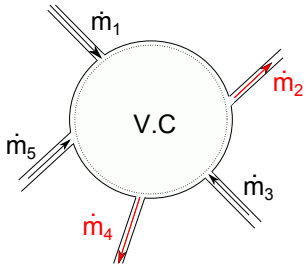
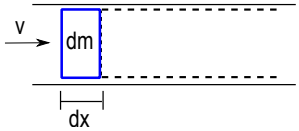
A 1 Lei para Volumes de Controle

1 Conservação de massa



$$\frac{\mathrm{d}m_{VC}}{\mathrm{d}t} = \sum_{entradas} \dot{m}_e - \sum_{saidas} \dot{m}_s$$

1.1 Vazão mássica (caso 1D)



$$\Delta m = \rho \Delta x A$$
$$\Delta m = \rho (\bar{V} \Delta t) A$$

$$\dot{m} = \lim_{\Delta t \longrightarrow 0} \left(\frac{\Delta m}{\Delta t} \right) = \rho \bar{V} A$$

Para perfis e propriedades não uniformes: $\dot{m} = \int_A \rho \vec{V} \cdot \mathrm{d}\vec{A}$

Para SC estacionária, regime permanente e propriedades uniformes nas entradas e saídas:

$$\sum_e \rho_e V_e A_e = \sum_s \rho_s V_s A_s$$

2 A 1ª Lei em termos de fluxo

1ª Lei para Sistemas

$$Q - W = \Delta U + \Delta EC + \Delta EP$$

Dividindo por Δt:

$$\frac{Q}{\Delta t} - \frac{W}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta EC}{\Delta t} + \frac{\Delta EP}{\Delta t}$$

Tomando o limite $\Delta t \longrightarrow 0$

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}U}{\mathrm{d}t} + \frac{\mathrm{d}EC}{\mathrm{d}t} + \frac{\mathrm{d}EP}{\mathrm{d}t} = \frac{\mathrm{d}E_{\text{sist}}}{\mathrm{d}t}$$

3 A 1ª Lei para Volumes de Controle

Variação de energia no volume de controle pode ocorrer por:

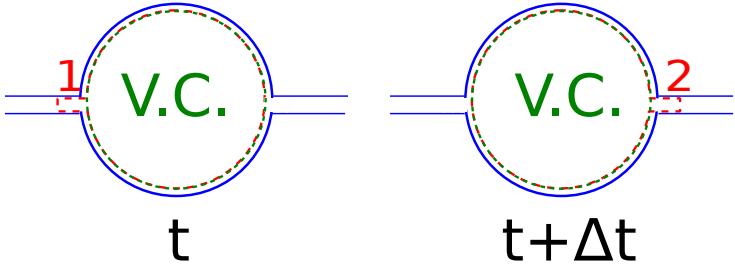
a) Transferência de calor

b) Realização de trabalho

c) Transporte de energia devido ao fluxo de massa

3.1 Energia associada ao fluxo de massa

Energia por unidade de massa: $e = u + \frac{1}{2}V^2 + gz$



$$E_{\text{sist}}(t) = E_{\not{V}C}(t) + m_1 \left(u_1 + \frac{V_1^2}{2} + g z_1 \right)$$

$$E_{\text{sist}}(t + \Delta t) = E_{\not{V}C}(t + \Delta t) + m_2 \left(u_2 + \frac{V_2^2}{2} + g z_2 \right)$$

Subtraindo as equações e dividindo-as por Δt:

$$\frac{E_{\text{sist}}(t+\Delta t) - E_{\text{sist}}(t)}{\Delta t} = \frac{E_{\not{V}C}(t+\Delta t) - E_{\not{V}C}(t)}{\Delta t} +$$

$$+ \frac{m_2}{\Delta t}\left(u_2 + \frac{V_2^2}{2} + gZ_2\right) - \frac{m_1}{\Delta t}\left(u_1 + \frac{V_1^2}{2} + gZ_1\right)$$

Fazendo $\Delta t \longrightarrow 0$

$$\frac{\mathrm{d}E_{sist}}{\mathrm{d}t} = \frac{\mathrm{d}E_{\not{V}C}}{\mathrm{d}t} + \dot{m}_2\left(u_2 + \frac{V_2^2}{2} + gZ_2\right) - \dot{m}_1\left(u_1 + \frac{V_1^2}{2} + gZ_1\right)$$

Porém:

$$\frac{\mathrm{d}E_{\text{sist}}}{\mathrm{d}t} = \dot{Q}_{\text{sist}} - \dot{W}_{\text{sist}}$$

$$\dot{Q}_{\text{sist}} = \dot{Q}_{\not{V}C}, \qquad \dot{W}_{\text{sist}} = \dot{W}_{\not{V}C} + \dot{W}_{\text{mov fronteira}}$$

$$\delta W_{mf} = F\,\mathrm{d}x = pA\mathrm{d}x$$

$$\dot{W}_{mf} = \frac{\delta W_{mf}}{\mathrm{d}t} = pA\frac{\mathrm{d}x}{\mathrm{d}t} = pAV$$

$$\underbrace{\dot{W}_{mf} = p\dot{\not{V}} = pv\dot{m}}_{\text{Trabalho de Escoamento}}$$

$$\dot{Q}_{\not{V}C} - \dot{W}_{\not{V}C} + \dot{m}_1 p_1 v_1 - \dot{m}_2 p_2 v_2 = \frac{\mathrm{d}E_{\not{V}C}}{\mathrm{d}t} +$$

$$+ \dot{m}_2\left(u_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}_1\left(u_1 + \frac{V_1^2}{2} + gz_1\right)$$

Rearranjando

$$\frac{\mathrm{d}E_{\not{V}C}}{\mathrm{d}t} = \dot{Q}_{\not{V}C} - \dot{W}_{\not{V}C} + \dot{m}_1 \left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + g z_1 \right) + \\ - \dot{m}_2 \left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + g z_2 \right)$$

Lembrando que $h = u + pv$ e generalizando para múltiplas entradas e saídas:

$$\boxed{\begin{aligned} \frac{\mathrm{d}E_{\not{V}C}}{\mathrm{d}t} &= \dot{Q}_{\not{V}C} - \dot{W}_{\not{V}C} + \\ &+ \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e \right) - \sum \dot{m}_s \left(h_s + \frac{V_s^2}{2} + g z_s \right) \end{aligned}}$$