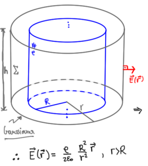
Solução Roteiro 3 – Continuação

09 Calculemos, também, $V(\frac{R}{2}) - V(2R)$. Para tanto, precisamos do campo exterior ao cilindro.



$$\oint_{\Sigma} \vec{E} \cdot d\vec{a} = \frac{Q_{int}}{\varepsilon_0}$$

$$\bullet \oint_{\Sigma} \vec{E} \cdot d\vec{a} = \int_{LATERAL} \vec{E} \cdot d\vec{a} = 2\pi r h\, E(r)$$

$$\bullet\ Q_{int} = \rho \cdot \pi R^2 h$$

$$\Rightarrow 2\pi r h \cdot E(r) = \frac{\rho \cdot \pi R^2 h}{\varepsilon_0} \Rightarrow E(r) = \frac{\rho}{2\varepsilon_0} \frac{R^2}{r}$$

$$\therefore \vec{E}(\vec{r}) = \frac{\rho}{2\varepsilon_0} \frac{R^2}{r^2} \vec{r}, \quad r > R$$

Assim,

$$V\left(\frac{R}{2}\right) - V(2R) = -\int_{\Theta}^{R/2} \vec{E}(\vec{r}) \cdot d\vec{l} - \left(-\int_{\Theta}^{2R} \vec{E}(\vec{r}) \cdot d\vec{l}\right) = \int_{R/2}^{2R} \vec{E}(\vec{r}) \cdot d\vec{l}$$

$$= \int_{R/2}^{R} \vec{E} \cdot d\vec{l} + \int_{R}^{2R} \vec{E} \cdot d\vec{l}$$

$$\int_{R/2}^{R} \vec{E} \cdot d\vec{l} = \frac{\rho}{8\varepsilon_0} R^2 \leftarrow \text{Já fizemos!}$$

$$\int_{R/2}^{R} \vec{E} \cdot d\vec{l} = \int_{R/2}^{R} \frac{\rho}{2\varepsilon_0} \frac{R^2}{r} \hat{r} \cdot (dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{z}) = \frac{\rho}{2\varepsilon_0} R^2 \int_{R/2}^{R} \frac{1}{r} dr$$

$$= \frac{\rho}{2\varepsilon_0} R^2 \ln r \Big|_{R/2}^{R} = \frac{\rho R^2}{2\varepsilon_0} \ln 2$$

Assim,

$$V\left(\frac{R}{2}\right) - V(2R) = \frac{\rho R^2}{8\varepsilon_0} + \frac{\rho R^2}{2\varepsilon_0} \ln 2 = \frac{\rho R^2}{8\varepsilon_0}\left(1 + 4\ln 2\right)$$

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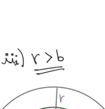
a) i) $0 \leq r < a \rightarrow \vec{E}(\vec{r}) = 0 \rightarrow$ Interior de um condutor

ii) $a < r < b$

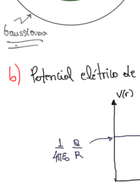


$$\oint_{\Sigma} \vec{E} \cdot d\vec{a} = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E(r)\, 4\pi r^2 = -\frac{1}{\varepsilon_0} Q$$

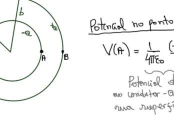
$$\Rightarrow E(r) = -\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \Rightarrow \boxed{\vec{E}(r) = -\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}}$$

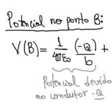
iii) $r > b$



No interior da gaussiana, a carga total é $(-Q) + (+Q) = 0$. Logo, pela Lei de Gauss, temos que não há campo elétrico nesta região.

b) Potencial elétrico de uma esfera condutora de raio R:





Potencial no ponto A:

$$V(A) = \underbrace{\frac{1}{4\pi\varepsilon_0} \frac{(-Q)}{a}}_{\text{Potencial devido ao condutor } -Q \text{ em sua superfície}} + \underbrace{\frac{1}{4\pi\varepsilon_0} \frac{+Q}{b}}_{\text{Potencial devido o condutor } +Q \text{ em seu interior.}}$$

Potencial no ponto B:

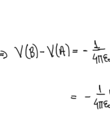
$$V(B) = \underbrace{\frac{1}{4\pi\varepsilon_0} \frac{(-Q)}{b}}_{\text{Potencial devido ao condutor } -Q} + \underbrace{\frac{1}{4\pi\varepsilon_0} \frac{(+Q)}{b}}_{\text{Potencial devido ao condutor } +Q \text{ em sua superfície}} = 0$$

Assim,

$$V(B) - V(A) = 0 - \frac{1}{4\pi\varepsilon_0} \frac{Q}{b} + \frac{1}{4\pi\varepsilon_0} \frac{Q}{a}$$

$$= \frac{1}{4\pi\varepsilon_0} Q \left(\frac{1}{a} - \frac{1}{b}\right)$$

2ª Solução:



$$V(B) - V(A) = -\int_{\Theta}^{B} \vec{E} \cdot d\vec{l} - \left(-\int_{\Theta}^{A} \vec{E} \cdot d\vec{l}\right)$$

$$= \int_{B}^{A} \vec{E} \cdot d\vec{l}$$

$$= \int_{B}^{A} -\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r} \cdot \underbrace{d\vec{l}}_{= dr\hat{r} + r d\phi\hat{\phi} + r \sin\theta d\theta}$$

$$\Rightarrow V(B) - V(A) = -\frac{1}{4\pi\varepsilon_0} Q \int_{b}^{a} \frac{1}{r^2} dr$$

$$= -\frac{1}{4\pi\varepsilon_0} Q \cdot \left(-\frac{1}{r}\right)\Big|_{b}^{a} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

c) $$U = \frac{\varepsilon_0}{2} \int_{Vol} E^2 dv' = \frac{\varepsilon_0}{2} \int_{r=a}^{b} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} k^2 \cdot \frac{1}{r^4} r^2 \sin\theta\, d\theta\, d\phi\, dr \qquad \left(k = -\frac{Q}{4\pi\varepsilon_0}\right)$$

$$\Rightarrow U = \frac{\varepsilon_0}{2} k^2 \cdot 4\pi \int_{a}^{b} \frac{1}{r^2} dr = \frac{\varepsilon_0}{2} \frac{Q^2}{16\pi^2 \varepsilon_0^2} \cdot 4\pi \cdot (-1) \cdot \frac{1}{r}\Big|_{a}^{b}$$

$$\Rightarrow \boxed{U = \frac{1}{8\pi\varepsilon_0} Q^2 \left(\frac{1}{a} - \frac{1}{b}\right)}$$

d) $$\boxed{C \equiv \frac{Q}{V(B) - V(A)} = 4\pi\varepsilon_0 \frac{ab}{b-a}}$$