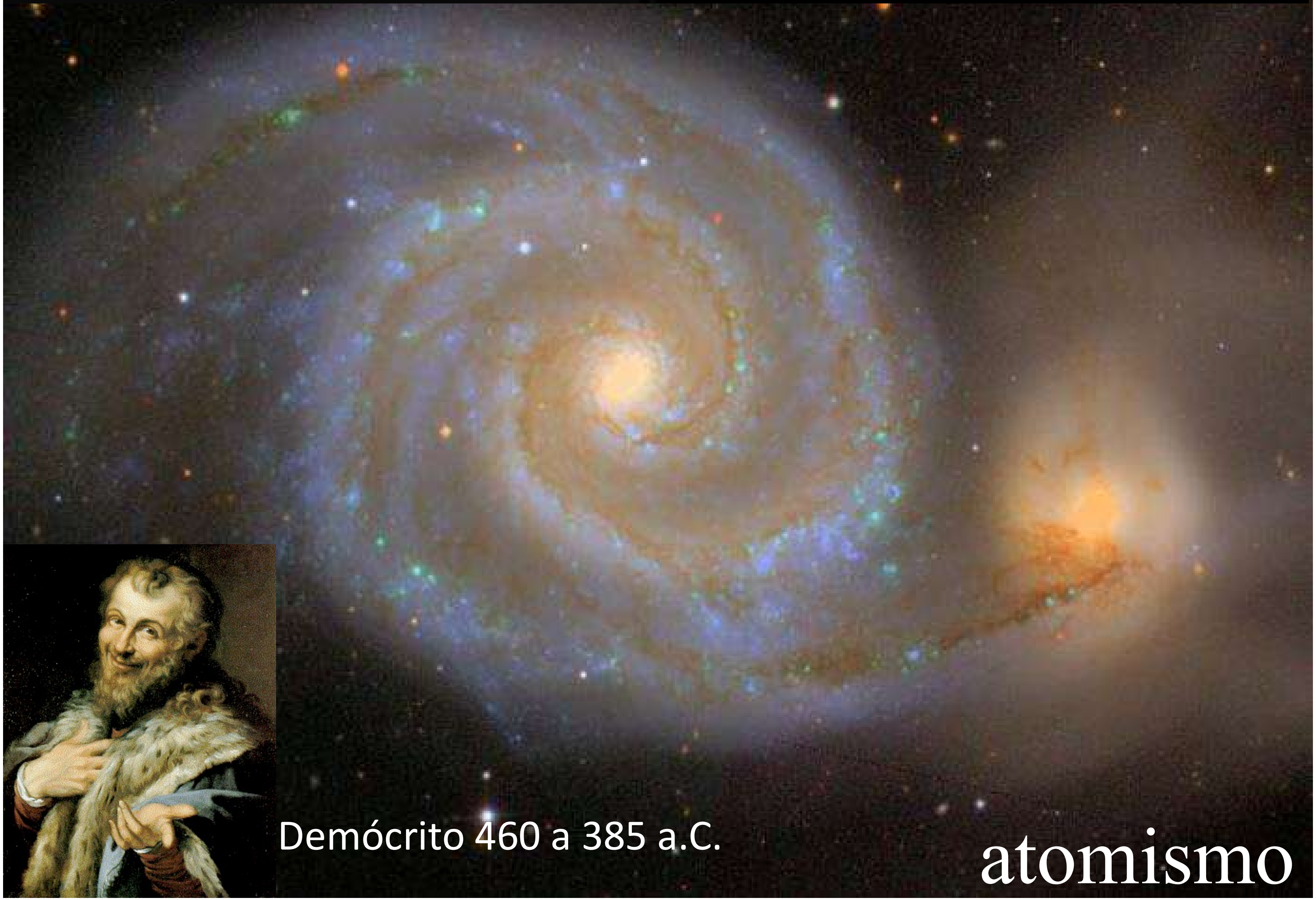
ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

# Turbulência fantástica

aantunha

titulo

# Tudo que existe no universo é fruto do acaso e da necessidade

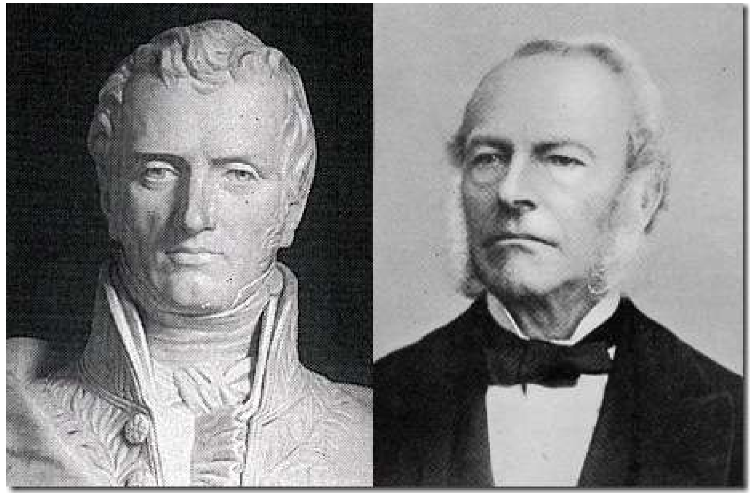


# da Vinci

**1452-1519**

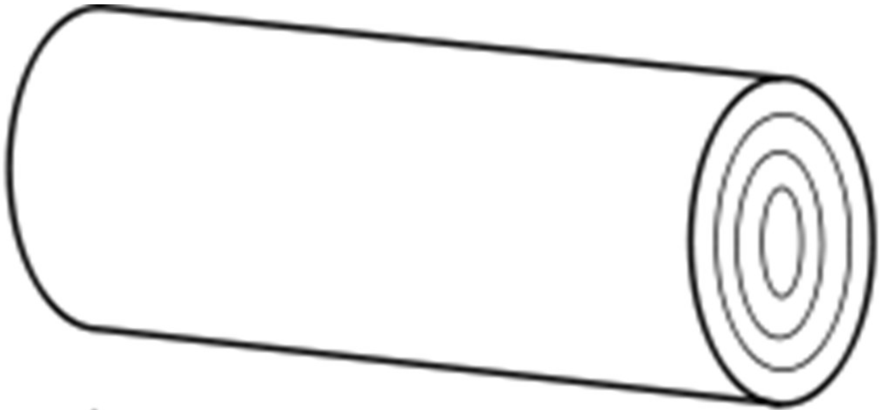
$$\underbrace{\frac{\partial(\rho\vec{v})}{\partial t}}_{\text{transiente}} = \underbrace{-\mathrm{div}(\rho\vec{v}\vec{v})}_{\text{convecção}} - \underbrace{\mathrm{div}\,\vec{\vec{\tau}}}_{\substack{\text{força}\\\text{contato}\\\text{irreversível}}} \underbrace{-\,\mathrm{gr\vec{a}d}\,p}_{\substack{\text{força}\\\text{contato}\\\text{reversivel}}} + \underbrace{\rho\vec{g}}_{\substack{\text{força}\\\text{campo}}}$$

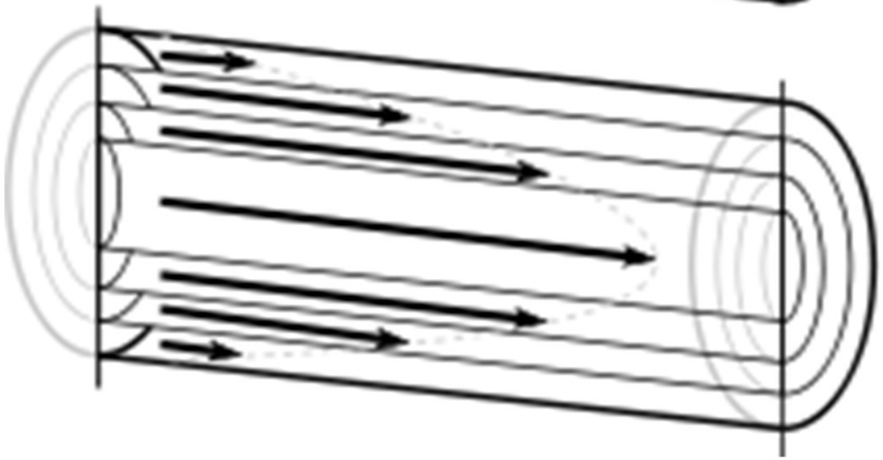
$$\vec{\vec{\tau}} = -\mu\frac{\partial v_z}{\partial r}$$

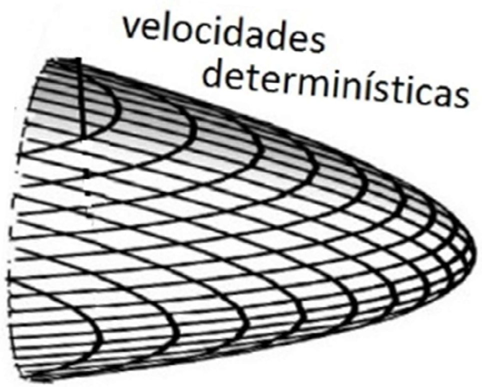


Navier 1785-1836

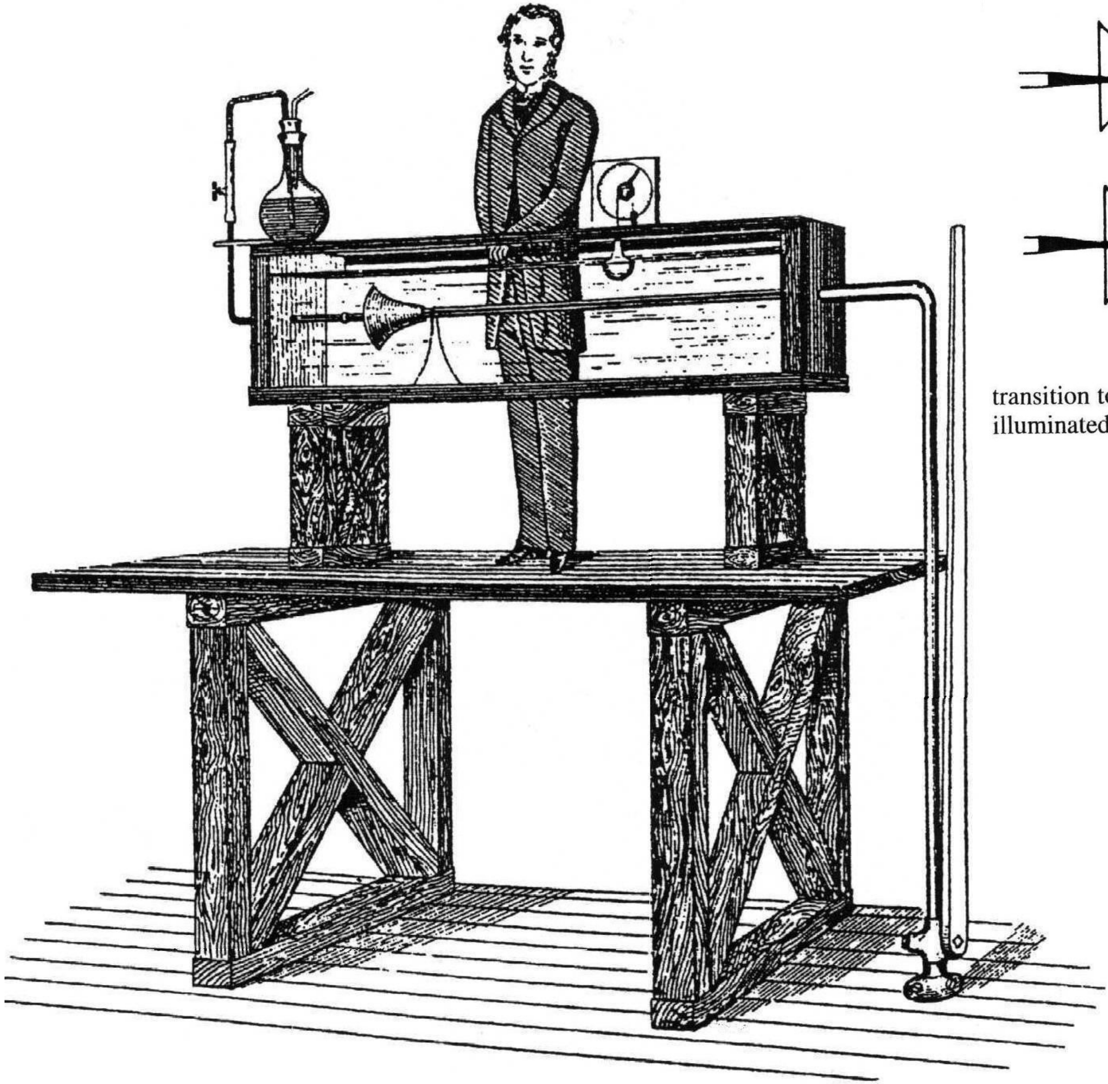
Stokes 1819 -1903



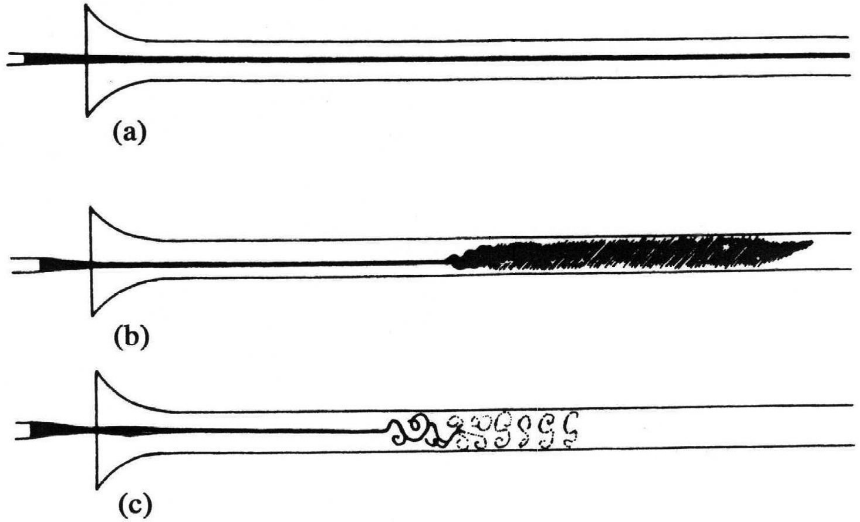




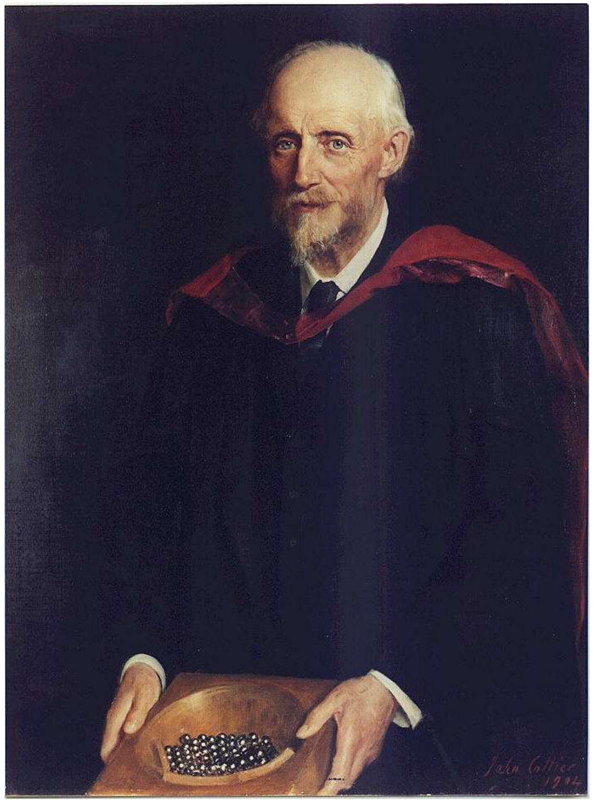
# Osborne Reynolds
# 1842 1912



The configuration of Reynolds's experiment on flow along a pipe.



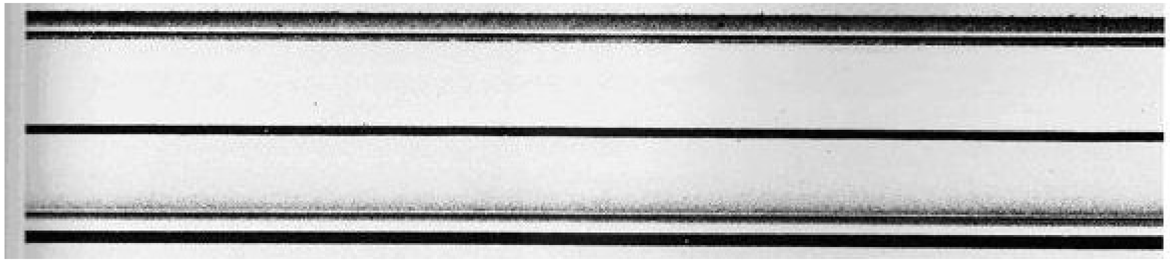

Sketches of (a) laminar flow in a pipe, indicated by a dye streak; (b) transition to turbulent flow in a pipe; and (c) transition to turbulent flow as seen when illuminated by a spark. (From Reynolds, 1883, Figs. 3, 4 and 5.)



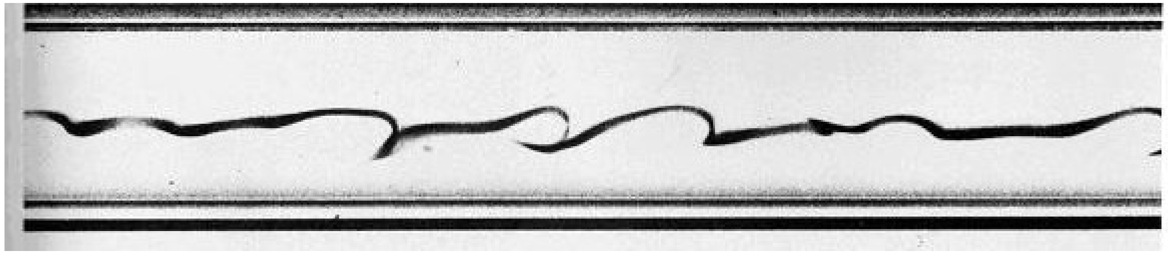
Reynolds

experimento

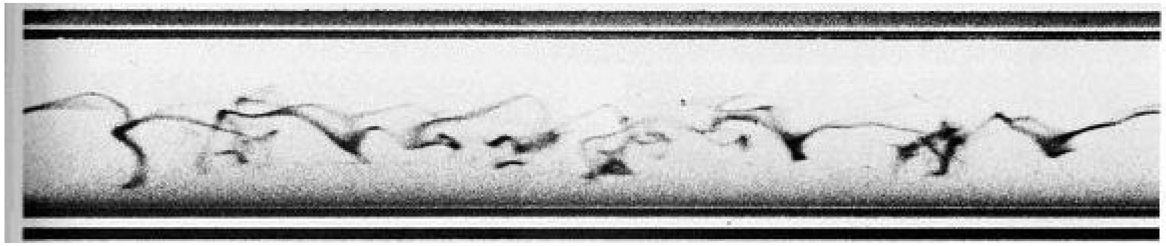
de 1883



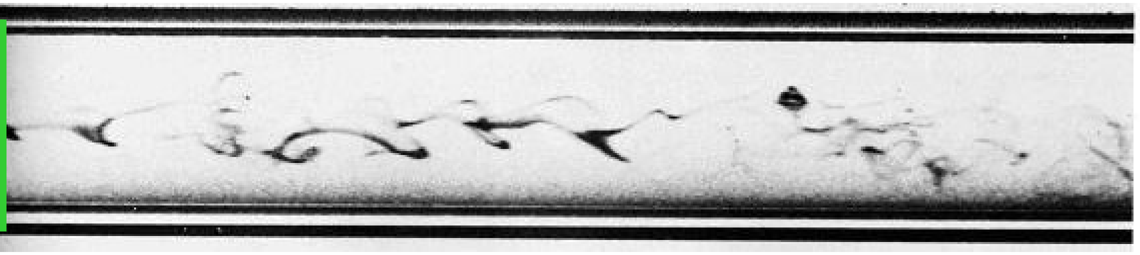
rotacional

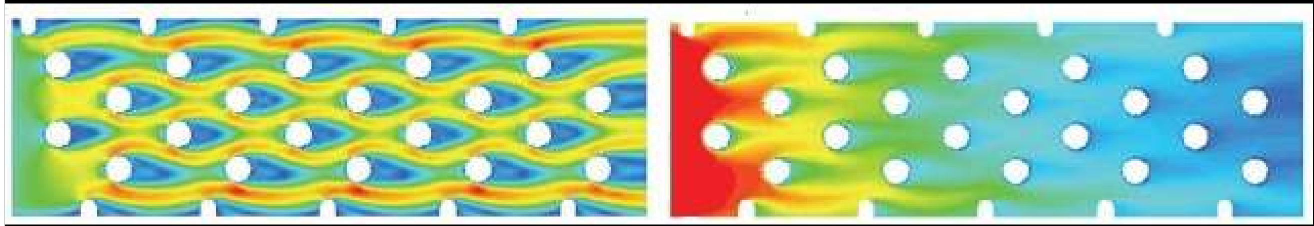


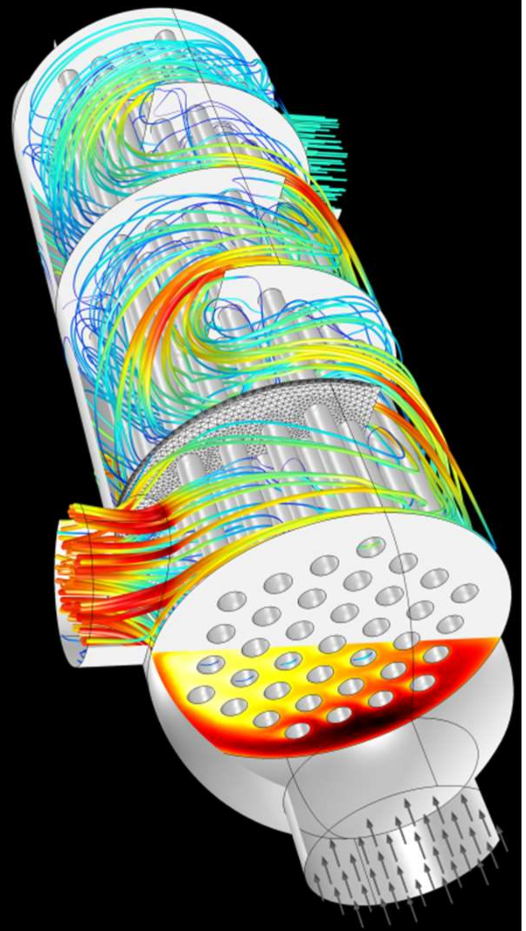
3D

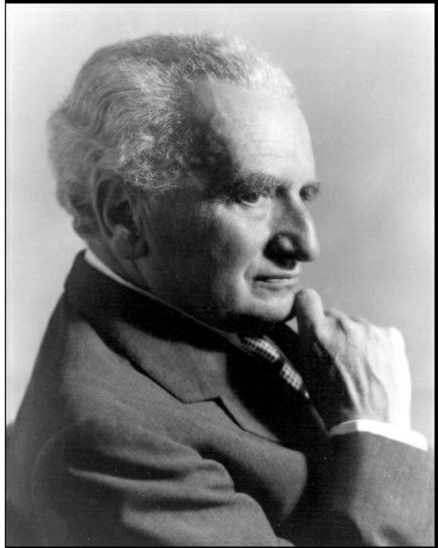


irreversível

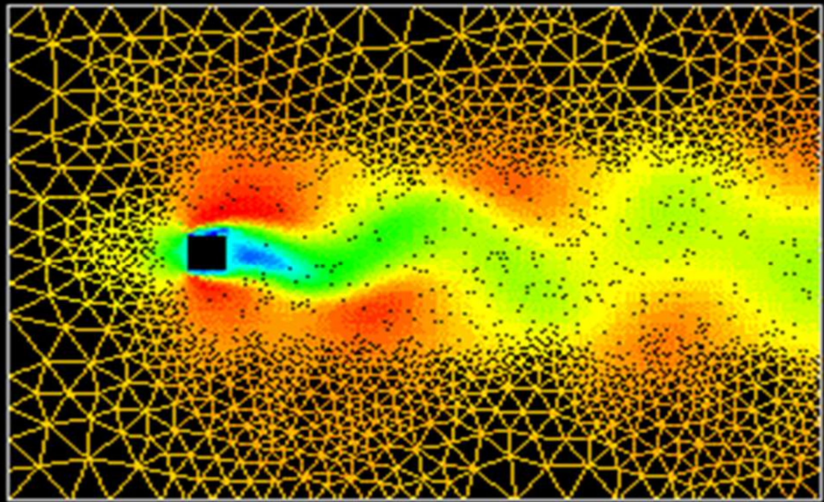


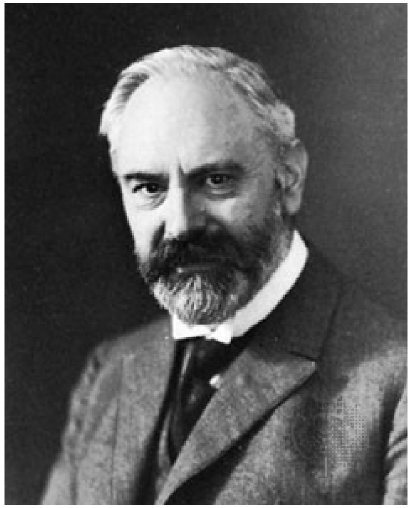






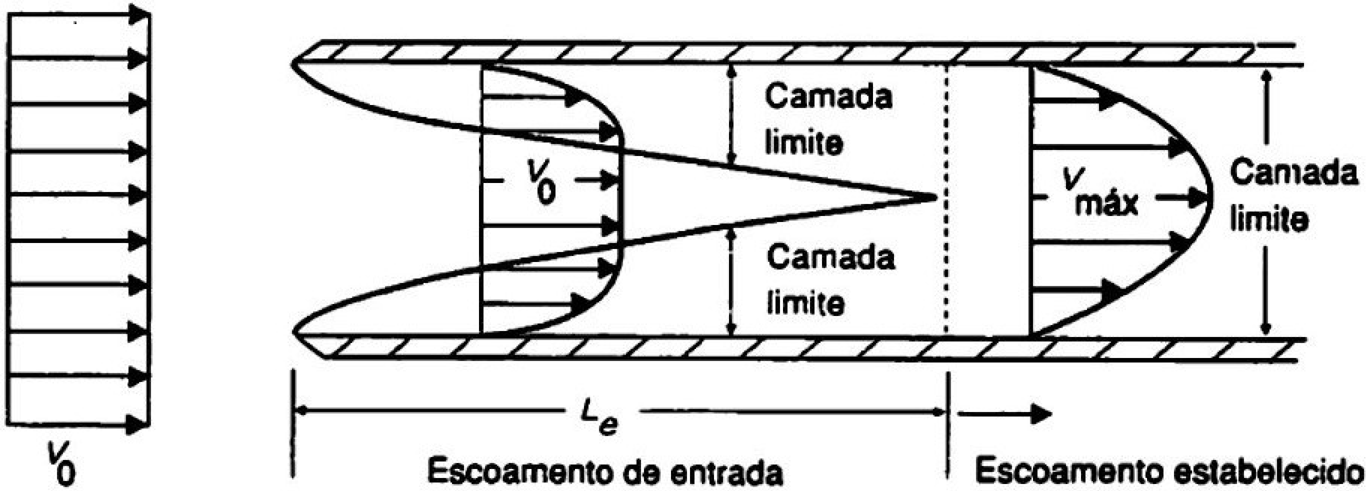
T. von Kármán
1881-1963





Prandtl
1875 –1953

$$\frac{\partial \vec{\bar{v}}}{\partial t} + \mathrm{div}\,\vec{\bar{v}}\,\vec{\bar{v}} = \mathrm{div}\left[\mu+\mu_T\right]\mathrm{gr}\vec{a}\mathrm{d}\,\vec{\bar{v}} + \vec{\bar{g}}$$




$$\mu_T = \ell\,\mathrm{gr}\vec{a}\mathrm{d}\,\vec{\bar{v}}$$

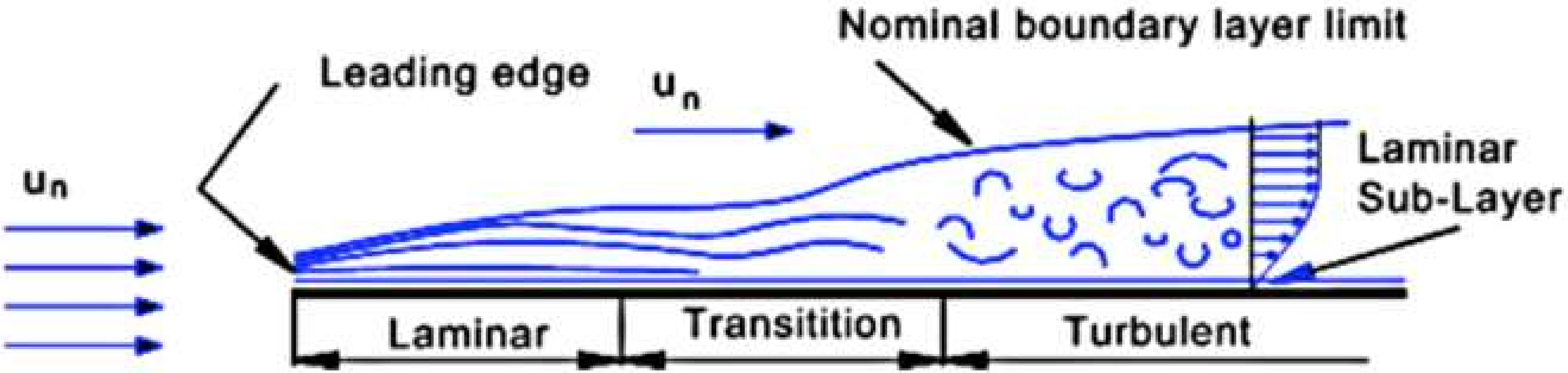
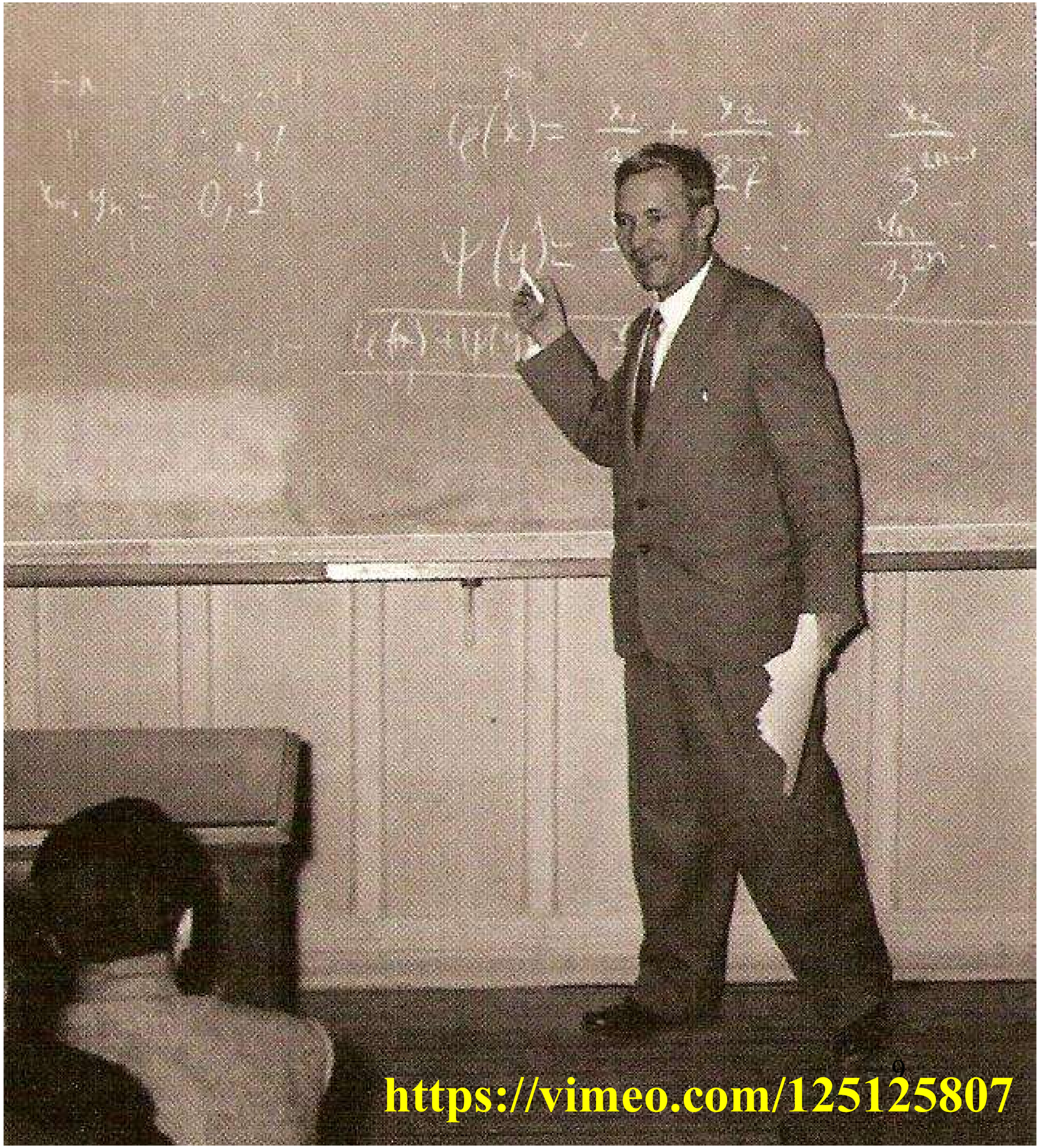
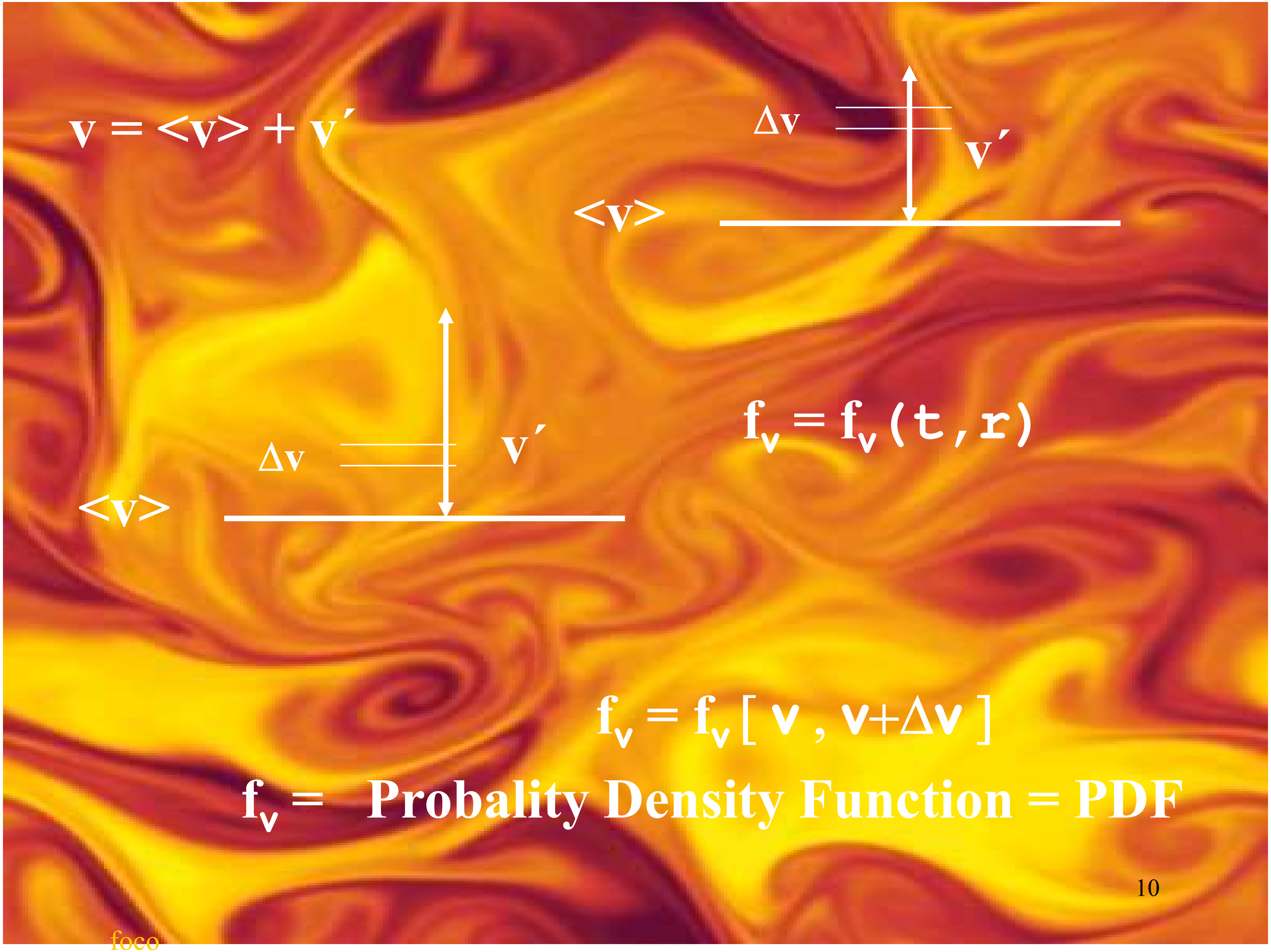



Image: www.roymech.co.uk/

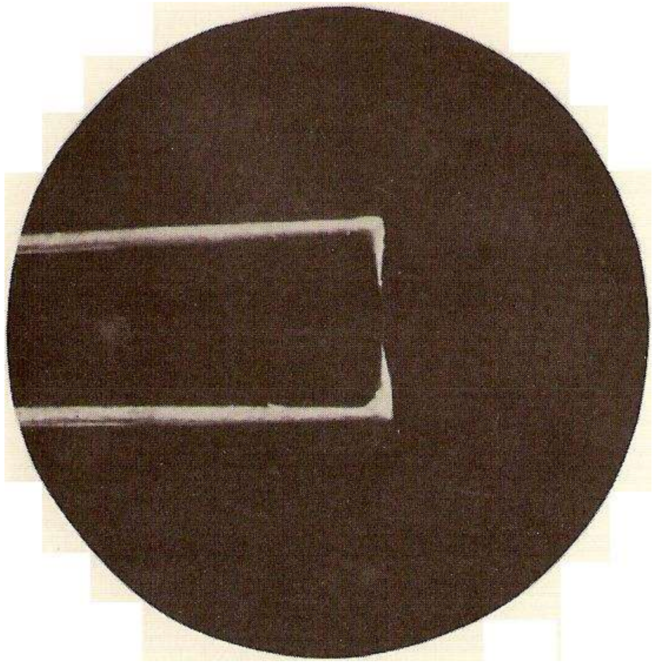
# Kolmogorov
# 1903 -1987

"meu interesse pelo estudo dos escoamentos turbulentos surgiu no fim dos anos 30. Pareceu-me evidente que a técnica matemática principal deveria ser a teoria das **funções aleatórias de diversas variáveis**, que estava então nascendo."



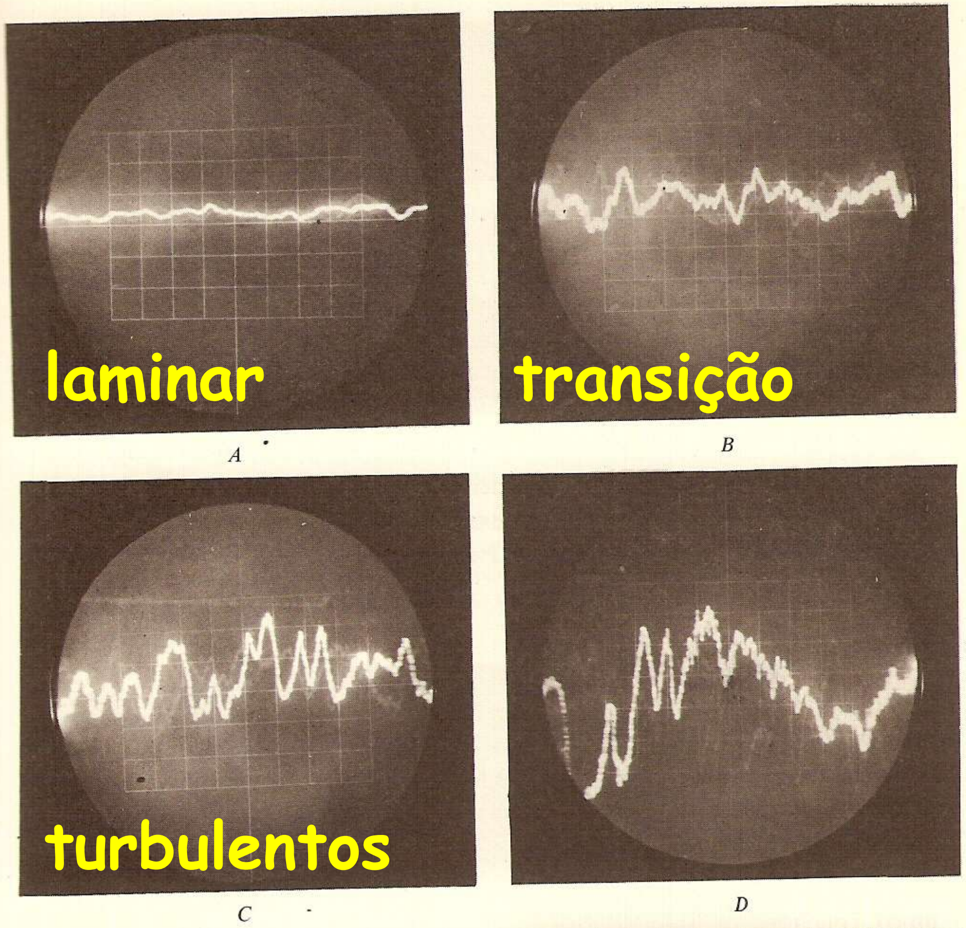

$v = \langle v \rangle + v'$
$\Delta v$
$v'$
$\langle v \rangle$
$\Delta v$
$v'$
$\langle v \rangle$
$f_v = f_v(t, r)$
$f_v = f_v[v, v + \Delta v]$
$f_v$ = Probality Density Function = PDF
10
foco

$$\pi = \pi' + \overline{\pi}$$



anemômetro de fio quente

$$\vec{v} = \vec{v}\,' + \overline{\vec{v}}$$




osciloscópio

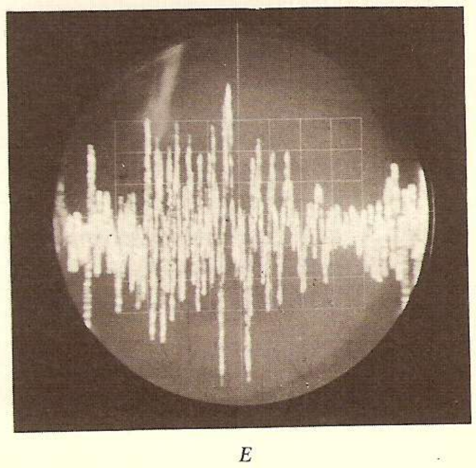
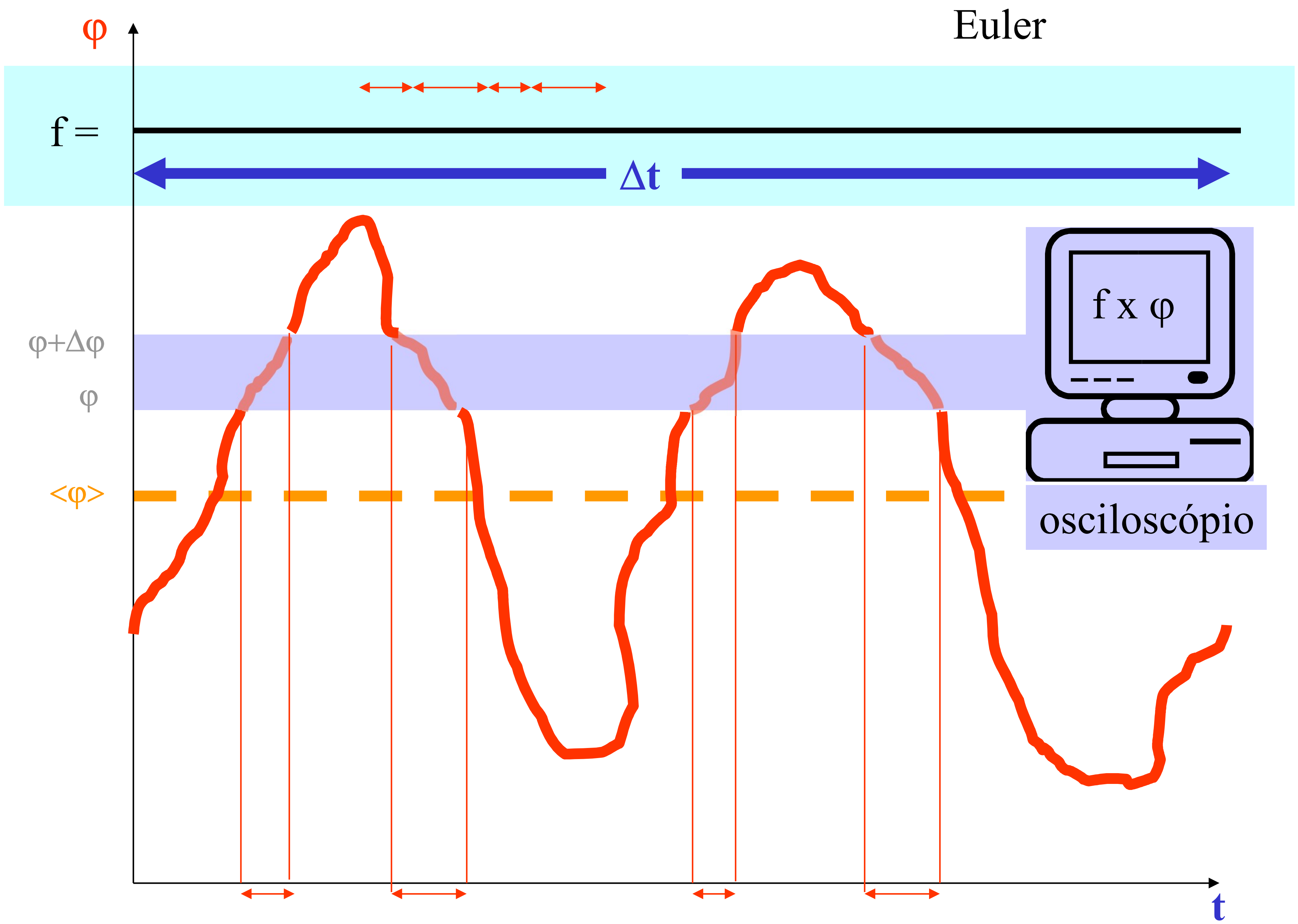



Fig. 34. Fotografias de flutuações da velocidade de perturbação $v'$ visualizadas na tela de um osciloscópio através da técnica da anemometria de fio quente.

A – regime laminar
B – transição
C, D e E – regime turbulento, com diversas intensidades de turbulência.

*A ordem de grandeza da intensidade de turbulência usual em escoamentos nas aplicações da Engenharia é de 0,05.*

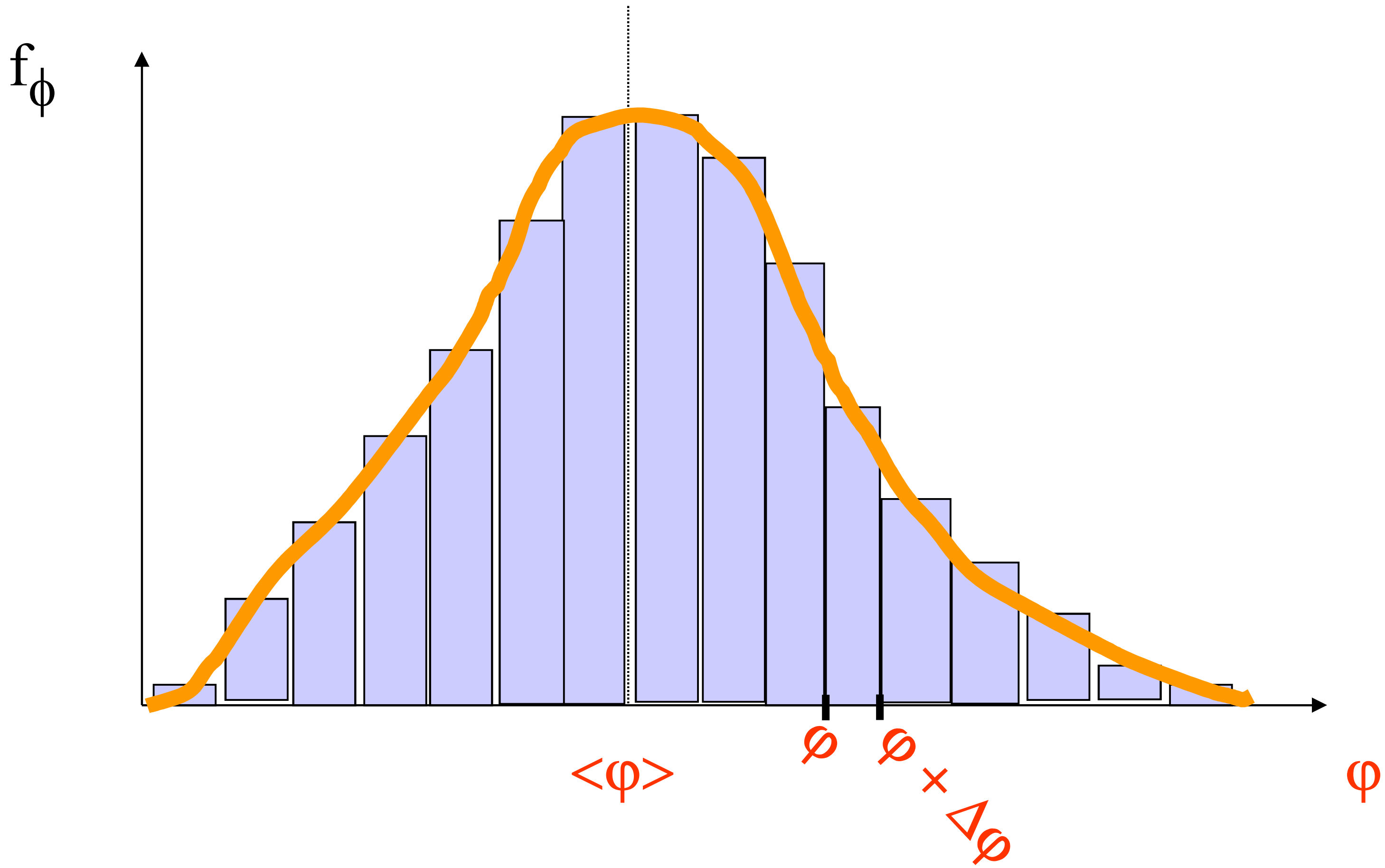


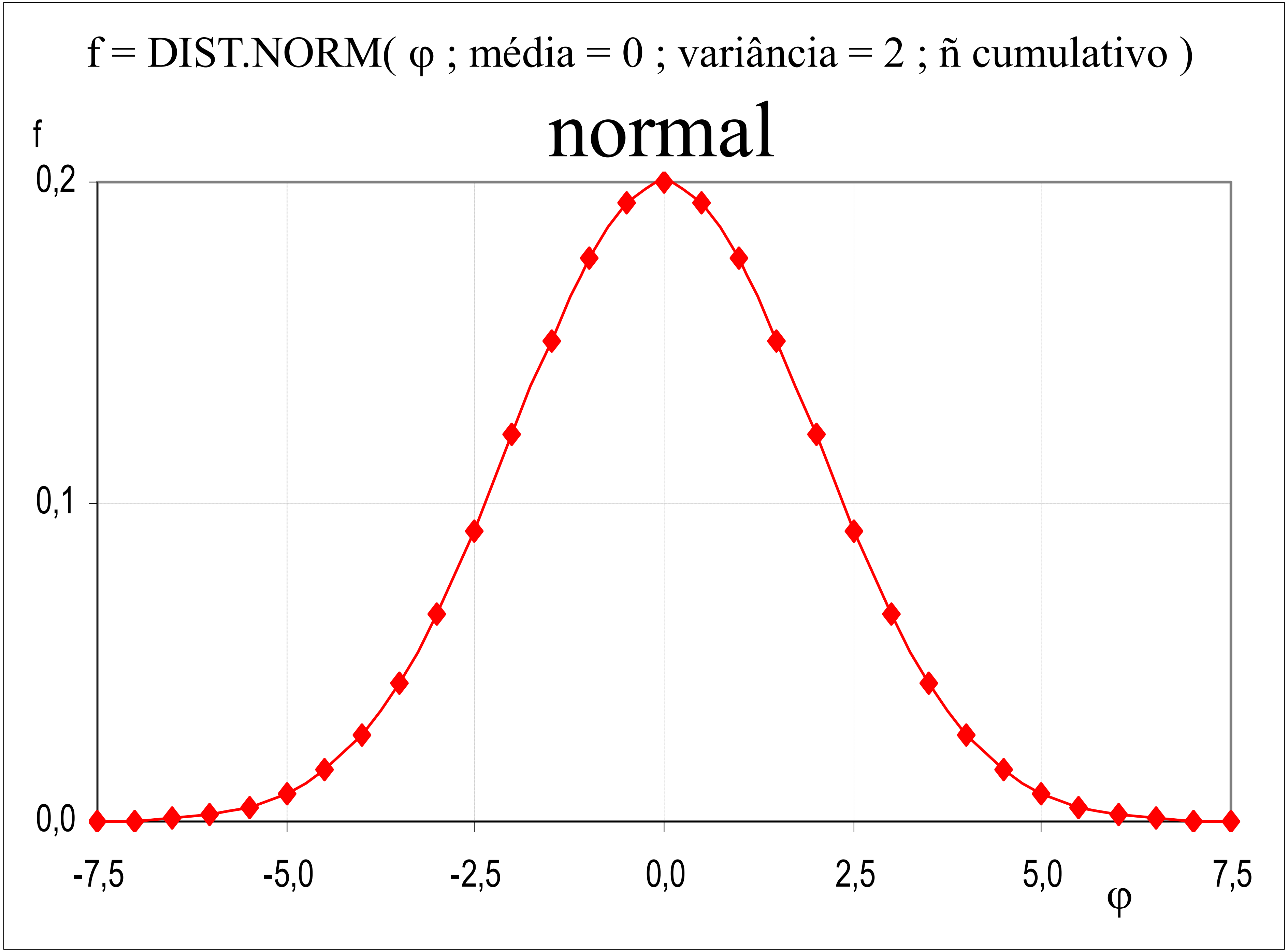

medidas

$f_\phi$ = Probality Density Function - PDF

PDF

$$f_\phi = f_\phi\,[\,\varphi\,,\,\varphi+\Delta\varphi\,] = f_\phi(\mathtt{t},\mathtt{r})$$




f = DIST.NORM( φ ; média = 0 ; variância = 2 ; ñ cumulativo )
normal
f
0,2
0,1
0,0
-7,5
-5,0
-2,5
0,0
2,5
5,0
7,5
φ

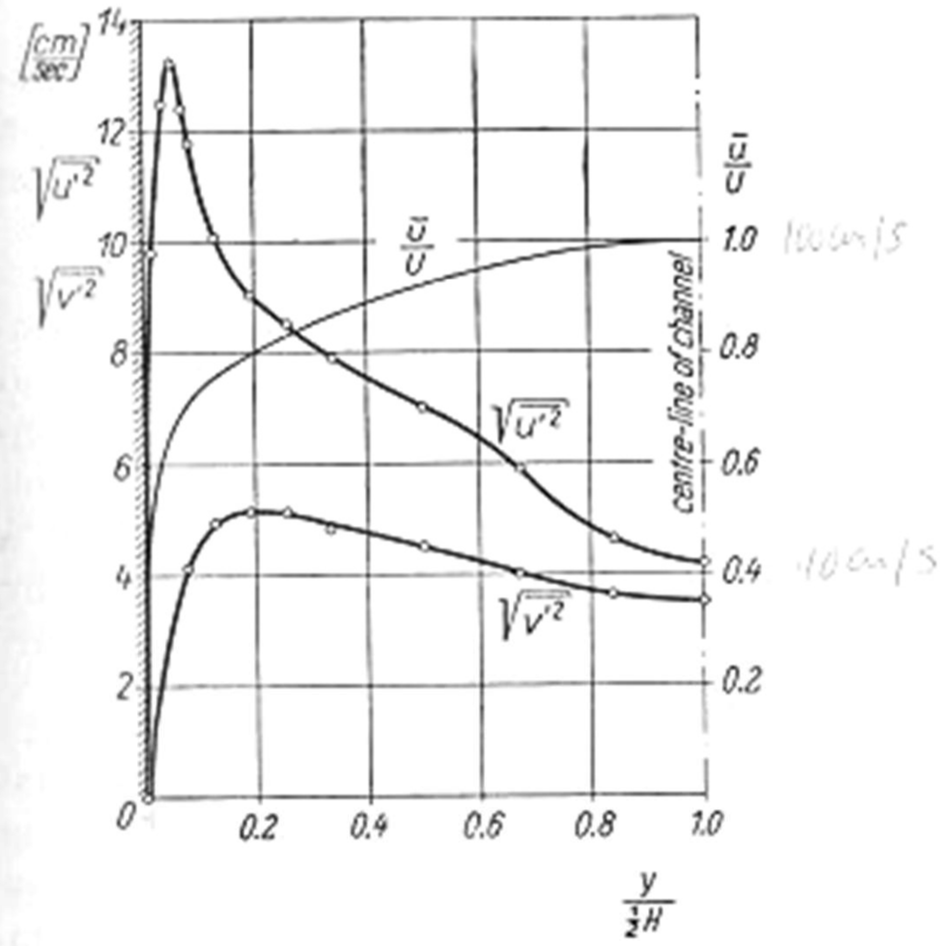



Fig. 18.3. Measurement of fluctuating turbulent components in a wind tunnel, at maximum velocity $U$ = 100 cm/sec after Reichardt [41]

Root-mean-square of longitudinal fluctuation $\sqrt{\overline{u'^2}}$, transverse fluctuation $\sqrt{\overline{v'^2}}$, mean velocity $\bar{u}$

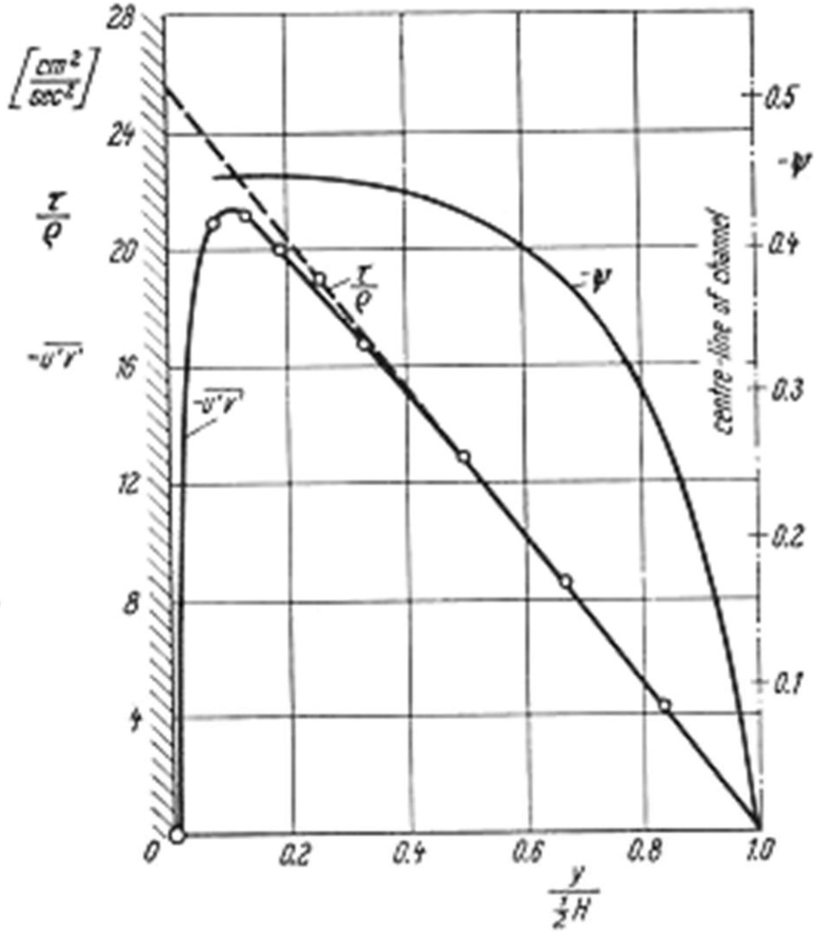
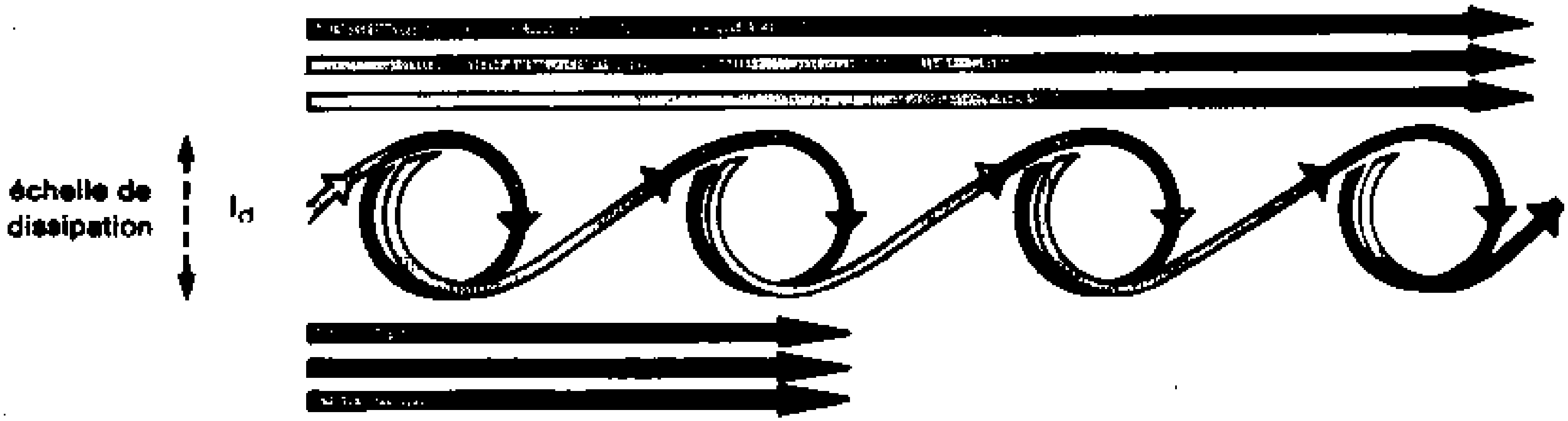
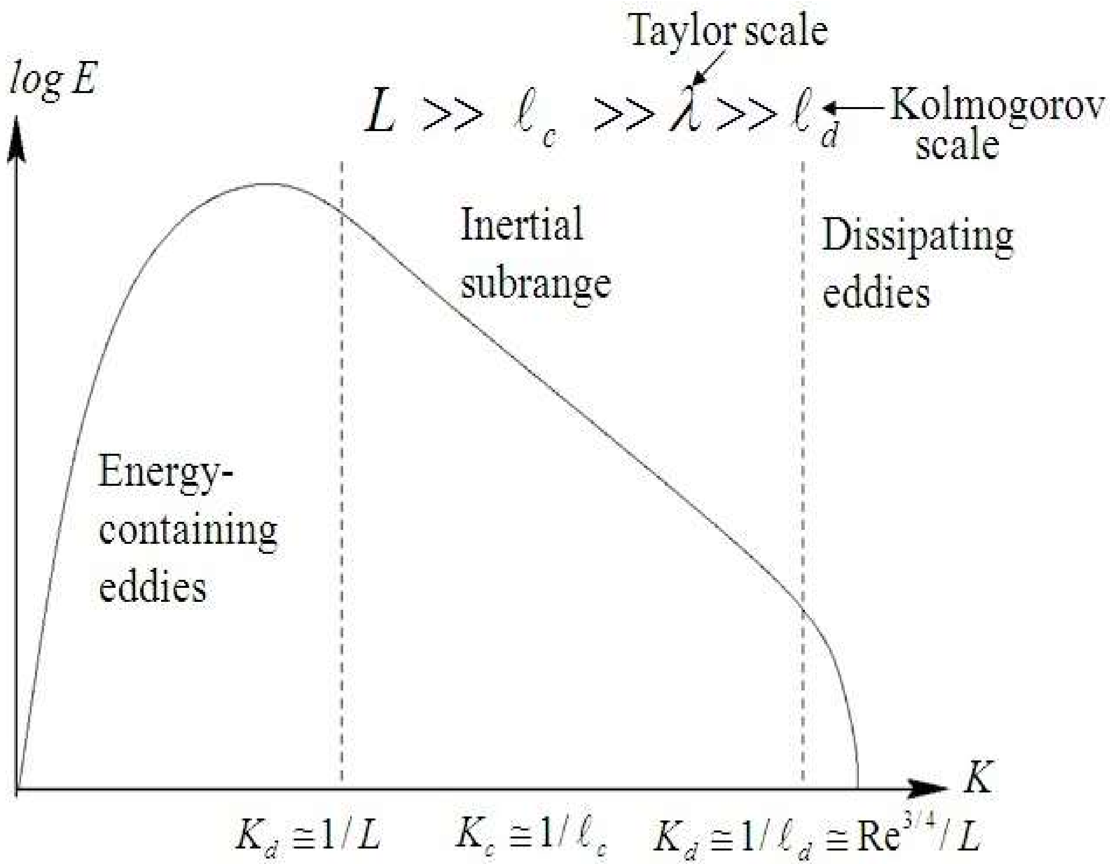


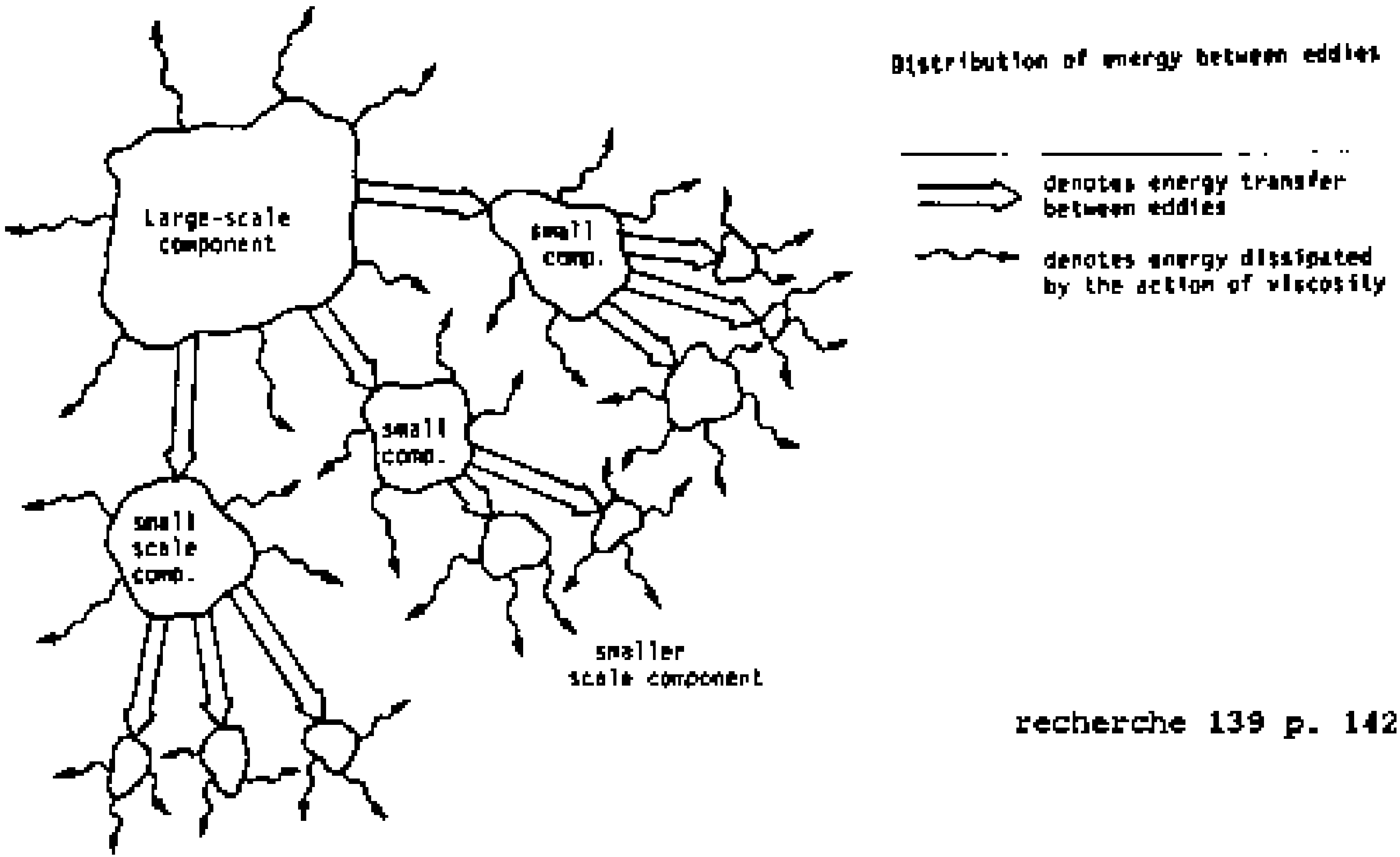

Fig. 18.4. Measurement of fluctuating components in a channel, after Reichardt [41]

The product $\overline{u'v'}$, the shearing stress $\tau/\varrho$ and the correlation coefficient $\psi$

## Intensidade de Turbulência

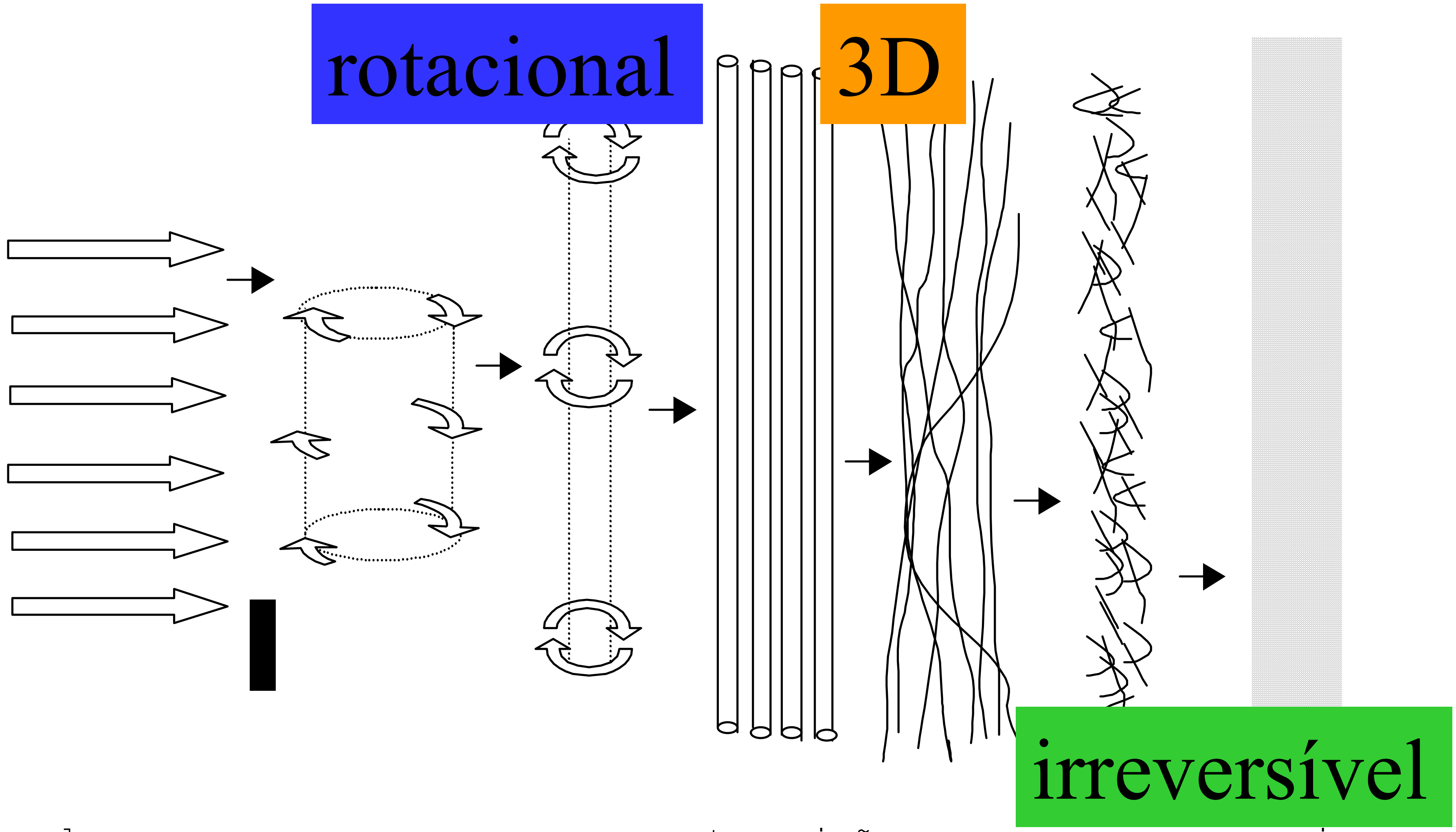

échelle de dissipation
l_d


Taylor scale
log E
L >> ℓ_c >> λ >> ℓ_d ← Kolmogorov scale
Inertial subrange
Dissipating eddies
Energy-containing eddies
K
K_d ≅ 1/L
K_c ≅ 1/ℓ_c
K_d ≅ 1/ℓ_d ≅ Re^3/4/L




recherche 139 p. 1422

# Kolmogorov



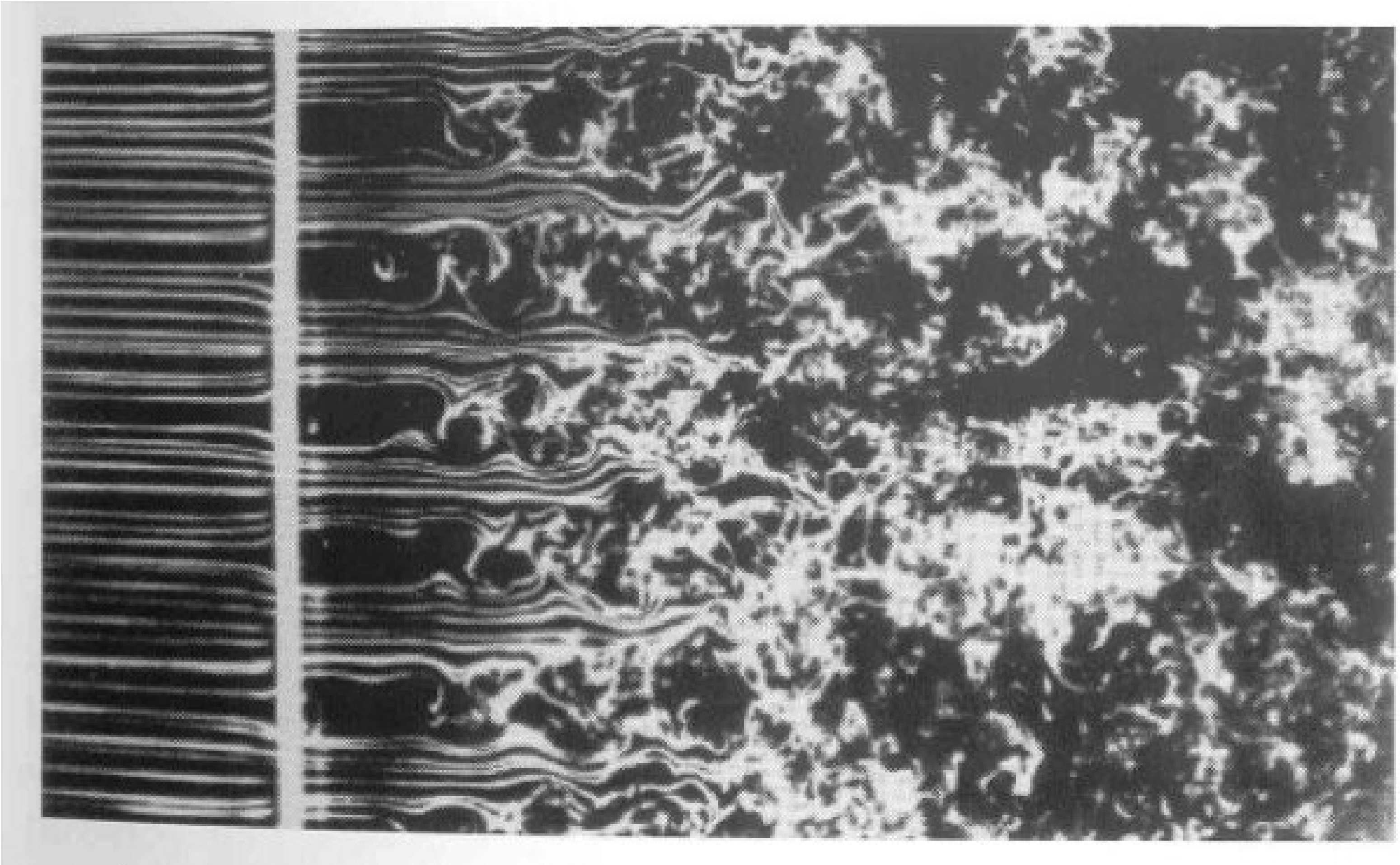

```
escala:        macro                        transição                       micro
energia cinética macro                    turbulenta                        micro
proporcional: v      <-------------- k ---------------->                     T
```

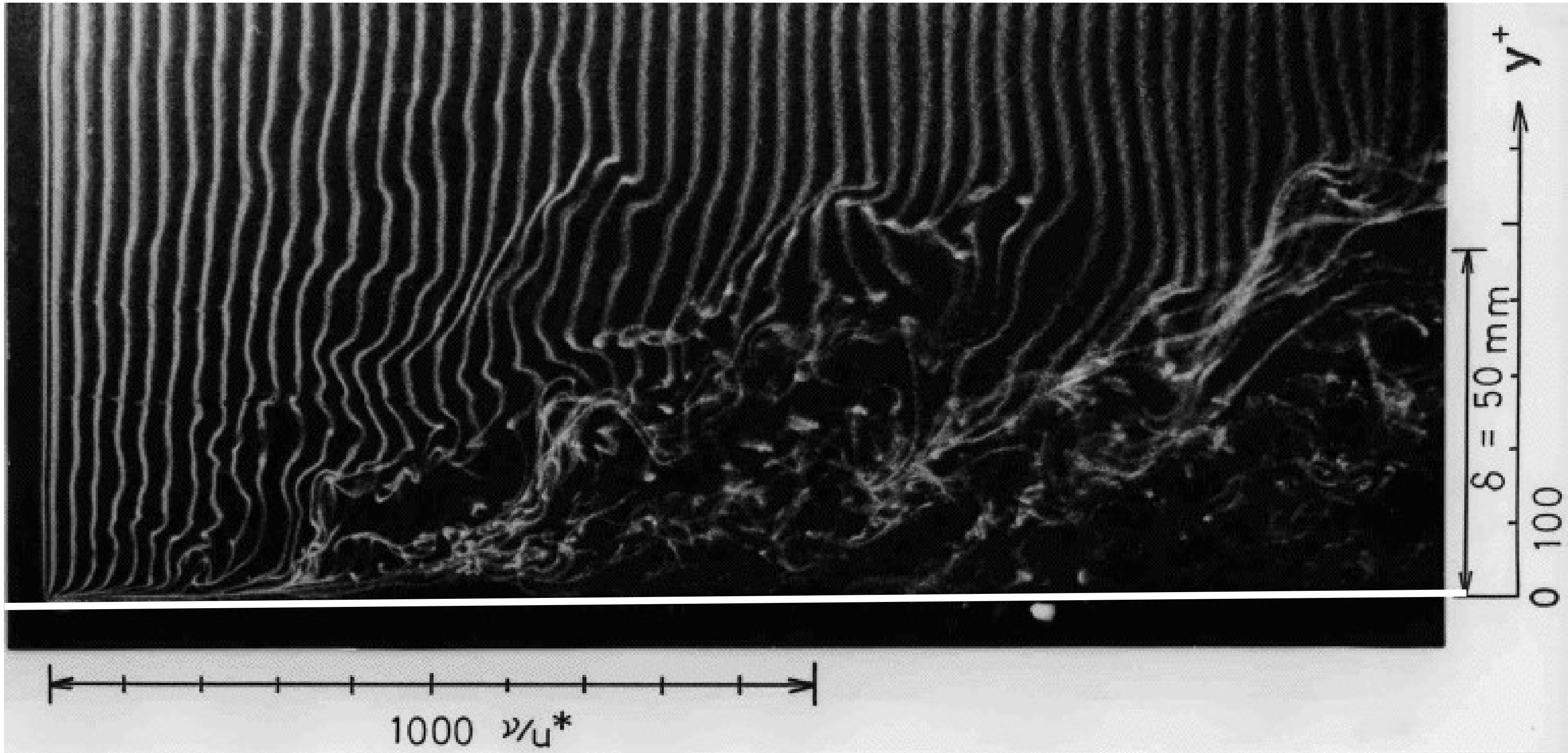
# escalas de turbulências

# Escalas de Kolmogorov – menores escalas de turbulência

- Comprimento $\eta = \left( \frac{\nu^3}{\varepsilon} \right)^{1/4}$ $\frac{\eta}{l_0} = \mathrm{Re}^{-3/4}$
- Velocidade $u_\eta = (\nu\varepsilon)^{1/4}$ $\frac{u_\eta}{u_0} = \mathrm{Re}^{-1/4}$
- Tempo $\tau_\eta = \left( \frac{\nu}{\varepsilon} \right)^{1/2}$ $\frac{\tau_\eta}{\tau_0} = \mathrm{Re}^{-1/2}$
- Reynolds $\mathrm{Re}_\eta = \frac{u_\eta \eta}{\nu} = 1$


transition to turbulence
©2005 Pal & Basu





y⁺
0
100
δ = 50 mm
1000 ν/u*

sendo a difusividade turbulenta >> laminar:

$$\left[\lambda_{\phi} + \lambda_{\phi T}\right] \approx \lambda_{\phi T}$$

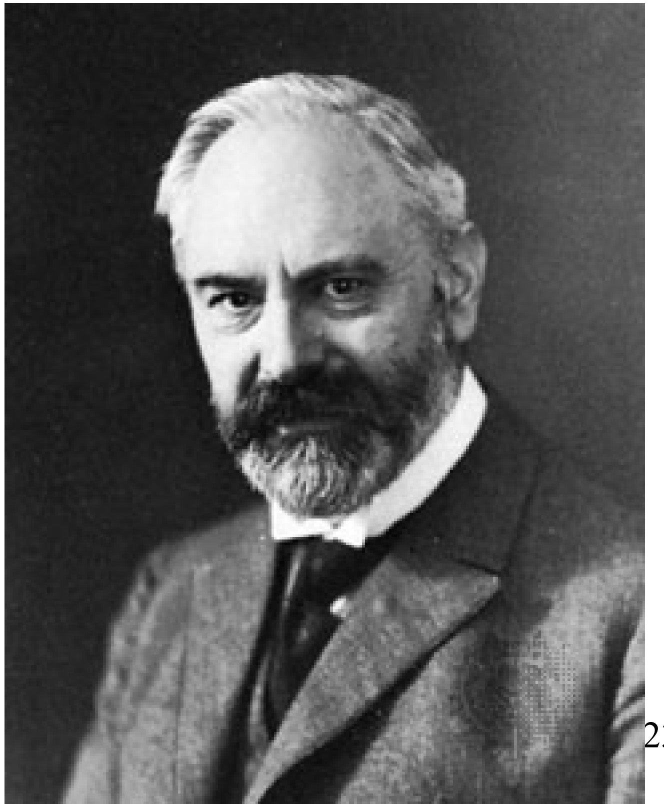
$$\frac{\partial \overline{\varphi}}{\partial t} + \operatorname{div} \overline{\vec{v}}\,\overline{\varphi} = \operatorname{div} \lambda_{\phi T}\, \operatorname{gr\vec{a}d} \overline{\varphi} + \overline{\dot{\sigma}_{M_{\phi}}}$$

voltou à poderosa

agora super-poderosa

$$\lambda_{\vec{v}T} = \ell_m^2 \left| \operatorname{gr\vec{a}d} \overline{\vec{v}} \right|$$

Prandtl mixing lenght



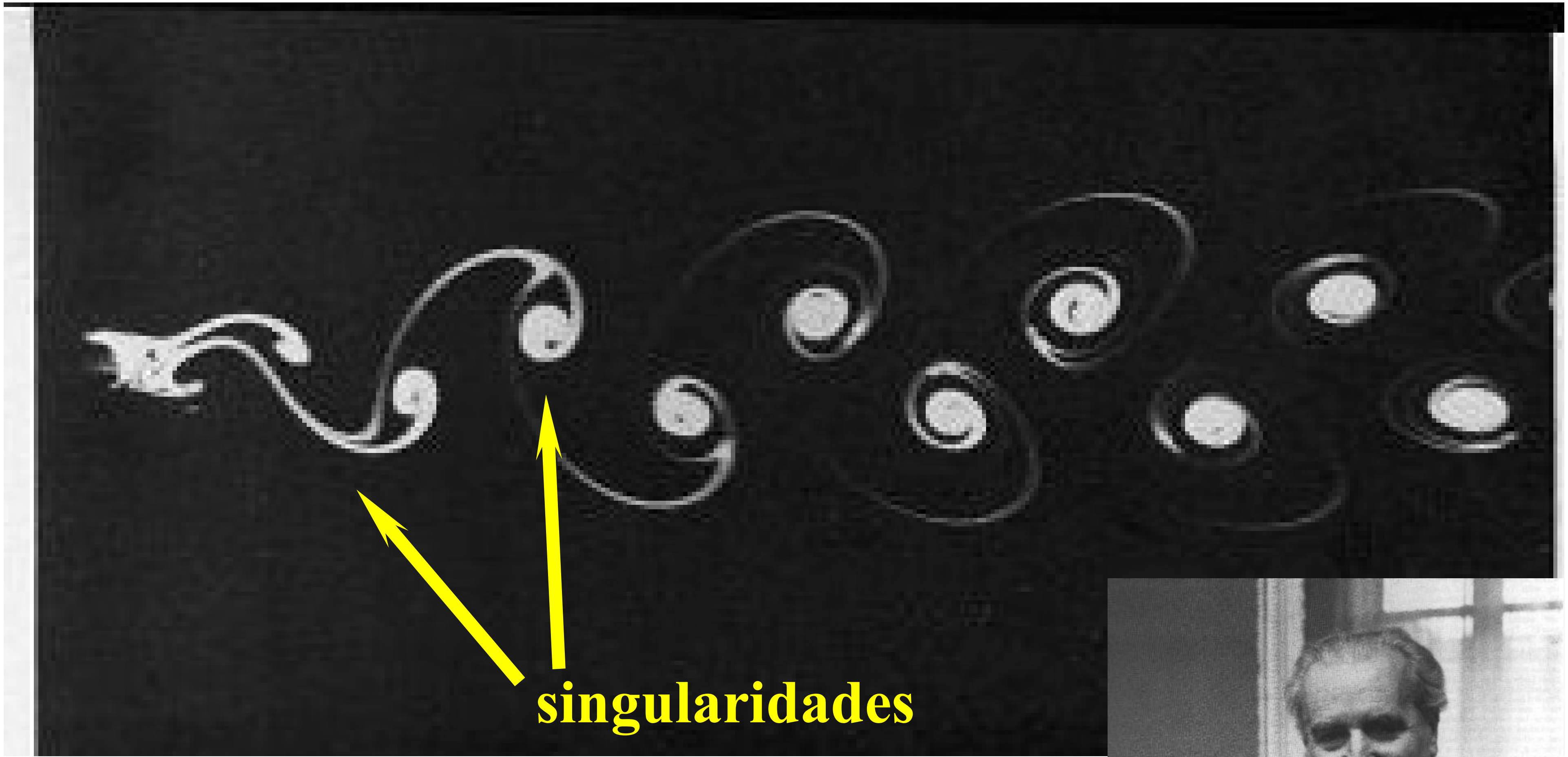


hipótese

sendo $\varphi = v$ :

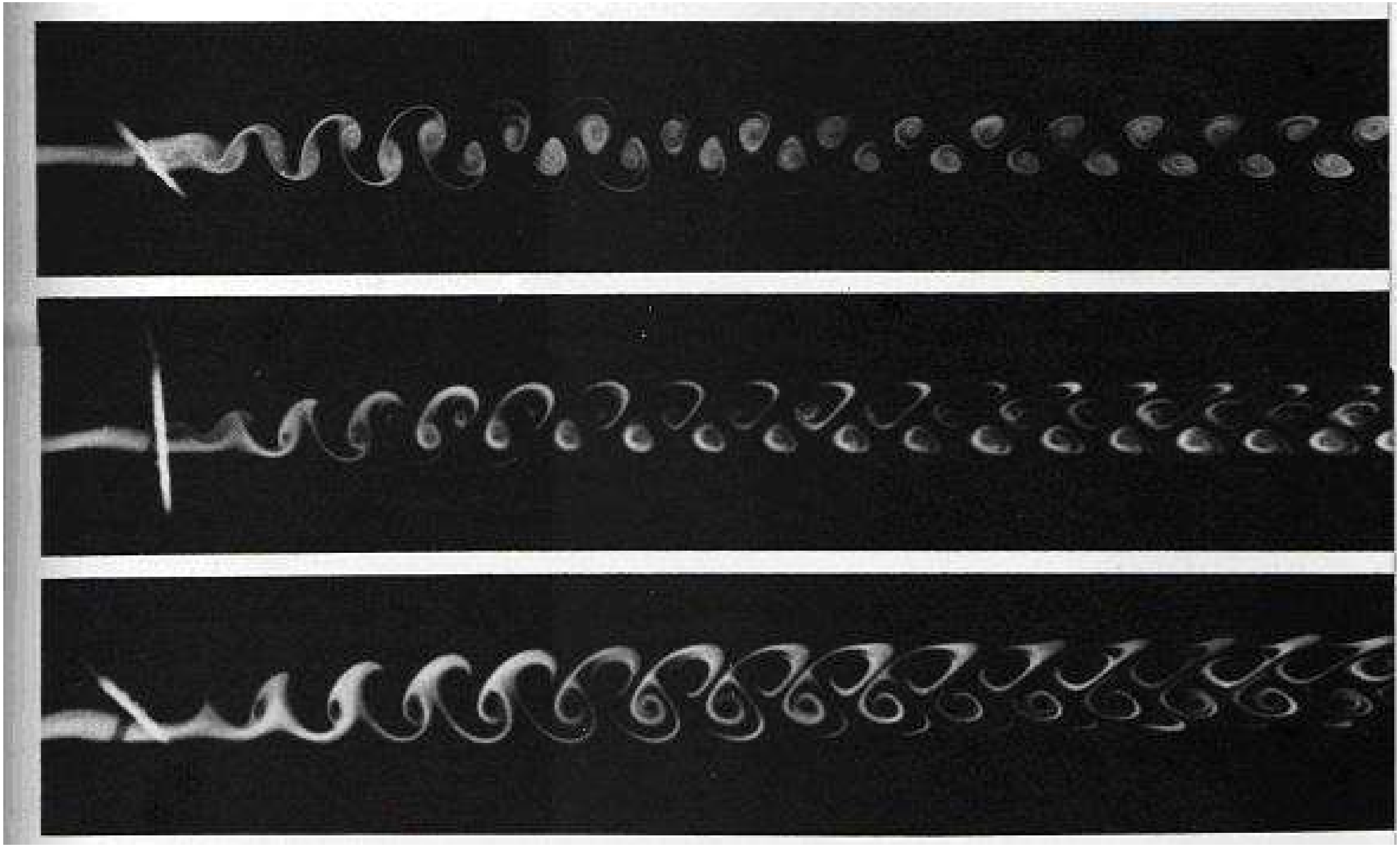
$$\frac{\partial \overline{\vec{v}}}{\partial t} + d\vec{i}v\, \overline{\vec{v}}\ \overline{\vec{v}} = d\vec{i}v\ \nu_T\, gr\vec{a}d\, \overline{\vec{v}} - \frac{gr\vec{a}d\, \overline{p}}{\overline{\rho}} + \overline{\vec{g}}$$

| | |
|---|---|
| Prandtl mixing lenght | $\lambda_{\vec{v}T} = \nu_T = \frac{\mu_T}{\rho} = \ell_m^2 \left| gr\vec{a}d\ \overline{\vec{v}} \right|$ |
| similaridade Prandtl | $\nu_T = c_\mu \ell \sqrt{K}$ |
| k | K → $k = \overline{e_{c_T}} = \frac{\overline{\vec{v}'.\vec{v}'}}{2}$ <br> $\frac{Dk}{Dt} = div\left(\frac{\nu_T}{\sigma_k}\, gr\vec{a}d\ k\right) + \nu_T \left| gr\vec{a}d\ \vec{v} \right|^2 - c_D \frac{k^{3/2}}{\ell}$ |
| k ε | $\nu_T = C_\mu\, K^2/\varepsilon$ |

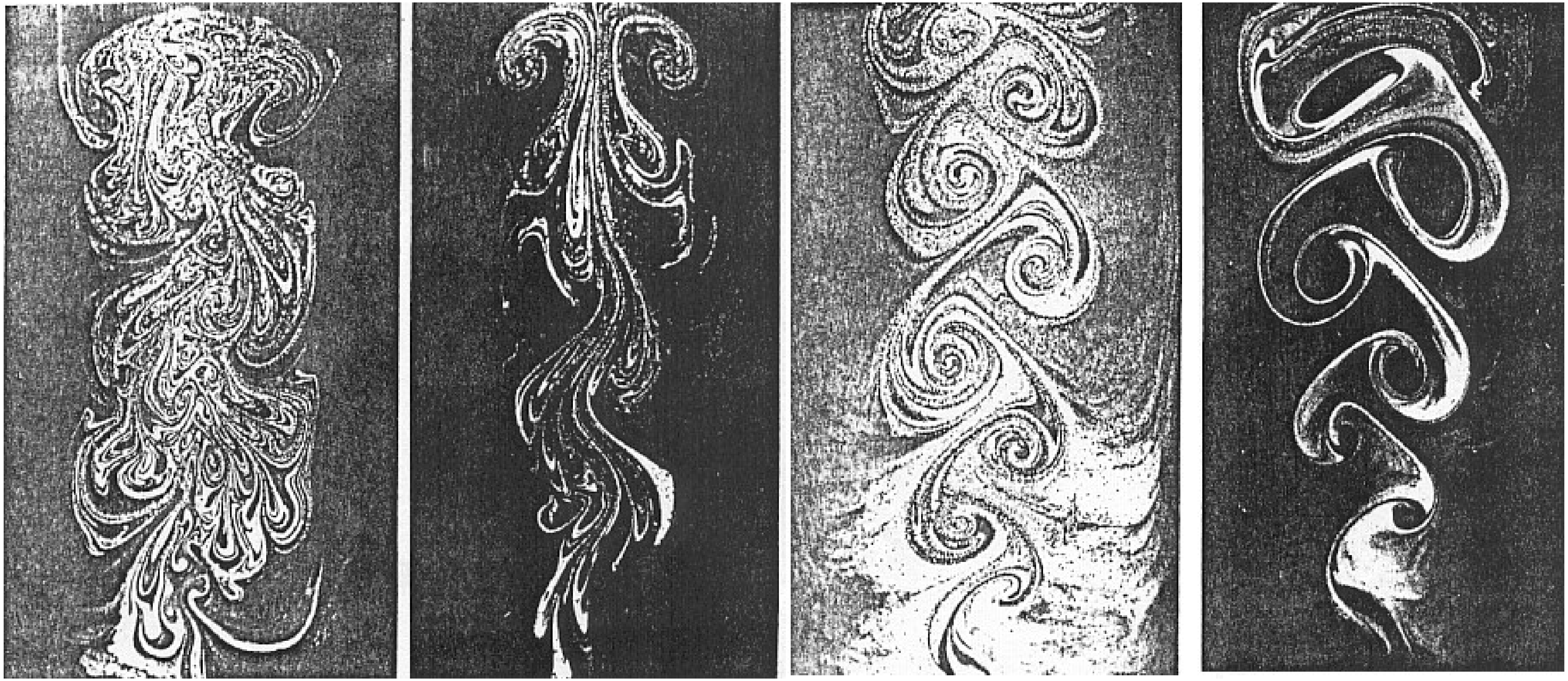


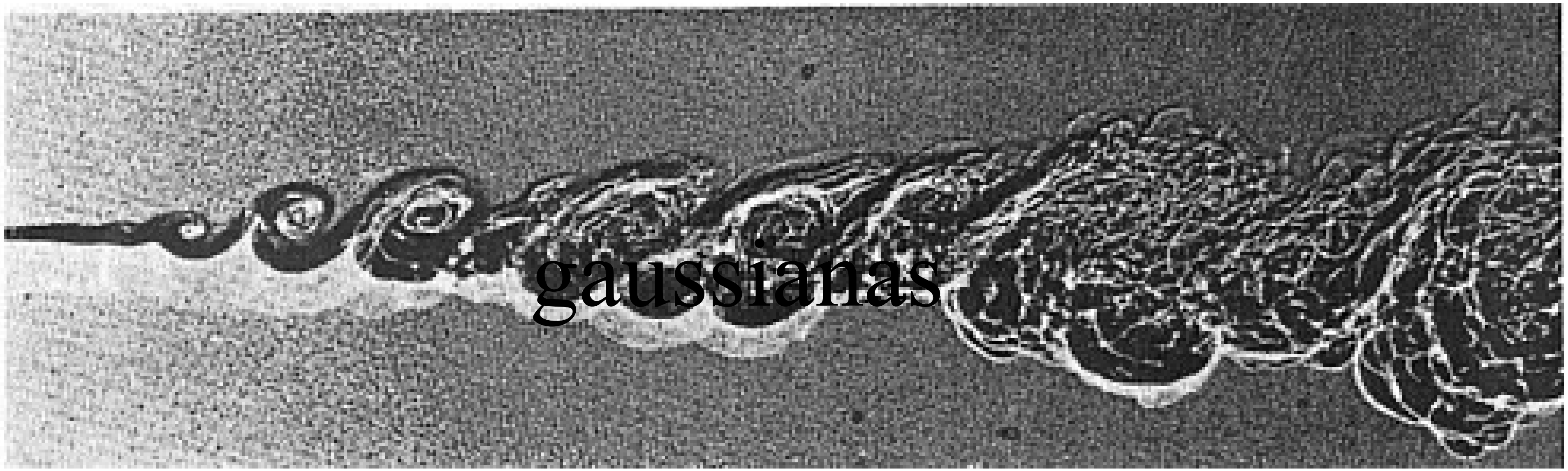

cilindro
Re = 105

Von Karman
vortex

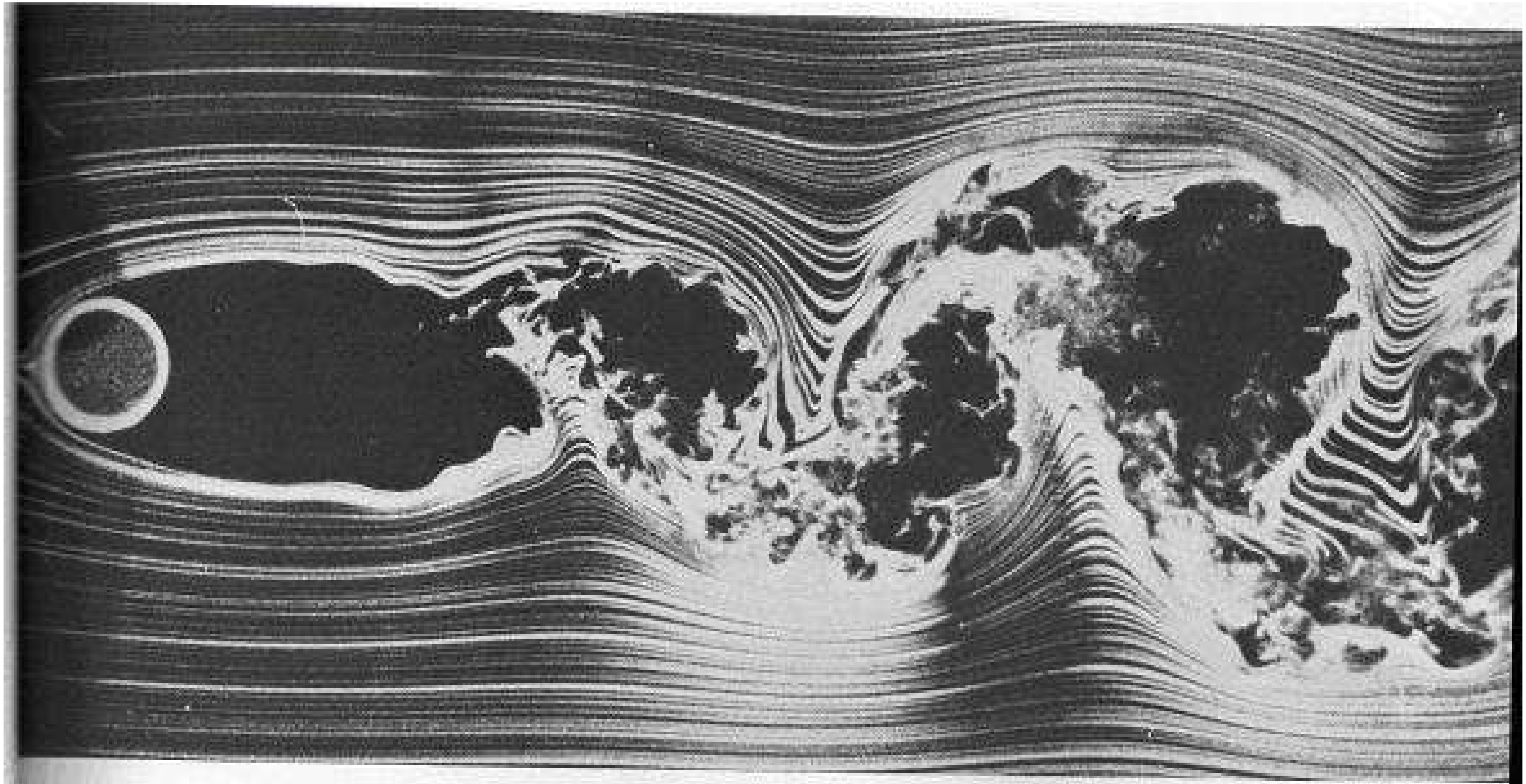


vortex Re = 100



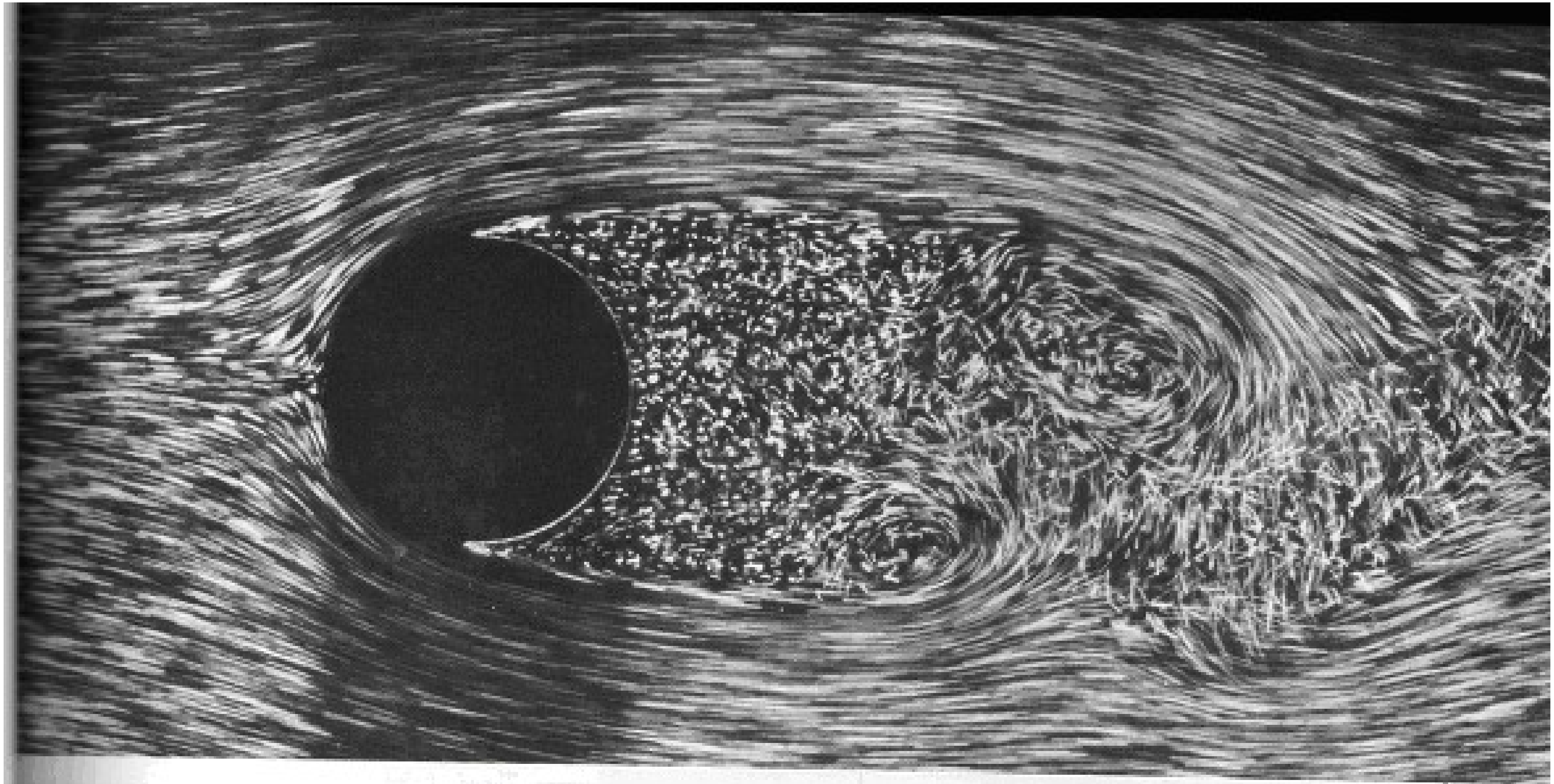

gaussianas

# cilindro $Re = 10.000$



**48. Circular cylinder at $R$=10,000.** At five times the speed of the photograph at the top of the page, the flow pattern is scarcely changed. The drag coefficient consequently remains almost constant in the range of Reynolds number spanned by these two photographs. It drops later when, as in figure 57, the boundary layer becomes turbulent at separation. *Photograph by Thomas Corke and Hassan Nagib*

# cilindro Re = 2000



**47. Circular cylinder at** $R = 2000$. At this Reynolds number one may properly speak of a boundary layer. It is laminar over the front, separates, and breaks up into a turbulent wake. The separation points, moving forward as the Reynolds number is increased, have now attained their upstream limit, ahead of maximum thickness. Visualization is by air bubbles in water. *ONERA photograph, Werlé & Gallon 1972*

$$v_z = \frac{\Delta p R^2}{2\mu L}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$\frac{v_z}{v_o} = 1-\left(\frac{r}{R}\right)^2$$

**$v_{bulk}$ =** $\dot{m}v_b = \rho v_b \pi R^2 = \int_0^R \rho v_z 2\pi r\,dr$

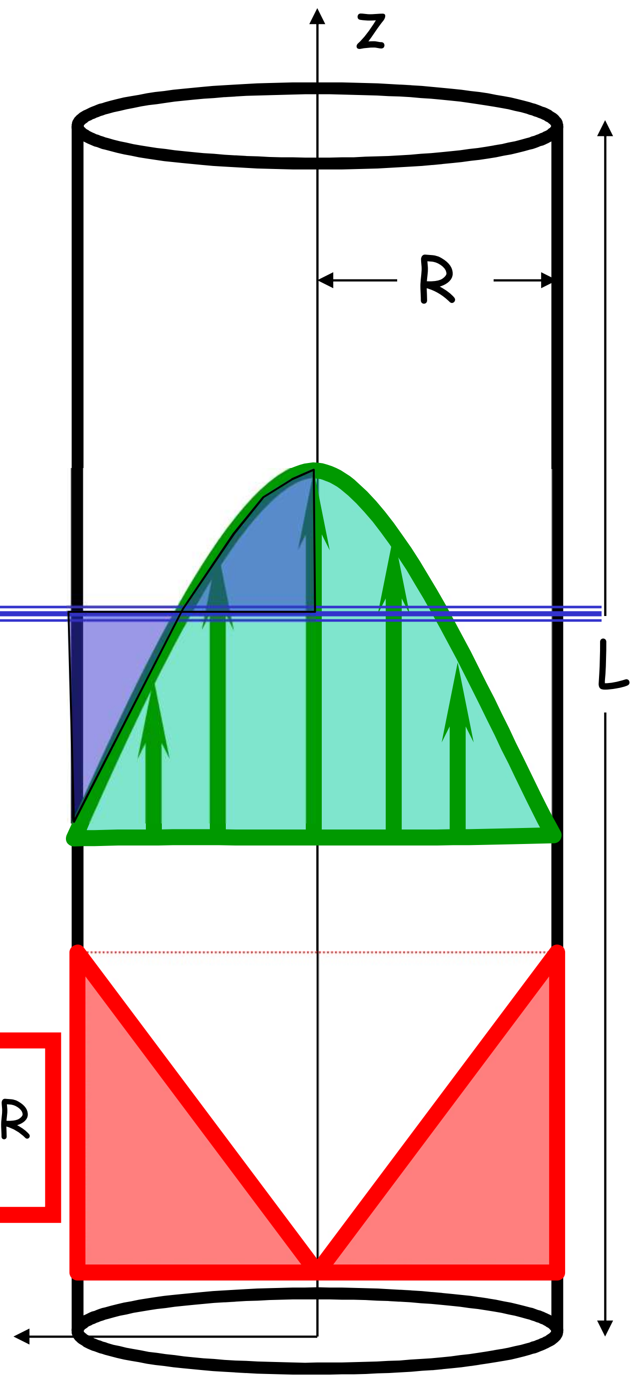
$$v_b = \frac{\rho 2\pi}{\rho \pi R^2}\int_0^R v_z r\,dr = \frac{2}{R^2}\frac{\Delta p R^2}{2\mu L}\int_0^R \left(1-\left(\frac{r}{R}\right)^2\right) r\,dr$$

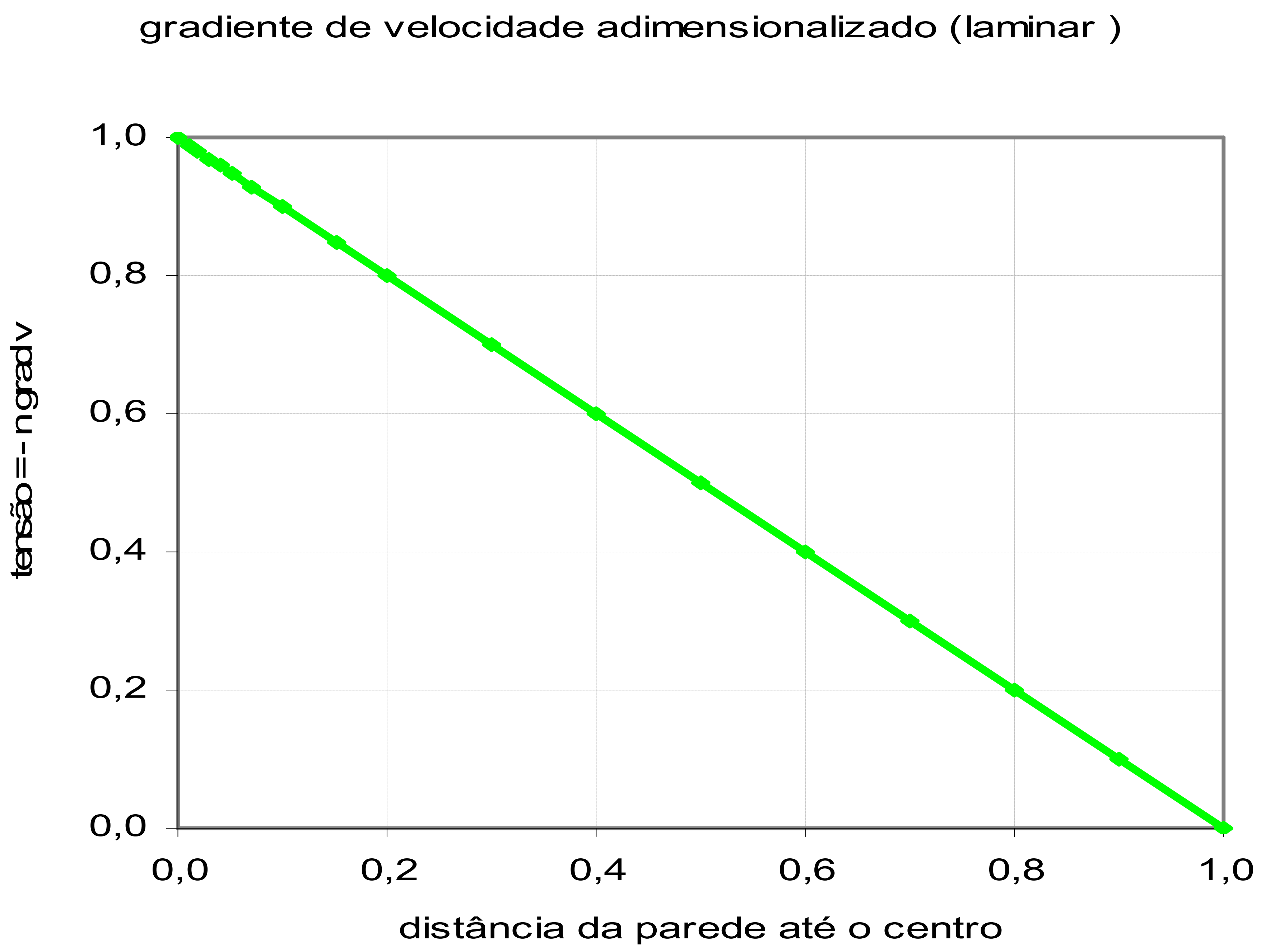
$$v_b = \frac{R^2 \Delta p}{4\mu L}$$

tensão $\zeta_{rz} = -\mu\frac{\partial v_z}{\partial r} = -\mu\frac{\partial}{\partial r}\left[\frac{\Delta p}{2\mu L}\left(R^2 - r^2\right)\right]$

$$\zeta_{rz} = \frac{\Delta p}{L} r$$

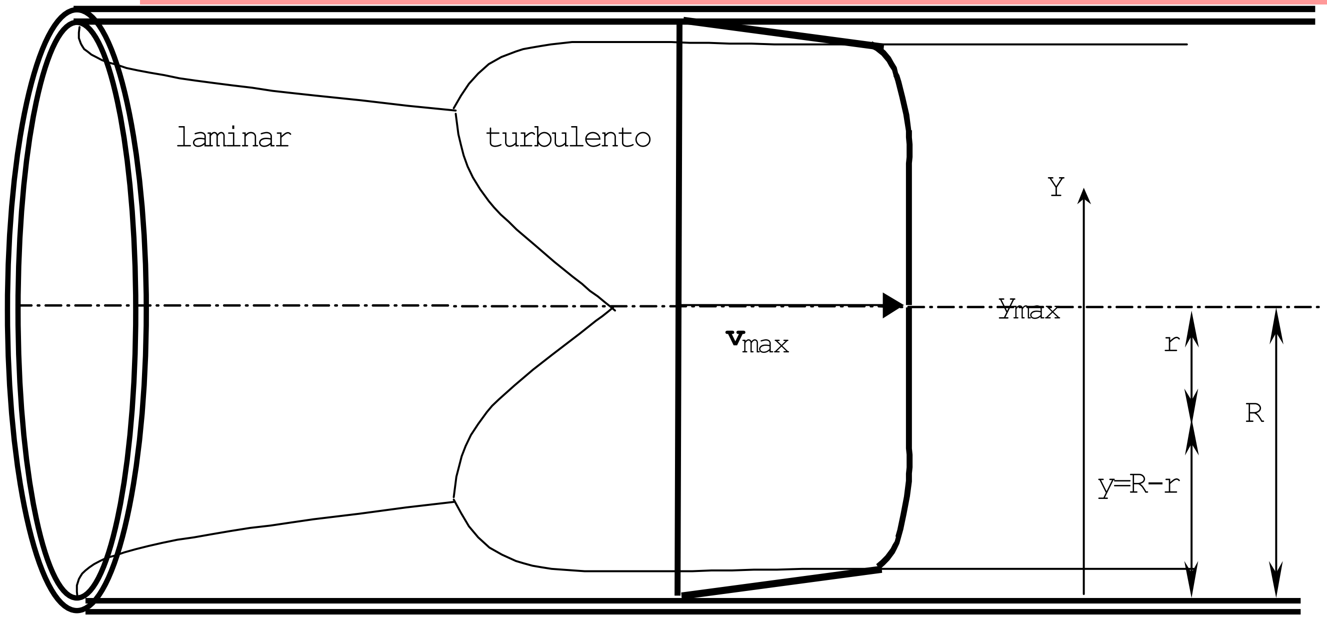
$$\zeta_w = \frac{\Delta p}{L} R$$




gradiente de velocidade adimensionalizado (laminar )
1,0
0,8
0,6
0,4
0,2
0,0
tensão=-ngradv
0,0
0,2
0,4
0,6
0,8
1,0
distância da parede até o centro

# Turbulento

**Escoamento unidimensional em estado estacionário sobre uma superfície (parede plana, tubo escoamento desenvolvido)**




$$\frac{\partial \bar{\varphi}}{\partial t} + \operatorname{div} \bar{\vec{v}}\, \bar{\varphi} = \operatorname{div}\left[\lambda_\phi + \lambda_{\phi T}\right] \operatorname{gr\vec{a}d}\, \bar{\varphi} + \overline{\dot{\sigma}_{M_\phi}}$$

$$\lambda_{\vec{v}T} = \nu_T = \ell_m^2 \left|\operatorname{gr\vec{a}d}\, \bar{\vec{v}}\right| >> \lambda_{\vec{v}} = \nu$$

**lm = comprimento de mistura de Prandtl**

$$\operatorname{div} \bar{\vec{v}}\, \bar{\vec{v}} = \operatorname{div}\left\{\ell_m^2 \left|\operatorname{gr\vec{a}d}\, \bar{\vec{v}}\right| \operatorname{gr\vec{a}d}\, \bar{\vec{v}}\right\}$$

$$\mathrm{div}\,\overline{\vec{v}}\,\overline{\vec{v}} = \mathrm{div}\left\{\ell_m^2 \left|\mathrm{gr}\vec{a}\mathrm{d}\,\overline{\vec{v}}\right| \mathrm{gr}\vec{a}\mathrm{d}\,\overline{\vec{v}}\right\}$$

Próximo à parede (w)

$$\ell_m = \alpha\, y$$

$$\left\{\mathrm{d}\vec{\mathrm{i}}\mathrm{v}\,\overline{\vec{v}}\,\overline{\vec{v}}\right\}_w = \mathrm{d}\vec{\mathrm{i}}\mathrm{v}\left\{\alpha^2 y^2 \left(\frac{\partial \overline{v_z}}{\partial y}\right)^2\right\} = \frac{1}{\rho}\,\mathrm{d}\vec{\mathrm{i}}\mathrm{v}\,\vec{\vec{\tau}}_w$$

$$\alpha^2 y^2 \left(\frac{\partial \overline{v_z}}{\partial y}\right)^2 = \frac{\tau_w}{\rho} \quad \rightarrow \quad y\,\frac{\partial \overline{v_z}}{\partial y} = \frac{1}{\alpha}\sqrt{\frac{\tau_w}{\rho}}$$

$$d\,\overline{v}_z = \frac{1}{\alpha}\sqrt{\frac{\tau_w}{\rho}}\,\frac{dy}{y} \qquad \overline{v}_z = \frac{\sqrt{\tau_w/\rho}}{\alpha}\ln y + cte'$$

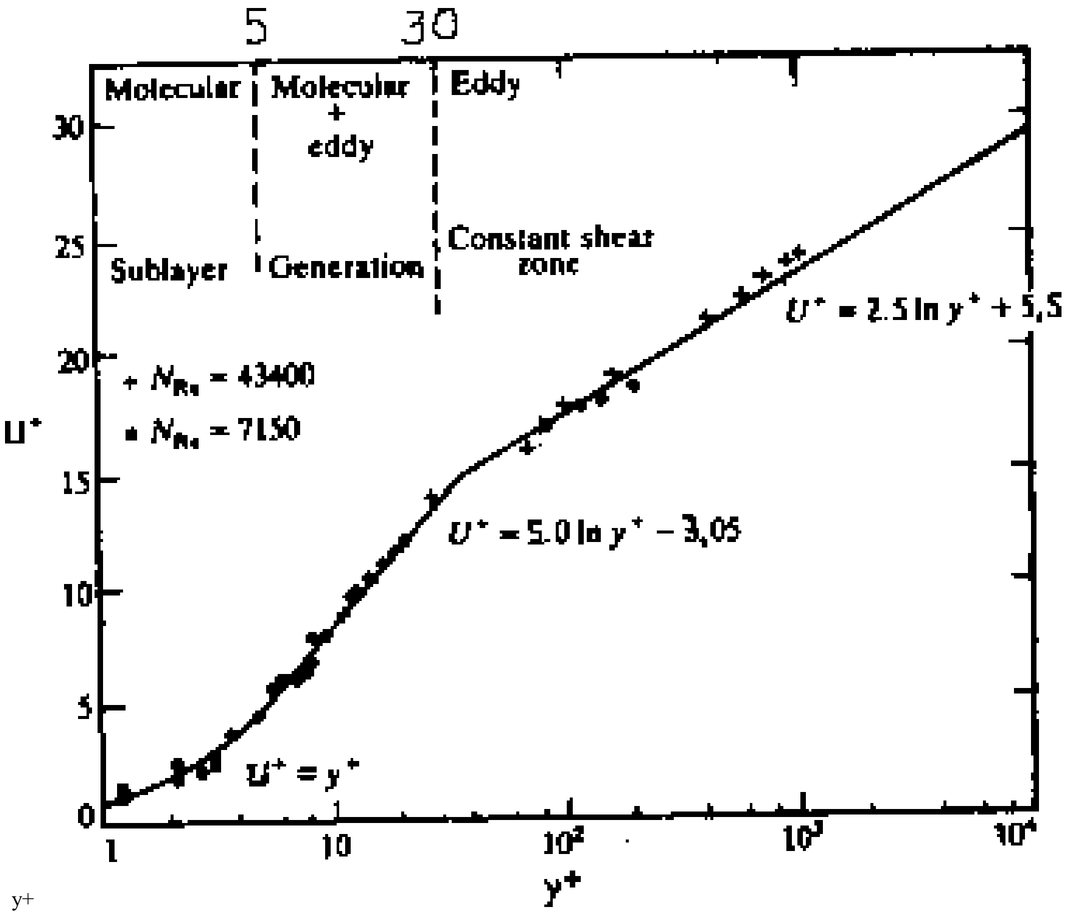
$$\overline{v}_z^+ = \frac{\overline{v}_z}{\sqrt{\tau_w/\rho}} \qquad y^+ = \frac{y}{\nu}\sqrt{\frac{\tau_w}{\rho}} = \frac{y}{\mu}\sqrt{\rho\,\tau_w} \qquad u_t = \sqrt{\tau_w/\rho}$$

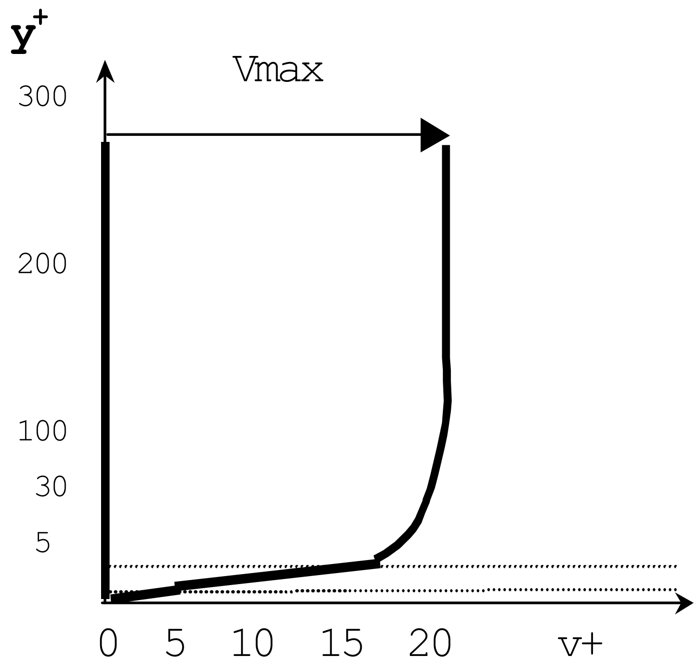
"drift velocity"

$$\overline{v}_z^+ = \frac{1}{\alpha}\ln y^+ + cte$$

v+

| Região | subcamada | transição | "Core" turbulento |
|---|---|---|---|
| Mecanismo | molecular | molecular + eddy | eddy |
| propriedade | $\lambda_\Phi$ | $\lambda_\Phi + \lambda_{\Phi T}$ | $\lambda_{\Phi T}$ |
| $V^+$ | $y^+$ | $5 \ln y^+ - 3{,}05$ | $2{,}5 \ln y^+ + 5{,}5$ |
| $y^+$ min | 0 | 5 | 30 |
| $y^+$ max | 5 | 30 | $\infty$ |




y+

# Perfil Universal de Velocidades

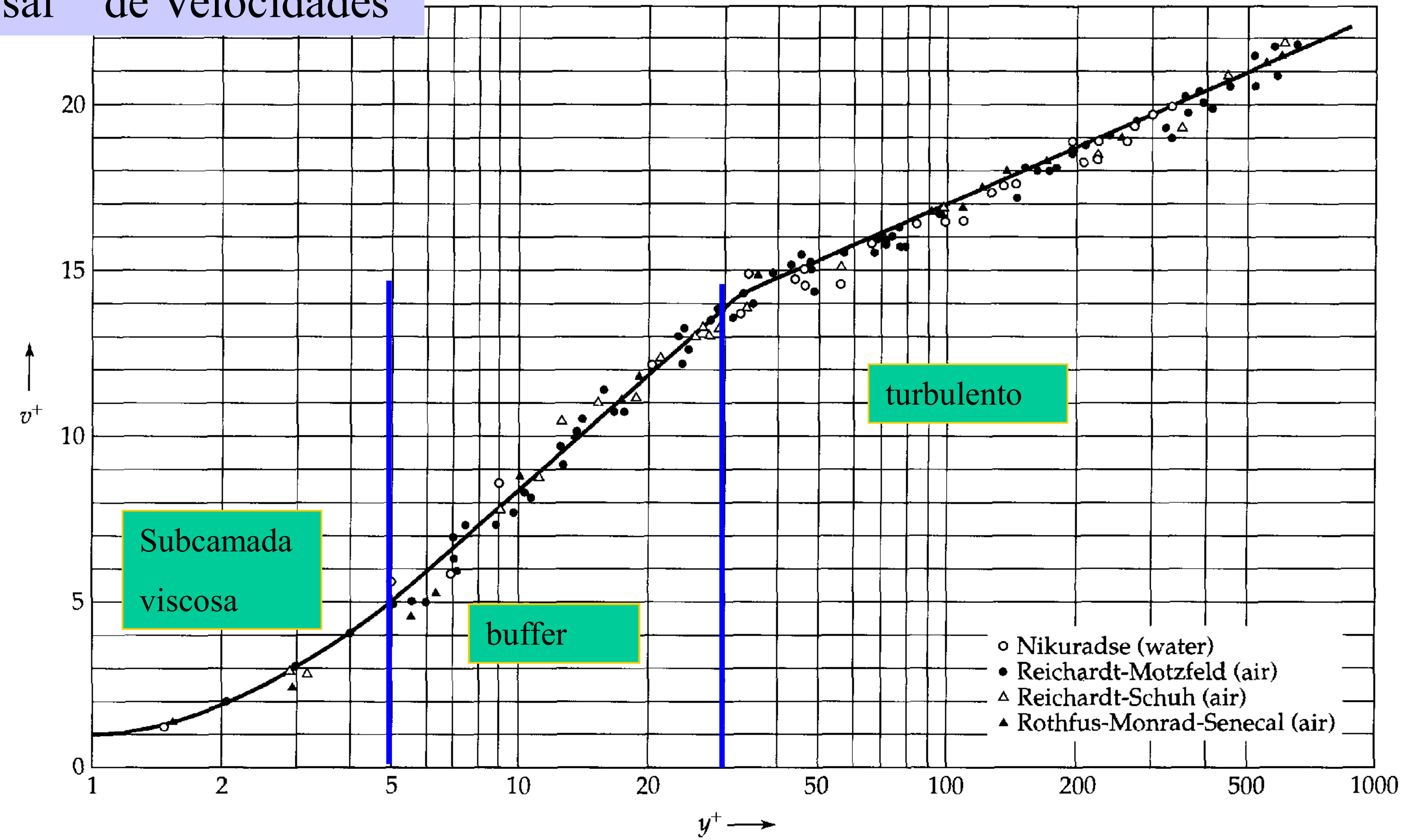



**Fig. 5.5-3.** Dimensionless velocity distribution for turbulent flow in circular tubes, presented as $v^+ = \bar{v}_z/v_*$ vs. $y^+ = yv_*\rho/\mu$, where $v_* = \sqrt{\tau_0/\rho}$ and $\tau_0$ is the wall shear stress. The solid curves are those suggested by Lin, Moulton, and Putnam [*Ind. Eng. Chem.*, **45**, 636–640 (1953)]:

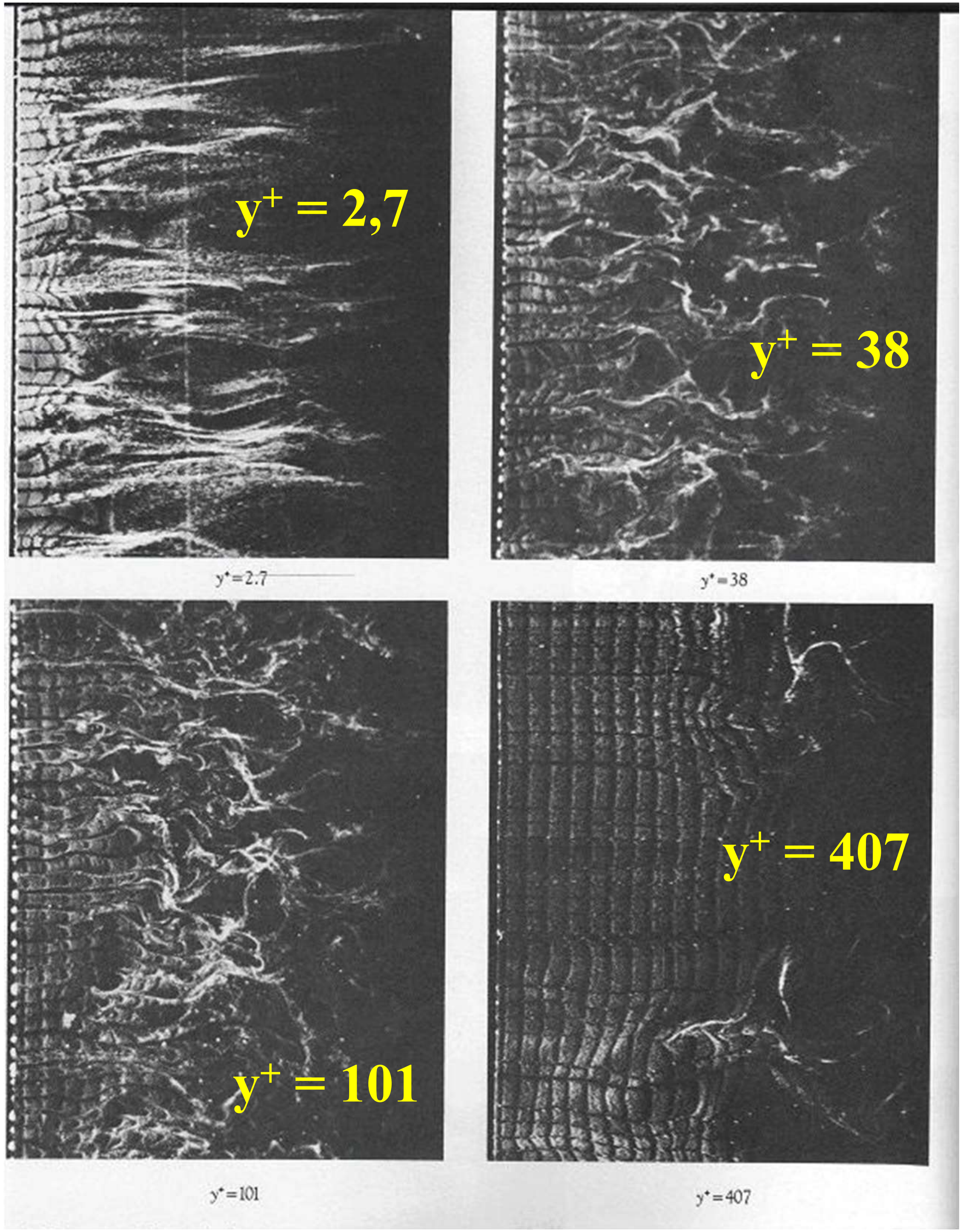
$$0 < y^+ < 5: \quad v^+ = y^+[1 - \tfrac{1}{4}(y^+/14.5)^3]$$

$$5 < y^+ < 30: \quad v^+ = 5 \ln(y^+ + 0.205) - 3.27$$

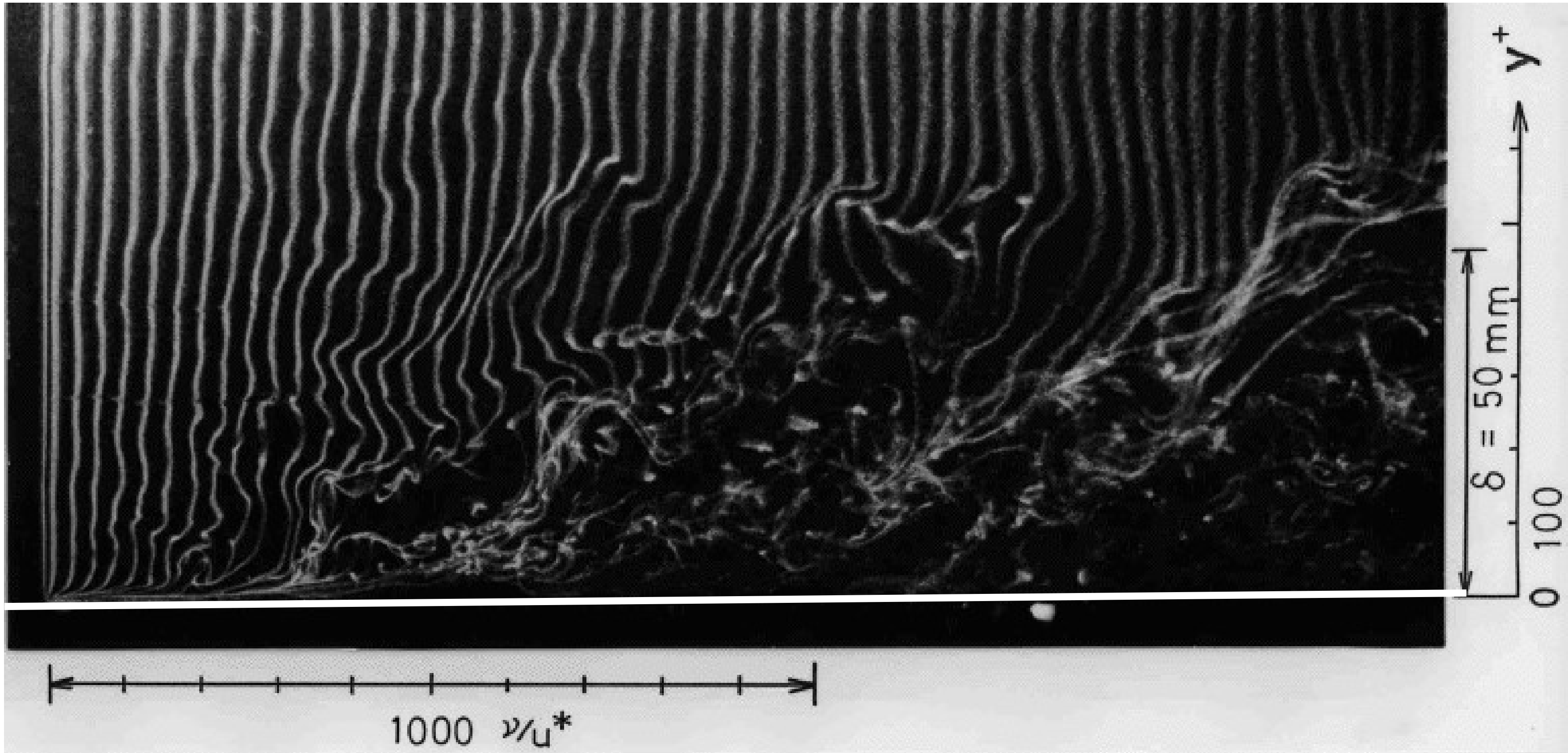
$$30 < y^+: \quad v^+ = 2.5 \ln y^+ + 5.5$$

The experimental data are those of J. Nikuradse for water (○) [*VDI Forschungsheft*, **H356** (1932)]; Reichardt and Motzfeld for air (●); Reichardt and Schuh (△) for air [H. Reichardt, NACA Tech. Mem. 1047 (1943)]; and R. R. Rothfus, C. C. Monrad, and V. E. Senecal for air (■) [*Ind. Eng. Chem.*, **42**, 2511–2520 (1950)].

**161. Structure of a turbulent boundary layer.** Successive layers of the flow near a flat plate in a water channel are shown by tiny hydrogen bubbles released periodically from a thin platinum wire seen at the left. The height $y^{+} = y\,u_{\tau}/\nu$ of the wire above the plate is shown in wall variables, where $u_{\tau} = (\tau_w/\varrho)^{1/2}$ is the friction velocity. The characteristic low- and high-speed streaks shown in the viscous sublayer at $y^{+} = 2.7$ become less noticeable farther away, and have disappeared in the logarithmic region at $y^{+} = 101$. In the wake region at $y^{+} = 407$ the turbulence is seen to be intermittent and of larger scale. *Kline, Reynolds, Schraub & Runstadler 1967*



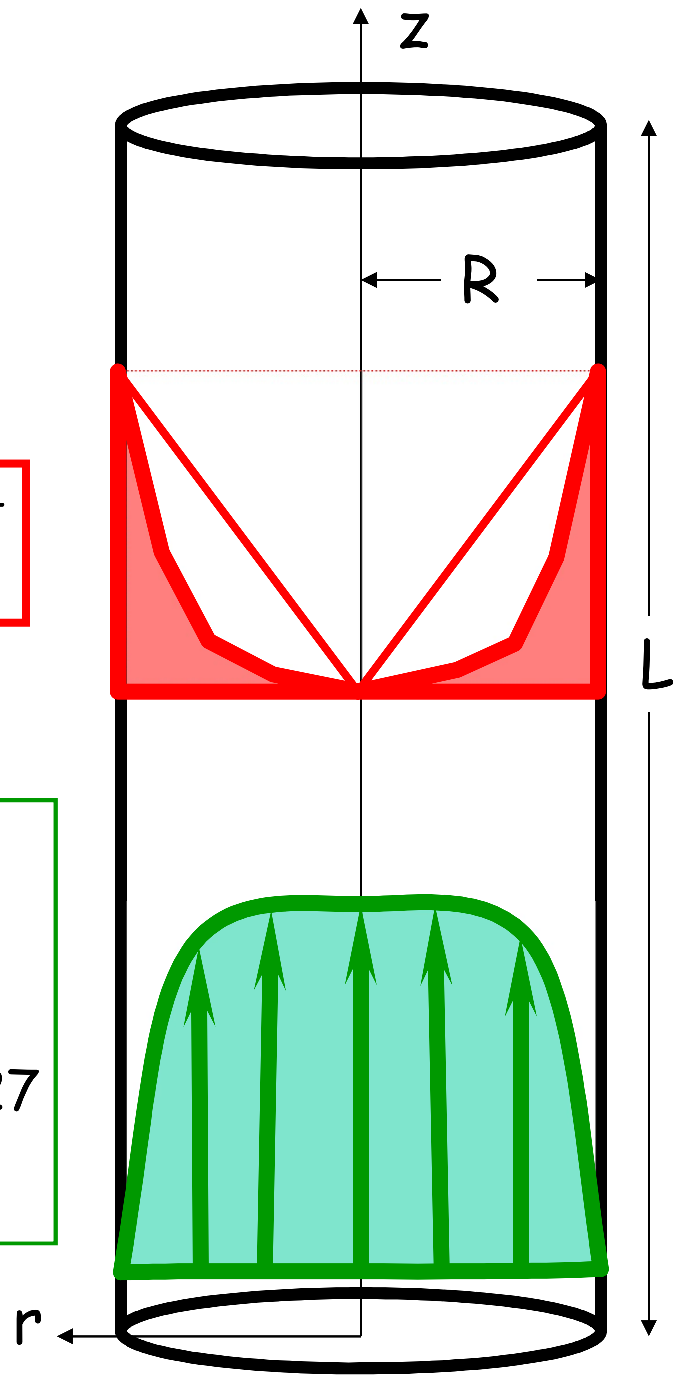

# y+


y+
δ = 50 mm
0 100
1000 ν/u*

# Turbulento

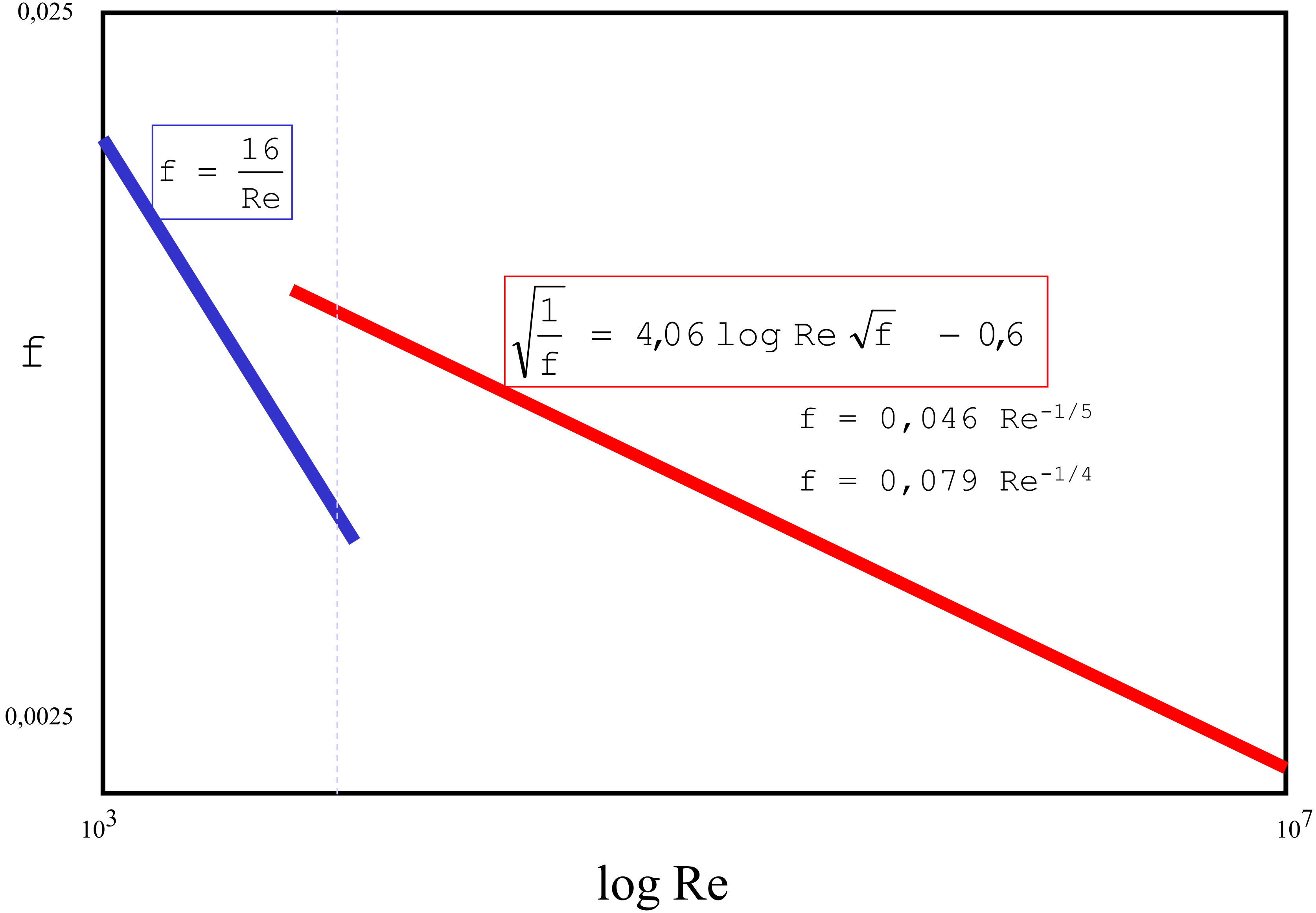
$$\vec{\vec{\zeta}} = \nu \, \mathrm{gr\vec{a}d} \, \overline{\vec{v}} - \overline{\vec{v}'\vec{v}'} = \nu_T \, \mathrm{gr\vec{a}d} \, \overline{\vec{v}}$$

| | |
|---|---|
| $0 > y^+ > 5$ | $\overline{v}^+ = y^+ \left[ 1 - \frac{1}{4} \left( \frac{y^+}{14{,}5} \right)^3 \right]$ |
| $5 > y^+ > 30$ | $\overline{v}^+ = 5 \ln\left(y^+ + 0{,}205\right) - 3{,}27$ |
| $y^+ > 30$ | $\overline{v}^+ = 2{,}5 \ln y^+ + 5{,}5$ |

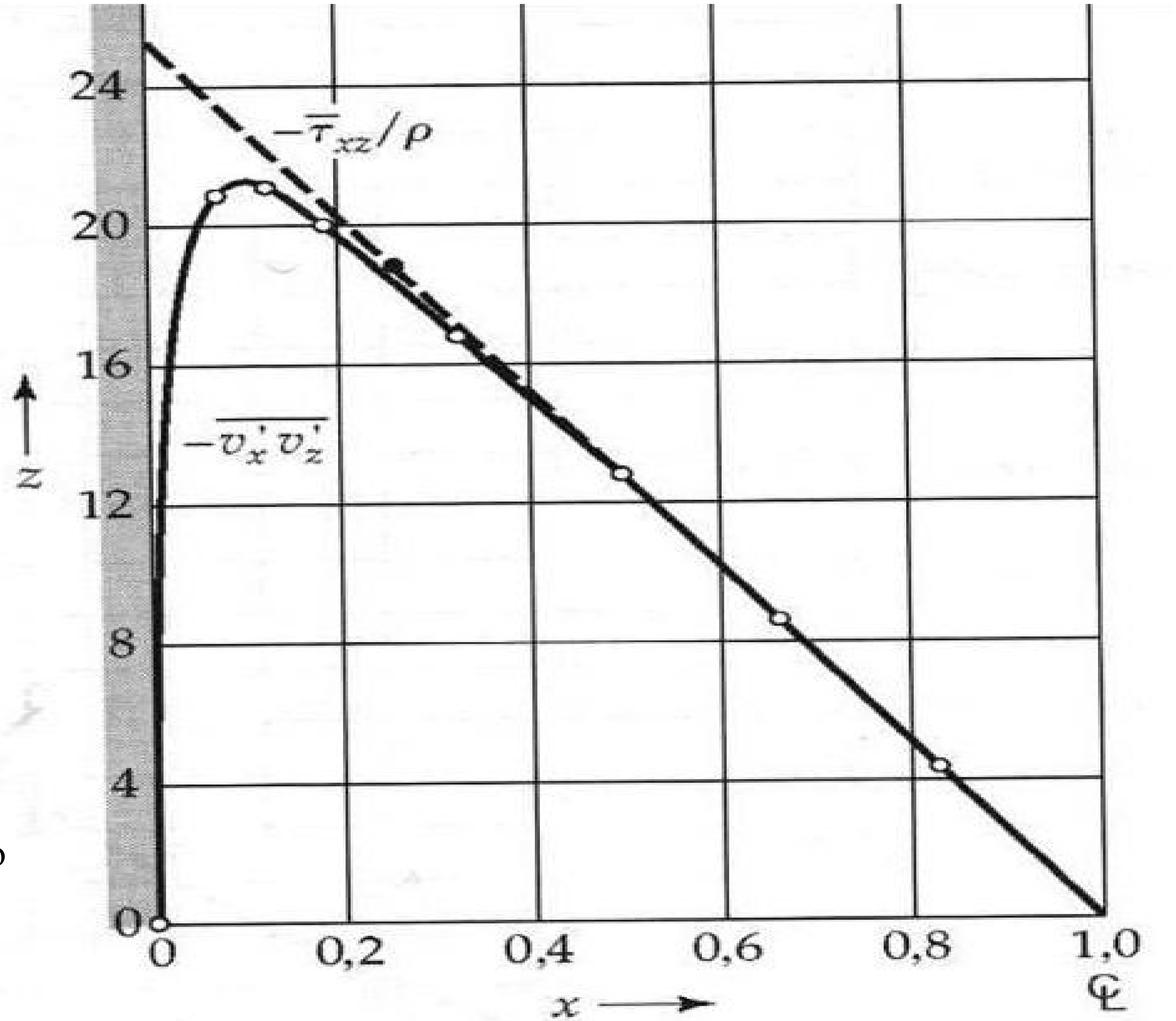


# fator de atrito

$$\tau_w = \frac{f}{2} \rho \, v_b^2$$



Tensão




Escoamento
turbulento

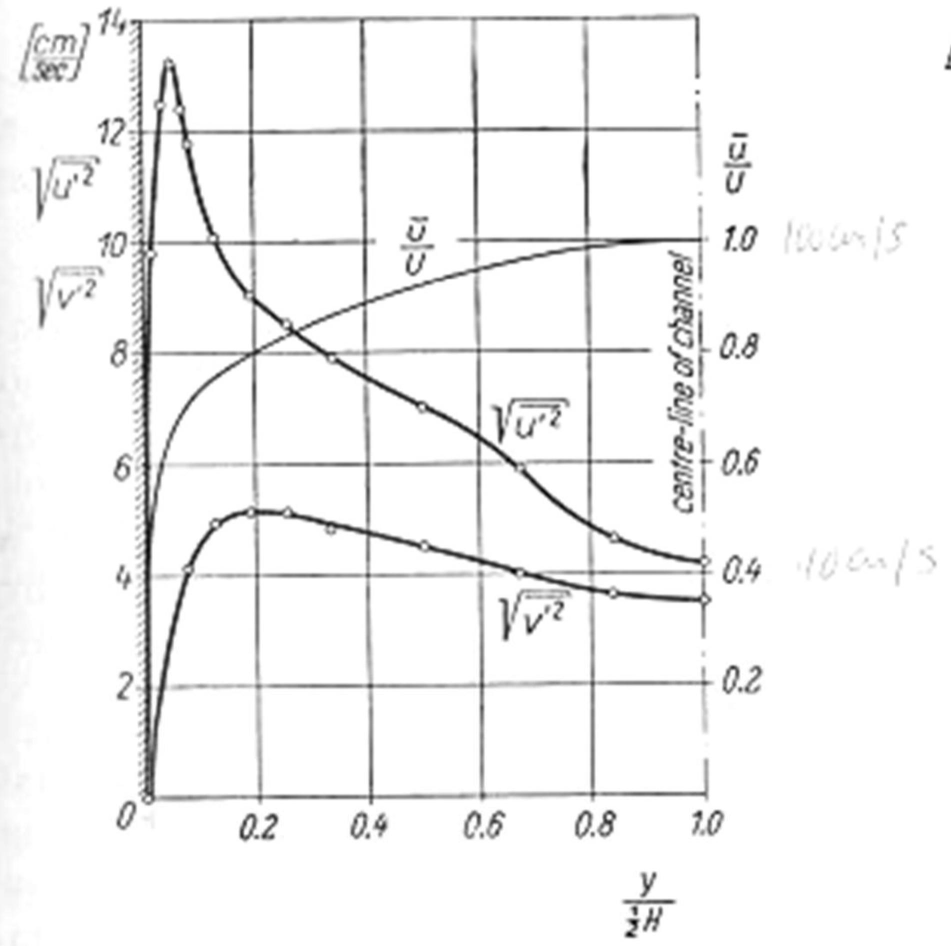



Fig. 18.3. Measurement of fluctuating turbulent components in a wind tunnel, at maximum velocity $U = 100$ cm/sec after Reichardt [41]

Root-mean-square of longitudinal fluctuation $\sqrt{\overline{u'^2}}$, transverse fluctuation $\sqrt{\overline{v'^2}}$, mean velocity $\bar{u}$

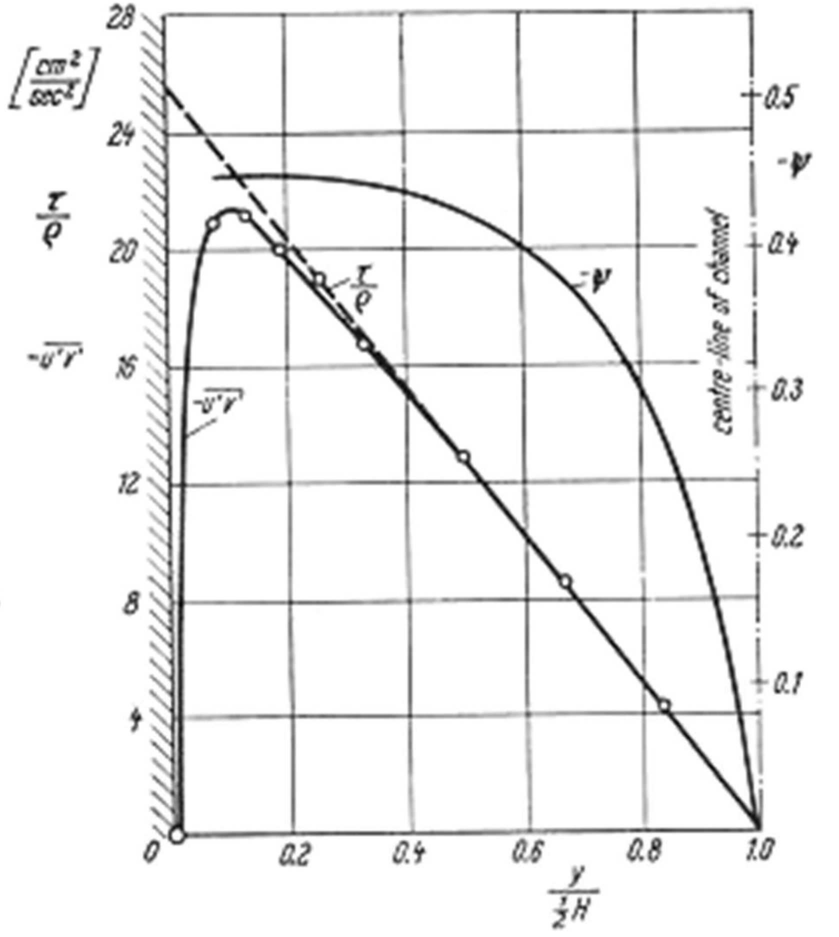



Fig. 18.4. Measurement of fluctuating components in a channel, after Reichardt [41]

The product $\overline{u'v'}$, the shearing stress $\tau/\varrho$ and the correlation coefficient $\psi$

Intensidade de Turbulência

Comprimento

Comprimento de Mistura - Prandtl

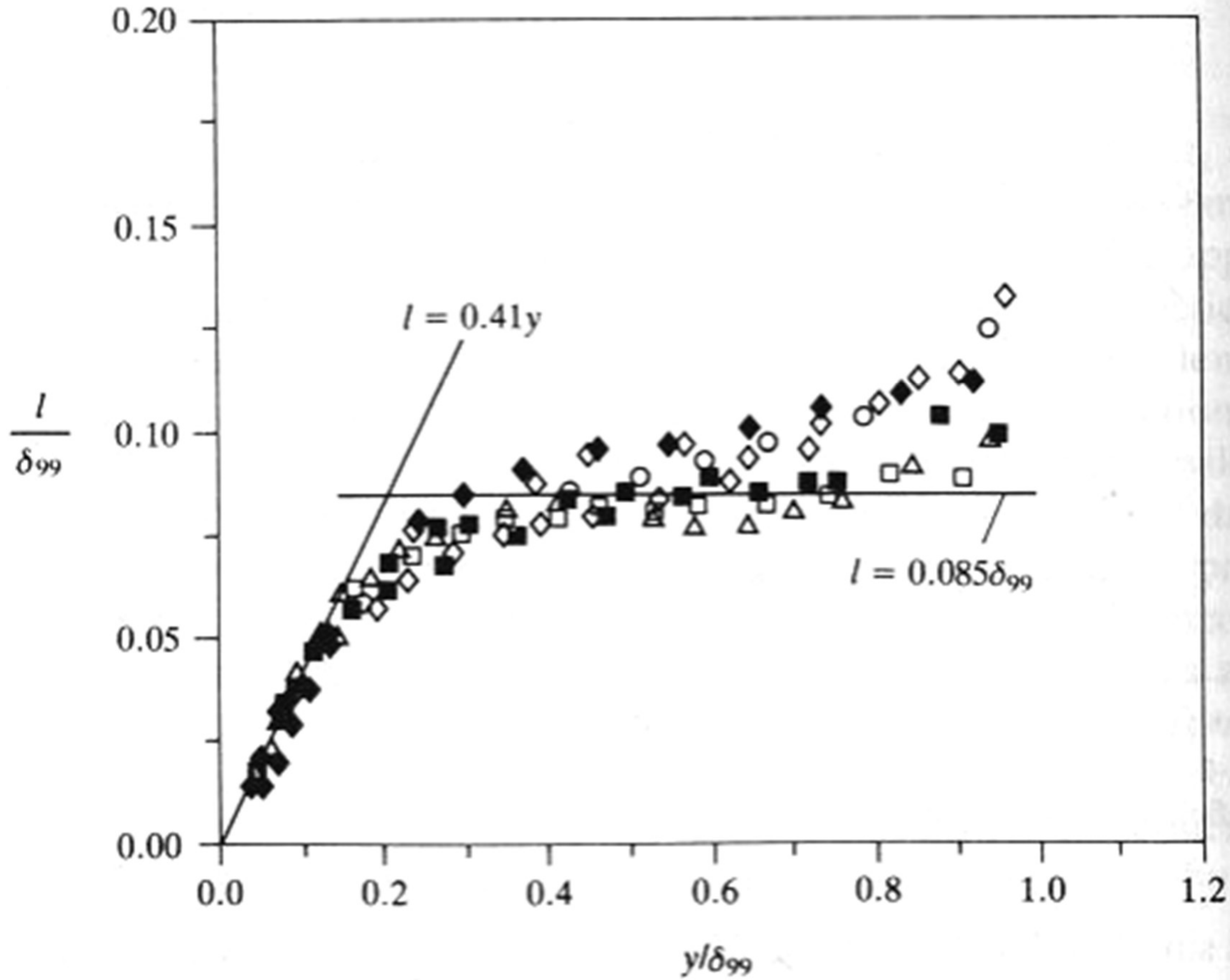



**FIGURE 11-2**

Mixing-length measurements of Andersen[1] for no pressure gradient, advers gradient, blowing, and suction.

# Comprimento de Mistura - Prandtl

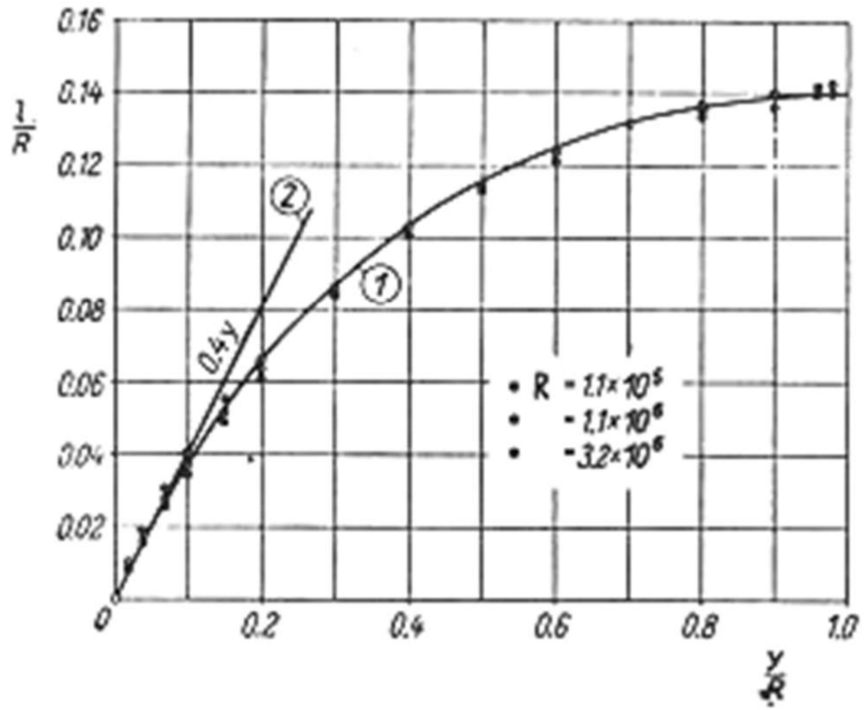



Fig. 20.5. Variation of mixing length over pipe diameter for smooth pipes at different Reynolds numbers

Curve (1) from eqn. (20.18)

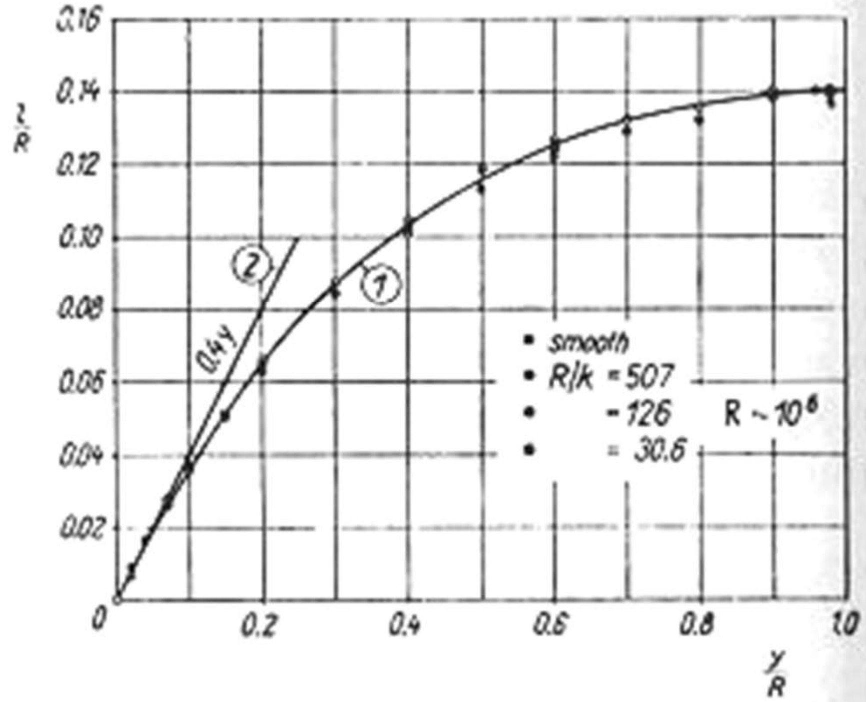
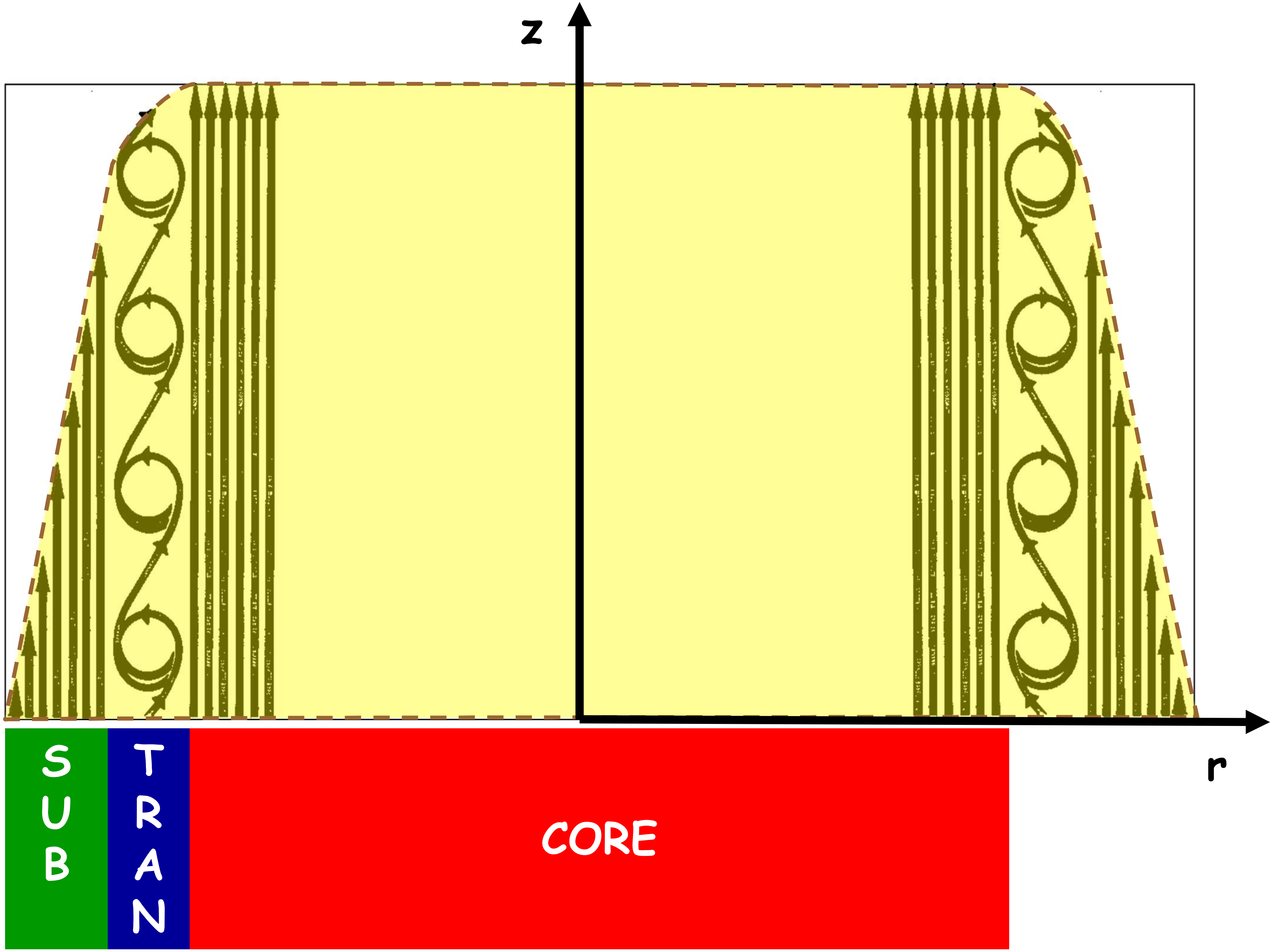



Fig. 20.6. Variation of mixing length over pipe diameter for rough pipes

Curve (1) from eqn. (20.18)


z
r
SUB
TRAN
CORE

# Modelos de Turbulência

**Reynolds Stress** $\vec{\vec{R}} = \overline{\vec{v}'.\vec{v}'}$

| | |
|---|---|
| sete equações diferenciais | $\frac{D\vec{\vec{R}}}{Dt} = -\,\vec{div}\left[\vec{\vec{\vec{j}}}_{\vec{\vec{R}}} + \vec{\vec{\vec{\pi}}}_{\vec{\vec{R}}} + \vec{\vec{\vec{\Omega}}}_{\vec{\vec{R}}}\right] + \overline{\vec{\vec{\sigma}}_{M_{\vec{\vec{R}}}}} - \overline{\vec{\vec{\varepsilon}}_{M_{\vec{\vec{R}}}}}$ |
| algébrico | $\vec{\vec{R}} = \frac{2}{3} K\,\vec{\vec{\delta}} + \left[\frac{C_D}{C_1 - 1 + p/\varepsilon}\right]\left(\overline{\vec{\vec{\sigma}}_{M_{\vec{\vec{R}}}}} - \frac{2}{3} p\,\vec{\vec{\delta}}\right)\frac{K}{\varepsilon}$ |
| 2 eq. K ε | $\nu_T = C_\mu\, K^2/\varepsilon$ |
| 0 eq. Prandtl mixing lenght | $\nu_T = \ell_m^2 \left\lvert \vec{grad}\,\vec{v} \right\rvert$ |
| large eddy simulation LES | por hora apenas fornecem parâmetros |



tabela

Rayleight

$$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] + \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]^2 - \frac{2}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]^2$$

## Dissipação da energia cinética de turbulência - ε

$$\varepsilon = \nu\left[2\left\langle\left(\frac{\partial \mathrm{v'_x}}{\partial \mathrm{x}}\right)^2\right\rangle + 2\left\langle\left(\frac{\partial \mathrm{v'_y}}{\partial \mathrm{y}}\right)^2\right\rangle + 2\left\langle\left(\frac{\partial \mathrm{v'_z}}{\partial \mathrm{z}}\right)^2\right\rangle + \left\langle\left(\frac{\partial \mathrm{v'_y}}{\partial \mathrm{x}} + \frac{\partial \mathrm{v'_x}}{\partial \mathrm{y}}\right)^2\right\rangle + \left\langle\left(\frac{\partial \mathrm{v'_z}}{\partial \mathrm{y}} + \frac{\partial \mathrm{v'_y}}{\partial \mathrm{z}}\right)^2\right\rangle + \left\langle\left(\frac{\partial \mathrm{v'_z}}{\partial \mathrm{x}} + \frac{\partial \mathrm{v'_x}}{\partial \mathrm{z}}\right)^2\right\rangle\right]$$

## Energia cinética de turbulência - k

$$k = \frac{1}{2}\left(\mathrm{v}_x^{'\,2} + \mathrm{v}_y^{'\,2} + \mathrm{v}_z^{'\,2}\right)$$

$$\frac{\partial \bar{k}}{\partial t} + div\,\bar{\vec{v}}\,\bar{k} = div\,\frac{\nu_T}{\sigma_k}\,gr\vec{a}d\,\bar{k} + P_k - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} + div\,\bar{\vec{v}}\,\varepsilon = div\,\frac{\nu_T}{\sigma_\varepsilon}\,gr\vec{a}d\,\varepsilon + C_{\varepsilon 1}\frac{\varepsilon}{\bar{k}}P_k - C_{\varepsilon 2}\frac{\varepsilon^2}{\bar{k}}$$

$$P_k = -\bar{\rho}\,\nu_T\left(gr\vec{a}d\,\bar{\vec{v}} : gr\vec{a}d\,\bar{\vec{v}}\right)$$

$$\nu_T = C_\mu \frac{\bar{k}^2}{\varepsilon}$$

$\sigma_k = 1{,}0$ ; $\sigma_\varepsilon = 1{,}217$ ;
$C_{\varepsilon 1} = 1{,}44$ ; $C_{\varepsilon 2} = 1{,}92$ ; $C_\mu = 0.09$

constantes experimentais

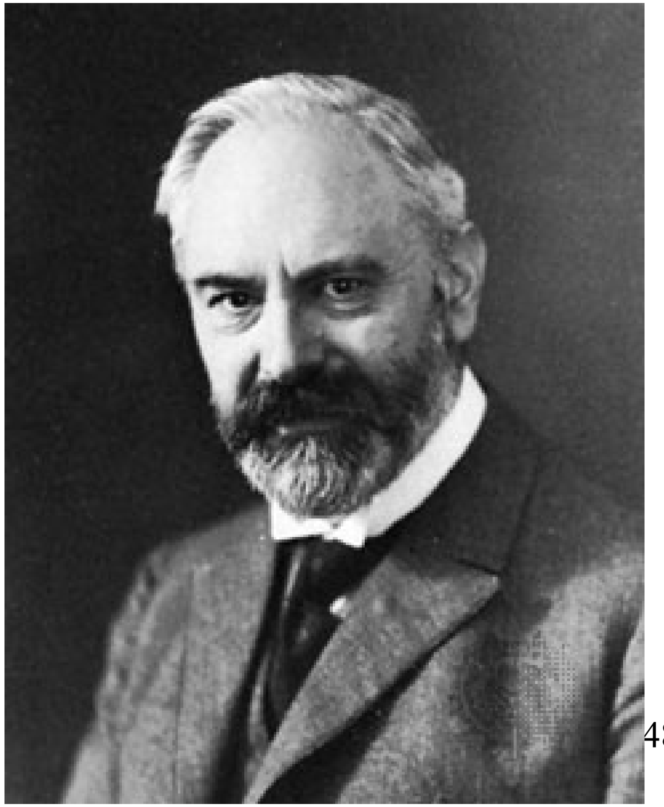
sendo a difusividade turbulenta >> laminar:

$$\left\lfloor \lambda_{\phi} + \lambda_{\phi T} \right\rfloor \approx \lambda_{\phi T}$$

$$\frac{\partial \overline{\varphi}}{\partial t} + \operatorname{div} \overline{\vec{v}}\,\overline{\varphi} = \operatorname{div} \lambda_{\phi T}\, \operatorname{gr\vec{a}d} \overline{\varphi} + \overline{\dot{\sigma}_{M_{\phi}}}$$

voltou à poderosa

agora super-poderosa



$$\lambda_{\vec{v}T} = \ell_m^2 \left| \operatorname{gr\vec{a}d} \overline{\vec{v}} \right|$$

Prandtl mixing lenght



hipótese

sendo $\varphi = v$ :

$$\frac{\partial \vec{\bar{v}}}{\partial t} + \mathrm{d\vec{i}v}\, \vec{\bar{v}}\ \vec{\bar{v}} = \mathrm{d\vec{i}v}\ \ \nu_T\ \mathrm{gr\vec{a}d}\, \vec{\bar{v}} - \frac{\mathrm{gr\vec{a}d}\, \bar{p}}{\bar{\rho}} + \vec{\bar{g}}$$

| | |
|---|---|
| Prandtl mixing lenght | $\lambda_{\vec{v}T} = \nu_T = \dfrac{\mu_T}{\rho} = \ell_m^2 \left\| \mathrm{gr\vec{a}d}\ \vec{\bar{v}} \right\|$ |
| similaridade Prandtl | $\nu_T = c_\mu \ell \sqrt{K}$ |
| k | K → $k = \overline{e_{c_T}} = \dfrac{\overline{\vec{v}'.\vec{v}'}}{2}$ <br> $\dfrac{Dk}{Dt} = \mathrm{div}\left(\dfrac{\nu_T}{\sigma_k}\mathrm{gr\vec{a}d}\ k\right) + \nu_T \left\| \mathrm{gr\vec{a}d}\ \vec{v} \right\|^2 - c_D \dfrac{k^{3/2}}{\ell}$ |
| k ε | $\nu_T = C_\mu\, K^2 / \varepsilon$ |

# Modelos de Turbulência

**Reynolds Stress** $\quad \vec{\vec{R}} = \overline{\vec{v}'.\vec{v}'}$

| | |
|---|---|
| sete equações diferenciais | $\frac{D\vec{\vec{R}}}{Dt} = -\,\vec{\mathrm{div}}\left[\vec{\vec{\vec{j}}}_{\vec{\vec{R}}} + \vec{\vec{\vec{\pi}}}_{\vec{\vec{R}}} + \vec{\vec{\vec{\Omega}}}_{\vec{\vec{R}}}\right] + \overline{\dot{\vec{\vec{\sigma}}}_{M_{\vec{\vec{R}}}}} - \overline{\dot{\vec{\vec{\varepsilon}}}_{M_{\vec{\vec{R}}}}}$ |
| algébrico | $\vec{\vec{R}} = \frac{2}{3}K\vec{\vec{\delta}} + \left[\frac{C_D}{C_1 - 1 + P/\varepsilon}\right]\left(\overline{\dot{\vec{\vec{\sigma}}}_{M_{\vec{\vec{R}}}}} - \frac{2}{3}p\,\vec{\vec{\delta}}\right)\frac{K}{\varepsilon}$ |
| 2 eq. $\quad K\ \varepsilon$ | $\nu_T = C_\mu\, K^2/\varepsilon$ |
| 0 eq. Prandtl mixing lenght | $\nu_T = \ell_m^2 \left\| \vec{\mathrm{grad}}\ \vec{v} \right\|$ |
| large eddy simulation LES | por hora apenas fornecem parâmetros |

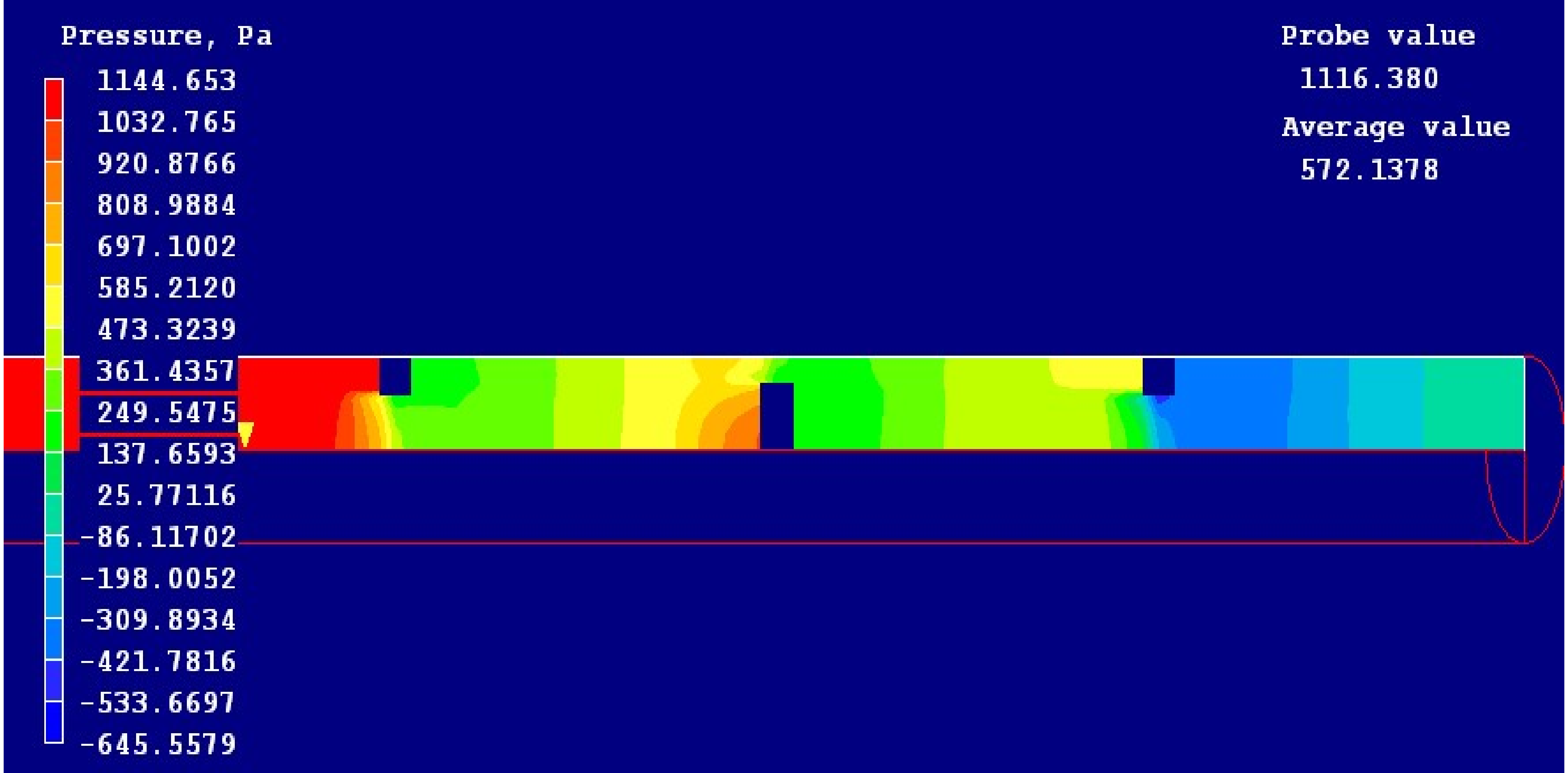
**Rayleight** $P = \mu \Phi_v = \rho \nu \left( \vec{\vec{grad}}\, \vec{v} : \vec{\vec{grad}}\, \vec{v} \right)$

turbulento $\vec{\vec{\zeta}}_T = -\rho\ \nu_T\ \ \vec{\vec{grad}}\, \overline{\vec{v}}$

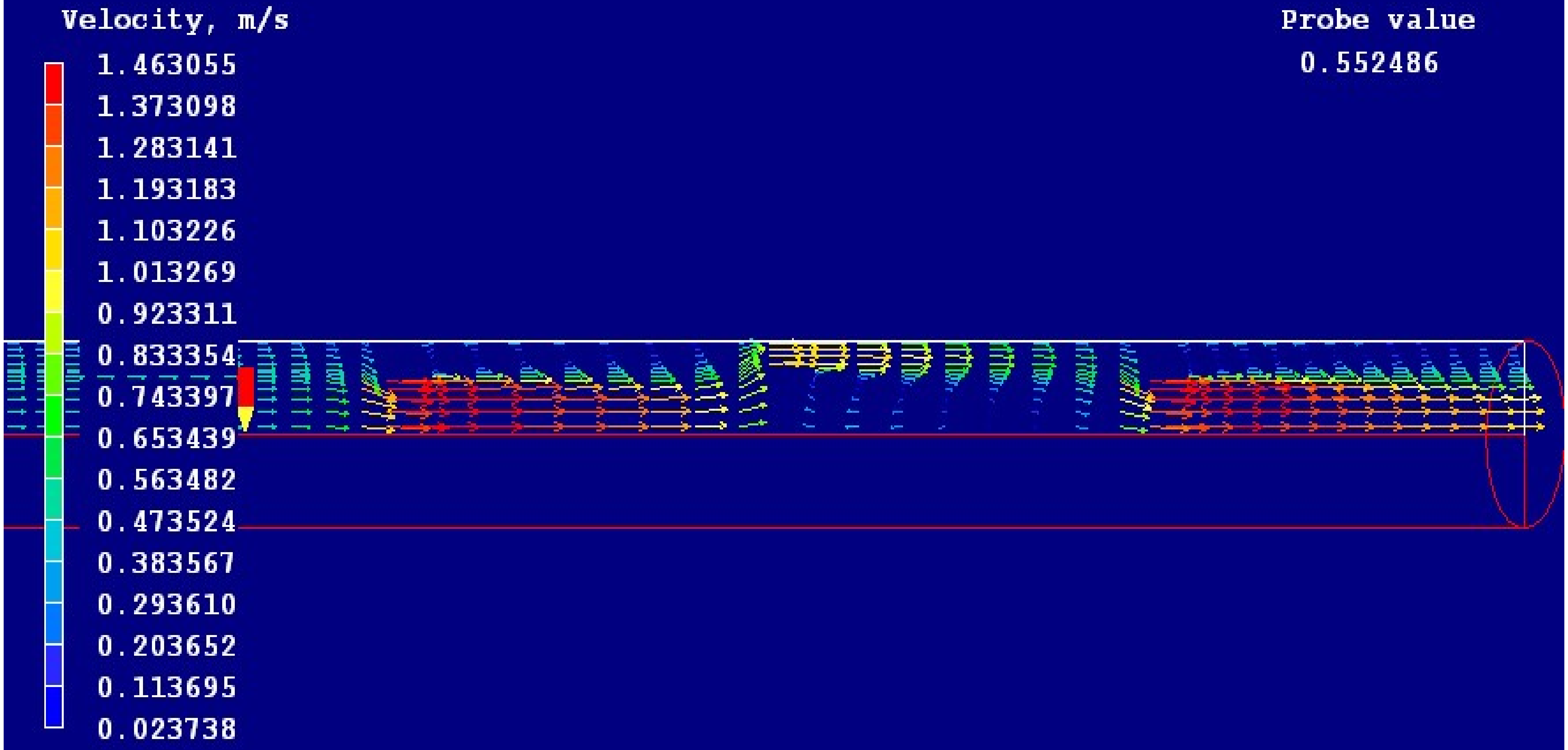
comprimento de mistura de Prandtl $\nu_T = \ell_m^2 \left| \vec{\vec{grad}}\, \overline{\vec{v}} \right|$

dissipação de energia cinética de turbulência

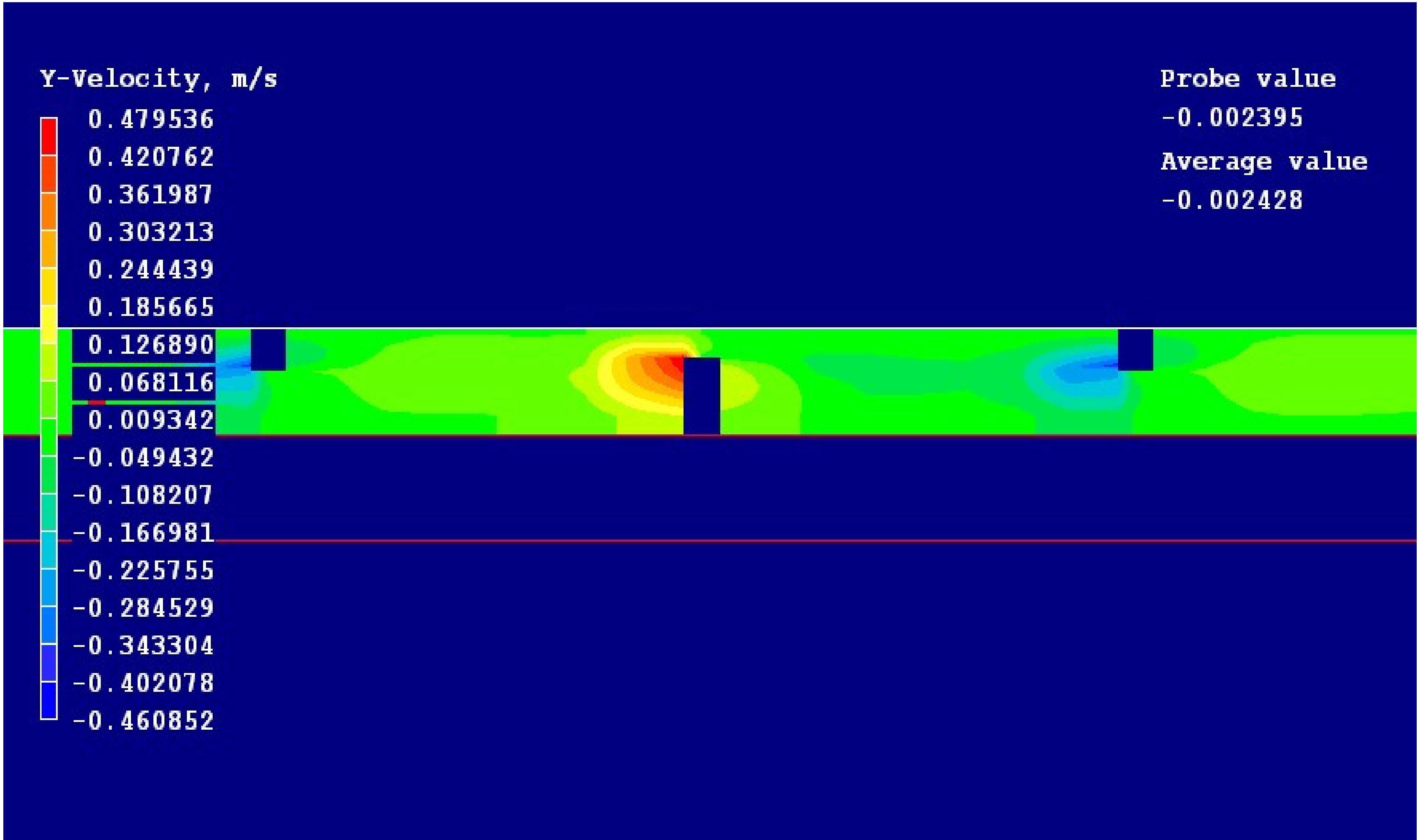
$$P_k = -\overline{\rho}\, \ell_m^2 \left( \vec{\vec{grad}}\, \overline{\vec{v}} : \vec{\vec{grad}}\, \overline{\vec{v}} \right)$$


Pressure, Pa
1144.653
1032.765
920.8766
808.9884
697.1002
585.2120
473.3239
361.4357
249.5475
137.6593
25.77116
-86.11702
-198.0052
-309.8934
-421.7816
-533.6697
-645.5579
Probe value
1116.380
Average value
572.1378

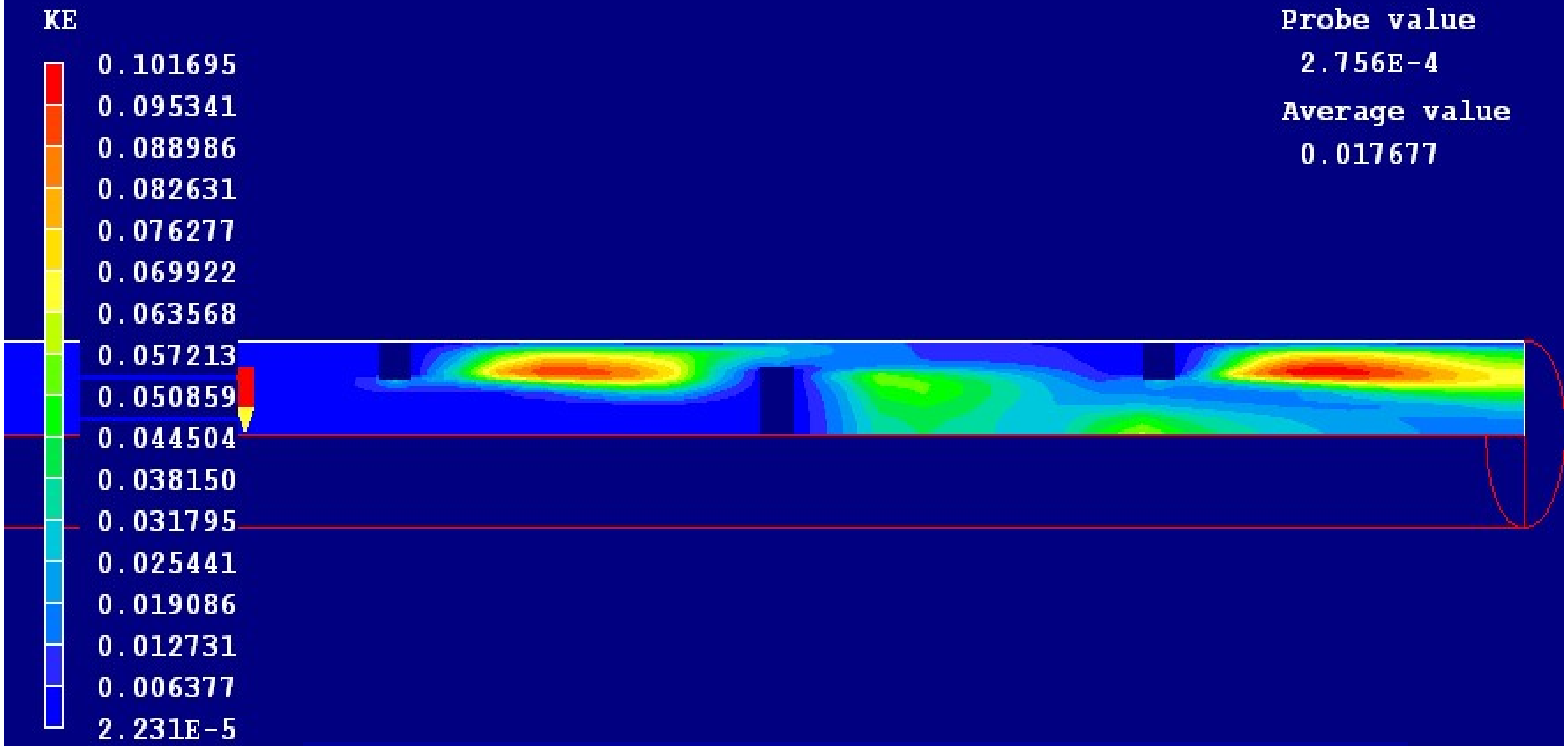
pressão



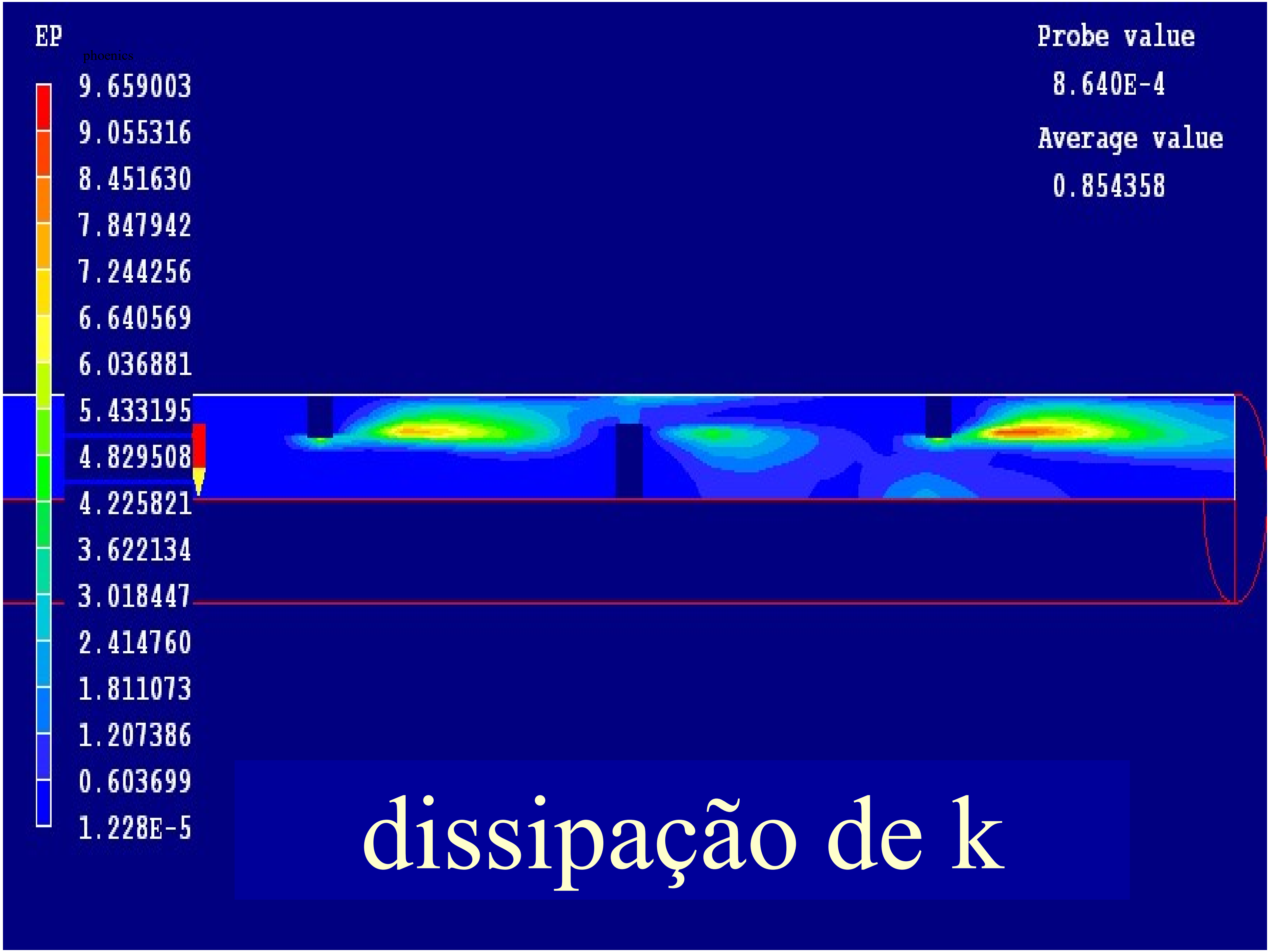

velocidades




# velocidade radial




# energia cinética de turbulência k


EP
phoenics
9.659003
9.055316
8.451630
7.847942
7.244256
6.640569
6.036881
5.433195
4.829508
4.225821
3.622134
3.018447
2.414760
1.811073
1.207386
0.603699
1.228E-5
Probe value
8.640E-4
Average value
0.854358
dissipação de k