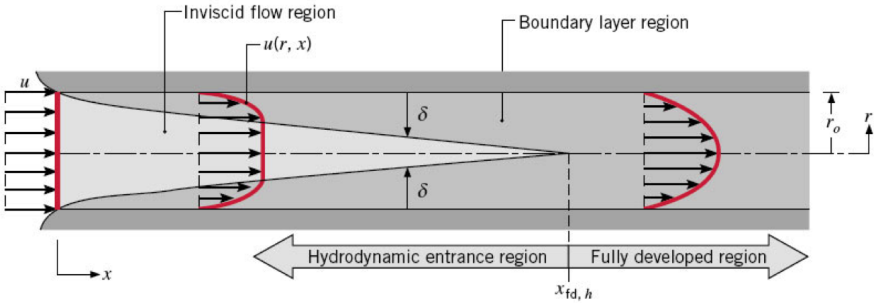
Convecção Forçada – Escoamento Interno

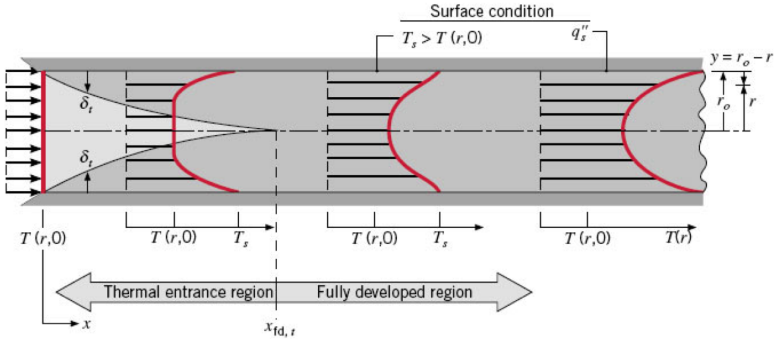
1 Fluidodinâmica



$$Re_D \equiv \frac{\bar{V}D}{\nu} \quad \begin{cases} Re_D < 2100 & \Rightarrow \text{laminar} \\ 2100 < Re_D < 4000 & \Rightarrow \text{transição} \\ Re_D > 4000 & \Rightarrow \text{turbulento} \end{cases}$$

$$\text{Comprimento de entrada:} \quad \begin{cases} \text{laminar} & \Rightarrow \frac{l_e}{D} = 0{,}06 Re_D \\ \text{turbulento} & \Rightarrow \frac{l_e}{D} = 4{,}4 Re_D^{\frac{1}{6}} \end{cases}$$

2 Características Térmicas



Casos estudados: T_s constante e q_s'' constante.

Comprimento de entrada térmico: $\begin{cases} \text{laminar} & \Rightarrow \frac{l_{e_T}}{D} = 0{,}06 Re_D Pr \\ \text{turbulento} & \Rightarrow \frac{l_{e_T}}{D} = 10 \end{cases}$

2.1 Temperatura média

$$\dot{m} c_p T_m = \int_{A_T} \rho U c_p T \, \mathrm{d}A \quad \Rightarrow \quad T_m = \frac{\int_{A_T} \rho U c_p T \, \mathrm{d}A}{\dot{m} c_p}$$

2.2 Lei do resfriamento de Newton

$$q_s'' = h \left(T_s - T_m \right)$$

2.3 Condições plenamente desenvolvidas

$$\frac{\mathrm{d}T_m}{\mathrm{d}x} \neq 0, \quad \frac{\partial T}{\partial x} \neq 0, \quad \text{Porém, podemos ter: } \frac{\partial}{\partial x} \left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)} \right] = 0$$

Condições plenamente desenvolvidas são atingidas com T_s constante <u>ou</u> q_s'' constante.

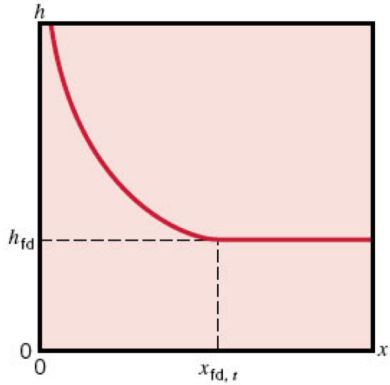
Características importantes:

$$\frac{\partial}{\partial r}\frac{\partial}{\partial x}\left[\frac{T_s - T}{T_s - T_m}\right]_{r=R} = \frac{\partial}{\partial x}\frac{\partial}{\partial r}\left[\frac{T_s - T}{T_s - T_m}\right]_{r=R} = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial x}\left[\frac{-\frac{\partial T}{\partial r}\Big|_{r=R}}{T_s - T_m}\right] = 0, \qquad \text{Mas} \quad q_s'' = -k\,\frac{\partial T}{\partial y}\bigg|_{y=0} = k\,\frac{\partial T}{\partial r}\bigg|_{r=R}$$

$$\text{Assim:} \quad \frac{\partial}{\partial x}\left[\frac{-q_s''}{k(T_s - T_m)}\right] = \frac{\partial}{\partial x}\left[-\frac{h}{k}\right] = 0$$

$$\boxed{\text{Portanto, } h \text{ não depende de } x.}$$



$$\frac{\partial}{\partial x}\left[\frac{T_s - T}{T_s - T_m}\right] = \frac{\frac{\partial(T_s - T)}{\partial x}(T_s - T_m) - \frac{\partial(T_s - T_m)}{\partial x}(T_s - T)}{(T_s - T_m)^2} = 0$$

$$\frac{\partial T}{\partial x} = \frac{\partial T_s}{\partial x} - \frac{(T_s - T)}{(T_s - T_m)}\frac{\partial T_s}{\partial x} + \frac{(T_s - T)}{(T_s - T_m)}\frac{\partial T_m}{\partial x} \tag{1}$$

- Fluxo uniforme (q_s'' constante)

$$\frac{\partial q_s''}{\partial x} = 0 \quad \Rightarrow \quad \frac{\partial[h(T_s - T_m)]}{\partial x} = (T_s - T_m)\cancel{\frac{\partial h}{\partial x}} + h\frac{\partial(T_s - T_m)}{\partial x} = 0$$

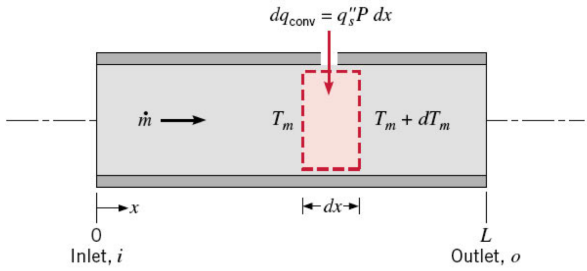
$$\therefore \frac{\partial T_s}{\partial x} = \frac{\partial T_m}{\partial x}$$

Substituindo na eq. (1): $\dfrac{\partial T}{\partial x} = \dfrac{\partial T_s}{\partial x} = \dfrac{\partial T_m}{\partial x}$

- Temperatura uniforme (T_s constante) $\quad \Rightarrow \quad \dfrac{\partial T_s}{\partial x} = 0$

Substituindo na eq. (1): $\dfrac{\partial T}{\partial x} = \dfrac{(T_s - T)}{(T_s - T_m)}\dfrac{\partial T_m}{\partial x}$

3 Balanço de Energia



$$\mathrm{d}q_{\text{conv}} = \dot{m}c_p\left[(T_m + \mathrm{d}T_m) - T_m\right] = \dot{m}c_p \mathrm{d}T_m$$

$$\mathrm{d}q_{\text{conv}} = q_s'' P \mathrm{d}x \qquad (P = \pi D, \text{perímetro})$$

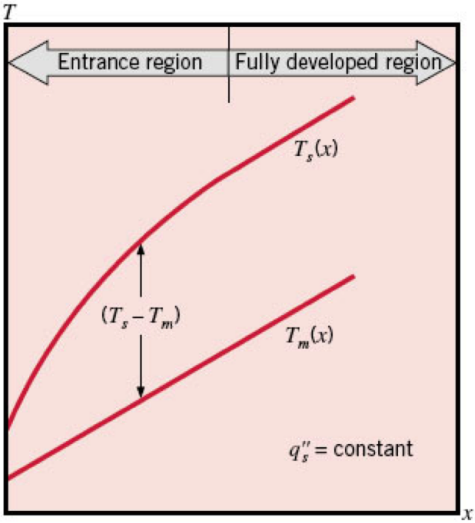
$$\therefore \frac{\mathrm{d}T_m}{\mathrm{d}x} = \frac{q_s'' P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p} h\left(T_s - T_m\right) \tag{2}$$

3.1 Fluxo térmico na superfície constante

Integrando $\dfrac{\mathrm{d}T_m}{\mathrm{d}x} = \dfrac{q_s'' P}{\dot{m} c_p}$:

$$\Rightarrow \quad T_{m,x} = T_{m,e} + \frac{q_s'' P}{\dot{m} c_p} x$$

$$\overline{T_m} = \frac{T_{m,e} + T_{m,s}}{2}$$



3.2 Temperatura superficial constante

Definindo: $\Delta T_\mu = T_s - T_m$, substituindo na eq. (2):

$$\frac{\mathrm{d}T_m}{\mathrm{d}x} = -\frac{\mathrm{d}(\Delta T_\mu)}{\mathrm{d}x} = \frac{P}{\dot{m}c_p} h \Delta T_\mu$$

Integrando: $$\int_{\Delta T_{\mu,e}}^{\Delta T_{\mu,x}} \frac{\mathrm{d}\left(\Delta T_\mu\right)}{\Delta T_\mu} = -\frac{P}{\dot{m}c_p} \int_0^x h \, \mathrm{d}x$$

$$\ln\left(\frac{\Delta T_{\mu,x}}{\Delta T_{\mu,e}}\right) = -\frac{Px}{\dot{m}c_p}\left(\frac{1}{x}\int_0^L h \, \mathrm{d}x\right) = -\frac{Px}{\dot{m}c_p}\bar{h}$$

$$\therefore \frac{\Delta T_{\mu,x}}{\Delta T_{\mu,e}} = \frac{T_s - T_{m,x}}{T_s - T_{m,e}} = e^{\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right)}$$

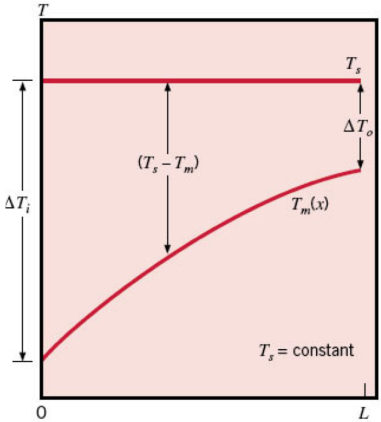
Assim, $q_{\text{conv}} = \bar{h} A_s \Delta T_{ml}$, onde $\Delta T_{ml} \equiv \dfrac{\Delta T_{\mu,x} - \Delta T_{\mu,e}}{\ln\left(\Delta T_{\mu,x}/\Delta T_{\mu,e}\right)}$

Se T_∞ for especificado ao invés de $T_{s,i}$:

$$\frac{\Delta T_{\mu,x}}{\Delta T_{\mu,e}} = \frac{T_\infty - T_{m,x}}{T_\infty - T_{m,e}} = \mathrm{e}^{\left(-\frac{\bar{U} P x}{\dot{m} c_p}\right)}$$

$$\frac{\Delta T_{\mu,x}}{\Delta T_{\mu,e}} = \mathrm{e}^{\left(-\frac{1}{\dot{m} c_p R_{\text{tot}}}\right)}$$

$$q = \bar{U} A_s \Delta T_{ml} = \frac{\Delta T_{ml}}{R_{\text{tot}}}$$



$$\overline{T_m} = \frac{1}{L}\int_0^L T_{m,x}\,\mathrm{d}x = T_s - \Delta T_{ml}$$

4 Correlações

Condições	Correlação
Laminar; completamente desenvolvido; fluxo uniforme; $Pr \geq 0{,}6$	$Nu_D = 4{,}36$
Laminar; completamente desenvolvido; temperatura uniforme; $Pr \geq 0{,}6$	$Nu_D = 3{,}66$
Laminar; comprimento de entrada térmico (ou comprimento de entrada combinado com $Pr \geq 5$; temperatura uniforme	$\overline{Nu}_D = 3{,}66 + \dfrac{0{,}0668(D/L)Re_D Pr}{1 + 0{,}04[(D/L)Re_D Pr]^{2/3}}$

Condições	**Correlação**
Laminar; comprimento de entrada combinado; temperatura uniforme; $0{,}6 < Pr < 5$; $0{,}0044 < (\mu/\mu_{\text{sup}}) < 9{,}75$	$\overline{Nu}_D = 1{,}86 \left(\dfrac{Re_D Pr}{L/D}\right)^{1/3} \left(\dfrac{\mu}{\mu_{\text{sup}}}\right)^{0{,}14}$
Turbulento; completamente desenvolvido; $0{,}6 \leq Pr \leq 160$; $Re_D \geq 10000$; $(L/D) \geq 10$; $n = 0{,}4$ para $T_s > T_m$ e $n = 0{,}3$ para $T_s < T_m$	$Nu_D = 0{,}023 Re_D^{4/5} Pr^n$

Exercício 1

Óleo de motor é aquecido ao escoar através de um tubo circular com diâmetro $D = 50\,\mathrm{mm}$ e comprimento $L = 25\,\mathrm{m}$, cuja superfície é mantida em $150\,^{\circ}\mathrm{C}$. Se a vazão de escoamento e a temperatura do óleo na entrada do tubo forem de $0{,}5\,\mathrm{kg/s}$ e $20\,^{\circ}\mathrm{C}$, qual será a temperatura do óleo na saída do tubo? Qual é a taxa de transferência de calor total no tubo?