Table 2-1. LAPLACE TRANSFORM PAIRS

	$f(t)$	$F(s)$
1	unit impulse $\delta(t)$	1
2	unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	e^{-at}	$\frac{1}{s+a}$
5	te^{-at}	$\frac{1}{(s+a)^2}$
6	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
8	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
11	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
12	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
13	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
14	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
15	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
16	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
17	$\frac{-1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin (\omega_n \sqrt{1-\zeta^2}\, t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
18	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin (\omega_n \sqrt{1-\zeta^2}\, t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$

Table 2-2. PROPERTIES OF LAPLACE TRANSFORMS

1	$\mathscr{L}[Af(t)] = AF(s)$
2	$\mathscr{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathscr{L}_\pm\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathscr{L}_\pm\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathscr{L}_\pm\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}\overset{(k-1)}{f}(0\pm)$ where $\overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$
6	$\mathscr{L}_\pm\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{\left[\int f(t)\,dt\right]_{t=0\pm}}{s}$
7	$\mathscr{L}_\pm\left[\int\int f(t)\,dt\,dt\right] = \frac{F(s)}{s^2} + \frac{\left[\int f(t)\,dt\right]_{t=0\pm}}{s^2} + \frac{\left[\int\int f(t)\,dt\,dt\right]_{t=0\pm}}{s}$
8	$\mathscr{L}_\pm\left[\int \cdots \int f(t)\,(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n} \frac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)\,(dt)^k\right]_{t=0\pm}$
9	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$
10	$\mathscr{L}[f(t-a)\,1(t-a)] = e^{-as}F(s)$
11	$\mathscr{L}[tf(t)] = -\frac{dF(s)}{ds}$
12	$\mathscr{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds$
13	$\mathscr{L}\left[f\left(\frac{t}{a}\right)\right] = a\,F(as)$

If $f_1(t)$ and $f_2(t)$ are piecewise continuous and of exponential order, then the Laplace transform of

$$\int_0^t f_1(t-\tau)f_2(\tau)\,d\tau$$

can be obtained as follows:

$$\mathscr{L}\left[\int_0^t f_1(t-\tau)f_2(\tau)\,d\tau\right] = F_1(s)F_2(s) \tag{2-7}$$

where

$$F_1(s) = \int_0^\infty f_1(t)e^{-st}\,dt = \mathscr{L}[f_1(t)]$$
$$F_2(s) = \int_0^\infty f_2(t)e^{-st}\,dt = \mathscr{L}[f_2(t)]$$

To prove Eq. (2-7), note that $f_1(t-\tau)1(t-\tau) = 0$ for $\tau > t$. Hence