m+1}(v'c)} \tag{4}$$

where

$$\hat{Z}_m(x) = \frac{Z_m(x)}{x Z_{m+1}(x)} \tag{5}$$

(Z representing the Bessel functions J, Y, I, or K) and

$$c = \frac{a}{b} = \frac{1}{S}.$$

III. Cutoff Properties

A. Cutoff Equations

At cutoff, $\beta = k_o n$. The radial propagation constants take the values

$$\begin{cases} u = V & r < a \\ u' = V_2 (\Delta n' > 0) \quad \text{or} \quad v' = V_2 (\Delta n' < 0) & a < r < b \\ v = 0 & r > b. \end{cases} \tag{6}$$

Close to cutoff, the radial propagation constant v decreases to zero. We find the cutoff conditions by calculating the limiting form of the dispersion equation for $v \to 0$. We use the limiting form of $K_m(v)$ for small arguments [7]

$$\text{for } v \to 0, \quad K_o(v) \sim -\ln(v)$$

$$K_m(v) \sim \frac{2^{m-1}\Gamma(m)}{v^m}. \tag{7}$$

1) For $\Delta n' > 0$, the LP01 mode is always guided. $\hat{K}_m(v)$ is equivalent to $1/2m$ $(m \geqslant 1)$, and the brackets containing $\hat{K}_m(v)$ in (3) become $-J_{m-1}(V_2)/2m\, J_{m+1}(V_2)$ at numerator and $-Y_{m-1}(V_2)/2m\, Y_{m+1}(V_2)$ at denominator. We then have the cutoff conditions for higher order modes $(m \geqslant 1)$

$$\frac{\hat{J}_m(V) - \hat{Y}_m(V_2 c)}{\hat{J}_m(V) - \hat{J}_m(V_2 c)} = \frac{J_{m+1}(V_2 c)\,Y_{m-1}(V_2)}{Y_{m+1}(V_2 c) J_{m-1}(V_2)}. \tag{8}$$

2) For $\Delta n' < 0$, the LP01 mode may be leaky. $\hat{K}_o(v)$ is equivalent to $-\ln(v)$ and the terms containing $\hat{K}_m(v)$ cancel out. For $m \geqslant 1$, they become $I_{m-1}(V_2)/2m\, I_{m+1}(V_2)$ (numerator) and $K_{m-1}(V_2)/2m\, K_{m+1}(V_2)$ (denominator). Finally, taking into account the formal identities $I_{-1}(x) = I_1(x)$ and $K_{-1}(x) = K_1(x)$, we obtain for $\Delta n' < 0$ and for all cases ($m = 0$ and $m \geqslant 1$)

$$\frac{\hat{J}_m(V) - \hat{K}_m(V_2 c)}{\hat{J}_m(V) + \hat{I}_m(V_2 c)} = \frac{I_{m+1}(V_2 c) K_{m-1}(V_2)}{K_{m+1}(V_2 c) I_{m-1}(V_2)}. \tag{9}$$

B. Cutoff Curves

Equations (8) and (9) contain V, V_2, and $V_2 c$, which can be expressed as functions of $V = k_o a\sqrt{2n(\Delta n + \Delta n')}$, $R = \Delta n'/\Delta n$, and $S = b/a$, since $c = 1/S$ and $V_2 = VS(|R|/1 + R)^{1/2}$. We deduce relations $f_m(V_{cm}, R, S) = 0$ for modes with azimuthal order m, V_{cm} being the normalized frequency at cutoff.

1) LP01 Mode: As pointed out by Kawakami and Nishida [1] and Sammut [3], the fundamental mode has not necessarily a nonzero cutoff as soon as $\Delta n' < 0$. The refractive index gap must be large enough ($b/a \gg 1$ or $|\Delta n'| \simeq \Delta n$) for the field to be modified. Fig. 2 shows the limit between guiding and leaky areas for the LP01 mode in the (b/a, $\Delta n'/\Delta n$) diagram. This curve is the exact one, computed numerically from (9).

However, it is possible to find in a simple way the equation of this curve. Kawakami and Nishida [1] speak of the average index effectively seen by the wave. Using a stationary expression allows us to quantify this idea and effectively leads to the exact equation, but we prefer here to give a direct demonstration. We shall look at the condition leading to $V = 0$ at cutoff of the LP01 mode. Starting from (9) and taking its limiting form for $V \to 0$, we have

$$\hat{J}_o(V) \sim 2\,V^{-2}, \quad \hat{K}_o(V_2 c) \sim -\ln(V_2 c), \quad \hat{I}_o(V_2 c) \sim 2(V_2 c)^{-2}$$

the right term being equal to c^2. We then obtain

$$c^2 + \left(\frac{V}{V_2}\right)^2 = 1, \quad \text{or} \quad \frac{b}{a} = \left(\frac{|\Delta n'|}{\Delta n}\right)^{-1/2}. \tag{10}$$

This is the equation of the curve shown in Fig. 2. As a matter of fact, this is the condition for the LP01 mode to have no cutoff, but it does not give the cutoff value of V when it is different from zero.

It must be well understood that we speak here of the mathematical cutoff, defined as $\beta = k_o n$. Obviously, the LP01 mode will be physically guided whatever $\Delta n'$ may be, as soon as b/a is sufficiently large, its attenuation being very low [2].

2) LP01, LP11, and LP02 Modes: We show in Figs. 3 and 4 the cutoff characteristics of the first propagating modes. Fig. 3 represents the V_c value of the normalized frequency V at cutoff as a function of $\Delta n'/\Delta n$ for two values of b/a. Curiously, one can see that if b/a is sufficiently large then there is a range of positive $\Delta n'/\Delta n$ for which the LP02 mode is guided, whereas the LP11 mode is still leaky. This seems easily understandable since the LP02 mode takes advantages of the core index difference (maximum field value for $r = 0$), whereas the LP11 mode is principally affected by the inner cladding refractive index (slightly above cutoff, the oscillating part of the field spreads up to $r = b$).

Fig. 4 shows V_c versus b/a for various $\Delta n'/\Delta n$ for the LP01 and LP11 modes.

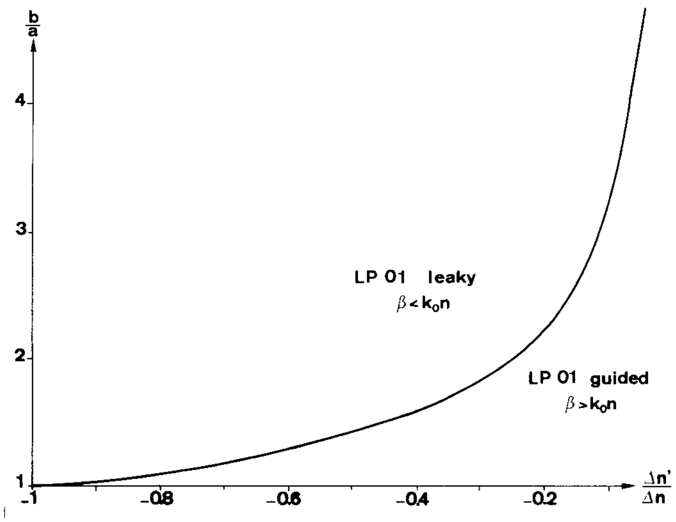


Fig. 2. Cutoff limit for the LP01 mode.

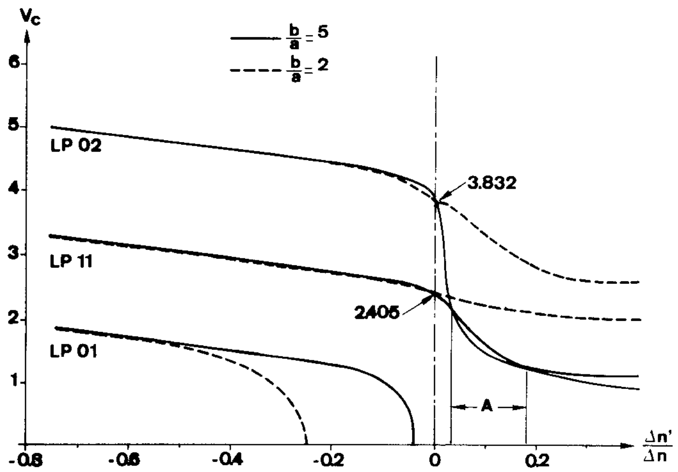


Fig. 3. Normalized frequency at cutoff V_c versus $\Delta n'/\Delta n$ for $b/a = 2$ and $b/a = 5$. Note that in area A for $b/a = 5$, the LP02 mode is guided, whereas the LP11 mode is leaky.

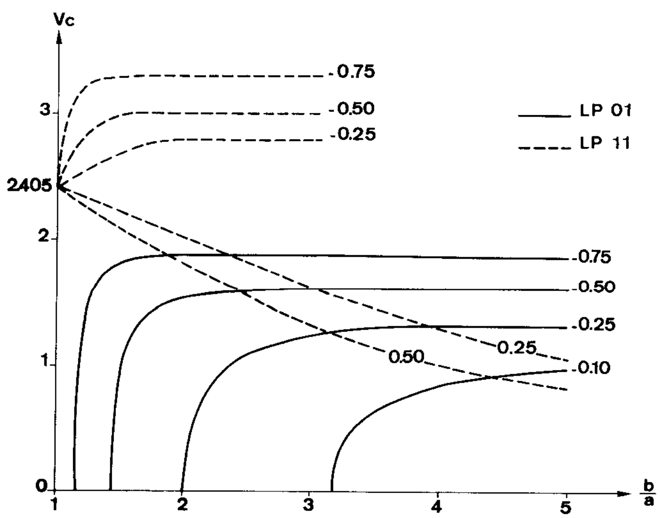


Fig. 4. Normalized frequency at cutoff V_c as a function of b/a for various $\Delta n'/\Delta n$ values.

IV. Normalized Propagation and Dispersion Parameters

A. Normalized Propagation Constant B(V)

It is possible to express u, u', v', and v as functions of $B, R = \Delta n'/\Delta n$, and $S = b/a$. We have from Section II-A

$$u = V(1 - B)^{1/2}$$

$$u' = VS\left(\frac{R}{1+R} - B\right)^{1/2}$$

$$v' = VS\left(B - \frac{R}{1+R}\right)^{1/2}$$

$$v = VS\sqrt{B}.$$

It is thus possible to compute $B(V)$ for given values of R and S. Fig. 5(a)-(c) shows the curves $B(V)$, respectively, for $b/a = 1.5$, 2, and 5, for various values of $\Delta n'/\Delta n$. The curvature sign change around $V = 3$ (Fig. 5(c), LP11 mode, $\Delta n' > 0$) approximately occurs for the V value for which the oscillating part of the field spreads in the inner cladding.

B. Normalized Mode Dispersion Parameters $d(VB)/dV$ and $V(d^2(VB)/dV^2)$

Defining the group delay of the LP01 mode as $\tau = d\beta/d\omega$, we obtain the derivative of the group delay with respect to the wavelength λ as (see Appendix II)

$$\frac{d\tau}{d\lambda} = M\left[1 + \Delta\frac{d(VB)}{dV}\right] - \frac{N\Delta}{\lambda c}\left[V\frac{d^2(VB)}{dV^2} - p\frac{d(VB)}{dV}\right] \tag{11}$$

where $N = n - \lambda(dn/d\lambda)$ is the group index of the outer cladding, $M = (1/c)\,(dN/d\lambda)$ its material dispersion, p the mean profile dispersion parameter, and Δ the relative core index difference $(\Delta n + \Delta n')/n$. It is then necessary to compute $d(VB)/dV$ and $V(d^2(VB)/dV^2)$ to know how to cancel the total (material + modal) mode dispersion. For $\Delta n'/\Delta n = 0$ (singly clad fiber) simple enough analytic expressions are known for these two quantities [8]. As the similar expressions for the doubly clad fibers must likely be quite complex, we merely did a numerical computation of $d(VB)/dV$ and $V(d^2(VB)/dV^2)$ from $B(V)$ for three neighboring values of V. The relative accuracy on B is 10^{-6}, and the computed mode dispersion parameters for $\Delta n' = 0$ are within 10^{-3} compared to the exact analytical results in the case of the singly clad fiber. However, some numerical difficulties appear close to cutoff, when the LP01 has a nonzero V_c. In this case, the $B(V)$ curve is strongly affected and its derivatives vary very rapidly. It can be shown easily that at cutoff, for all guided modes, $d(VB)/dV$ and $V(d^2(VB)/dV^2)$ tends to zero when V approaches V_c.

Fig. 6 shows what occurs in the vicinity of $B = 0$ for $R = -0.5$. The left vertical scale $B_2 = \beta - k_o n_2/k_o(n_1 - n_2) = B(1 + R) - R$ is associated with the lower horizontal scale $V_{12} = V(1 + R)^{-1/2}$ and is valid for the broken line, showing the normalized propagation parameter without outer cladding. The right horizontal scale B, associated with the upper hori-

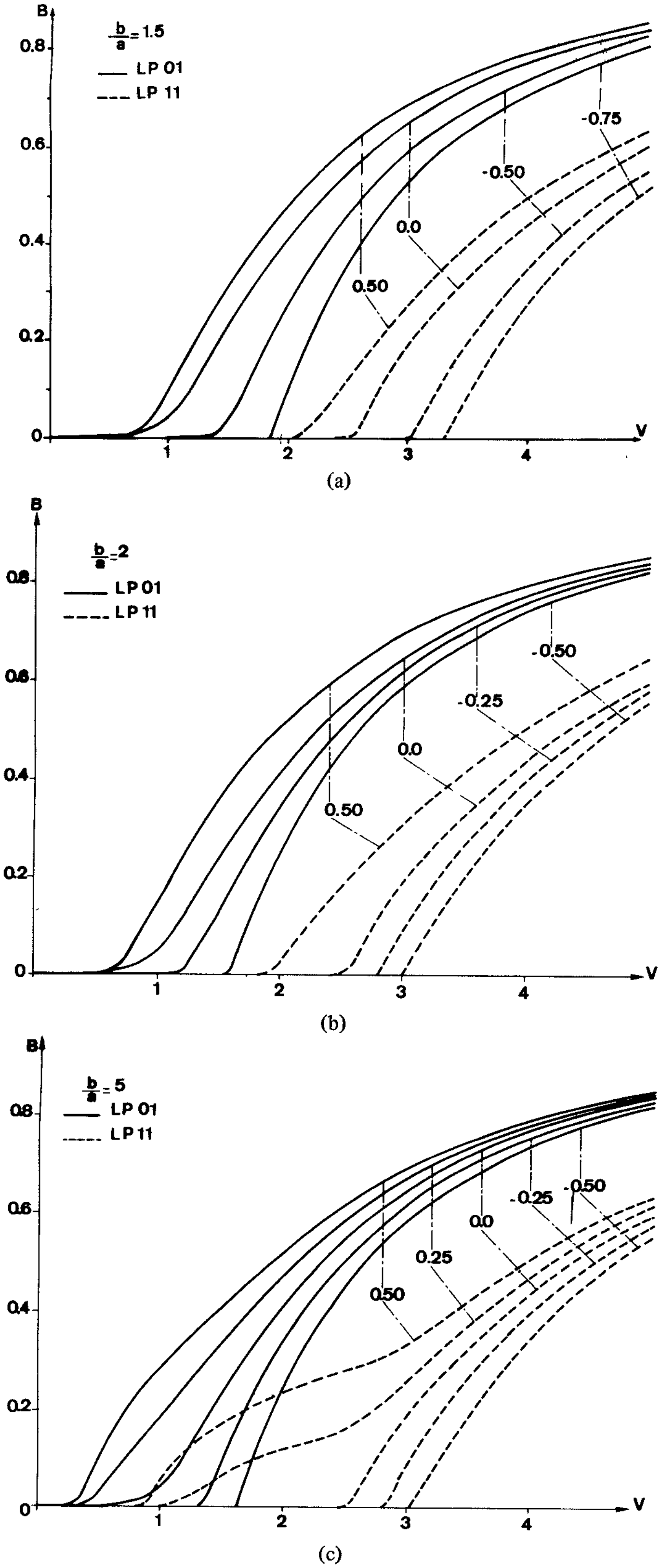


Fig. 5. (a) $B(V)$ for various $\Delta n'/\Delta n$ for $b/a = 1.5$, (b) $b/a = 2$, and (c) $b/a = 5$.

zontal scale V, is valid for full lines and shows $B(V)$ for $b/a = 1.5$ and 5. It is clear that B will vary rapidly the larger b/a is,

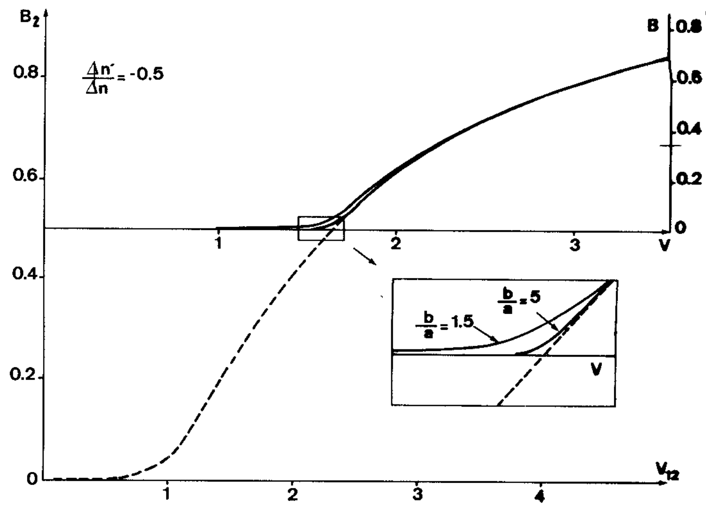


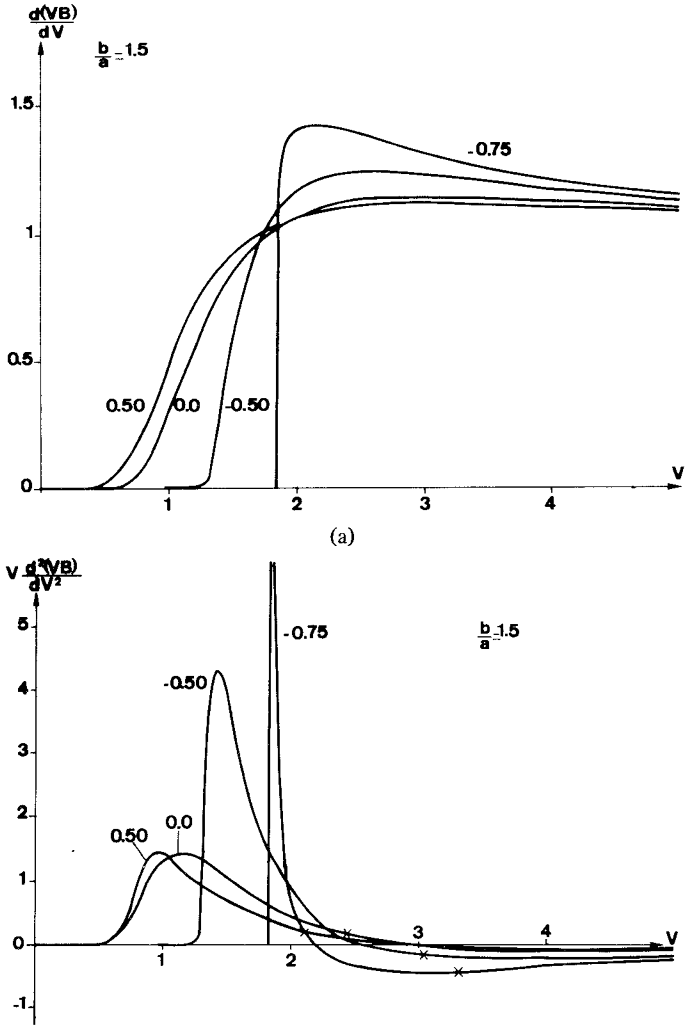
Fig. 6. Depressed inner cladding influence on the $B(V)$ curve. See explanations in the main text.

leading to very fast changes of $d(VB)/dV$ and $V(d^2(VB)/dV^2)$. Roughly speaking, there will be a sharp peak for $d^2(VB)/dV^2$ when V_c will approach the asymptotic value of the curves $V_c(b/a)$ in Fig. 4. This limiting value corresponds to the crossing between $B_2(V_{12})$ and the V axis in Fig. 6.

Figs. 7 and 8 show $d(VB)/dV$ (a) and $V(d^2(VB)/dV^2)$ (b), respectively, for $b/a = 1.5$ and $b/a = 2$ for various $\Delta n'/\Delta n$. One can see the above-mentioned curve features, especially the high values of $d^2(VB)/dV^2$ when V_c is close to its asymptotic value for $b/a \to \infty$.

C. Zero Total Mode Dispersion

To obtain a zero total dispersion at a wavelength between 1.3 and 1.7 μm, it is necessary to have a high negative modal dispersion to cancel the high positive material dispersion coefficient M of silica (22 ps/nm/km at 1.55 μm, 31 ps/nm/km at 1.7 μm). $V(d^2(VB)/dV^2)$ has a maximum value of 1.4 for singly clad single-mode fiber, leading to high core/cladding index differences for dispersion-free operation: $\Delta n = 10.10^{-3}$ at $\lambda = 1.55$ μm, $\Delta n = 15.10^{-3}$ at $\lambda = 1.7$ μm [9]. These high values lead to excess propagation loss, since the scattering losses increase with increasing dopant concentration in germanium doped silica fibers [4], [10]. Using doubly clad fibers avoids this drawback. It would not be advisable to use the very high obtainable peak values of $V(d^2(VB)/dV^2)$ since tolerances would be very stringent. Moreover, the cutoff region would bring extra bending losses. However, it seems possible to ensure $V(d^2(VB)/dV^2) = 2\text{-}3$ without disadvantage. It would then not be convenient to work at the top of the curve, but rather on a side in order to relax the manufacturing parameters by taking advantage of compensation phenomena between different successive fibers. Of course, we shall choose the right side to avoid cutoff. Many combinations between Δn, $\Delta n'/\Delta n$, and b/a can be found to cancel the total mode dispersion. We shall not give exhaustive relations between these three parameters, but only some results concerning the "best" fibers having the minimum total loss taking into account microbendings, bends, lateral and angular misalign-



(b)

Fig. 7. (a) Dispersion parameters $d(VB)/dV$ and (b) $V(d^2(VB)/dV^2)$ for $b/a = 1.5$ for various $\Delta n'/\Delta n$ values. Crosses indicate the limits of single-mode operation.

ments at splices, absorption, and scattering. The best equivalent step-index singly clad fiber has a core/cladding index difference equal to 6.5 10^{-3} when working at 1.55 μm (optimization for terrestrial and submarine applications), this result being not affected by the V value for $1.6 < V < 2.4$ [11].

From the equivalence relations established by Matsumura [12], we compute the W-fiber equivalent to this best step-index singly clad fiber. For $\Delta n'/\Delta n = -0.5$ and $b/a = 1.5$, this leads to $\Delta n = 12.6\ 10^{-3}$. For $\Delta n'/\Delta n = -0.25$ and $b/a = 2.0$, we obtain $\Delta n = 7.7\ 10^{3}$. Using (12) and curves of Figs. 7 and 8 we find that $N\Delta\ V(d^2(VB)/dV^2)$ must be equal to 10.10^{-3} for $\lambda = 1.55\ \mu$m and 16.10^{-3} for $\lambda = 1.7\ \mu$m, leading to the following results for zero total dispersion ($d\tau/d\lambda = 0$):

$$\frac{\Delta n'}{\Delta n} = -0.50, \quad \frac{b}{a} = 1.5,$$

$$\Delta n + \Delta n' = 6.3\ 10^{-3}$$

$$\begin{cases} \lambda = 1.55\ \mu\text{m} \Rightarrow V\dfrac{d^2(VB)}{dV^2} = 1.6, & V = 1.8 \\ \lambda = 1.7\ \ \mu\text{m} \Rightarrow V\dfrac{d^2(VB)}{dV^2} = 2.5, & V = 1.65 \end{cases}$$

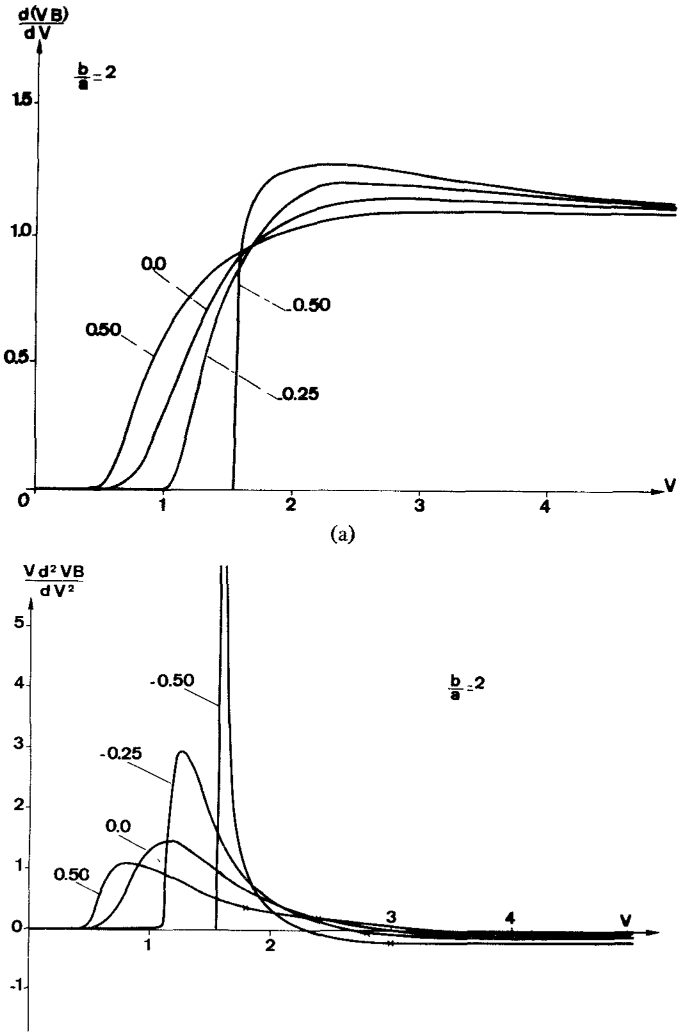
$$\frac{\Delta n'}{\Delta n} = -0.25, \quad \frac{b}{a} = 2,$$

$$\Delta n + \Delta n' = 5.8\ 10^{-3}:$$

$$\begin{cases} \lambda = 1.55\ \mu\text{m} \Rightarrow V\dfrac{d^2(VB)}{dV^2} = 1.7, & V = 1.55 \\ \lambda = 1.7\ \ \mu\text{m} \Rightarrow V\dfrac{d^2(VB)}{dV^2} = 2.8, & V = 1.3. \end{cases}$$

These fibers will have very similar total attenuations for a given wavelength since they correspond to the same equivalent step-index singly clad fiber (same V_{eq} and $w_{o\text{eq}}$ for the same wavelength). The second data set (for $b/a = 2$) will perhaps even show less scattering losses since the germanium doping level is lower in the core than for the singly clad fiber. Fig. 9 shows the three structures and their dispersion properties (their attenuations being very close) as seen above.

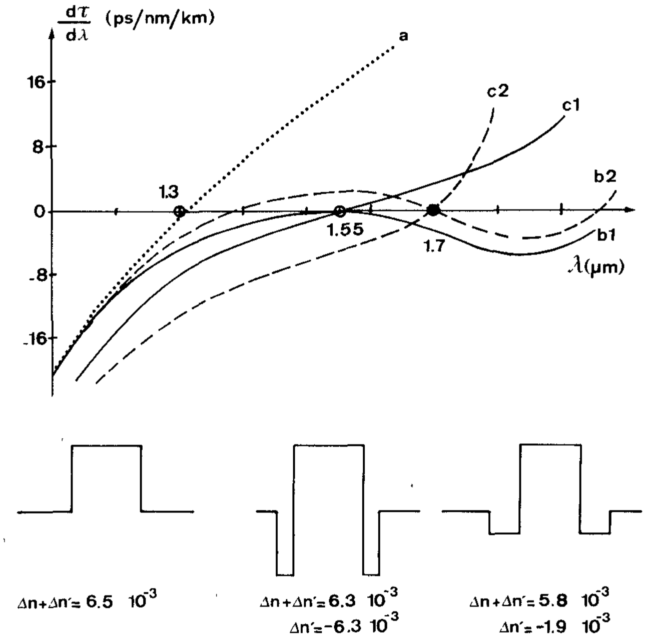
This particular example shows the very large possibilities offered by the doubly clad fibers in order to obtain low-loss dispersion-free fibers between 1.3 and 1.7 μm. The upper part of Fig. 9 gives an idea of what can be done with W-type fibers.



(a)

(b)

Fig. 8. (a) Dispersion parameters $d(VB)/dV$ and (b) $V(d^2(VB)/dV^2)$ for $b/a = 2$ for various $\Delta n'/\Delta n$ values. Crosses indicate the limits of single-mode operation.



(a) (b) (c)

Fig. 9. (b) and (c) Possible low-loss dispersion-free W fibers. (a) shows the "best" singly clad fiber [11]. The upper part shows the total dispersion for the three types of fibers: a) step index, core diameter $2a = 6.5$ μm, b) $b/a = 1.5$ $b1$: $2a = 6.5$ μm $b2$: $2a = 6.6$ μm, c) $b/a = 2.0$ $c1$: $2a = 5.7$ μm $c2$: $2a = 5.4$ μm.

We did not take into account the dopant contribution to material dispersion because various solutions can be found for a same index profile (pure silica, germanium or fluorine and/or phosphorus-doped silica). In all cases the dispersion contribution of the dopants will be less than 2 ps/nm/km between 1.2 and 1.9 μm. We also ignored the dispersion contribution of profile imperfections; for example, a core dip only has a very small effect on waveguide dispersion. Finally, the dispersion curves of Fig. 9 are correct within a few ps/nm/km, the correction depending on the exact dopant concentration and on the real profile.

V. Conclusion

By using the weakly guiding approximation, we solved numerically the dispersion equation of all doubly clad fibers with core index higher than inner cladding index. We gave universal propagation curves $B(V)$ for LP01 and LP11 modes and cutoff properties as well.

Concerning the important point of dispersion, we show how it is possible to obtain dispersion-free propagation with optimum doping levels lower than those required by dispersion-free singly clad fibers. This seems to be attractive, especially at 1.55 μm (theoretical and experimental minimum attenuation for high silica fibers), where state of the art laser diodes operate monochromatically only with great difficulty when modulated. This new possibility could avoid the use of an external modulator, thus leading to an improved link power budget.

Appendix I
The Field Coefficients

A_1, A_2, A_3 as functions of A_o are found from (1) by solving the system below:

$$\begin{bmatrix} J_m(u) & J_m(u'c) & Y_m(u'c) & 0 \\ uJ'_m(u) & u'c\,J'_m(u'c) & u'c\,Y'_m(u'c) & 0 \\ 0 & J_m(u') & Y_m(u') & K_m(v) \\ 0 & u'J'_m(u') & u'Y'_m(u') & vK'_m(v) \end{bmatrix} \cdot \begin{bmatrix} A_o \\ -A_1 \\ -A_2 \\ A_3 \end{bmatrix} = 0$$

where $Z' = dZ/dr$ (Z Bessel function).

Using the classical Bessel functions properties [7], we have

$$\begin{cases} A_1 = \dfrac{\pi A_o}{2}\,[u\,J_{m+1}(u)\,Y_m(u'c) - u'c\,J_m(u)\,Y_{m+1}(u'c)] \\[2ex] A_2 = \dfrac{\pi A_o}{2}\,[u'c\,J_{m+1}(u'c)\,J_m(u) - u\,J_{m+1}(u)\,J_m(u'c)] \\[2ex] A_3 = \dfrac{1}{K_m(v)}\,[A_1\,J_m(u') + A_2\,Y_m(u')]. \end{cases}$$

For $\beta > k_o n_2$, we must solve from (2)

$$\begin{bmatrix} J_m(u) & I_m(v'c) & K_m(v'c) & 0 \\ uJ'_m(u) & v'cI'_m(v'c) & v'cK'_m(v'c) & 0 \\ 0 & I_m(v') & K_m(v') & K_m(v) \\ 0 & v'I'_m(v') & v'K'_m(v') & vK'_m(v) \end{bmatrix} \begin{bmatrix} A'_o \\ -A'_1 \\ -A'_2 \\ A'_3 \end{bmatrix} = 0$$

leading to

$$\begin{cases} A'_1 = A'_o\,[v'c\,J_m(u)\,K_{m+1}(v'c) - u\,J_{m+1}(u)\,K_m(v'c)] \\ A'_2 = A'_o\,[v'c\,J_m(u)\,I_{m+1}(v'c) + u\,J_{m+1}(u)I_m(v'c)] \\ A'_3 = \dfrac{1}{K_m(v)}\,[A'_1 I_m(v') + A'_2 K_m(v')]. \end{cases}$$

Appendix II
The Group Delay Derivative

We start from the approximation and basic formula of Gloge [13, equations (12) and (13)].

$$\frac{n_1 - n}{n} \simeq \frac{N_1 - N}{N} \ll 1 \tag{A.II-1}$$

and

$$\frac{d\beta}{dk} = N + (N_1 - N)\frac{d(VB)}{dV} \tag{A.II-2}$$

where $N_i = n_i - \lambda(dn_i/d\lambda)$ is the group index. We then have

$$\tau = \frac{1}{c}\frac{d\beta}{dk} = \frac{N}{c}\left[1 + \frac{N_1 - N}{N}\frac{d(VB)}{dV}\right]. \tag{A.II-3}$$

Using (A.II-1), this leads to

$$\tau = \frac{N}{c}\left[1 + \Delta\frac{d(VB)}{dV}\right] \tag{A.II-4}$$

with

$$\Delta = \frac{N_1 - N}{N} = \frac{\Delta n + \Delta n'}{n} \quad \text{in our case.}$$

Differentiating with respect to the wavelength gives

$$\frac{d\tau}{d\lambda} = M\left[1 + \Delta\frac{d(VB)}{dV}\right] + \frac{N}{c}\frac{d(VB)}{dV}\frac{d\Delta}{d\lambda} + \frac{N\Delta}{c}\frac{dV}{d\lambda}\frac{d^2(VB)}{dV^2}$$

where $M = 1/c\; dN/d\lambda$ is the material dispersion.

Introducing the classical mean profile dispersion parameter $p = \lambda/\Delta\; d\Delta/d\lambda$ ($p \simeq 0.1$ at $\lambda = 1.55$ μm for germanium doped graded-index fibers [14]) and using again (A.II-1), we obtain (12)

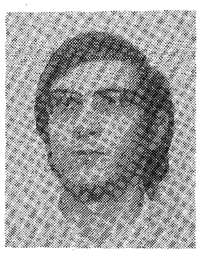
$$\frac{d\tau}{d\lambda} = M\left[1 + \Delta\frac{d(VB)}{dV}\right] - \frac{N\Delta}{\lambda c}\left[V\frac{d^2(VB)}{dV^2} - p\frac{d(VB)}{dV}\right].$$

Acknowledgment

The author wishes to thank Dr. L. Jeunhomme and Dr. C. Vassallo for fruitful discussions.

References

[1] S. Kawakami and S. Nishida, "Characteristics of a doubly clad optical fiber with a low-index inner cladding," *IEEE J. Quantum Electron.*, vol. QE-10, pp. 879–887, Dec. 1974.
[2] —, "Perturbation theory of a doubly clad optical fiber with a low-index inner cladding," *IEEE J. Quantum Electron.*, vol. QE-11, pp. 130–138, Apr. 1975.
[3] R. A. Sammut, "Range of monomode operation of W-fibres," *Opt. Quantum Electron.*, vol. 10, pp. 509–514, 1978.
[4] B. J. Ainslie, K. J. Beales, C. R. Day, and J. D. Rush, "Interplay of design parameters and fabrication conditions on the performance of monomode fibers made by MCVD," *IEEE J. Quantum Electron.*, vol. QE-17, pp. 854–857, June 1981.
[5] H. G. Unger, *Planar Optical Waveguides and Fibres.* Oxford, England: Clarendon, 1977, pp. 312–313.
[6] A. W. Snyder and W. R. Young, "Modes of optical waveguides," *J. Opt. Soc. Amer.*, vol. 68, pp. 297–309, Mar. 1978.
[7] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, Nat. Bureau of Standards, June 1964.
[8] H. Hung-Chia and W. Zi-Hua, "Analytical approach to prediction of dispersion properties of step-index single mode optical fibres," *Electron. Lett.*, vol. 17, pp. 202–204, 1981.
[9] L. Jeunhomme, "Dispersion minimisation in single-mode fibres between 1.3 μm and 1.7 μm," *Electron. Lett.*, vol. 15, pp. 478–479, July 1979.
[10] T. Miya, Y. Terunuma, T. Hosaka, and T. Miyashita, "Ultimate low-loss single-mode fibre at 1.55 μm," *Electron. Lett.*, vol. 15, pp. 106–108, Feb. 1979.
[11] L. Jeunhomme, "Single mode fiber design for long haul transmission," *IEEE J. Quantum Electron.*, this issue.
[12] H. Matsumura and T. Suganuma, "Normalization of single-mode fibers having an arbitrary index profile," *Appl. Opt.*, vol. 19, pp. 3151–3158, Sept. 1980.
[13] D. Gloge, "Dispersion in weakly guiding fibers," *Appl. Opt.*, vol. 10, pp. 2442–2445, Nov. 1971.
[14] F.M.E. Sladen, D. N. Payne, and M. J. Adams, "Profile dispersion measurements for optical fibres over the wavelength range 350 nm to 1900 nm," in *Proc. 4th European Conf. Opt. Commun.*, Genova, Italy, Sept. 12–15, 1978, paper II-1, pp. 48–57.



Michel Monerie was born in France, on July 14, 1948. He received the diplome of Engineer from Ecole Polytechnique, Paris, in 1971, and from Ecole Nationale Supérieure des Télécommunications, Paris, in 1973, and the Doctorat d'Etat from the University of Paris XI in 1977.

Since 1973 he has been with the Centre National d'Etudes des Télécommunications, Lannion, France, working in the fields of integrated optics and laser diodes. Since 1979 his research interest has been propagation in single-mode fibers.