Feedback Linearization

SEL 0364 Controle Não Linear

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Sistema

$$q = \begin{bmatrix} x & y & z & \psi \end{bmatrix}^T$$

$$\begin{aligned} \ddot{q} &= -f_2\dot{q} + f_1 u \\ f_2 &= NR^T \\ f_1 &= M \end{aligned}$$

$$M = \begin{bmatrix} \gamma_1 \cos\psi & -\gamma_3 \sin\psi & 0 & 0 \\ \gamma_1 \sin\psi & \gamma_3 \cos\psi & 0 & 0 \\ 0 & 0 & \gamma_5 & 0 \\ 0 & 0 & 0 & \gamma_7 \end{bmatrix}$$

$$N = \begin{bmatrix} \gamma_2 \cos\psi & -\gamma_4 \sin\psi & 0 & 0 \\ \gamma_2 \sin\psi & \gamma_4 \cos\psi & 0 & 0 \\ 0 & 0 & \gamma_6 & 0 \\ 0 & 0 & 0 & \gamma_8 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Linearização por realimentação

Definindo a lei de controle

$$u = f_1^{-1}(\nu + f_2\dot{q})$$

com

$$\nu = \ddot{q}_d + K_p\tilde{q} + K_d\dot{\tilde{q}}$$

O sistema é

$$\ddot{q} = -f_2\dot{q} + f_1 u$$

Substituindo o controle, temos

$$\ddot{q} = -f_2\dot{q} + f_1 f_1^{-1}(\nu + f_2\dot{q})$$

finalmente

$$\ddot{q} = \nu$$

Malha fechada

Como

$$\nu = \ddot{q}_d + K_p\tilde{q} + K_d\dot{\tilde{q}}$$

a dinâmica de malha fechada fica

$$\ddot{q} = \ddot{q}_d + K_p\tilde{q} + K_d\dot{\tilde{q}} \Longleftrightarrow$$

$$\ddot{\tilde{q}}_d + K_p\tilde{q} + K_d\dot{\tilde{q}} = 0$$

Onde

$$K_p = \begin{bmatrix} K_{px} & & & \\ & K_{py} & & \\ & & K_{pz} & \\ & & & K_{p\psi} \end{bmatrix}$$

$$K_d = \begin{bmatrix} K_{dx} & & & \\ & K_{dy} & & \\ & & K_{dz} & \\ & & & K_{d\psi} \end{bmatrix}$$

são projetados para estabilidade (elementos diagonais positivos)

Lyapunov

Função candidata

$$V = \frac{1}{2}\tilde{q}^T K_p \tilde{q} + \frac{1}{2}\dot{\tilde{q}}^T \dot{\tilde{q}} \geq 0$$

Tomando a primeira derivada

$$\begin{aligned} \dot{V} &= \tilde{q}^T K_p \dot{\tilde{q}} + \dot{\tilde{q}}^T \ddot{\tilde{q}} \\ &= \tilde{q}^T K_p \dot{\tilde{q}} + \dot{\tilde{q}}^T (-K_p \tilde{q} - K_d \dot{\tilde{q}}) \\ &= -\dot{\tilde{q}}^T K_d \dot{\tilde{q}} \leq 0 \end{aligned}$$