



Risers – Flexible Pipes and Umbilicals Local Structural Analysis

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Risers



- **Umbilical cables**: control signals, electrical power, fluid injection to the submarine equipment at the well head.
- *Flexible pipes:* conveying oil, gas, from the well head to the production floating system or to another storage and offloading vessel after processing.





Umbilicals







Flexible Pipes







Cylindrical Layers and Central Core (umbilicals)





Deformed Configurations Ramos Jr, R., 2001 (Anexo C)







Efforts on cylindrical layers



Tension:	F_{i}
Torsion:	$M_{_{t,i}}$
Bending:	${M}_{f,i}$
Inter-layer pressure:	$p_{c,i}$ and $p_{c,i-1}$





Deformed Cylindrical Layer



Elongation: $\Delta L/L$ Rotation around the central axis: $\Delta \varphi/L$ Curvature imposed to the central axis:KVariation of the mean layer radius: ΔR_i Variation of the layer thickness: Δt_i





Thickness variation

$$\frac{\Delta t_i}{t_i} = -v_i \cdot \frac{\Delta L}{L} - \left(1 - \frac{1}{2} \frac{t_i}{R_i}\right) \cdot \left[\frac{1}{2}(1 - v_i^2) + v_i(1 + v_i) \frac{R_i}{t_i}\right] \cdot \frac{\left(p_{c,i-1} + \mu_{in} \cdot p_{in}\right)}{E_i} - \left(1 + \frac{1}{2} \frac{t_i}{R_i}\right) \cdot \left[\frac{1}{2}(1 - v_i^2) - v_i(1 + v_i) \frac{R_i}{t_i}\right] \cdot \frac{\left(p_{c,i} + \mu_o \cdot p_o\right)}{E_i}$$

 μ_{in} : "flag": '1' if a central tube or '0' if not. μ_o : "flag": '1' if the last cylindrical layer, or '0' if not.

Theory of elasticity: thick wall pressure vessels





Thickness variation

$$\frac{\Delta t_i}{t_i} = -v_i \cdot \frac{\Delta L}{L} - \left[v_i (1+v_i) \frac{R_i}{t_i} \right] \frac{\left(p_{c,i-1} - p_{c,i} + \mu_{in} \cdot p_{in} - \mu_o \cdot p_o\right)}{E_i}$$

μ_{in} : "flag": '1' if a central tube or '0' if not. μ_o : "flag": '1' if the last cylindrical layer, or '0' if not.

Shell theory: thin wall pressure vesses





Mean radius variation

$$\frac{\Delta R_i}{R_i} = -v_i \cdot \frac{\Delta L}{L} + \left(1 - \frac{1}{2} \frac{t_i}{R_i}\right) \cdot \left[(1 - v_i^2) \frac{R_i}{t_i} + \frac{1}{2} v_i (1 + v_i)\right] \cdot \frac{\left(p_{c,i-1} + \mu_{in} \cdot p_{in}\right)}{E_i} - \left(1 + \frac{1}{2} \frac{t_i}{R_i}\right) \cdot \left[(1 - v_i^2) \frac{R_i}{t_i} - \frac{1}{2} v_i (1 + v_i)\right] \cdot \frac{\left(p_{c,i} + \mu_o \cdot p_o\right)}{E_i}$$

 μ_{in} : "flag": '1' if a central tube or '0' if not. μ_o : "flag": '1' if the last cylindrical layer, or '0' if not.

Theory of elasticity: thick wall pressure vessels





Mean radius variation

$$\frac{\Delta R_{i}}{R_{i}} = -v_{i} \cdot \frac{\Delta L}{L} + \left[(1 - v_{i}^{2}) \frac{R_{i}}{t_{i}} \right] \frac{\left(p_{c,i-1} - p_{c,i} + \mu_{in} \cdot p_{in} - \mu_{o} \cdot p_{o} \right)}{E_{i}}$$

μ_{in} : "flag": '1' if a central tube or '0' if not. μ_o : "flag": '1' if the last cylindrical layer, or '0' if not.

Shell theory: thin wall pressure vesses





Normal force supported by the cylindrical layer

$$\frac{F_i}{E_i A_i} = \frac{\Delta L}{L} + \frac{1}{2} \left(1 - \frac{1}{2} \frac{t_i}{R_i} \right) \cdot v_i \left(2 \frac{R_i}{t_i} - 1 \right) \frac{\cdot (p_{c,i-1} + \mu_{in} \cdot p_{in})}{E_i} - \frac{1}{2} \left(1 + \frac{1}{2} \frac{t_i}{R_i} \right) \cdot v_i \left(2 \frac{R_i}{t_i} + 1 \right) \frac{(p_{c,i} + \mu_o \cdot p_o)}{E_i}$$

 μ_{in} : "flag": '1' if a central tube or '0' if not. μ_o : "flag": '1' if the last cylindrical layer, or '0' if not.

Theory of elasticity: thick wall pressure vessels





Normal force supported by the cylindrical layer

$$\frac{F_{i}}{E_{i}A_{i}} = \frac{\Delta L}{L} + v_{i}\frac{R_{i}}{t_{i}}\frac{(p_{c,i-1} - p_{c,i} + \mu_{in}.p_{in} - \mu_{o}.p_{o})}{E_{i}}$$

μ_{in} : "flag": '1' if a central tube or '0' if not. μ_o : "flag": '1' if the last cylindrical layer, or '0' if not.

Shell theory: thin wall pressure vesses





Torsion and Bending Moments supported by the cylindrical layer

$$M_{t,i} = (GJ_p)_i \cdot \frac{\Delta \varphi}{L}$$

$$M_{f,i} = (EI)_i . K$$

Linear Theory of rods





Radius variation of a solid core and efforts

$$\frac{\Delta R_i}{R_i} = -v_i . \frac{\Delta L}{L} - (1 + v_i) . (1 - 2v_i) . \frac{p_{c,i}}{E_{i_{eq}}}$$

$$\frac{F_{i}}{E_{i_{eq}}\pi R_{i}^{2}} = \frac{\Delta L}{L} - 2v_{i} \frac{p_{c,i}}{E_{i_{eq}}}$$



$$M_{t,i} = (GJ_p)_{i_{eq}} \cdot \frac{\Delta \varphi}{L}$$

$$M_{f,i} = (EI)_{i_{eq}}.K$$

Linear Theory of rods





Umbilicals

Equilibrium Equations Stiffness Parameters





Cylindrical Elements. Central Core.







Incognitae

Incognitae	Symbol	Number
Contact pressure or gap between layers i and i+1	p_{ci} or g_i	n+m-1
Axial load supprted by layer i	Fi	n+m
Twist moment supported by layer i	M_{ti}	n+m
Bending moment supported by layer i	M _{fi}	n+m
Mean radius variation, layer i	ΔR_i	n+m
Tichness variatiation of layer i (*)	Δt_i	n+m
Laying angle variation (helical layer i)	$lpha_i$	п
Axial elongation	∆L/L	1
Twist per unit length	$\Delta \varphi/L$	1
Bending curvature	K	1
Tota	al	7n+6m+2

(*) Except if the first layer is na umbilical central core.





Equilibrium equations – all layers

$$\sum_{i=1}^{n+m} F_i = F$$

$$\sum_{i=1}^{n+m} M_{t,i} = M_t$$

$$\sum_{i=1}^{n+m} M_{f,i} = M_f$$

Algebraic equation system composed by *thirteen groups*, solvable numerically to obtain :

- 7n + 6m + 1 incognitae if the first layer is an electrical cable core
- 7n + 6m + 2 incognitae if the first layer is a pipe or equivalente pipe

n: number of helical layers;

m: number of cylindrical layers.





Bending stiffness for an umbilical

$$M_f = (EI)_{eq}$$
.K

$$(EI)_{eq} = \sum_{i=1}^{m} E_{i}I_{i} + \sum_{i=1}^{n} n_{i}\cos\alpha_{i}\left[G_{i}I_{t,i} + \frac{3}{2}\left(E_{i}I_{y,i} - G_{i}I_{t,i}\right)\cos^{2}\alpha_{i}\right]$$

Full slipping

$$(EI)_{eq} = \sum_{i=1}^{m} E_{i}I_{i} + \sum_{i=1}^{n} n_{i}\cos\alpha_{i}\left[\frac{1}{2}(EA)_{i}R_{i}^{2}\cos^{3}\alpha_{i} + G_{i}I_{t,i} + \frac{3}{2}\left(E_{i}I_{y,i} - G_{i}I_{t,i}\right)\cos^{2}\alpha_{i}\right]$$

No slipping





Stiffness matrix for a umbilical

$$\begin{bmatrix} F \\ M_{t}/R \\ M_{f}/R \end{bmatrix} = \begin{bmatrix} c_{hh} & c_{h\varphi} & 0 \\ c_{\varphi h} & c_{\varphi \varphi} & 0 \\ 0 & 0 & c_{\kappa \kappa} \end{bmatrix} \begin{bmatrix} \varepsilon_{h} \\ \varepsilon_{\varphi} \\ \kappa \end{bmatrix}$$

Depends on loading!

$$\varepsilon_{h} = \frac{\Delta L}{L}; \varepsilon_{\varphi} = \frac{\Delta \varphi R}{L}; \kappa = KR$$

$$c_{hh} = (EA)_{eq}; c_{\varphi\varphi} = \frac{(GJ)_{eq}}{R^2}; c_{\kappa\kappa} = \frac{(EI)_{eq}}{R^2}$$





Stiffness parameters of an umbilical cable – tension and torsion

For each loading condition (say, A and B, not linearly dependent), numerical values of the cable elongation and twist are determined as part of the system solution.

$$(F;M_{t})_{A} \Longrightarrow (\mathcal{E}_{h};\mathcal{E}_{\varphi})_{A}$$
$$(F;M_{t})_{B} \Longrightarrow (\mathcal{E}_{h};\mathcal{E}_{\varphi})_{B}$$





Stiffness parameters of an umbilical cable – tension and torsion

Coefficients c_{hh} and $c_{h\phi}$ are determined by solving the equations:

$$\begin{bmatrix} \mathcal{E}_{hA} & \mathcal{E}_{\varphi A} \\ \mathcal{E}_{hB} & \mathcal{E}_{\varphi B} \end{bmatrix} \cdot \begin{bmatrix} C_{hh} \\ C_{h\varphi} \end{bmatrix} = \begin{bmatrix} F_A \\ F_B \end{bmatrix}$$

Depend on loading!

Analogously for $c_{\varphi h}$ and $c_{\varphi \phi}$:

$$\begin{bmatrix} \mathcal{E}_{hA} & \mathcal{E}_{\varphi}_{A} \\ \mathcal{E}_{hB} & \mathcal{E}_{\varphi}_{B} \end{bmatrix} \cdot \begin{bmatrix} C_{\varphi h} \\ C_{\varphi \varphi} \end{bmatrix} = \begin{bmatrix} M_{tA} \\ M_{tB} \end{bmatrix}$$





Stiffness parameters for a umbilical subjected to axial tension, bending and torsion with tubes pressurization, without slippage between layers

$$\begin{bmatrix} F \\ M_{t}/R \\ M_{f}/R \\ p_{p}R^{2} \\ \dots \\ p_{q}R^{2} \end{bmatrix} = \begin{bmatrix} c_{hh} & c_{h\varphi} & c_{h\kappa} \\ c_{\varphi h} & c_{\varphi \varphi} & c_{\varphi \kappa} \\ c_{\kappa h} & c_{\kappa \varphi} & c_{\kappa \kappa} \\ c_{ph} & c_{p\varphi} & c_{p\kappa} \\ \dots & \dots & \dots \\ c_{qh} & c_{q\varphi} & c_{q\kappa} \end{bmatrix}$$

"Stiffness matrix" depends on loading; solution not a trivial task!

$$\varepsilon_{h} = \frac{\Delta L}{L}; \varepsilon_{\varphi} = \frac{\Delta \varphi R}{L}; \kappa = KR$$

$$c_{hh} = (EA)_{eq}; c_{\varphi\varphi} = \frac{(GJ)_{eq}}{R^2}; c_{\kappa\kappa} = \frac{(EI)_{eq}}{R^2}$$





Stiffness parameters for a umbilical to axial tension, bending and torsion with tubes pressurization, with slippage between layers

$$\begin{bmatrix} F \\ M_t / R \\ M_f / R \\ p_p R^2 \\ \dots \\ p_q R^2 \end{bmatrix} = \begin{bmatrix} c_{hh} & c_{h\varphi} & 0 \\ c_{\varphi h} & c_{\varphi \varphi} & 0 \\ c_{\kappa h} & c_{\kappa \varphi} & c_{\kappa \kappa} \\ c_{ph} & c_{p\varphi} & 0 \\ \dots & \dots & \dots \\ c_{qh} & c_{q\varphi} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_h \\ \varepsilon_{\varphi} \\ \kappa \end{bmatrix}$$

$$\varepsilon_{h} = \frac{\Delta L}{L}; \varepsilon_{\varphi} = \frac{\Delta \varphi R}{L}; \kappa = KR$$

$$c_{hh} = (EA)_{eq}; c_{\varphi\varphi} = \frac{(GJ)_{eq}}{R^2}; c_{\kappa\kappa} = \frac{(EI)_{eq}}{R^2}$$







Analytical Model vs Experiments

(Ref: OMAE2010-20895)

Steel Tube Umbilical testing in instrumented bench:

Experimental arrangement description

Stress results in the steel tubes

Comparisons to the mathematical model (OMAE2010-20892)





Umbilical Description

Non-armoured STU

1 central steel tube ID 38.1mm WT 3.0mm polyethylene coating t = 1.0mm;

3 steel tubes ID 25.4mm, WT 2.0mm;

9 steel tubes ID 12.7mm, WT 2.6mm;

low-density polyethylene fillers (suitable arrangement of the steel tubes and avoid direct contact between them);

high-density polyethylene outer sheath Heterogeneous layer type "B"







Experimental Arrangement – Test Bench

Test bench:

Total 4249mm sample;

Free umbilical length 3245mm;

Steel tubes can be pressurized

One fixed end plate, one active end plate

Tension

Bending







Experimental Arrangement – Test Bench

Fixed end-plate

Active end-plate







Experimental Arrangement – Instrumentation

- Several strain gauges were attached at certain points along the umbilical length.
- In these points, windows were opened on the external sheath to allow access to the tubes.









Experimental Arrangement – Instrumentation

Instrumented window detail:







Experimental Arrangement – Instrumentation

- Curvature assessed through LVDTs ٠ placed along the umbilical length.
- Curve fit with third-order polynomial. ٠

600 400 200

0

0

-200 -400 LVDT1

500

LVDT3

LVDT2







The stresses presented here were calculated from measurements of tubes A1 and A2.

They belong to window A, which presents larger curvature due to their position, for which larger bending stresses are expected.

The presented data refer to axial stress, since this is the main component on the external surface of the tube.

A representative subset of the complete test matrix was selected to illustrate the results encountered in the comparisons.

Load	Tension (kN)	Pressure (psi)	End Angle (deg)	Curvature * (1/m)
Minimum	10.0	0	-1.5	0
Maximum	90.0	3000	+1.5	0.023

*Due to end rotation.





Stresses for varying curvature, tension 60kN







Stresses for varying tension, no curvature







Stresses for varying tension, curvature 0.015m⁻¹







Stresses for varying pressure







The comparison of experimental data to model results indicates that the model can capture both qualitative and quantitative response aspects within the tested range.

Differences between experimental data and model results are within an acceptable level

Uncertainties and deviations that are intrinsic to any experiment

Hypotheses and simplifications included in the mathematical model





Summary

- Equilibrium and constitutive equations were stated for cylindrical layers and 'solid' cores.
- Equilibrium equations stated for umbilical cables with homogeneous layers (all elements in a given layer of the same type).
- Stiffness parameters were formulated.
- Experimental assesment made.





Flexible Pipes

Equilibrium Equations Axisymmetric loads Stiffness Parameters





Flexible Pipes





Recall equilibrium equations for helical layers



$$F_i = n_i . (EA)_i \left[(\cos^3 \alpha_i) . \frac{\Delta L}{L_1} + (R_i . \sin \alpha_i . \cos^2 \alpha_i) . \frac{\Delta \varphi}{L_1} + (\sin^2 \alpha_i . \cos \alpha_i) . \frac{\Delta R_i}{R_i} \right]$$

$$M_{t,i} = \frac{n_i}{R_i} \cdot \left[(GI_t)_i \cdot \cos\alpha_i \cdot \cos(2\alpha_i) + (EI_y)_i \cdot sen\alpha_i \cdot sen(2\alpha_i) \right] \Delta \alpha_i + \left[n_i (EA)_i R_i sen\alpha_i \cos^2\alpha_i \right] \frac{\Delta L}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot \cos\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n_i (EA)_i R_i^2 \cdot sen^2\alpha_i \right] \frac{\Delta \varphi}{L} + \left[n$$

$$+ n_i sen \alpha_i \left[(EA)_i sen^2 \alpha_i - \frac{(GI_t)_i \cos^2 \alpha_i}{R_i^2} - \frac{(EI_y)_i sen^2 \alpha_i}{R_i^2} \right] \Delta R_i$$

$$(p_{c,i} - p_{c,i-1})b_i = -\left[\frac{E_i A_i \sin^2 \alpha_i \cos^2 \alpha_i}{R_i}\right] \frac{\Delta L}{L_1} - \left[E_i A_i \sin^3 \alpha_i \cos \alpha_i\right] \frac{\Delta \varphi}{L_1} - \left[\frac{E_i A_i \sin^4 \alpha_i}{R_i^2}\right] \Delta R_i$$

$$\Delta t_i = -\frac{t_i}{2E_i} \cdot \left(p_{c,i} + p_{c,i-1} \right) - \left(v_i t_i \cos^2 \alpha_i \right) \frac{\Delta L}{L_1} - \left(v_i t_i \operatorname{R}_i \sin \alpha_i \cos \alpha_i \right) \cdot \frac{\Delta \varphi}{L_1} - \left(\frac{v_i t_i \sin^2 \alpha_i}{\operatorname{R}_i} \right) \Delta R_i$$

$$\Delta \alpha_i = \cos^2 \alpha_1 \cdot R_1 \frac{\Delta \varphi}{L_1} + \sin \alpha_1 \cdot \cos \alpha_1 \cdot \left(\frac{\Delta R}{R_1} - \frac{\Delta L}{L_1} \right)$$

(5*n* algebraic equations for the helical layers)





Compatibility equations and equibrium on the cross section

$$\Delta R_{i+1} = \Delta R_i + \frac{1}{2}(\Delta t_{i+1} + \Delta t_i) + g_i$$
 (n+m-1) eqs.

(*n*+*m*+1 additional equations)

Summing up:

- 4 m equations for the plastic layers;
- 5n equations for the helical layers;
- n+m+1 additional equations (geometric compatibility and equilibriumcompatibilidade e equilíbrio)

Total: 6n + 5m + 1 equations

Stiffness Matrix for a flexible pipe

$$\begin{cases} F \\ M_T \\ F_{ip,p} \\ F_{op,p} \end{cases} = \begin{bmatrix} k_{11,p} & k_{12,p} & k_{13,p} & k_{14,p} \\ k_{21,p} & k_{22,p} & k_{23,p} & k_{24,p} \\ k_{31,p} & k_{32,p} & k_{33,p} & k_{34,p} \\ k_{41,p} & k_{42,p} & k_{43,p} & k_{44,p} \end{bmatrix} \cdot \begin{cases} \frac{\Delta L}{L_0} \\ \frac{\Delta \varphi}{L_0} \\ \frac{\Delta R_{i,p}}{\Delta R_{o,p}} \end{cases}$$

Case study: Analytical Model vs Experiments

(Ramos Jr, et al. 2015)

Flexible Pipe: 2.5", 5 structural layers

Lab de Interação Fluido-Estrutura e Mecânica Offshore

Layer	Proprieties
1	$E_1 = 190$ GPa, $v_1 = 0.3$, $R_{i,1} = 33.37$ mm, $R_{o,1} = 35$ mm, $n_1 = 1$, $\alpha_1 = 85.8^\circ$, $t_1 = 1.63$ mm, $b_1 = 12$ mm, $A_1 = 19.56$ mm ²
2	$E_2 = 280 \text{ MPa}, v_2 = 0.4, R_{i,2} = 35 \text{mm}, R_{o,2} = 41 \text{mm}, t_2 = 6 \text{ mm}, A_2 = 1432.57 \text{mm}^2, J_2 = 2.082 \times 10^6 \text{ mm}^4$
3	$E_3 = 207$ GPa, $v_3 = 0.3$, $R_{i,3} = 41$ mm, $R_{o,3} = 43$ mm, $n_3 = 29$, $\alpha_3 = 55.5^\circ$, $t_3 = 2$ mm, $b_3 = 5$ mm, $A_3 = 10.0$ mm ²
4	$E_4 = 207$ GPa, $v_4 = 0.3$, $R_{i,4} = 43$ mm, $R_{o,4} = 45$ mm, $n_4 = 29$, $\alpha_4 = -55.5^\circ$, $t_4 = 2$ mm, $b_4 = 5$ mm, $A_4 = 10.0$ mm ²
5	$E_5 = 320$ MPa, $v_5 = 0.4$, $R_{i,5} = 45$ mm, $R_{o,5} = 50$ mm, $t_5 = 5$ mm, $A_5 = 1492.26$ mm ² , $J_5 = 3.376 \times 10^6$ mm ⁴

Experimental set up at IPT laboratory

Tensioning test

Tensioning test

Flexible Pipe Stiffness Matrix (IS):

K _{ij}	j = 1	j = 2	j = 3	j = 4
i = 1	1.443E+7	-3.137E+4	1.295E+8	5.122E+8
i = 2	-3.137E+4	8.632E+4	3.761E+6	-3.431E+4
i = 3	1.295E+8	3.761E+6	6.017E+10	-8.228E+9
i = 4	5.122E+8	-3.431E+4	-8.228E+9	2.998E+10

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