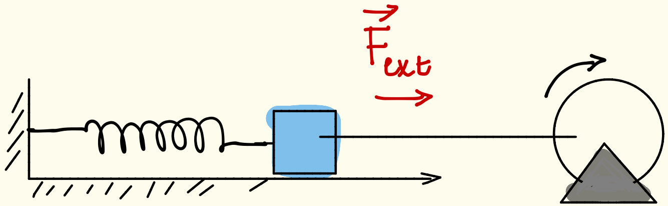
OSCILADOR FORÇADO

$$\vec{F}_{EXT} = F_0 \cos(\omega t)$$



BALANÇO DE FORÇAS:

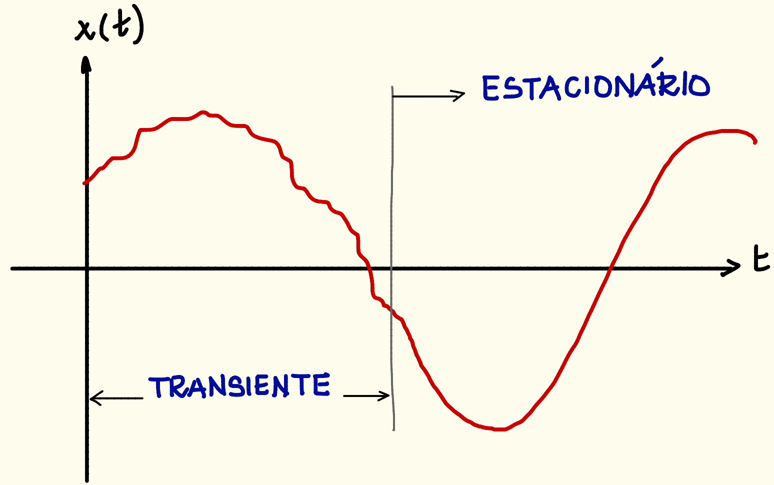
$$\vec{F}_{RES} = -kx - bv + F(t)$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega_0^2 x - \gamma x + \frac{F}{m}(t)}$$

$$\omega_0^2 = \frac{k}{m} \qquad \gamma = \frac{b}{m}$$

SE $F(t) = F_0 \cos \omega t \rightarrow$ FORÇA PERIÓDICA

SOLUÇÃO GERAL = MOVIMENTO TRANSIENTE + MOVIMENTO ESTACIONÁRIO



REGIME ESTACIONÁRIO: Movimento periódico com frequência angular igual à da força externa.

$$x(t) = A\cos(\omega t + \varphi)$$

NOTAÇÃO COMPLEXA: $\Longrightarrow$

$$z(t) = A\, e^{i(\omega t + \varphi)}$$

$$x(t) = \mathrm{Re}\left[z(t)\right]$$

VERIFICAÇÃO SE $z(t)$ SATISFAZ A EQUAÇÃO DIFERENCIAL

$$\frac{d^2 z}{dt^2} = -\omega_0^2 z - \gamma z + F_0 e^{i\omega t}$$

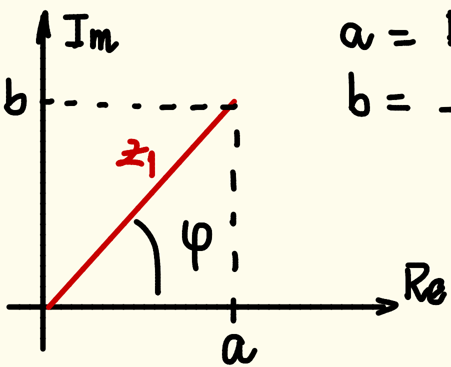
$$\frac{dz}{dt} = i\omega A e^{i(\omega t + \varphi)}$$

$$\frac{d^2 z}{dt^2} = -\omega^2 A e^{i(\omega t + \varphi)}$$

SUBSTITUINDO NA EQUAÇÃO DIFERENCIAL

$$-\omega^2 A \cancel{e^{i\omega t}}\, e^{i\varphi} = -\omega_0^2 \cancel{e^{i\omega t}}\, e^{i\varphi} A - i\omega\gamma A \cancel{e^{i\omega t}}\, e^{i\varphi} + \frac{F_0}{m} \cancel{e^{i\omega t}}$$

$$A e^{i\varphi} = \frac{F_0}{m}\left[\frac{1}{\omega_0^2 - \omega^2 + i\gamma\omega}\right] \rightarrow \boxed{\text{Imaginário}} \qquad z_1 = A e^{i\varphi}$$



$a = Re[z_1]$

$b = Im[z_1]$

A = módulo de z_1

$$A = \sqrt{a^2 + b^2}$$

$$tg\,\varphi = \frac{b}{a}$$

Parte real de $z_1 \Rightarrow$ $$a = \frac{F_0}{m} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

Parte imaginária de $z_1 \Rightarrow$ $$b = \frac{F_0}{m} \frac{-\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

$$A = \sqrt{a^2 + b^2}$$

$$\boxed{A = \frac{F_0}{m} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}}}$$

$$\operatorname{tg}\varphi = \frac{b}{a}$$

$$\boxed{\operatorname{tg}\varphi = \frac{-\gamma\omega}{\omega_0^2 - \omega^2}}$$

A e $\varphi \Rightarrow$ Dependem de ω!

ANÁLISE DA SOLUÇÃO ESTACIONÁRIA

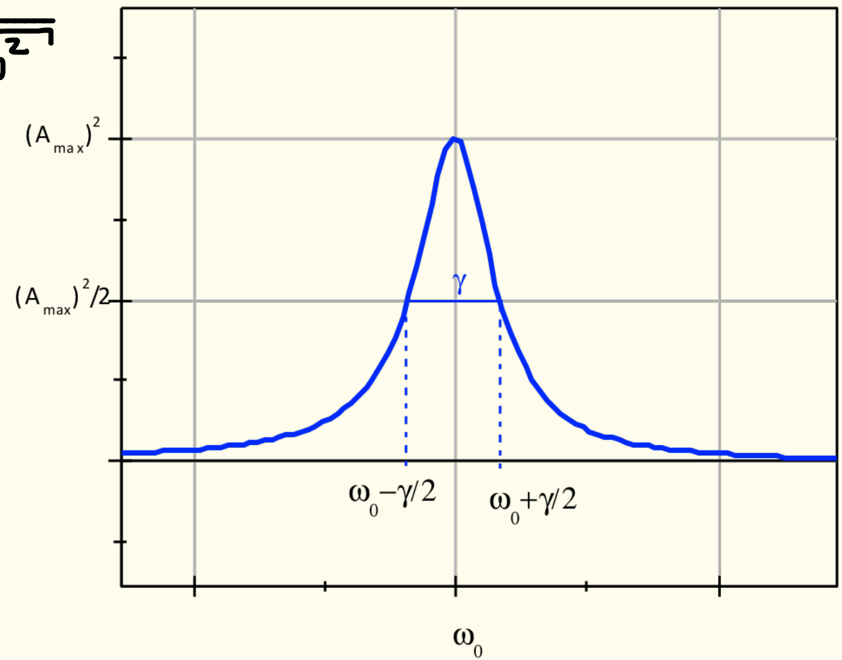
$$A = \frac{F_0}{m} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}}$$

com $\omega = \omega_0$

$$A(\omega_0) = \frac{F_0}{m\gamma\omega_0}$$

A_{MAX} para $\omega \to \omega_0$

$\boxed{\text{RESSONANCIA}}$

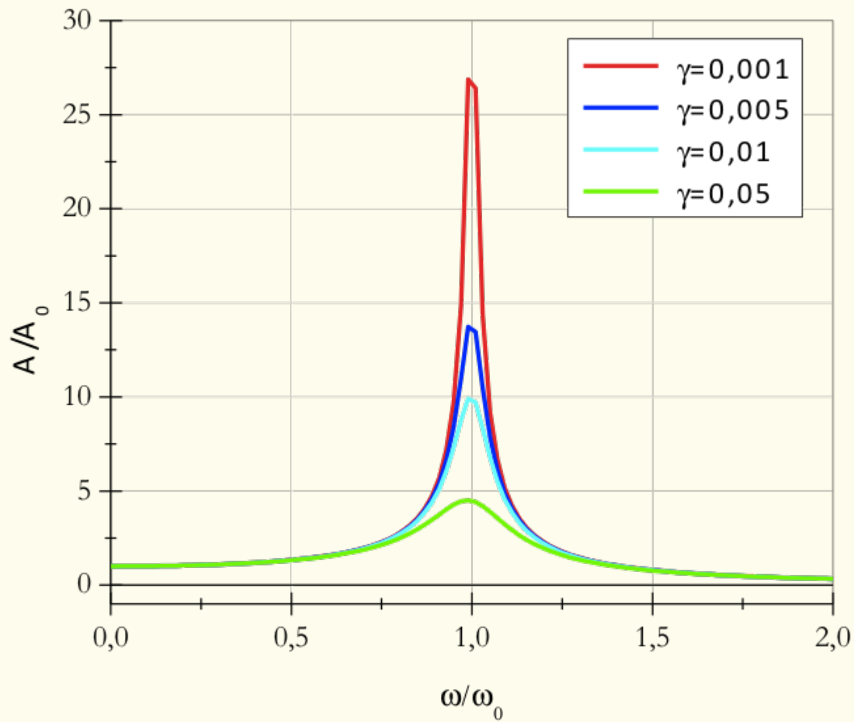


FATOR DE AMPLIFICAÇÃO

$$Q = \frac{A(\omega_0)}{A(0)} \qquad A(0) = A(\omega = 0) = \frac{F_0}{m\omega_0^2}$$

$$\boxed{Q = \frac{\omega_0}{\gamma}}$$

Quanto menor γ
+ estreito o pico
e maior o valor de
A_{max}



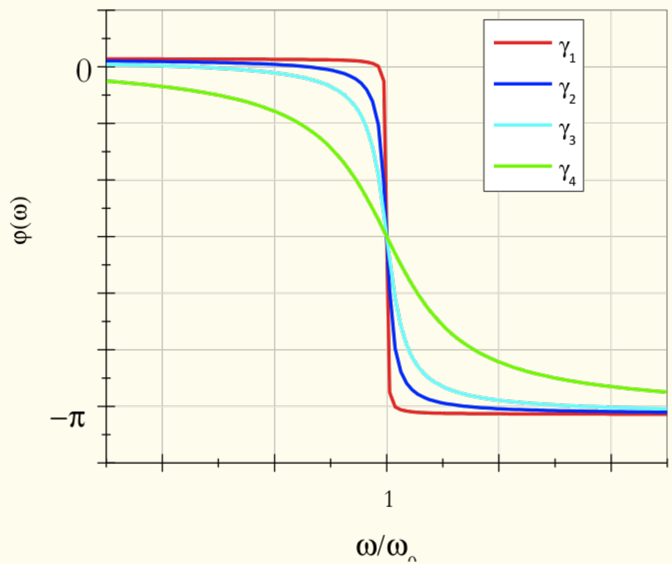
$$\gamma_4 > \gamma_3 > \gamma_2 > \gamma_1$$

$$\operatorname{tg}\varphi = \left[\frac{-\gamma\omega}{\omega_0^2 - \omega^2} \right]$$

$$\omega \to \omega_0 \quad \operatorname{tg}\varphi \to -\infty \quad \varphi \to \frac{\pi}{2}$$

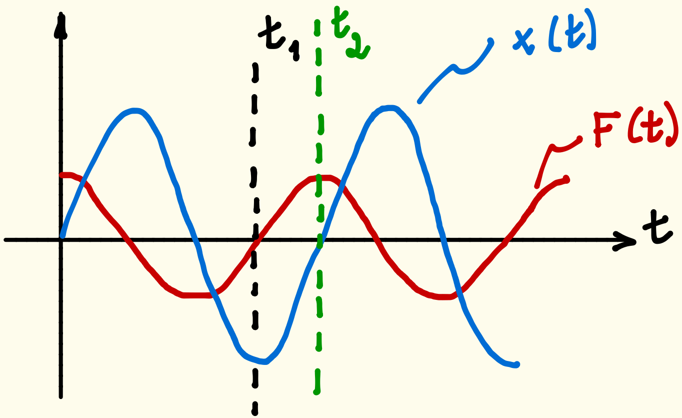
$$\omega \to 0 \quad \operatorname{tg}\varphi \to 0 \quad \varphi \to 0$$

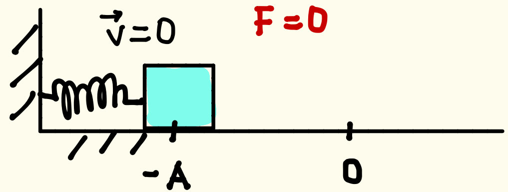
$$\omega \to \infty \quad \operatorname{tg}\varphi \to +\frac{1}{\omega} \to 0_+ \quad \varphi \to -\pi$$

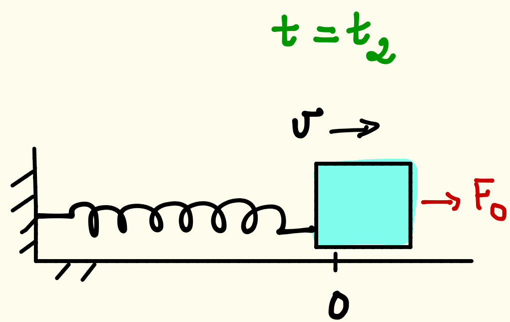


Na Ressonância: $\omega \cong \omega_0 \Rightarrow \varphi = -\pi/2$

$$x(t) = A(\omega_0)\cos(\omega_0 t - \pi/2) = A(\omega_0)\,\mathrm{sen}(\omega_0 t)$$



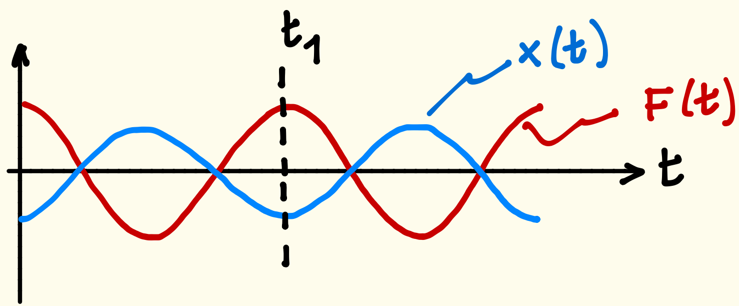




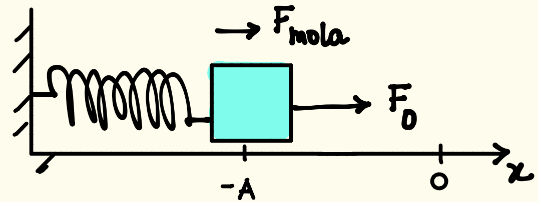
$F \Rightarrow$ máxima, no mesmo sentido de v para $x=0$

$$\boxed{\omega \gg \omega_0 \Rightarrow \varphi = -\pi}$$

$$x(t) = A(\omega)\cos(\omega t - \pi) = -A(\omega)\cos\omega t$$



$t = t_1$

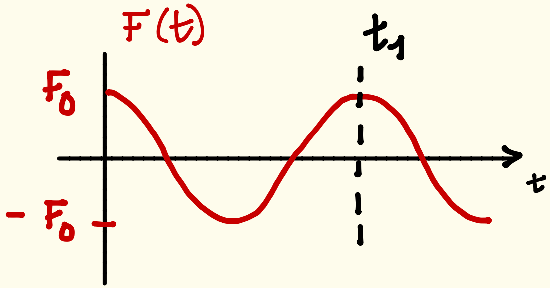


$F = 0$ para $x = 0$

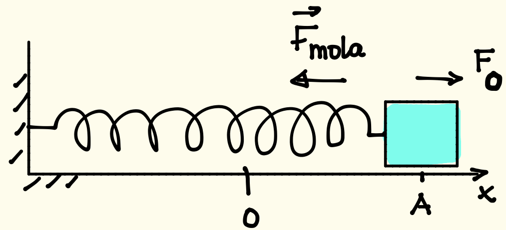
$F = F_0$ para $x = \pm A \rightarrow$ traz p/ posição de equilíbrio

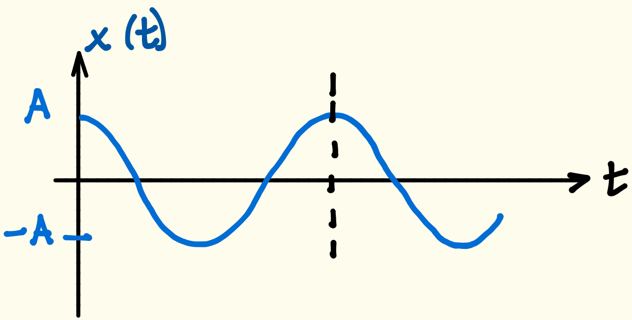
$$\omega \ll \omega_0 \Rightarrow \varphi = 0$$

se $F = F_0 \cos \omega t \Rightarrow x = A \cos \omega t$



$t = t_1$





Em $x = \pm A$, $F = F_0$ sentido oposto à F_{mola}

BALANÇO DE ENERGIA

$$E_{mec} = \frac{1}{2} k x^2 + \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$$

$$x(t) = A(\omega) \cos(\omega t + \varphi(\omega))$$

$$\frac{dx}{dt} = -\omega A \operatorname{sen}(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \varphi)$$

$$\frac{dE}{dt} = kx\frac{dx}{dt} + m\frac{dx}{dt}\left(\frac{d^2x}{dt^2}\right) \rightarrow \frac{\text{Variação da energia}}{\text{tempo}} = \text{Potência}$$

$$\left\langle \frac{dE}{dt} \right\rangle_T = k\left\langle x\frac{dx}{dt}\right\rangle + m\left\langle \frac{dx}{dt}\frac{d^2x}{dt^2}\right\rangle \rightarrow \text{Potência média em 1 período}$$

$$\left\langle \frac{dE}{dt} \right\rangle_T = k A^2 \omega \underbrace{\langle -\cos(\omega t + \varphi) \operatorname{sen}(\omega t + \varphi)\rangle_T}_{0} + m \underbrace{\langle \operatorname{sen}(\omega t + \varphi) \cos(\omega t + \varphi)\rangle_T}_{0}$$

$$\left\langle \frac{dE}{dt} \right\rangle_T = 0$$

Em um período a energia média não varia!

Voltando à expressão: $\frac{dE}{dt} = \frac{dx}{dt}\left[kx + m\frac{d^2x}{dt^2}\right]$

Equação diferencial: $m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F(t) \Rightarrow m\frac{d^2x}{dt^2} + kx = -b\frac{dx}{dt} + F(t)$

$$\frac{dE}{dt} = \frac{dx}{dt}\left[-b\frac{dx}{dt} + F(t)\right]$$

$P_{diss} = \left(-b\frac{dx}{dt}\right)\frac{dx}{dt} = \frac{\text{Trabalho força diss.}}{\text{unidade de tempo}}$

$P_{forn.} = F(t)\,dt = \frac{\text{Trabalho da força externa}}{\text{unidade de tempo}}$

$$\left\langle \frac{dE}{dt} \right\rangle_T = \langle P_{diss} \rangle_T + \langle P_{forn} \rangle_T = 0 \qquad \boxed{\langle P_{diss} \rangle = -\langle P_{forn} \rangle}$$

$$\langle P_{diss} \rangle_T = -b\left\langle \left(\frac{dx}{dt}\right)^2 \right\rangle = -b\omega^2A^2 \underbrace{\langle \text{sen}(\omega t + \varphi) \rangle}_{1/2} \rightarrow \langle P_{diss} \rangle_T = -\frac{b\omega^2A^2}{2} \qquad \gamma = \frac{b}{m}$$

$$\boxed{\langle P_{diss} \rangle_T = -\frac{\gamma m \omega^2 A^2}{2}}$$

Como $A(\omega)$ é máxima quando $\omega \cong \omega_0 \rightarrow$ Ressonância

A transferência de energia do motor p/ o oscilador é máxima quando $\omega \cong \omega_0$