Universidade de São Paulo
Escola de Engenharia de São Carlos

SEM0551 Fenômenos de Transporte

CONDUÇÃO UNIDIMENSIONAL DE CALOR: CIRCUITOS EQUIVALENTES

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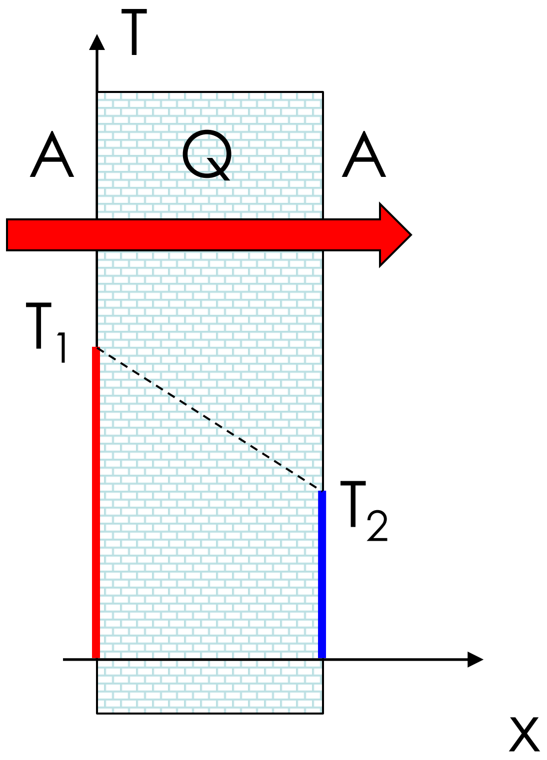
CONDUÇÃO → Lei de Fourier

CONVECÇÃO → Lei de Newton

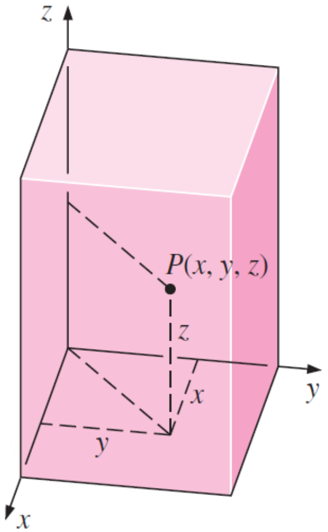
RADIAÇÃO → Lei de Stefan–Boltzmann

Lei de Fourier (condução de calor)

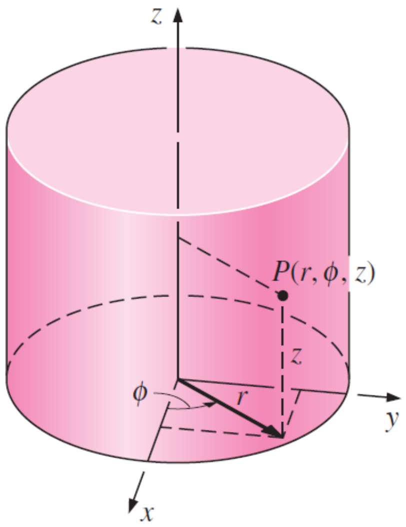


$$q = \frac{Q}{A} = -k\frac{dT}{dx}$$

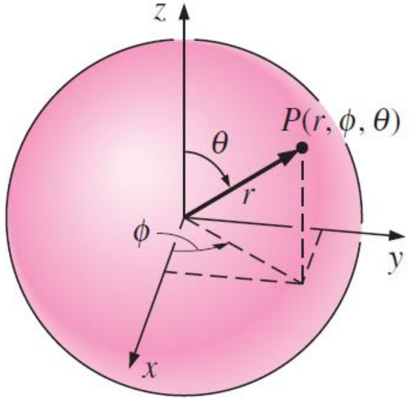
$$\vec{q} \overset{\text{Fourier}}{=} -k \cdot \vec{\nabla} T$$



$$\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$



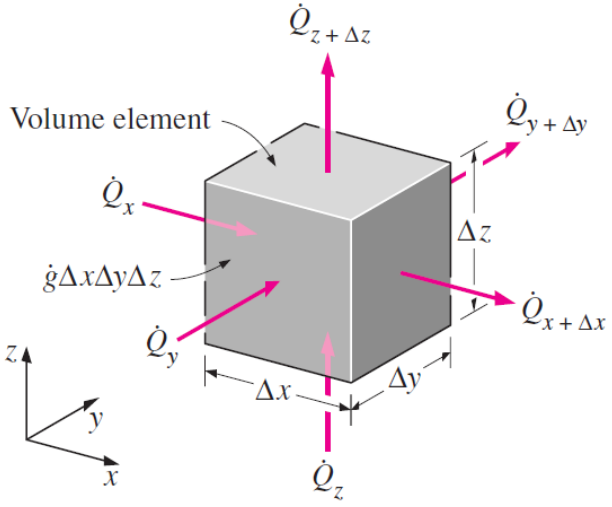
$$\vec{\nabla} T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$



$$\vec{\nabla} T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{1}{r\,\sin\phi}\frac{\partial T}{\partial \theta}\hat{\theta}$$

Inventário de energia: primeira lei da termodinâmica

$$+\left[\begin{array}{c}\text{taxa líquida de}\\ \text{condução de}\\ \text{calor entrando em}\\ \text{x, y e z}\end{array}\right] - \left[\begin{array}{c}\text{taxa líquida de}\\ \text{condução de}\\ \text{calor saindo em}\\ \text{x+dx, y+dy e z+dz}\end{array}\right] + \left[\begin{array}{c}\text{taxa de geração}\\ \text{de calor no}\\ \text{volume de}\\ \text{controle}\end{array}\right] = \left[\begin{array}{c}\text{taxa de variação}\\ \text{da energia no}\\ \text{volume de}\\ \text{controle}\end{array}\right]$$



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{\rho C_P}{k}\frac{\partial T}{\partial t}$$

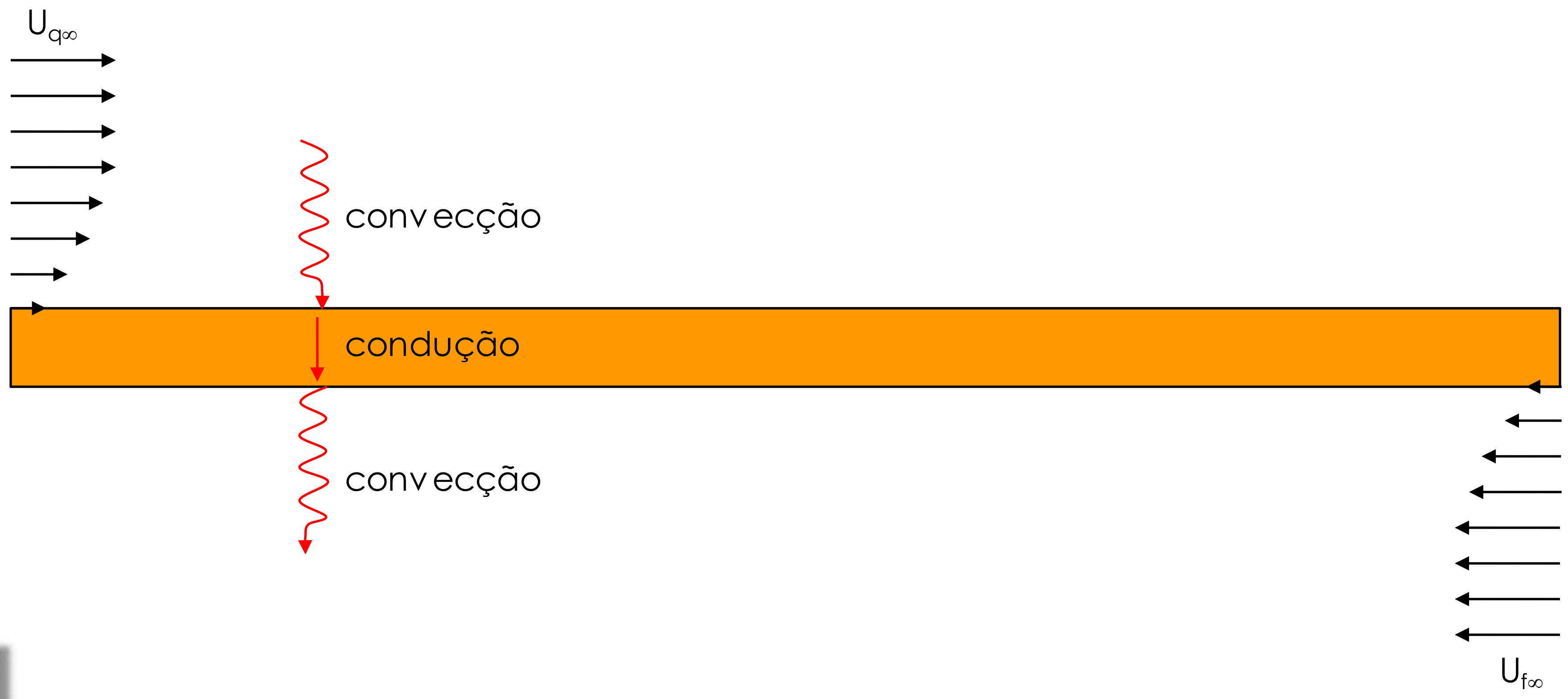
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T$$

$$\nabla^2 T + \frac{\dot{g}}{k} = \frac{\rho C_P}{k}\frac{\partial T}{\partial t}$$

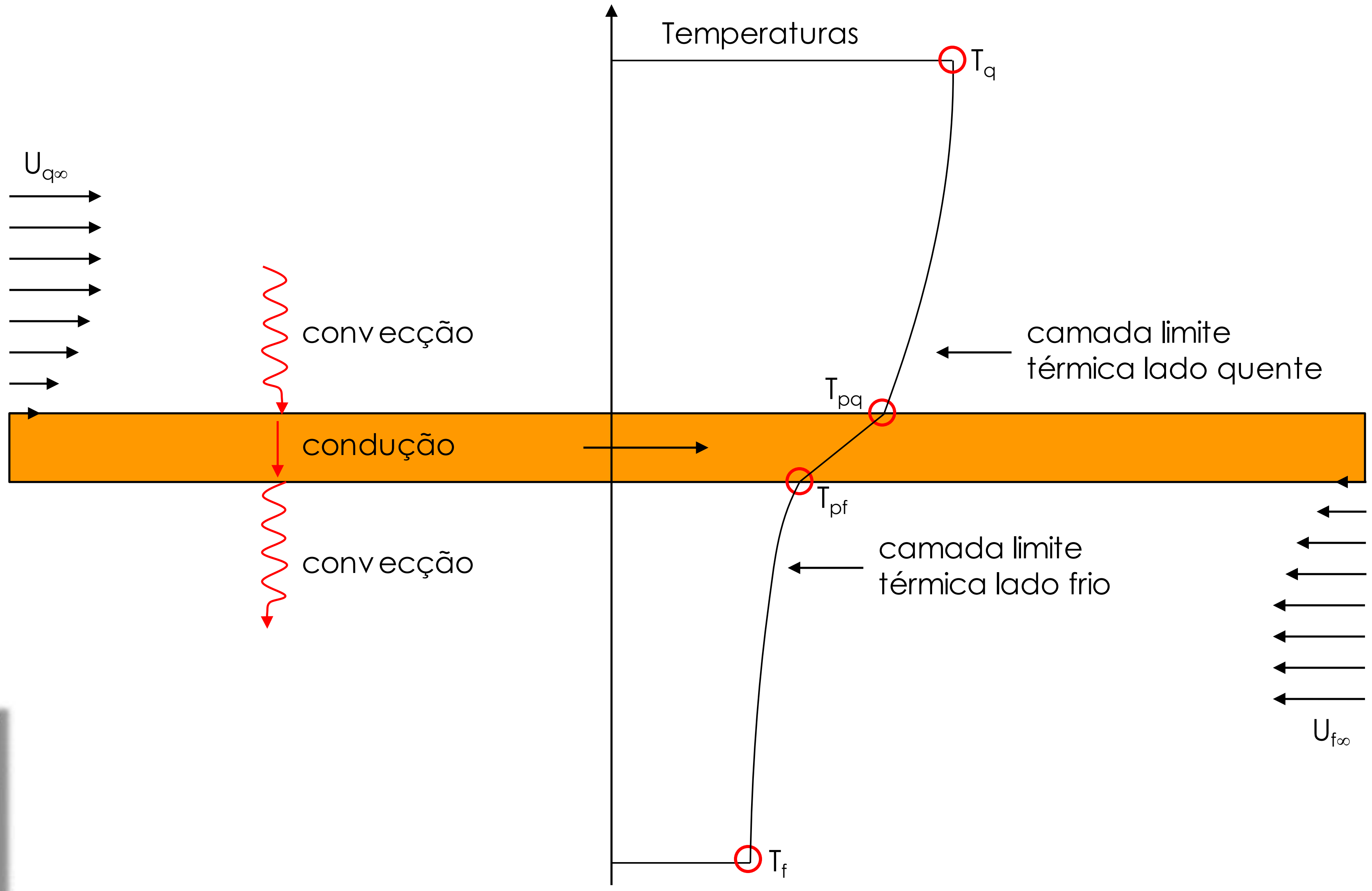
$$\alpha = \frac{k}{\rho C_P}$$

Problemas unidimensionais

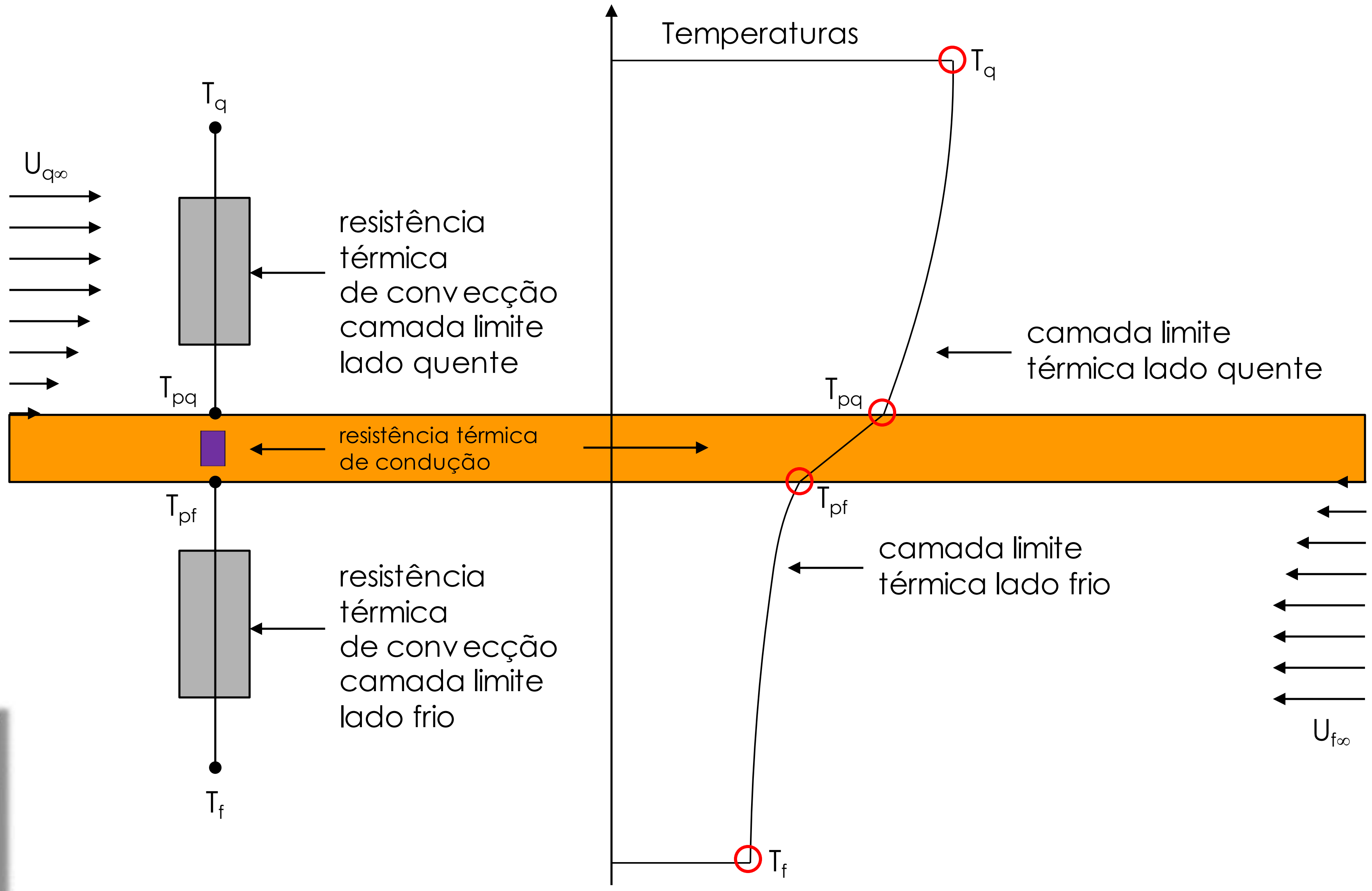
O conceito de resistência térmica:



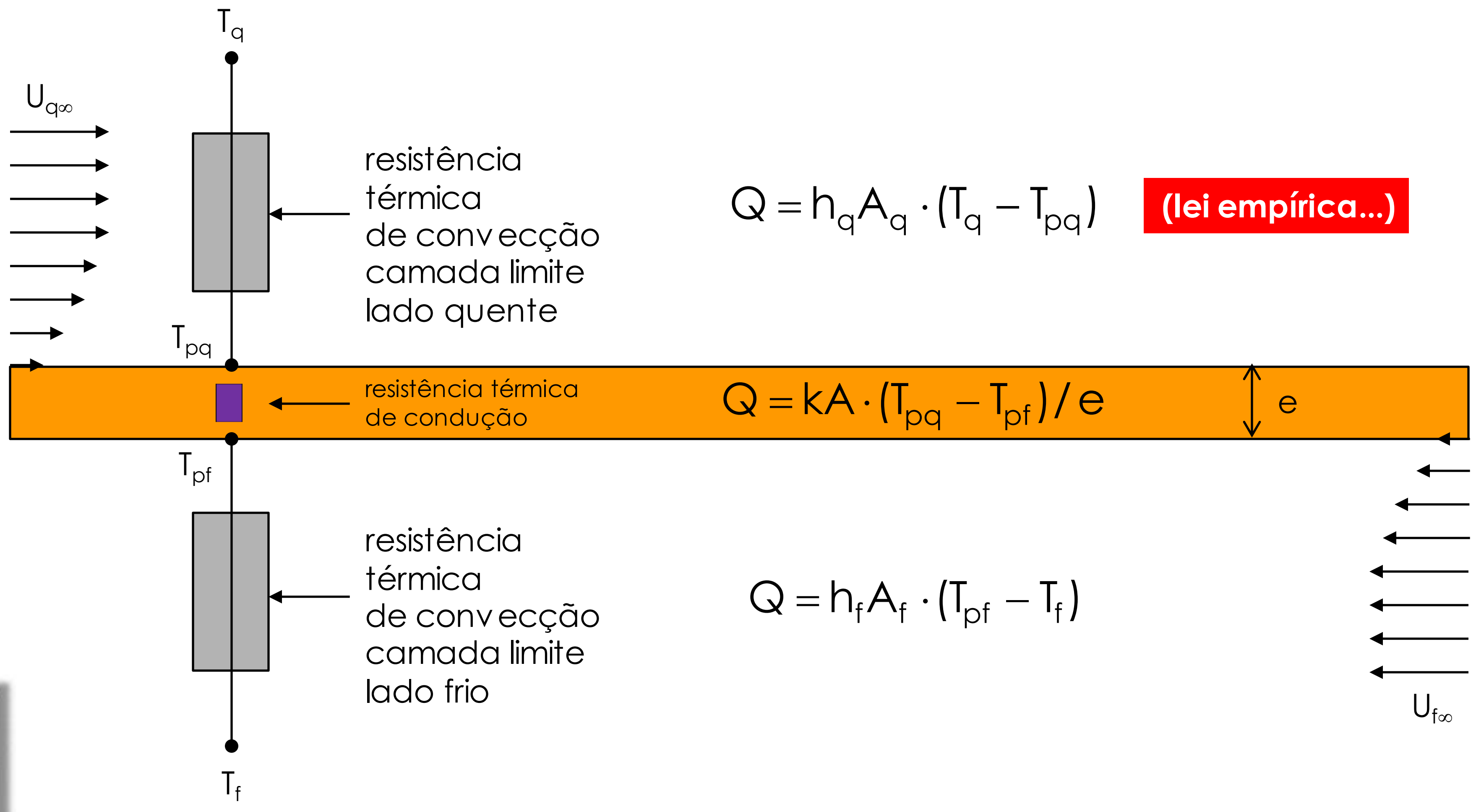
O conceito de resistência térmica:



O conceito de resistência térmica:



O conceito de resistência térmica:



$$Q = h_q A_q \cdot (T_q - T_{pq}) \rightarrow T_{pq} = T_q - \frac{Q}{h_q A_q}$$

$$Q = kA \cdot (T_{pq} - T_{pf}) / e$$

$$Q = h_f A_f \cdot (T_{pf} - T_f) \rightarrow T_{pf} = T_f + \frac{Q}{h_f A_f}$$

$$\rightarrow Q = \frac{KA_q}{e}\left[\left(T_q - \frac{Q}{h_q A_q}\right) - \left(T_f + \frac{Q}{h_f A_f}\right)\right] \qquad \rightarrow Q = \frac{T_q - T_f}{\frac{1}{h_q A_q} + \frac{e}{kA_q} + \frac{1}{h_f A_f}}$$

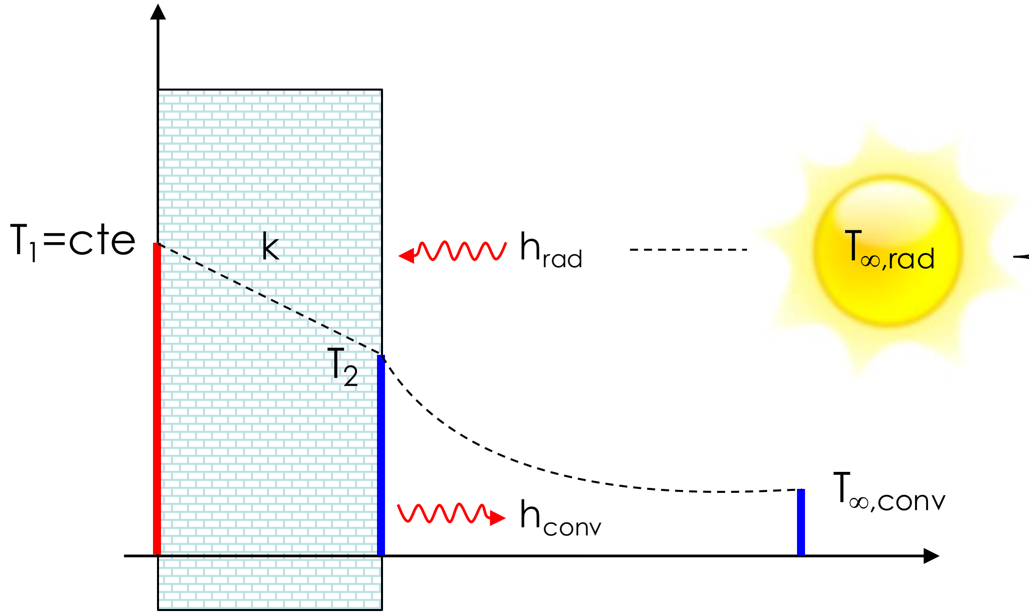
resistência térmica de convecção (q) → $\frac{1}{h_q A_q}$

resistência térmica de condução (p) → $\frac{e}{kA_q}$

resistência térmica de convecção (f) → $\frac{1}{h_f A_f}$

$$Q = \frac{T_q - T_f}{\dfrac{1}{h_q A_q} + \dfrac{e}{k A_q} + \dfrac{1}{h_f A_f}} = \frac{T_q - T_f}{R_{cq} + R_{cp} + R_{cf}} \rightarrow Q = \frac{\Delta T}{R_{total}}$$

$$Q_{cond} = \frac{kA}{e}(T_1 - T_2)$$



$$Q_{rad} = \varepsilon\sigma A(T_{\infty,rad}^4 - T_2^4)$$

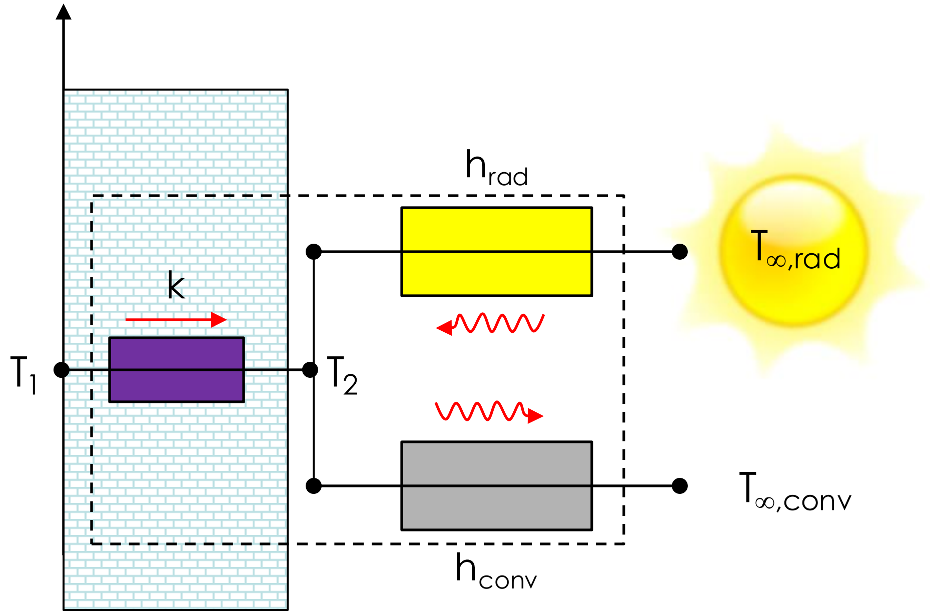
$$Q_{rad} = h_{rad} A(T_{\infty,rad} - T_2)$$

$$h_{rad} = \varepsilon\sigma (T_\infty + T)(T_\infty^2 + T^2)$$

$$Q_{conv} = h_{conv} A(T_2 - T_{\infty,conv})$$

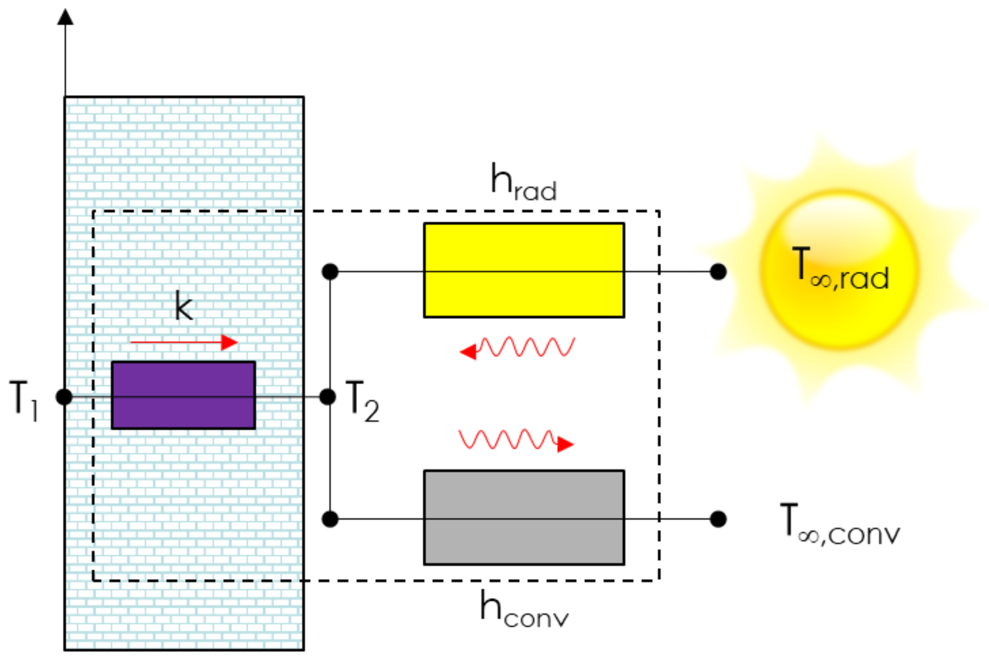
$$Q = \frac{T_q - T_f}{\frac{1}{h_q A_q} + \frac{e}{k A_q} + \frac{1}{h_f A_f}} = \frac{T_q - T_f}{R_{cq} + R_{cp} + R_{cf}} \rightarrow Q = \frac{\Delta T}{R_{total}}$$

$$Q_{cond} = \frac{kA}{e}(T_1 - T_2) = \frac{T_1 - T_2}{R_{cond}}$$



$$Q_{rad} = \frac{T_{\infty,rad} - T_2}{1/(h_{rad}A)} = \frac{T_{\infty,rad} - T_2}{R_{rad}}$$

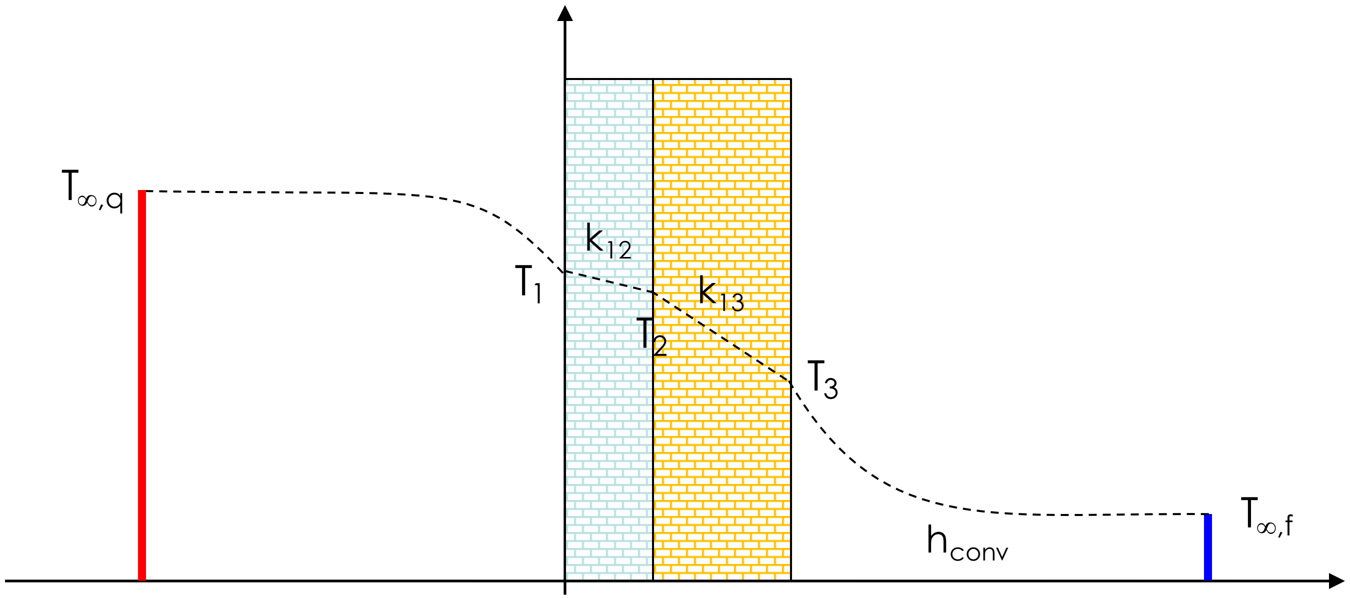
$$Q_{conv} = \frac{T_2 - T_{\infty,conv}}{1/(h_{conv}A)} = \frac{T_2 - T_{\infty,conv}}{R_{conv}}$$



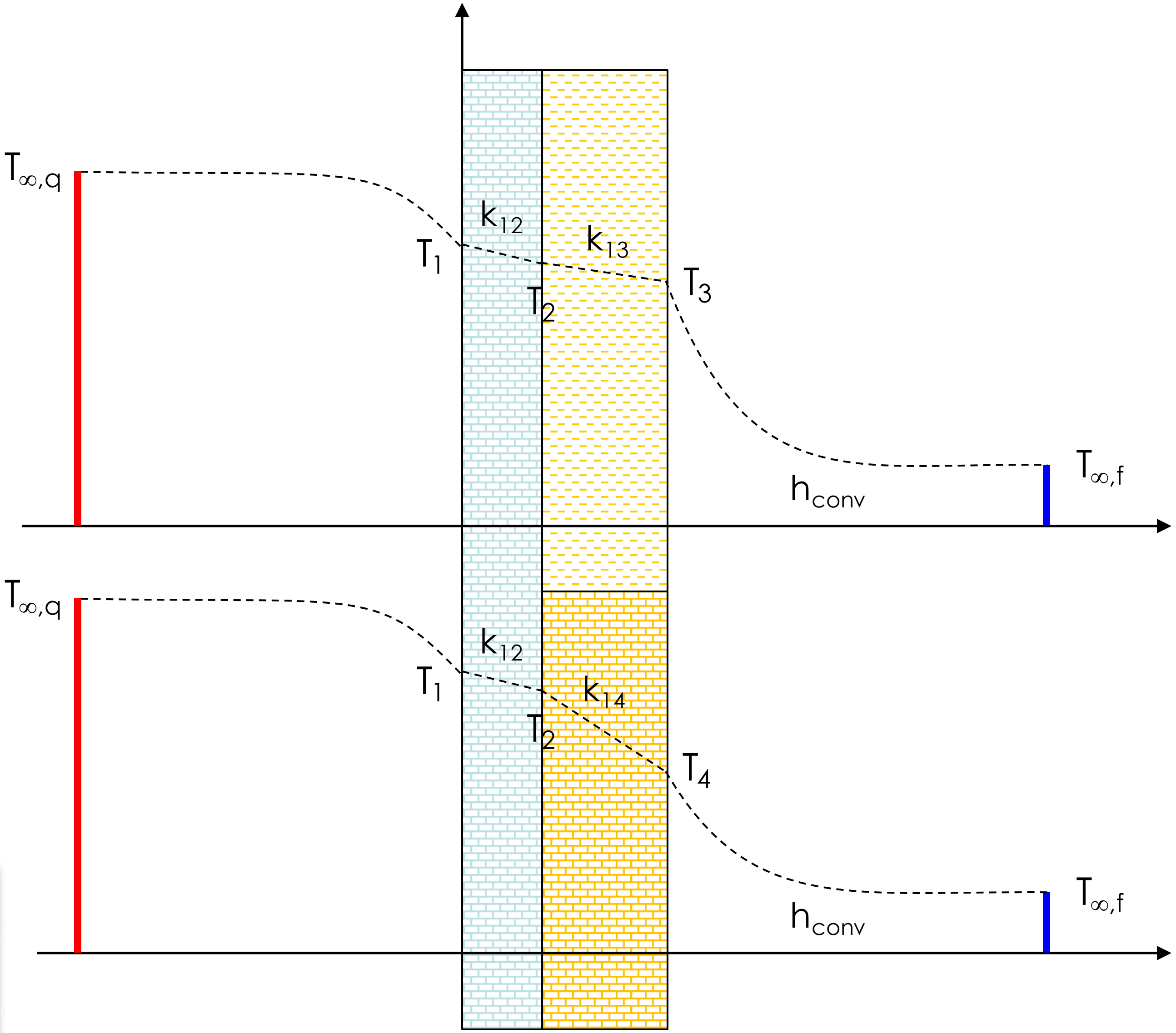
$$Q_{12} + Q_{rad} = Q_{conv}$$

$$\frac{T_1 - T_2}{R_{cond}} + \frac{T_{\infty,rad} - T_2}{R_{rad}} = \frac{T_2 - T_{\infty,conv}}{R_{conv}}$$

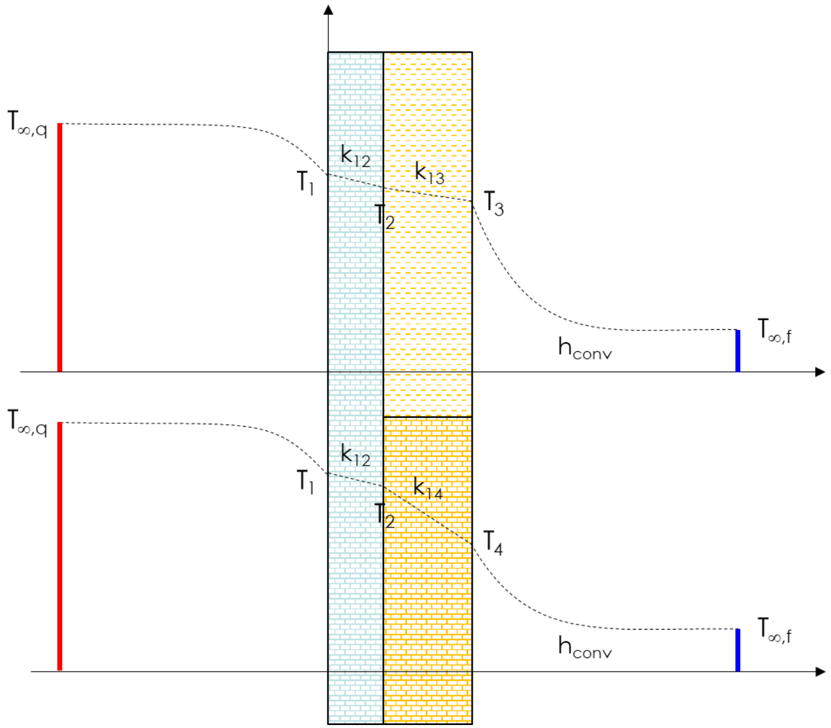
$$T_2 = \frac{T_1 / R_{cond} + T_{\infty,rad} / R_{rad} + T_{\infty,conv} / R_{conv}}{R_{cond} + R_{rad} + R_{conv}}$$



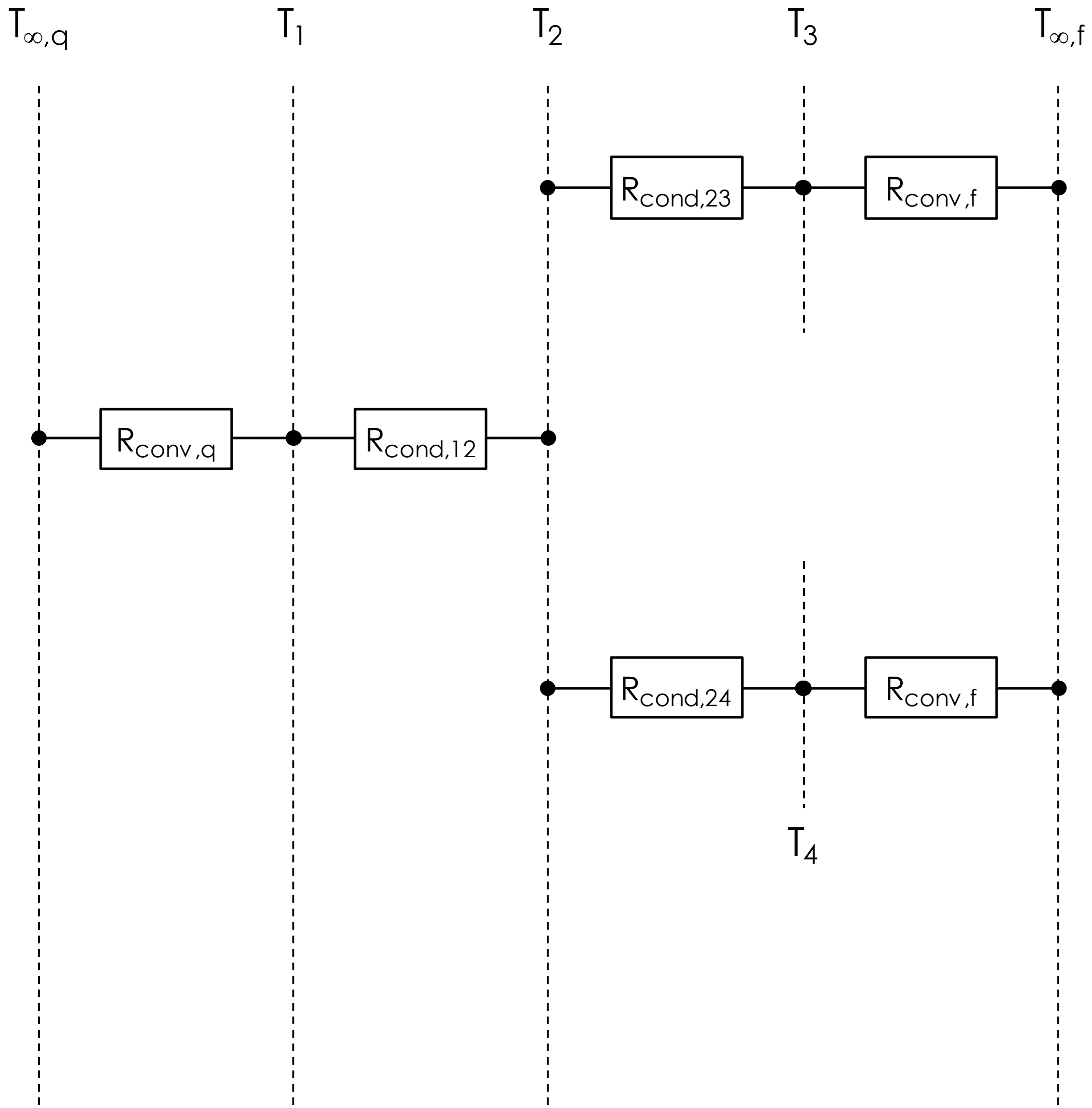
$$Q = \frac{T_{\infty,q} - T_{\infty,f}}{R_{total}} = \frac{T_{\infty,q} - T_{\infty,f}}{R_{conv,q} + \underbrace{R_{cond,12} + R_{cond,23}}_{\sum_k R_{cond,k}} + R_{conv,f}}$$



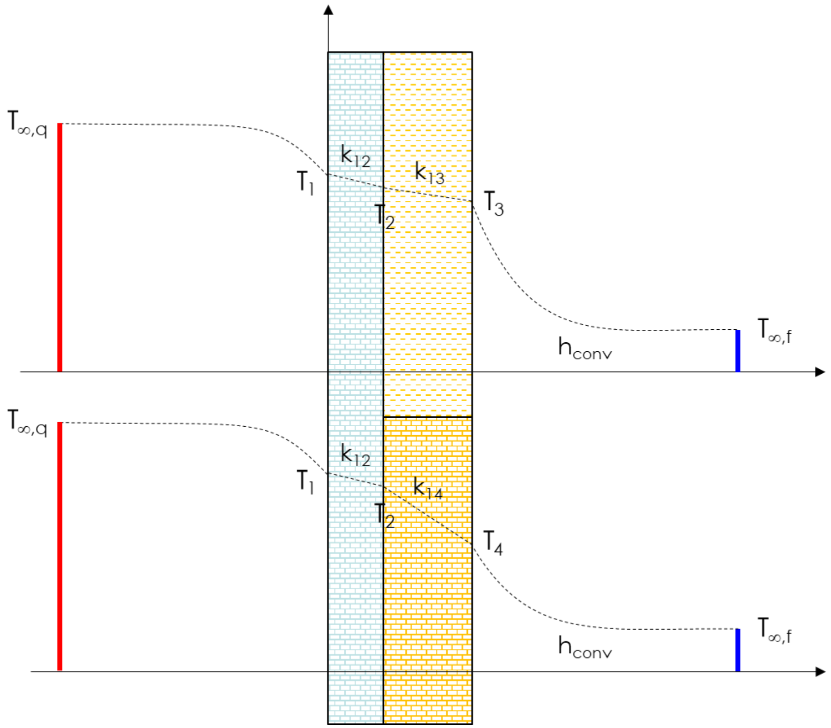
T∞,q
k12
k13
T1
T2
T3
T∞,f
hconv
T∞,q
k12
T1
k14
T2
T4
T∞,f
hconv

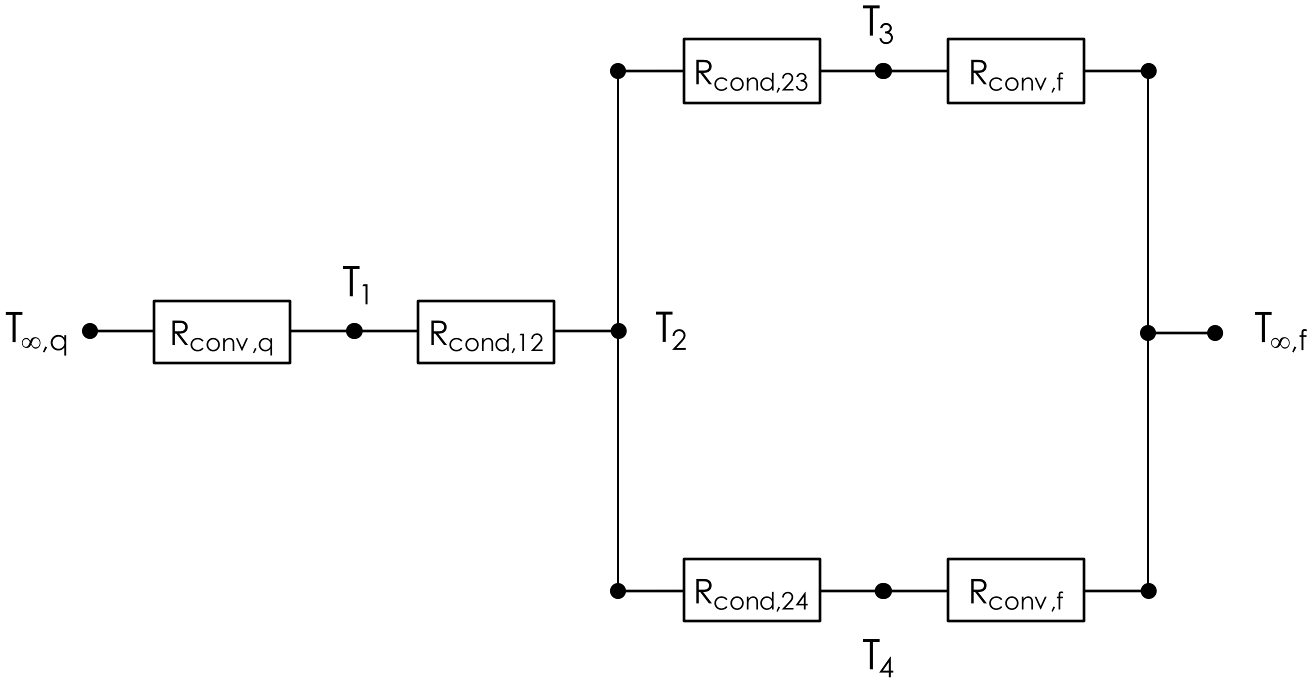


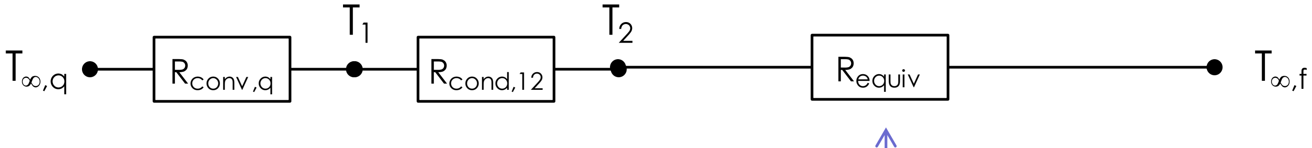
T∞,q
k12
k13
T1
T2
T3
T∞,f
hconv
T∞,q
k12
k14
T1
T2
T4
T∞,f
hconv



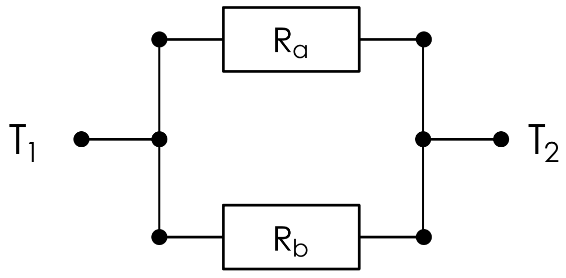
T∞,q
T1
T2
T3
T∞,f
Rcond,23
Rconv,f
Rconv,q
Rcond,12
Rcond,24
Rconv,f
T4







$$R_{equiv} = \left[\frac{1}{R_{cond,23} + R_{conv,f}} + \frac{1}{R_{cond,24} + R_{conv,f}} \right]^{-1}$$



$$Q = Q_a + Q_b$$

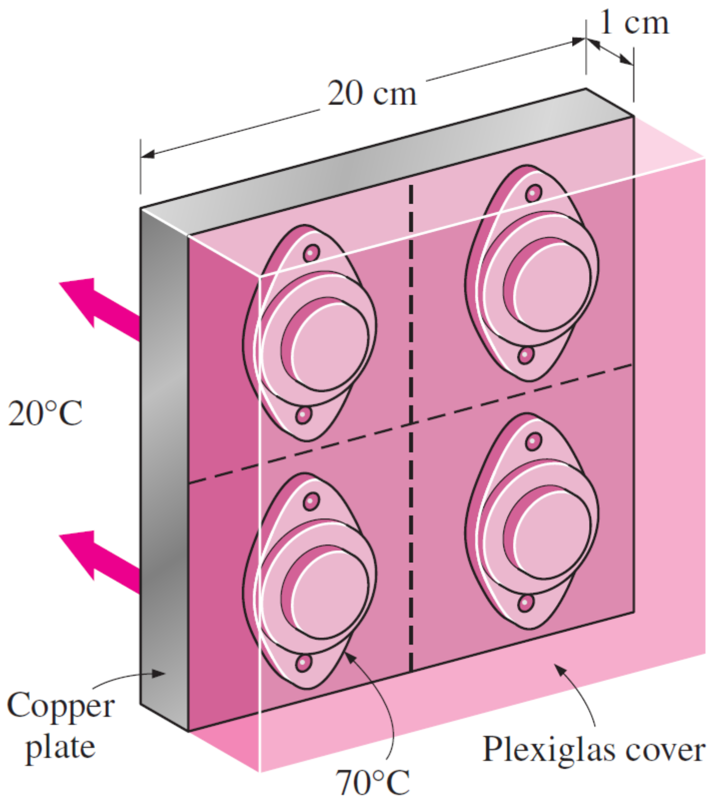
$$Q_a = \frac{T_1 - T_2}{R_a} \qquad Q_b = \frac{T_1 - T_2}{R_b}$$

$$Q = \frac{T_1 - T_2}{R_a} + \frac{T_1 - T_2}{R_b} = \frac{R_a(T_1 - T_2) + R_b(T_1 - T_2)}{R_a R_b} = \ldots$$

$$Q = \ldots = \frac{R_a + R_b}{R_a R_b}(T_1 - T_2) = \frac{1}{\frac{R_a R_b}{R_a + R_b}}(T_1 - T_2) = \frac{T_1 - T_2}{R_{eq}}$$

$$R_{eq} = \frac{R_a R_b}{R_a + R_b} \rightarrow R_{eq} = \left[\frac{1}{R_a} + \frac{1}{R_b}\right]^{-1}$$

Exemplo: dissipação de calor em transistores



Four identical power transistors with aluminum casing are attached on one side of a 1cm thick 20 x 20 cm square copper plate (k = 386 W/m/°C) by screws that exert an average pressure of 6 MPa. The base area of each transistor is 8 cm^2, and each transistor is placed at the center of a 10 x 10 cm quarter section of the plate. The interface roughness is estimated to be about 1.5 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 25 W/m2/°C. If the case temperature of the transistor is not to exceed 70°C, **determine the maximum power each transistor can dissipate safely**, and the temperature jump at the case-plate interface. (ÇG 3-5, pg. 142)

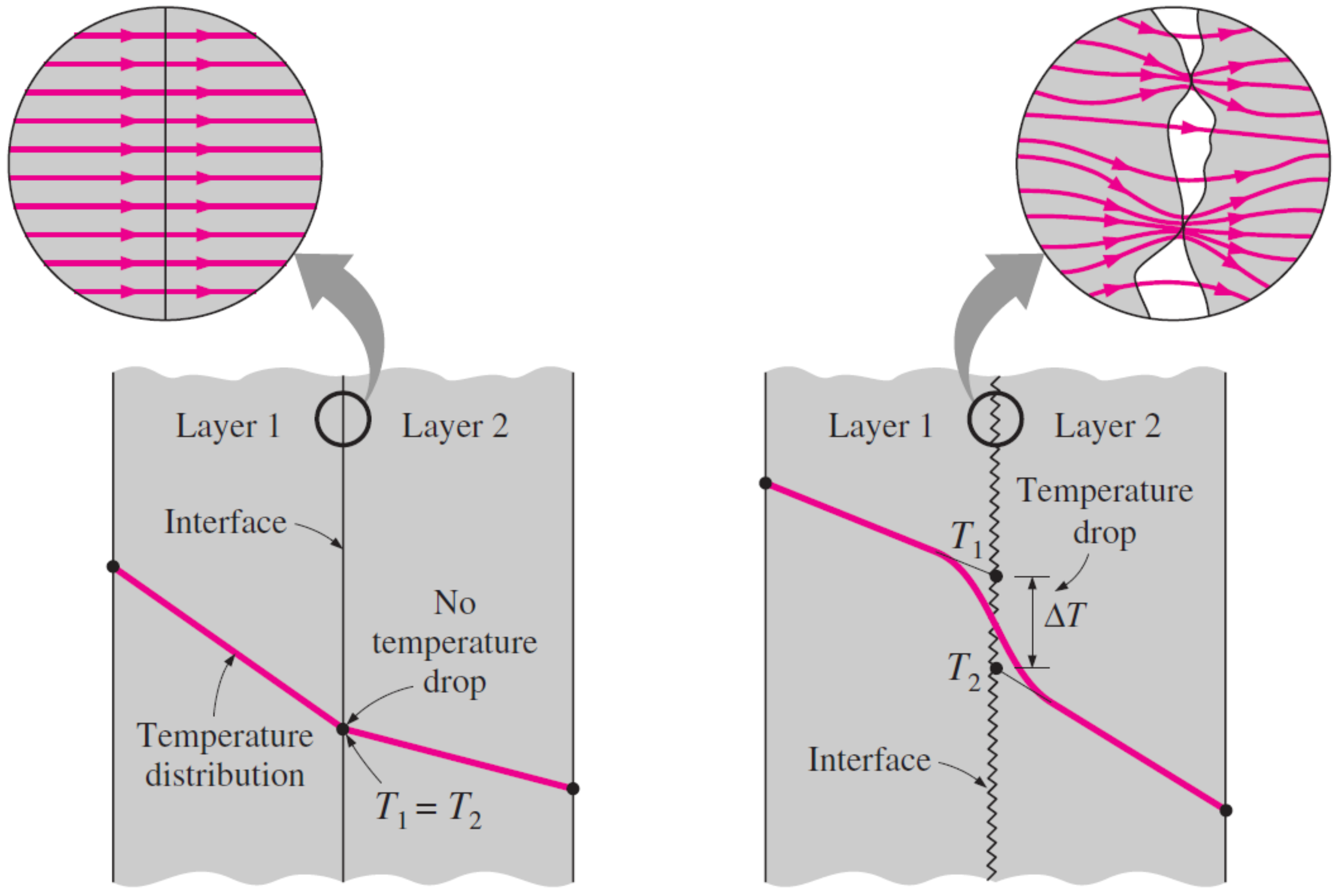
$$k_{cobre} = 386\, W/m/°C$$

$$h_{contato} = 42\ kW/m^2/°C$$

$$h_{conv} = 25\ W/m^2/°C$$

Resistência térmica de contato

Comprimindo uma camada contra a outra produz deformações no material e, por conseguinte, uma redução do volume vazio devido à diminuição da rugosidade superficial...
...ou usa-se uma material de preenchimento (**pasta térmica**).



(*a*) Ideal (perfect) thermal contact

(*b*) Actual (imperfect) thermal contact

Resistência térmica de contato

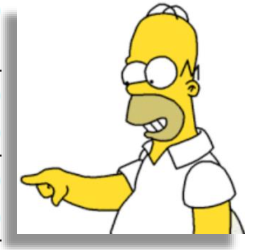
$$Q = h_{contato} \, A \, \Delta T$$

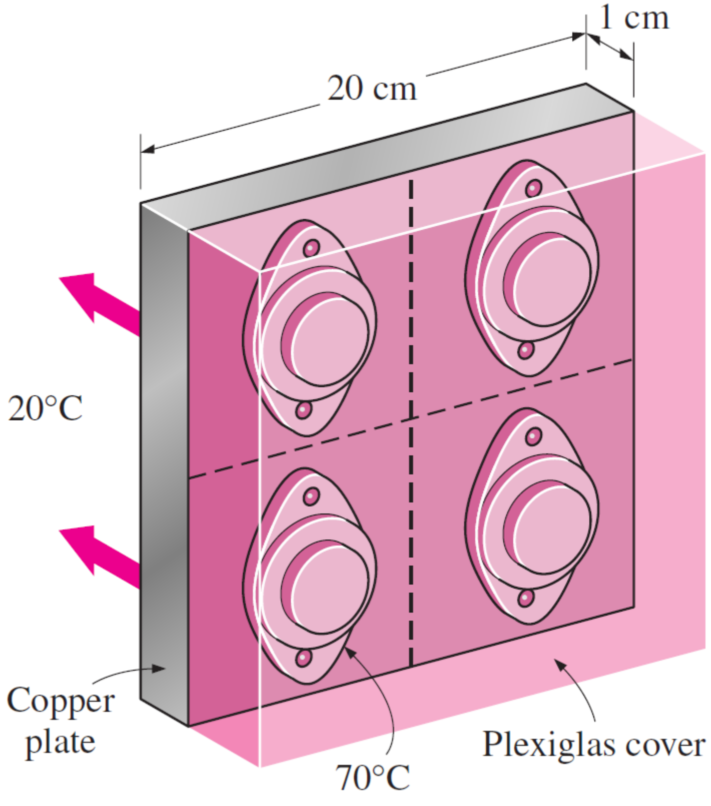
$$h_{contato} = \frac{Q}{A \, \Delta T}$$

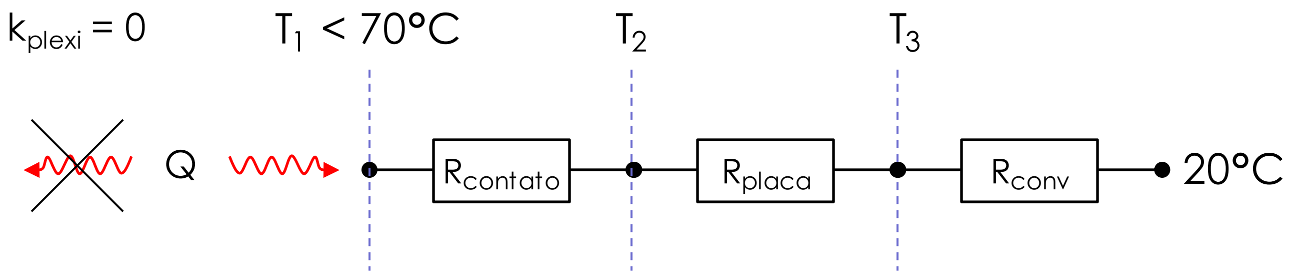
TABLE 3–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface Condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m² · °C
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.3–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.7–7	11,400
Dissimilar Metal Pairs					
Stainless steel–				10	2900
Aluminum		20–30	20	20	3600
Stainless steel–				10	16,400
Aluminum		1.0–2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4–2.0	20	15–35	59,000
Steel Ct-30–				10	4800
Aluminum	Milled	4.5–7.2	20	30	8300
				5	42,000
Aluminum-Copper	Ground	1.3–1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4–4.5	20	20–35	22,000



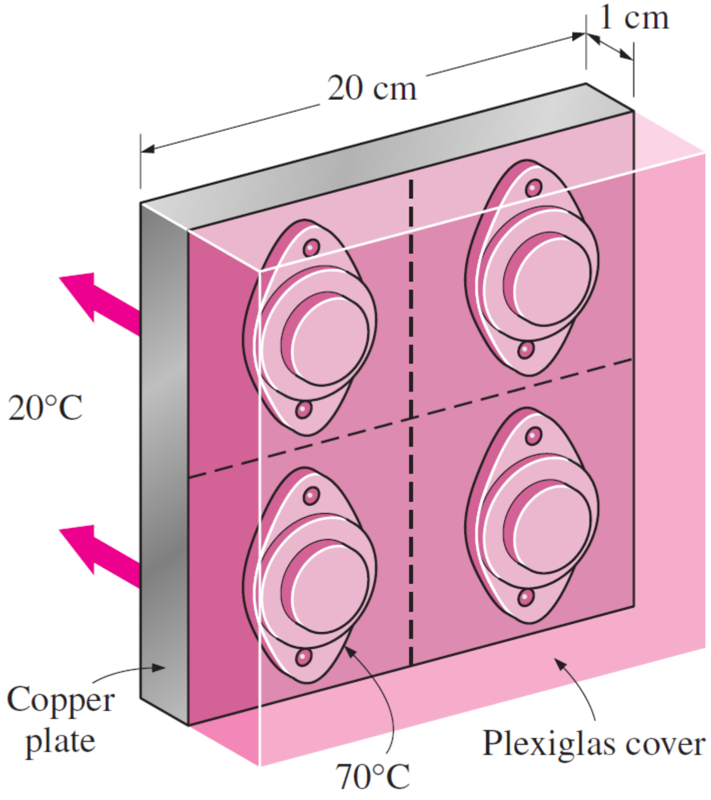


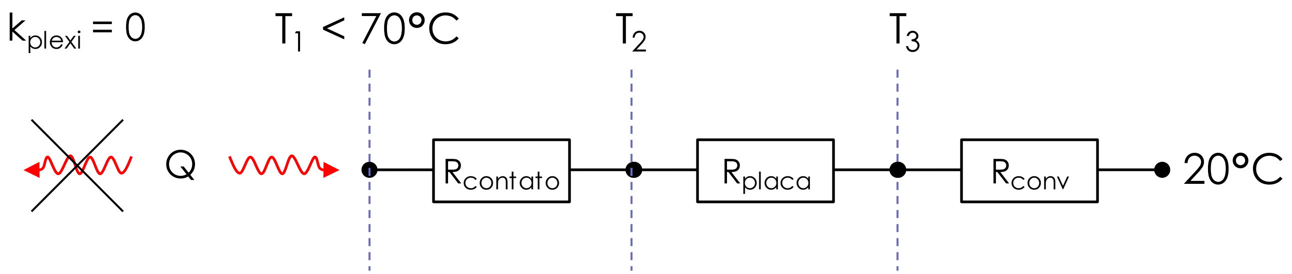


$$R_{contato} = \frac{1}{h_{contato}A} = \frac{1}{42\,\dfrac{kW}{m^2\,°C}\cdot 8\cdot 10^{-4}\,m^2} = 0.030\,°C/W$$

$$R_{placa} = \frac{e}{KA} = \frac{0.01m}{386\,\dfrac{W}{m°C}\cdot 0.01m^2} = 0.0026\,°C/W$$

$$R_{conv} = \frac{1}{h_{conv}A} = \frac{1}{25\,\dfrac{W}{m^2\,°C}\cdot 0.01m^2} = 4.0\,°C/W$$





$$R_{total} = R_{contato} + R_{placa} + R_{conv} = 4.0326\,°C/W$$

$$Q_{max} = \frac{T_{1,max} - T_{\infty}}{R_{total}} = \frac{70-20}{4.0326}\frac{°C}{°C/W} = 12.4\,W$$

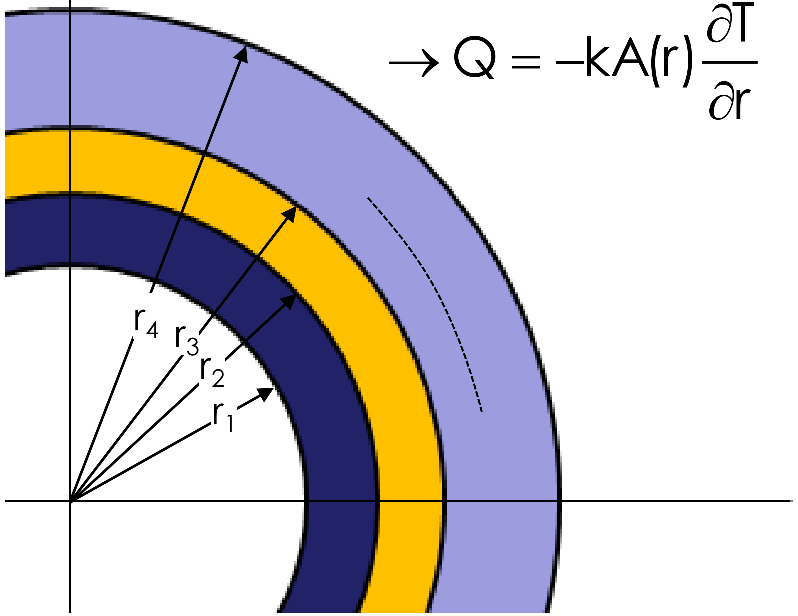
$$Q_{max} = \frac{\Delta T_{contato}}{R_{contato}} = \frac{T_{1,max} - T_2}{R_{contato}} \rightarrow 12.4\,W = \frac{\Delta T_{contato}}{0.030\,°C/W}$$

$$\rightarrow \Delta T_{contato} = 0.37\,°C$$

Geometrias cilíndricas

$$\vec{\nabla} T = \frac{\partial T}{\partial r}\hat{r} + \cancel{\frac{1}{r}\frac{\partial T}{\partial \phi}\hat{\phi}} + \cancel{\frac{\partial T}{\partial z}\hat{z}}$$

$$\rightarrow Q = -kA(r)\frac{\partial T}{\partial r}$$



$$\frac{Q}{A(r)} = -k\frac{dT}{dr} \quad \rightarrow \int_{r_k}^{r_{k+1}} \frac{Q}{2\pi r L}dr = -\int_{T_k}^{T_{k+1}} k\, dT$$

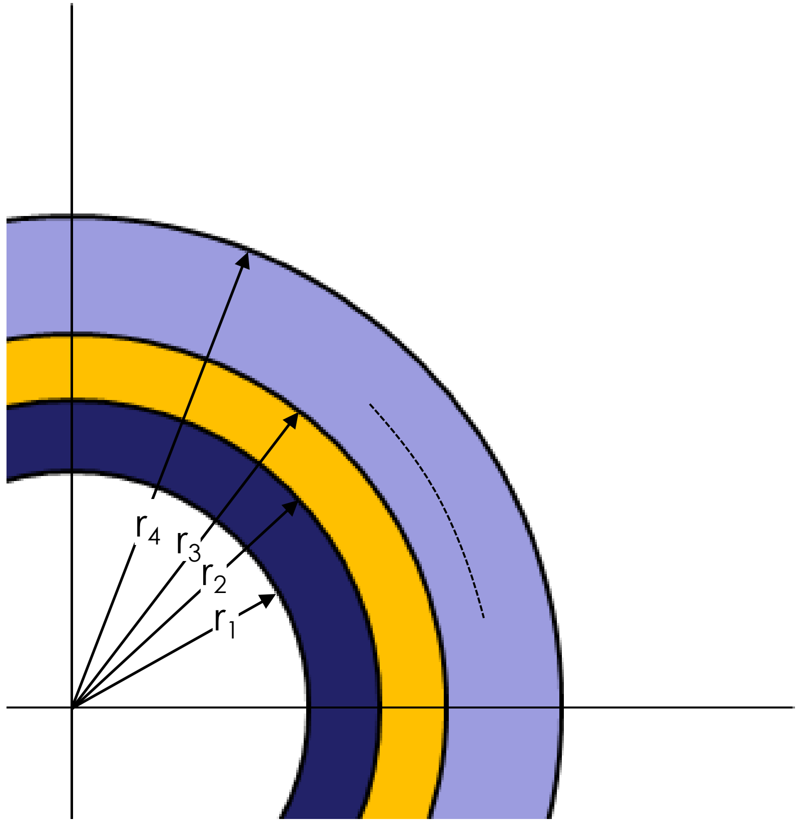
$$\frac{Q}{2\pi L}\int_{r_k}^{r_{k+1}} \frac{1}{r}dr = -k_k \int_{T_k}^{T_{k+1}} dT \quad \rightarrow \frac{Q}{2\pi L}\ln\left(\frac{r_{k+1}}{r_k}\right) = -k_k \cdot (T_{k+1} - T_k)\ldots$$

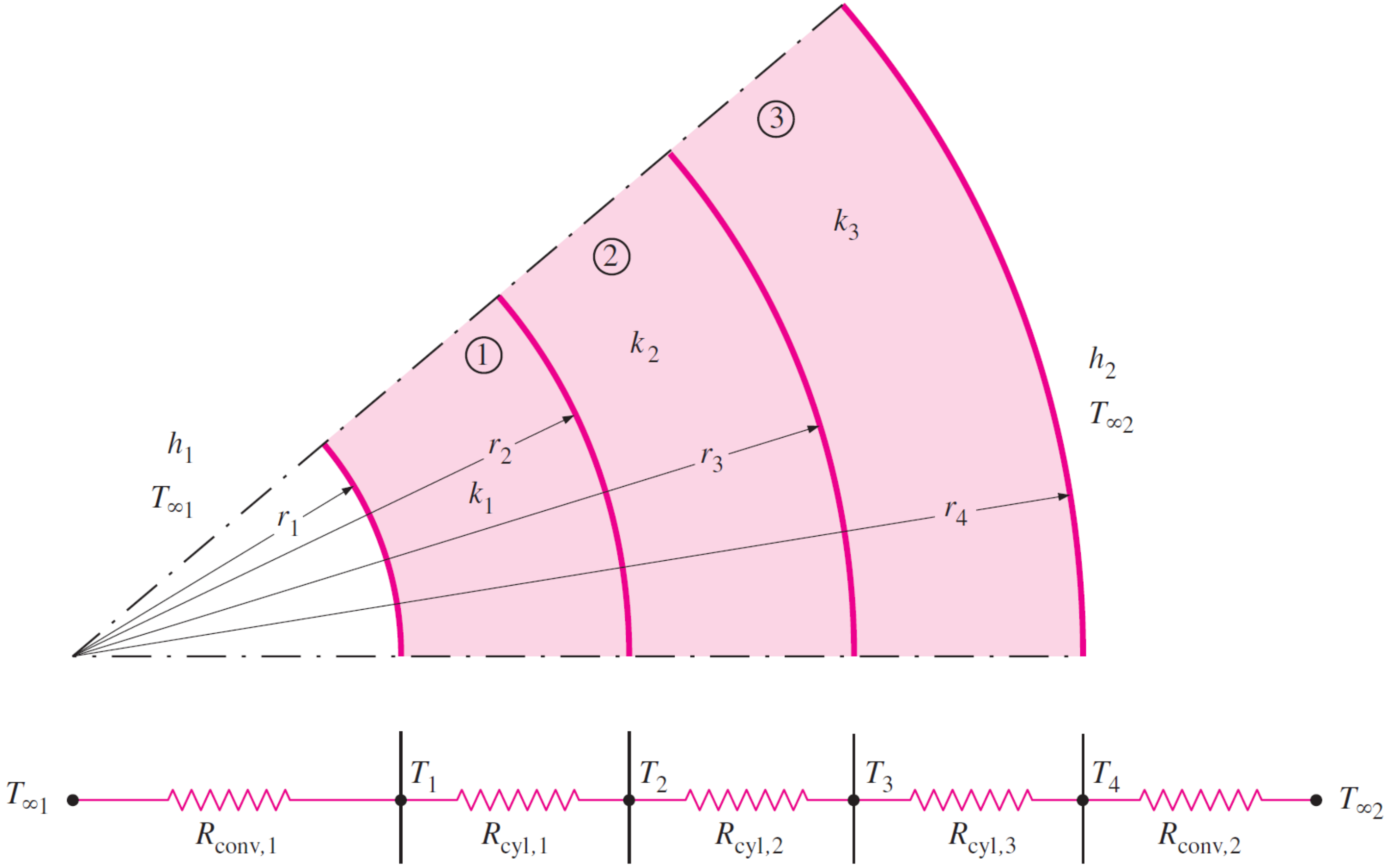
$$\ldots Q = \frac{2\pi L k_k}{\ln(r_{k+1}/r_k)} \cdot (T_k - T_{k+1}) = \frac{T_k - T_{k+1}}{\ln(r_{k+1}/r_k)/(2\pi L k_k)}$$

$R_{cond,k}$

$$R_k = \frac{\ln(r_{k+1}/r_k)}{2\pi L k_k}$$

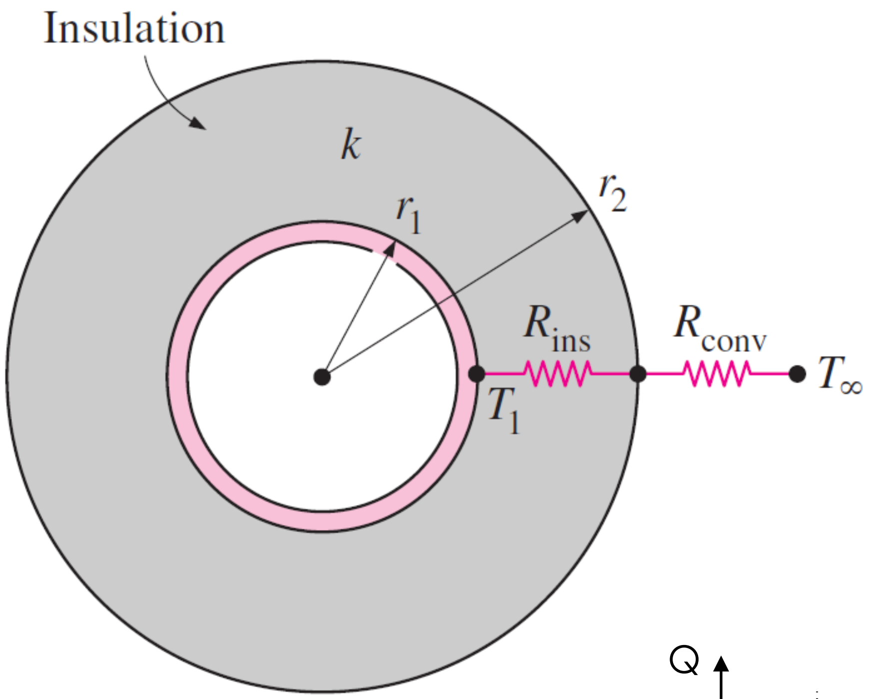
Geometrias cilíndricas





$$R_k = \frac{\ln(r_{k+1}/r_k)}{2\pi L k_k}$$

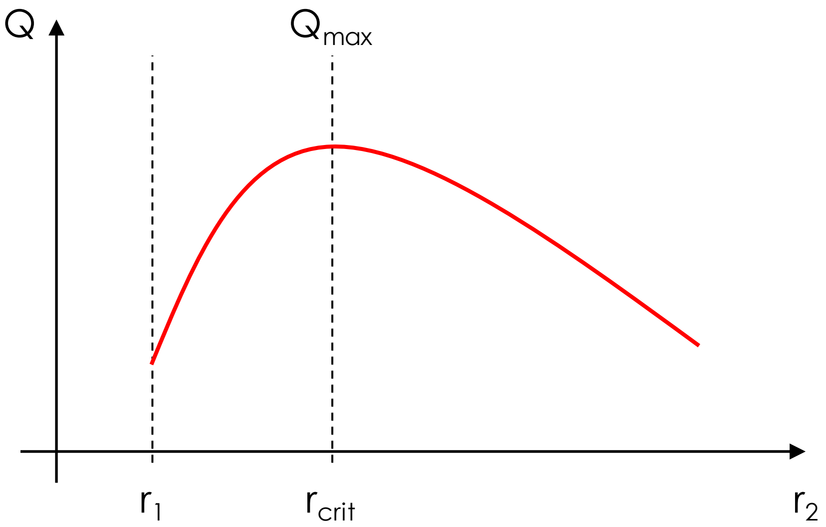
Raio crítico de isolamento...



$$Q = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi L k} + \dfrac{1}{h \cdot (2\pi r_2 L)}}$$

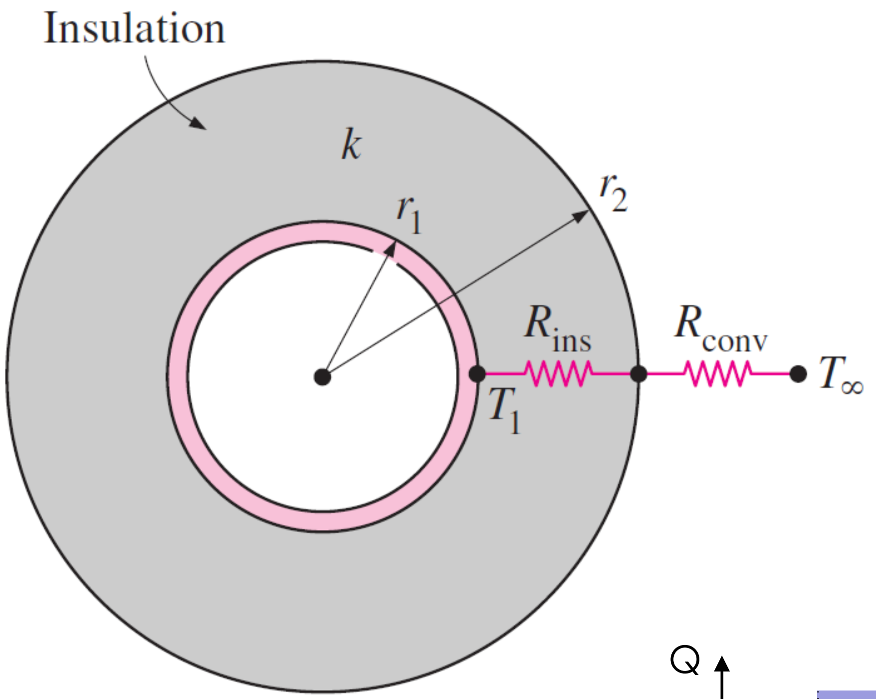
Aumentando a espessura de isolamento ($r_2 \uparrow$)...

$$\frac{\ln(r_2/r_1)}{2\pi L k}\uparrow \qquad \frac{1}{h \cdot (2\pi r_2 L)}\downarrow$$



$$\frac{dQ}{dr} = 0 \rightarrow r_{crit} = \frac{k}{h}$$

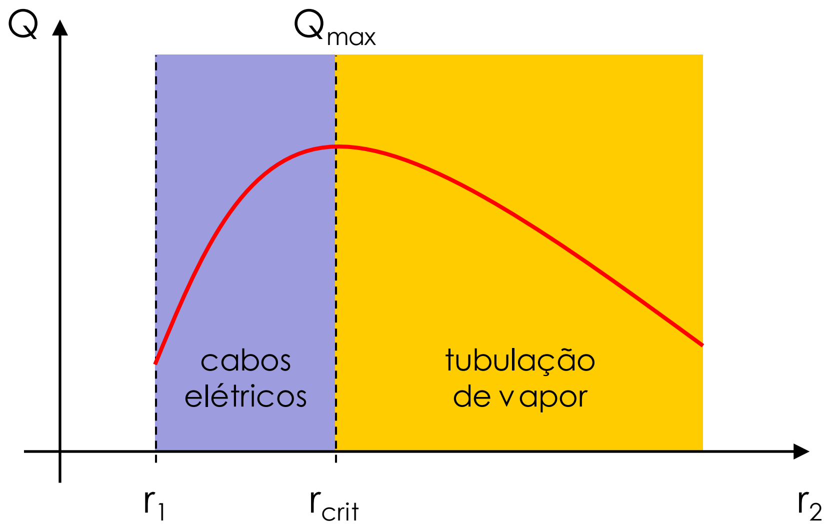
Raio crítico de isolamento...



$$Q = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi L k} + \dfrac{1}{h\cdot(2\pi r_2 L)}}$$

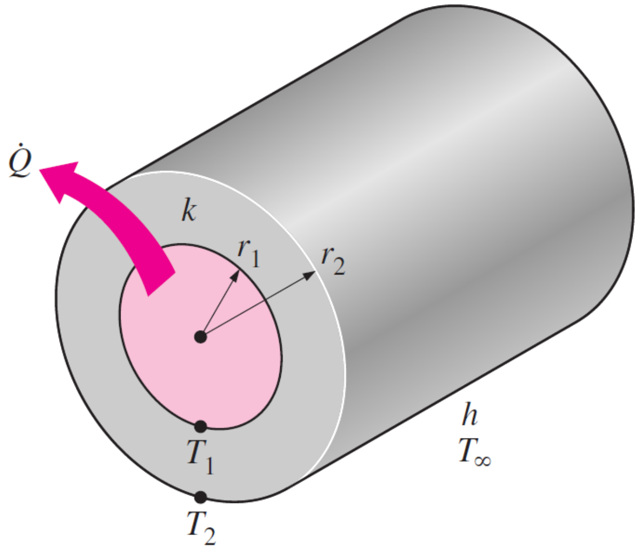
Aumentando a espessura de isolamento ($r_2 \uparrow$)...

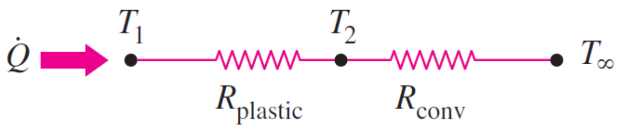
$$\frac{\ln(r_2/r_1)}{2\pi L k}\uparrow \qquad \frac{1}{h\cdot(2\pi r_2 L)}\downarrow$$



$$\frac{dQ}{dr} = 0 \rightarrow r_{crit} = \frac{k}{h}$$

Exemplo: dissipação de calor em um cabo elétrico





A 3 mm diameter and 5 m long electric wire is tightly wrapped with a 2 mm thick plastic cover whose thermal conductivity is k = 0.15 W/m/°C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at T_∞ = 30°C with a heat transfer coefficient of h = 12 W/m2/°C, **determine the temperature at the interface** of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.(ÇG 3-9, pg. 154)

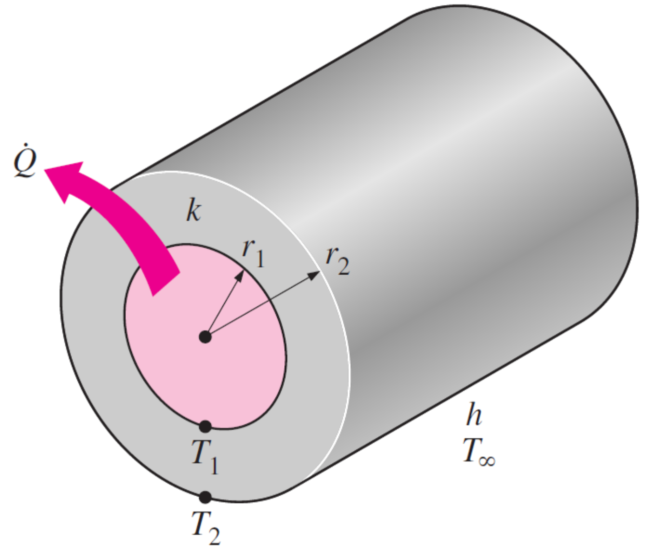
$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

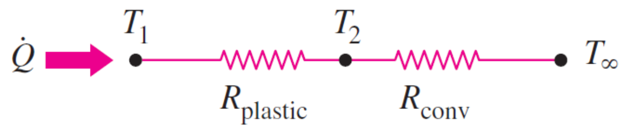
$$A_2 = (2\pi r_2)L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot{}^\circ\text{C})(0.110\text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot{}^\circ\text{C})(5\text{ m})} = 0.18^\circ\text{C/W}$$

Exemplo: dissipação de calor em um cabo elétrico





$$R_{total} = R_{plastic} + R_{conv} = 0.76 + 0.18 = 0.94°C/W$$

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$$

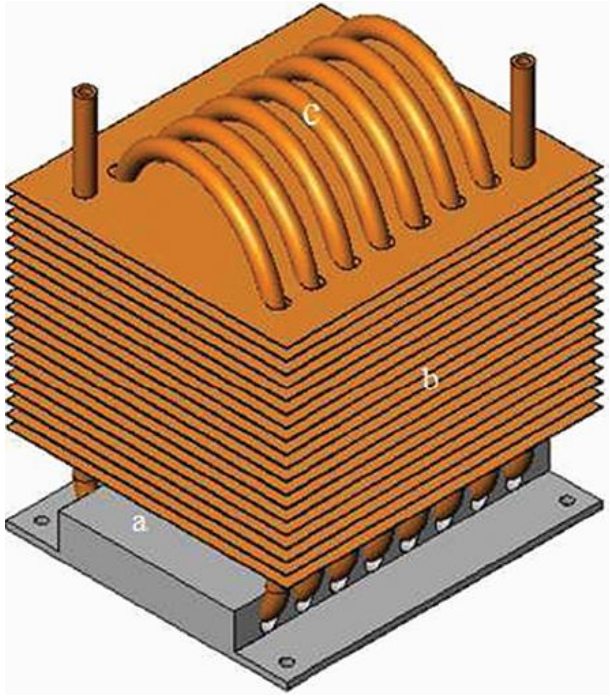
$$T_1 = T_\infty + \dot{Q}R_{total}$$
$$= 30°C + (80\ W)(0.94°C/W) = \mathbf{105°C}$$

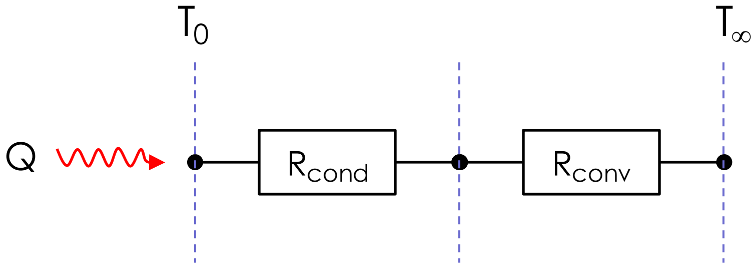
$$r_{cr} = \frac{k}{h} = \frac{0.15\ W/m \cdot °C}{12\ W/m^2 \cdot °C} = 0.0125\ m = 12.5\ mm$$

Obs.: $r_2 = 4$ mm $\rightarrow T_1 = 90.6°C$

Obs.: $r_2 = 12.5$ mm $\rightarrow T_1 = 83.0$ °C

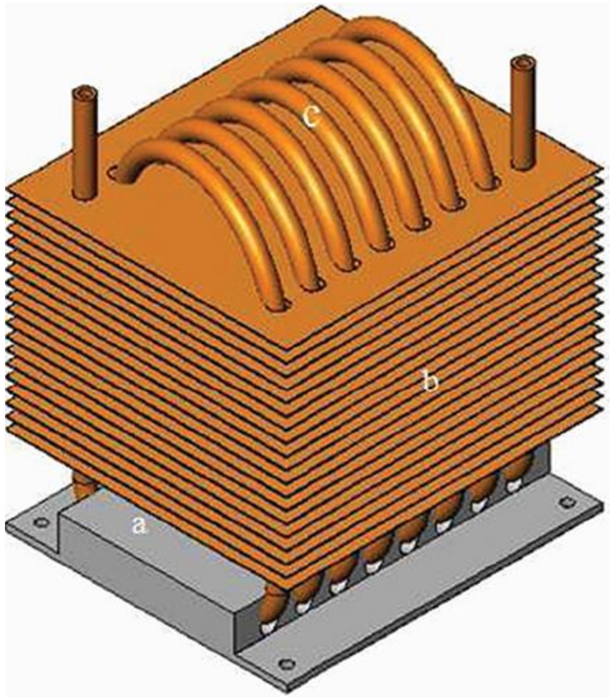
Superfícies aletadas: maximizando a remoção de calor

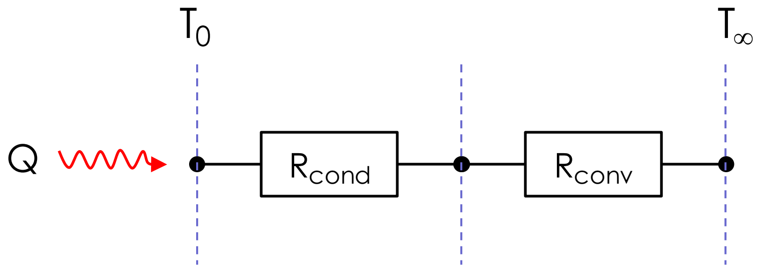




$$Q = \frac{T_{sup} - T_{\infty}}{\dfrac{L}{kA_{cond}} + \dfrac{1}{hA_{conv}}}$$

Superfícies aletadas: maximizando a remoção de calor





$$Q = \frac{T_{sup} - T_{\infty}}{\dfrac{L}{kA_{cond}} + \dfrac{1}{hA_{conv}}}$$

Extensão condução (L)

Condutibilidade térmica do material da aleta. Cobre ou alumínio. (k)

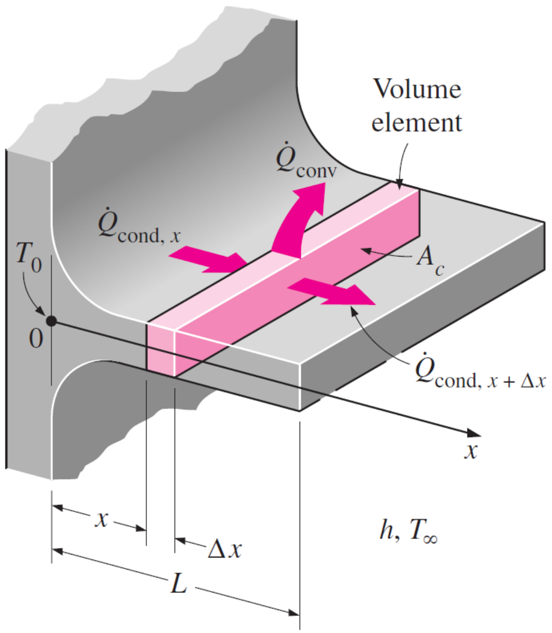
Área transversal de condução. (A_{cond})

Área externa de contato com o escoamento. Pode ser aumentada (A_{conv})

Coeficiente de convecção do escoamento. Dificuldades para aumenta-lo. (h)

$$L \uparrow \Rightarrow \begin{cases} R_{cond} \uparrow \\ R_{conv} \downarrow \end{cases} \Rightarrow R_{total} \downarrow$$

Superfícies aletadas: maximizando a remoção de calor



$$\frac{d}{dx}\left(kA_C\frac{dT}{dx}\right)-hp(T-T_\infty)=0$$

$$A,p=cte \quad\Rightarrow\quad \frac{d^2\theta}{dx^2}-a^2\theta=0 \quad \begin{cases}\theta=T-T_\infty\\ a^2=\dfrac{hp}{kA_c}\end{cases}$$

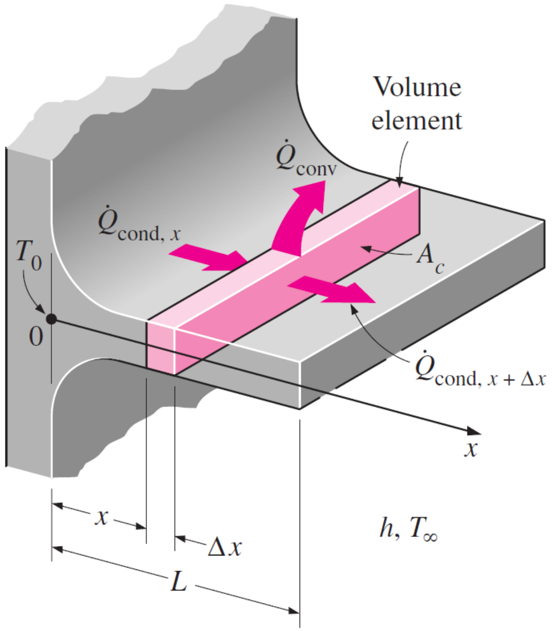
$$\theta(x)=C_1\exp(+ax)+C_2\exp(-ax)$$

$$\text{Caso 1: } L\to\infty \quad T(L)\to T_\infty\to\theta=0\to C_1=0$$

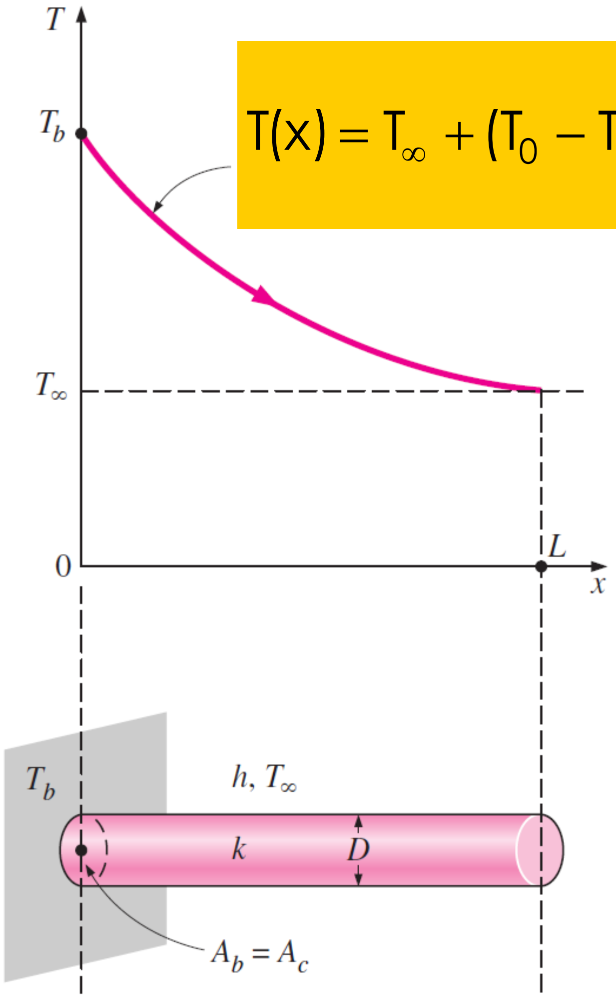
$$\theta(0)=\theta_0=T_0-T_\infty\to C_2=\theta_0\ldots$$

$$\theta(x)=\theta_b\exp(-ax)\to\frac{T(x)-T_\infty}{T_0-T_\infty}\exp\left(-\sqrt{\frac{hp}{kA_c}}\cdot x\right)$$

Superfícies aletadas: maximizando a remoção de calor



$$T(x) = T_\infty + (T_0 - T_\infty)\exp\left(-\sqrt{\frac{hp}{kA_c}}\cdot x\right)$$

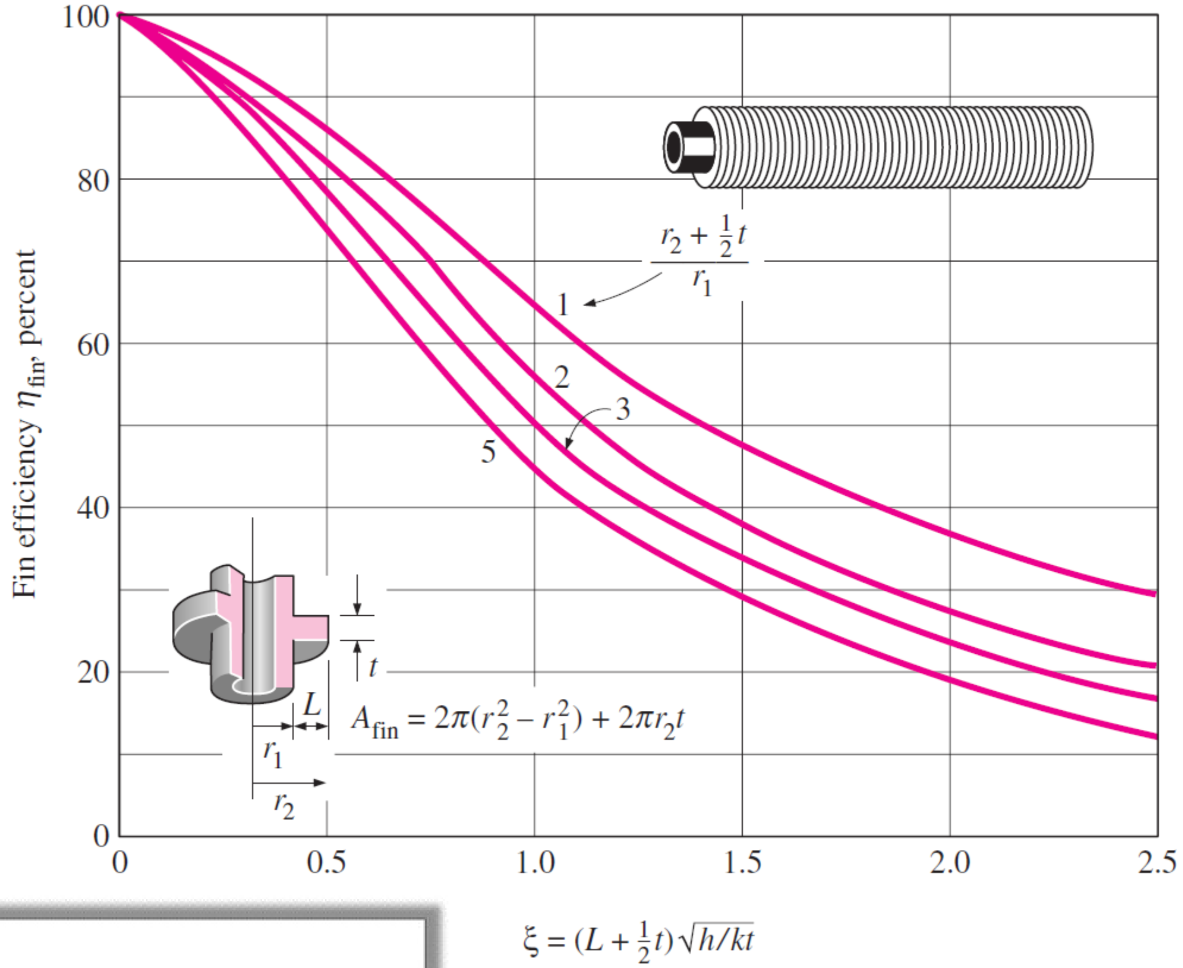


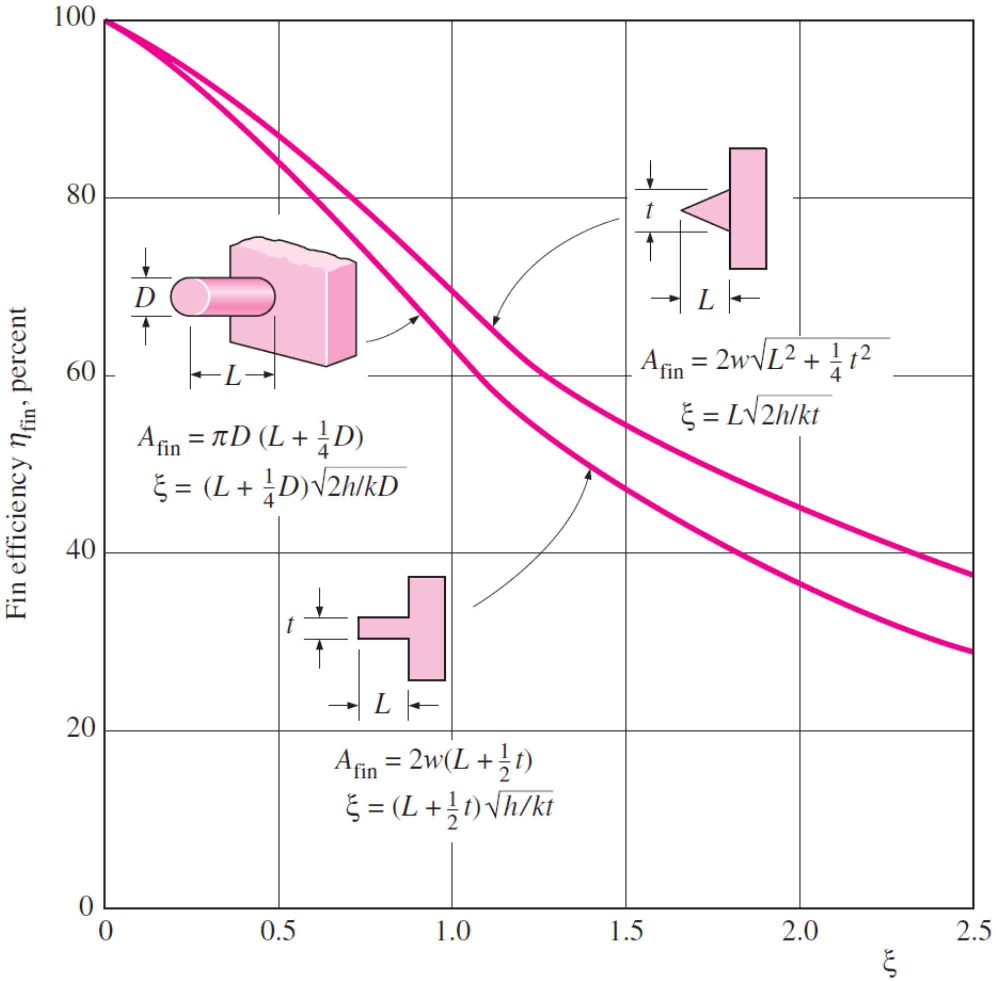
($p = \pi D$, $A_c = \pi D^2/4$ for a cylindrical fin)

Obs.: se $k \to 0 \to T(x) = T_0$

$$Q_{max} = hpL(T_0 - T_\infty)$$

$$\eta_{aleta} = \frac{Q_{aleta}}{Q_{max}} \rightarrow Q_{aleta} = \eta_{aleta}\, hpL(T_0 - T_\infty)$$





$$Q_{aleta} = \eta_{aleta}\, hpL(T_0 - T_\infty)$$

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