

Risers – Flexible Pipes and Umbilicals

Local Structural Analysis

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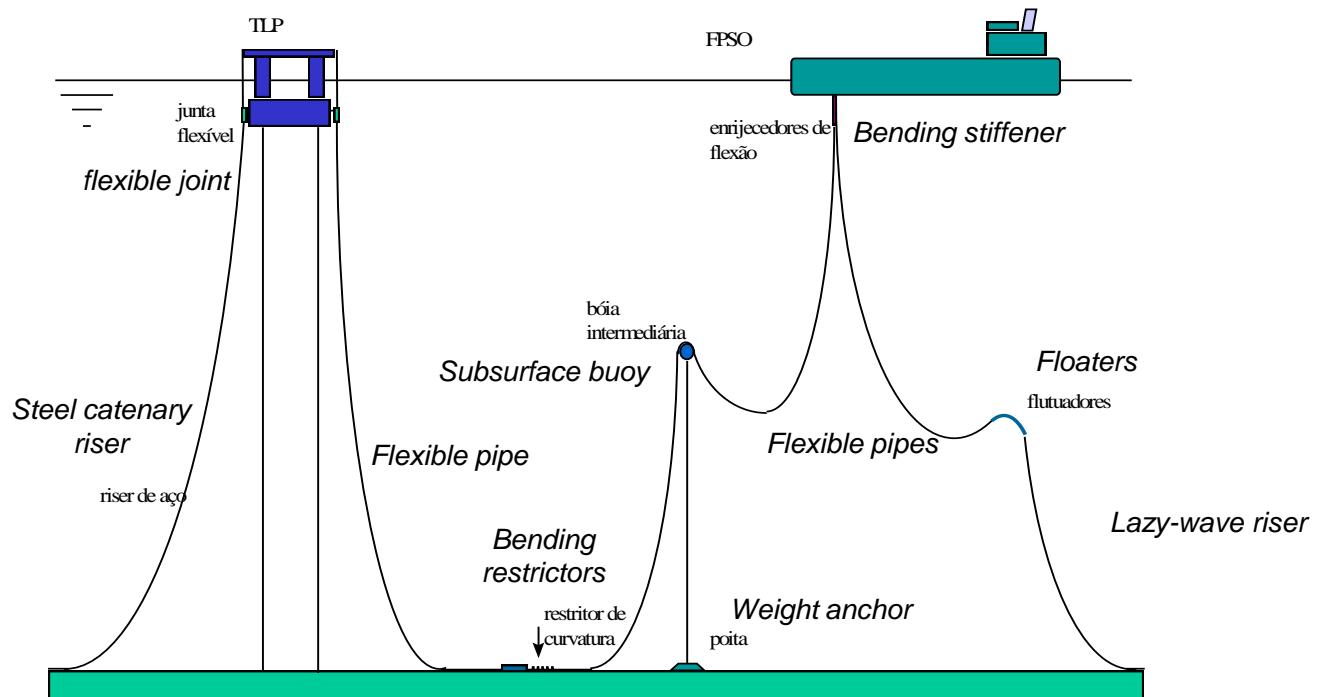
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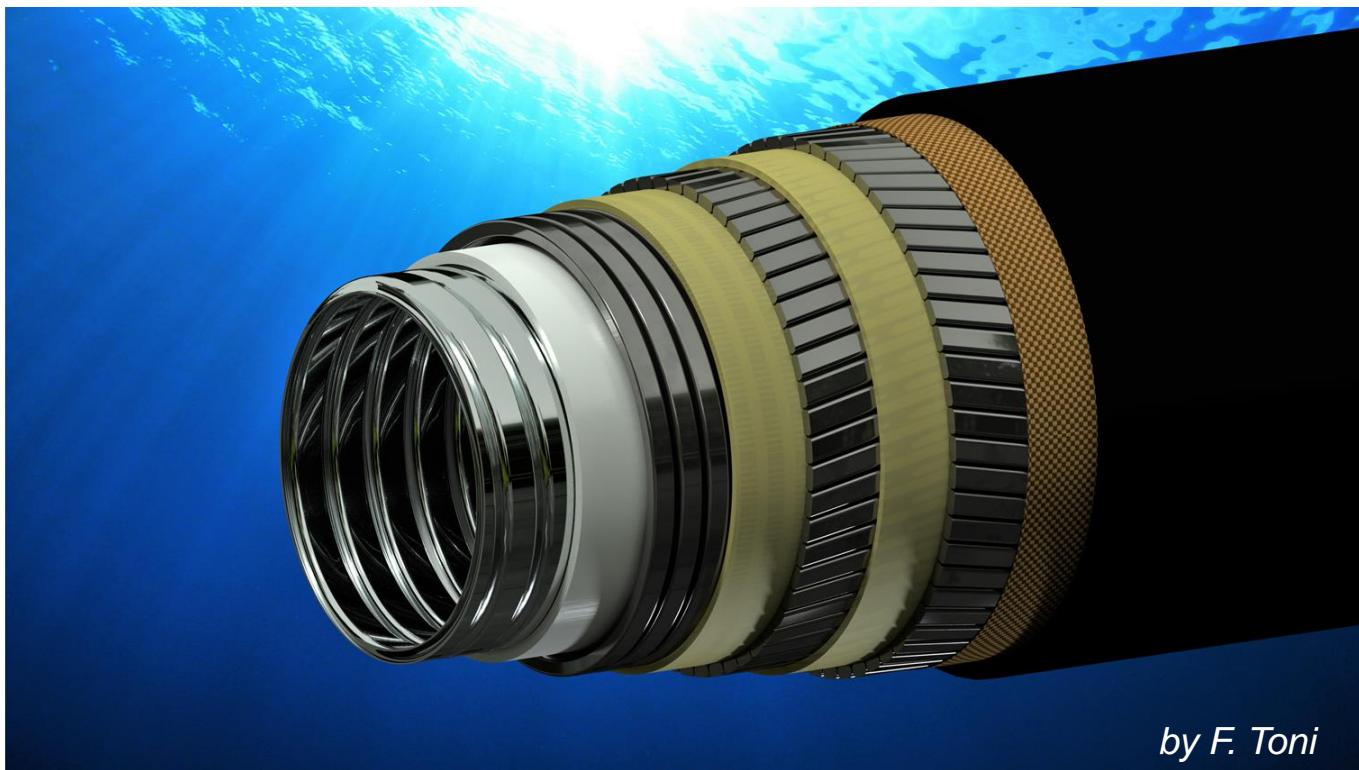
University of São Paulo

Risers



- **Umbilical cables:** control signals, electrical power, fluid injection to the submarine equipment at the well head.
- **Flexible pipes:** conveying oil, gas, from the well head to the production floating system or to another storage and offloading vessel after processing.

Typical Flexible Pipe



by F. Toni

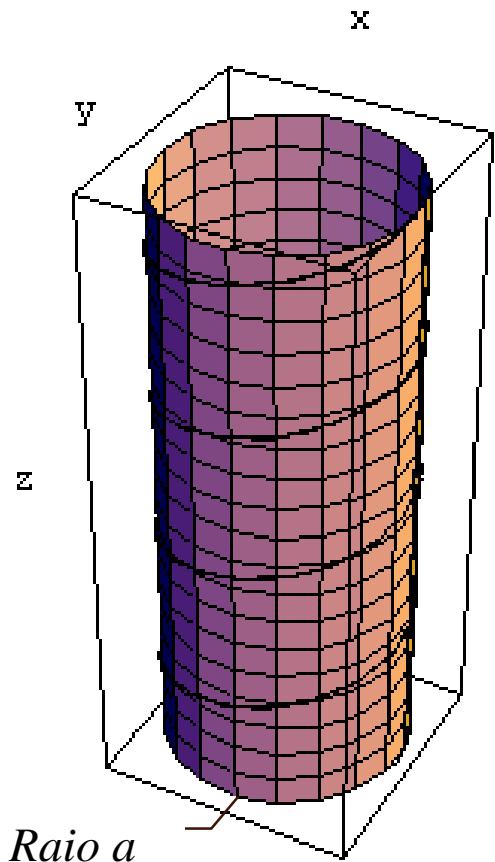
Steel Tube Umbilical



Fundaments: Differential Geometry of a Helix



Helix Geometry



$$x = a \cos \phi$$

$$y = a \sin \phi$$

$$z = \frac{h}{2\pi} \phi$$

Parametric equations

h : pitch

$$k = h/2\pi$$

k : normalized pitch

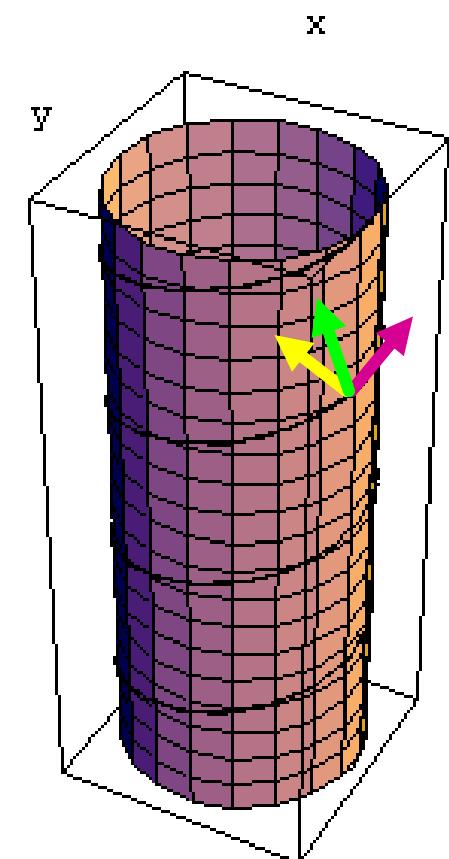
$$ds = \left(a^2 + k^2 \right)^{1/2} d\phi$$

Arch differential

$$ds = aKd\phi$$

$$K = \left(1 + \frac{k^2}{a^2} \right)^{1/2}$$

Helix Geometry



$$(P - O) = a \left(\cos \phi \mathbf{i} + \sin \phi \mathbf{j} + \frac{k}{a} \phi \mathbf{k} \right)$$

Position of a point
along the helix

$$\mathbf{t} = \frac{dP}{ds} = K^{-1} \left(-\sin \phi \mathbf{i} + \cos \phi \mathbf{j} + \frac{k}{a} \mathbf{k} \right)$$

Tangent unity vector

$$\frac{d\mathbf{t}}{ds} = \chi \mathbf{n}$$

Frenet-Serret formulae

$$\frac{d\mathbf{b}}{ds} = -\tau \mathbf{n}$$

where,

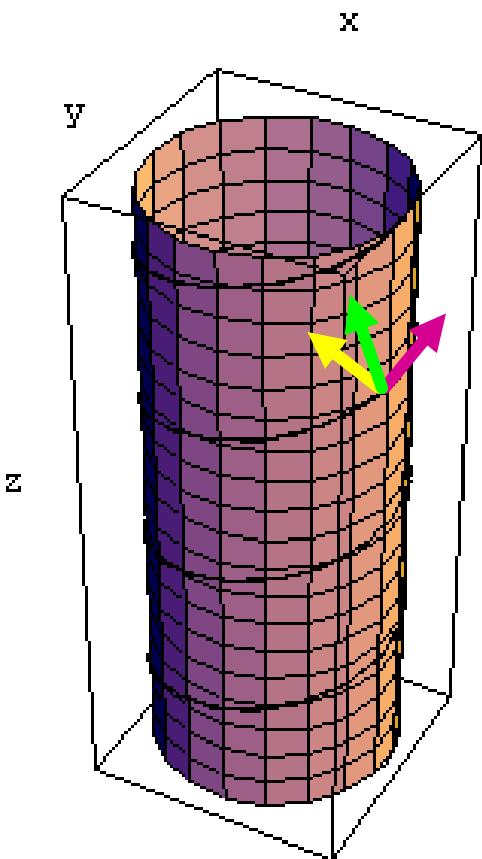
$$\frac{d\mathbf{n}}{ds} = \tau \mathbf{b} - \chi \mathbf{t}$$

$(\chi; \tau)$ are curvature and tortuosity

$$(\mathbf{t}, \mathbf{n}, \mathbf{b})$$

Frenet triad

Helix Geometry



$$\mathbf{t} = \frac{dP}{ds} = K^{-1} \left(-\sin \phi \mathbf{i} + \cos \phi \mathbf{j} + \frac{k}{a} \mathbf{k} \right)$$

Tangent

$$\mathbf{n} = -(\cos \phi \mathbf{i} + \sin \phi \mathbf{j})$$

Normal

$$\mathbf{b} = K^{-1} \left(\frac{k}{a} \sin \phi \mathbf{i} - \frac{k}{a} \cos \phi \mathbf{j} + \mathbf{k} \right)$$

Bi-normal

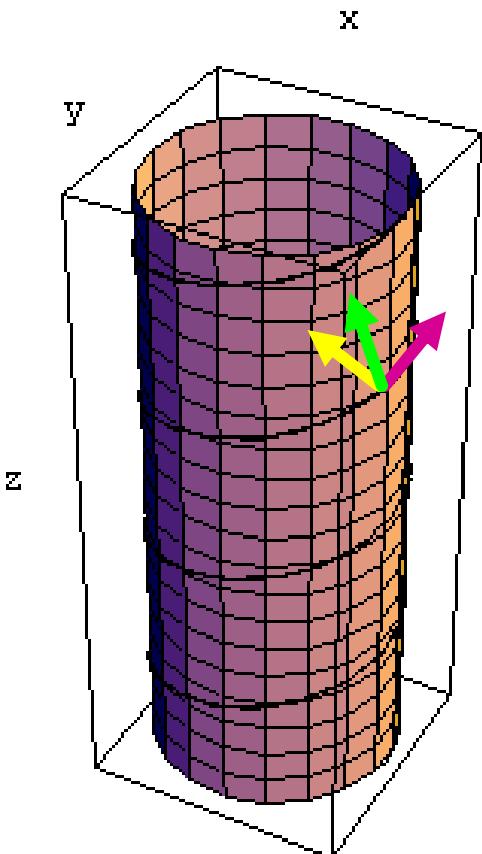
$$\tan \alpha = \left(\frac{t_x^2 + t_y^2}{t_z^2} \right)^{1/2} = \frac{a}{k} = \frac{2\pi a}{h}$$

$$K = \frac{1}{\sin \alpha}$$

$$\chi = \frac{\sin^2 \alpha}{a} = \frac{1}{K^2 a}$$

$$\tau = \frac{\sin \alpha \cos \alpha}{a} = \frac{k}{a} \chi$$

Helix Geometry: axially stretching the supporting cylinder



$$\mathbf{t} = -\sin \alpha \sin \phi \mathbf{i} + \sin \alpha \cos \phi \mathbf{j} + \cos \alpha \mathbf{k}$$

$$\mathbf{n} = -(\cos \phi \mathbf{i} + \sin \phi \mathbf{j})$$

$$\mathbf{b} = (\cos \alpha \sin \phi \mathbf{i} - \cos \alpha \cos \phi \mathbf{j} + \sin \alpha \mathbf{k})$$

Undeformed
condition

$$\mathbf{t}' = -\sin \alpha' \sin \phi \mathbf{i} + \sin \alpha' \cos \phi \mathbf{j} + \cos \alpha' \mathbf{k}$$

$$\mathbf{n}' = -(\cos \phi \mathbf{i} + \sin \phi \mathbf{j})$$

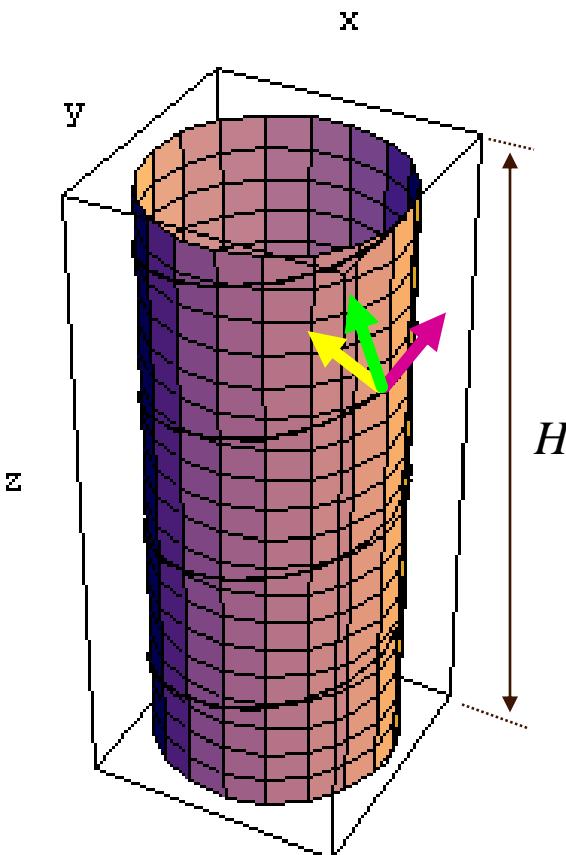
$$\mathbf{b}' = (\cos \alpha' \sin \phi \mathbf{i} - \cos \alpha' \cos \phi \mathbf{j} + \sin \alpha' \mathbf{k})$$

Deformed
condition

$$\alpha' = \alpha - \delta\alpha; \quad \delta\alpha \ll 1$$

$$a' = a - \delta a; \quad \frac{\delta a}{a} \ll 1$$

Helix Geometry: axially stretching the supporting cylinder



$$H = l \cos \alpha = 2\pi a m \cot \alpha$$

$$H' = l' \cos \alpha' = 2\pi a m \cot \alpha'$$

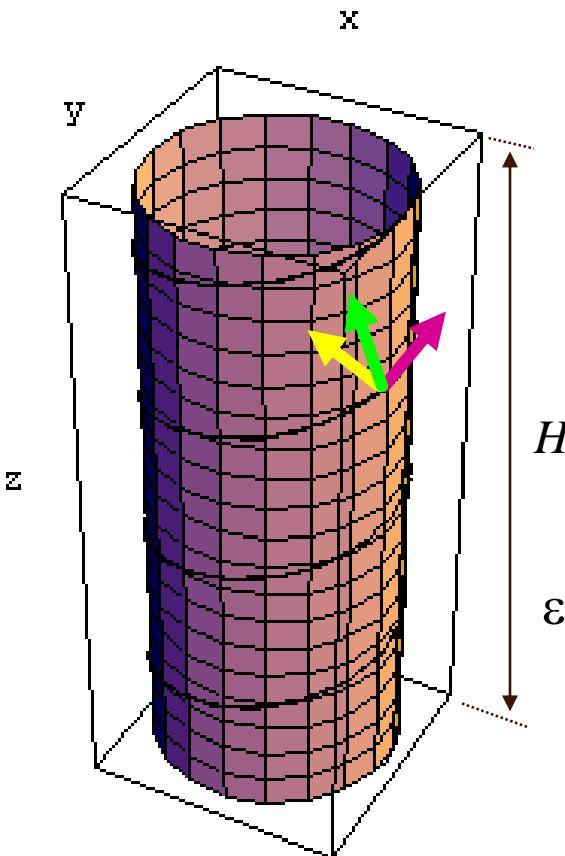
$$\frac{H'}{H} = (1 + \varepsilon_c) = \frac{a'}{a} \frac{\tan \alpha}{\tan \alpha'}$$

$$\varepsilon = \frac{\Delta l}{l} = \frac{l'}{l} - 1 = \frac{H'}{H} \frac{\cos \alpha}{\cos \alpha'} - 1 = \frac{ds'}{ds} - 1 = (1 + \varepsilon_c) \frac{\cos \alpha}{\cos \alpha'} - 1$$

$$\varepsilon_c = \frac{\Delta H}{H} = \frac{H'}{H} - 1$$

$$\varepsilon = \frac{a'}{a} \frac{\sin \alpha}{\sin \alpha'} - 1$$

Helix Geometry: axially stretching the supporting cylinder



$$\cos \alpha' \cong \cos \alpha + \delta \alpha \sin \alpha$$

$$\sin \alpha' \cong \sin \alpha - \delta \alpha \cos \alpha$$

$$\frac{\sin \alpha}{\sin \alpha'} \cong \left(1 - \frac{\delta \alpha}{\tan \alpha} \right)^{-1}$$

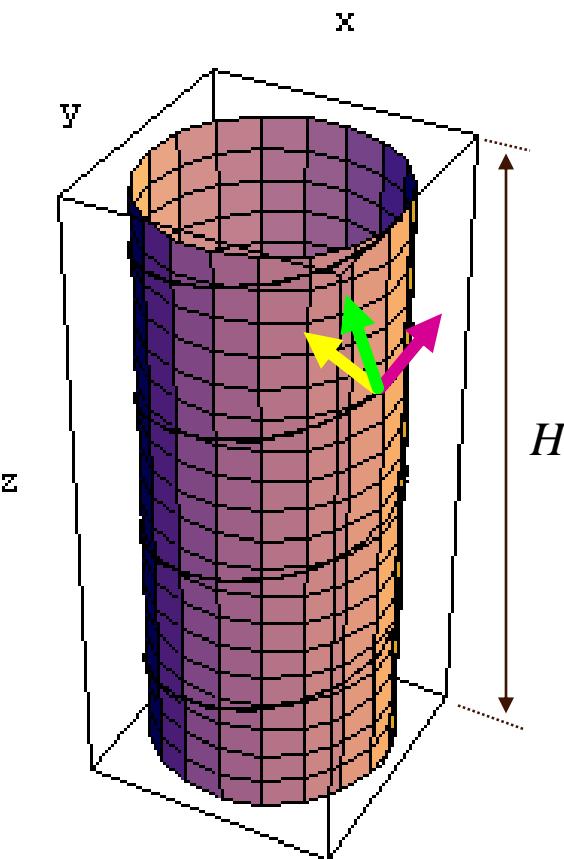
$$\frac{\tan \alpha}{\tan \alpha'} \cong \frac{\left(1 + \delta \alpha \tan \alpha \right)}{\left(1 - \frac{\delta \alpha}{\tan \alpha} \right)}$$

$$\varepsilon \cong \frac{\left(1 - \frac{\delta a}{a} \right)}{\left(1 - \frac{\delta \alpha}{\tan \alpha} \right)} - 1$$

$$\delta \alpha \cong \frac{\tan \alpha}{\left(1 + \varepsilon_c \right) + \left(1 - \frac{\delta a}{a} \right) \tan^2 \alpha} \left[\varepsilon_c + \frac{\delta a}{a} \right]$$

$$\boxed{\varepsilon = \varepsilon_c \cos^2 \alpha - \frac{\delta a}{a} \sin^2 \alpha}$$

Helix Geometry: axially stretching the supporting cylinder



$$\mathbf{t}' = \mathbf{t} + \delta\alpha \mathbf{b}$$

$$\mathbf{n}' = \mathbf{n}$$

$$\mathbf{b}' = \mathbf{b} - \delta\alpha \mathbf{t}$$

$$\chi' = \chi(1 - \varepsilon) - \tau\delta\alpha$$

$$\tau' = \tau(1 - \varepsilon) + \chi\delta\alpha$$

$$\mathbf{c} = \chi\mathbf{b} + \tau\mathbf{t}$$

$$\mathbf{c}' = (1 - \varepsilon)\mathbf{c}$$

$$\mathbf{c}' = \chi'\mathbf{b}' + \tau'\mathbf{t}'$$

$$\Delta\mathbf{c} = -\varepsilon\mathbf{c}$$

$$\delta\kappa_t = \kappa'_t - \kappa_t = \mathbf{t} \cdot \Delta\mathbf{c} = -\varepsilon\tau$$

$$\delta\kappa_n = \kappa'_n - \kappa_n = \mathbf{n} \cdot \Delta\mathbf{c} = 0$$

$$\delta\kappa_b = \kappa'_b - \kappa_b = \mathbf{b} \cdot \Delta\mathbf{c} = -\varepsilon\chi$$

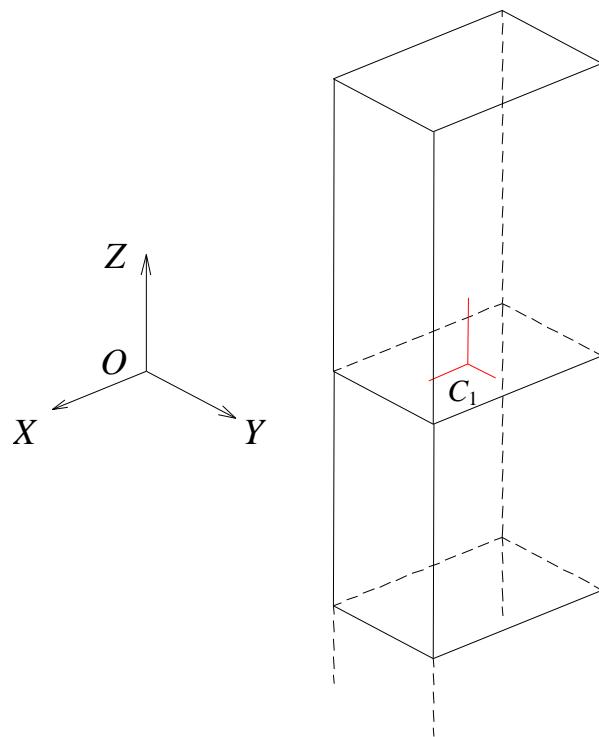
Differential Equilibrium Equations for a Curved Bar

References:

Ramos Jr., R., *Modelos analíticos no estudo do comportamento estrutural de tubos flexíveis e cabos umbilicais*. Escola Politécnica, Universidade de São Paulo. Tese (Doutorado). 367 p., São Paulo, 2001.

Ramos Jr., R., Pesce, C.P., “A Consistent Analytical Model to Predict the Structural Behaviour of Flexible Risers Subjected to Combined Loads”, *Journal of Offshore Mechanics and Arctic Engineering*, 126 (2), 141-146, 2004.

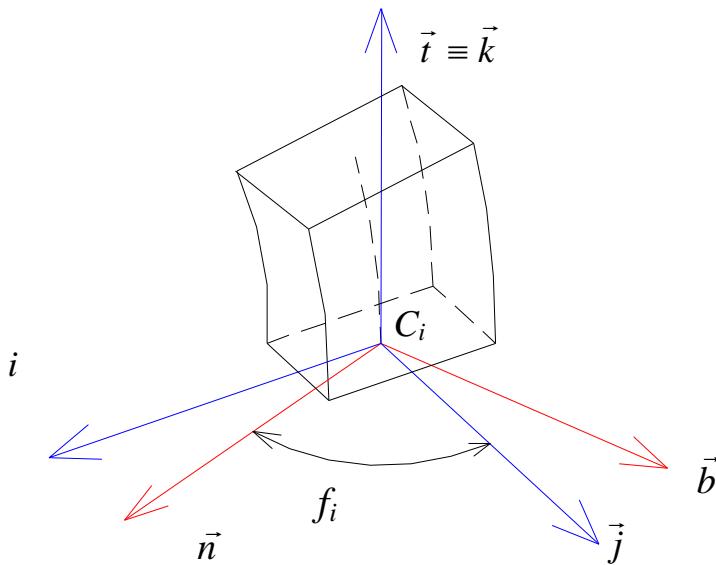
Principal flexural-torsional axes



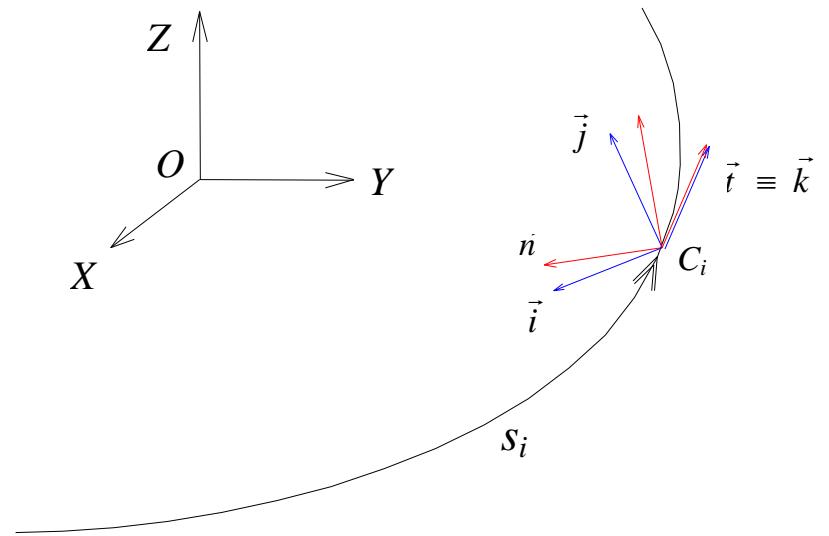
Undeformed prismatic bar

$$\kappa_{ti} = \lim_{\Delta S_i \rightarrow 0} \frac{\Delta f_i}{\Delta S_i} = \frac{df_i}{dS_i} \quad \text{twist}$$

Principal flexural-torsional axes



Central axis in the deformed configuration with principal flexure-torsional axis and Frenet triad



Central axis of the bar in the deformed configuration

$$\begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix} = \begin{bmatrix} \sin f_i & -\cos f_i & 0 \\ \cos f_i & \sin f_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \vec{n} \\ \vec{b} \\ \vec{t} \end{bmatrix}$$

Principal flexural-torsional axes

$$\begin{bmatrix} \vec{di} \\ \vec{dj} \\ \vec{dk} \\ \vec{dt} \end{bmatrix} = \frac{df_i}{dS_i} \cdot \begin{bmatrix} \cos f_i & \sin f_i & 0 \\ -\sin f_i & \cos f_i & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \vec{n} \\ \vec{b} \\ \vec{t} \end{bmatrix} + \begin{bmatrix} \sin f_i & -\cos f_i & 0 \\ \cos f_i & \sin f_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \vec{dn} \\ \vec{db} \\ \vec{dt} \end{bmatrix}$$

$$\left. \begin{array}{l} \frac{d\vec{n}}{dS_i} = \tau_i \cdot \vec{b} - \chi_i \cdot \vec{t} \\ \frac{d\vec{b}}{dS_i} = -\tau_i \cdot \vec{n} \\ \frac{d\vec{t}}{dS_i} = \chi_i \cdot \vec{n} \end{array} \right\} \Rightarrow \begin{bmatrix} \vec{di} \\ \vec{dj} \\ \vec{dk} \\ \vec{dt} \end{bmatrix} = \begin{bmatrix} \left(\frac{df_i}{dS_i} + \tau_i \right) \cos f_i & \left(\frac{df_i}{dS_i} + \tau_i \right) \sin f_i & -\chi_i \cdot \sin f_i \\ -\left(\frac{df_i}{dS_i} + \tau_i \right) \sin f_i & \left(\frac{df_i}{dS_i} + \tau_i \right) \cos f_i & -\chi_i \cdot \cos f_i \\ \chi_i & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \vec{n} \\ \vec{b} \\ \vec{t} \end{bmatrix}$$

Principal flexural-torsional axes

$$\vec{K}_i = \left(\frac{df_i}{dS_i} + \tau_i \right) \vec{t} + \chi_i \cdot \vec{b}$$

Generalized curvature vector

$$\kappa_{xi} = \vec{K}_i \cdot \vec{i} = -\chi_i \cdot \cos f_i$$

$$\kappa_{yi} = \vec{K}_i \cdot \vec{j} = \chi_i \cdot \sin f_i$$

Projections of the curvature vector
onto the principal directions

$$\kappa_{ti} = \vec{K}_i \cdot \vec{t} = \vec{K}_i \cdot \vec{k} = \kappa_{zi} = \frac{df_i}{dS_i} + \tau_i$$

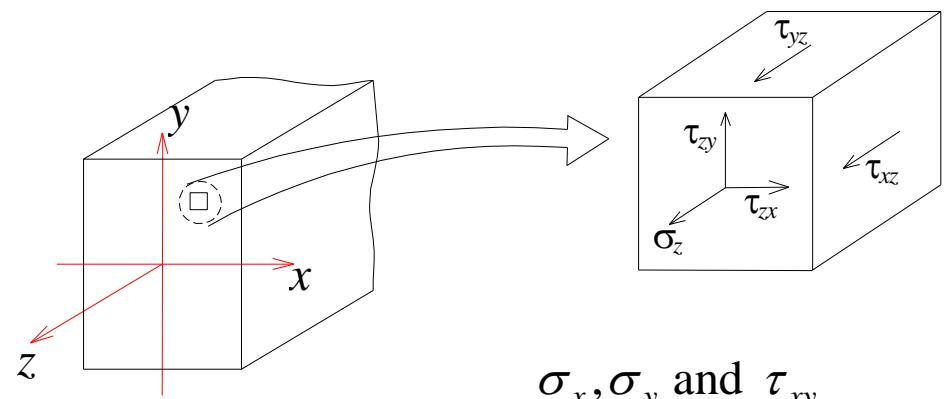
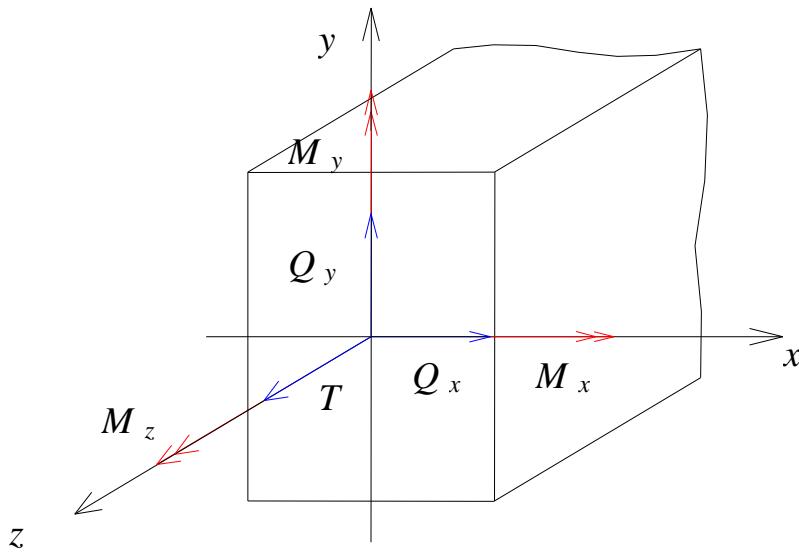
Torsion: twist + tortuosity

Principal flexural-torsional axes

$$\begin{bmatrix} \vec{n} \\ \vec{b} \\ \vec{t} \end{bmatrix} = \begin{bmatrix} \sin f_i & \cos f_i & 0 \\ -\cos f_i & \sin f_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix}$$

$$\begin{bmatrix} \vec{di} \\ \vec{dj} \\ \vec{dk} \end{bmatrix} = \begin{bmatrix} 0 & \kappa_{ti} & -\kappa_{yi} \\ -\kappa_{ti} & 0 & \kappa_{xi} \\ \kappa_{yi} & -\kappa_{xi} & 0 \end{bmatrix} \cdot \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix}$$

Differential Equilibrium Equations for a Curved Bar



σ_x, σ_y and τ_{xy}

neglectable

$$Q_x = \iint (\tau_{zx}) dx dy$$

$$M_x = \iint (\sigma_z \cdot y) dx dy$$

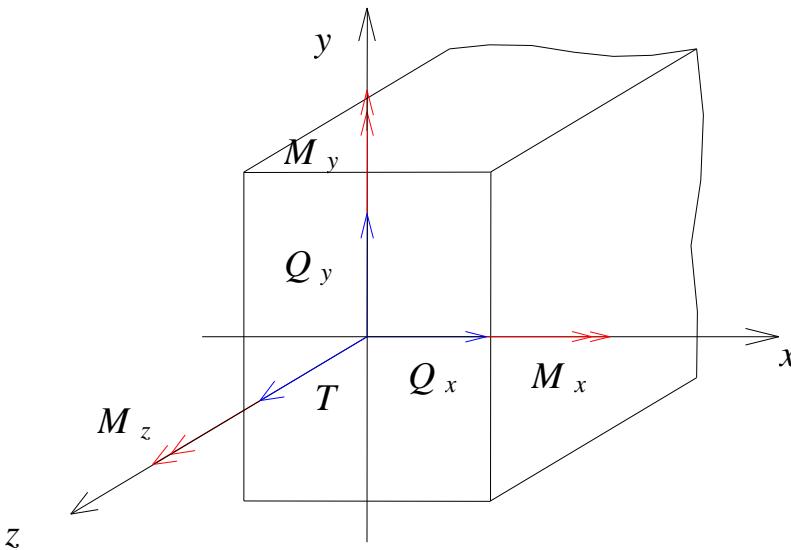
$$Q_y = \iint (\tau_{zy}) dx dy$$

$$M_y = - \iint (\sigma_z \cdot x) dx dy$$

$$T = \iint (\sigma_z) dx dy$$

$$M_z = \iint (\tau_{zy} \cdot x - \tau_{zx} \cdot y) dx dy$$

Kirschoff-Clebsh-Love Equilibrium Equations for a Curved Bar



$$(f_x; f_y; f_z) \\ (m_x; m_y; m_z)$$

External distributed forces and
couples applied to the bar

Static equilibrium leads to

$$\frac{\partial Q_x}{\partial S_i} - Q_y \cdot \kappa_{ti} + T \cdot \kappa_{yi} + f_x = 0$$

$$\frac{\partial Q_y}{\partial S_i} - T \cdot \kappa_{xi} + Q_x \cdot \kappa_{ti} + f_y = 0$$

$$\frac{\partial T}{\partial S_i} - Q_x \cdot \kappa_{yi} + Q_y \cdot \kappa_{xi} + f_z = 0$$

$$\frac{\partial M_x}{\partial S_i} - M_y \cdot \kappa_{ti} + M_z \cdot \kappa_{yi} - Q_y + m_x = 0$$

$$\frac{\partial M_y}{\partial S_i} - M_z \cdot \kappa_{xi} + M_x \cdot \kappa_{ti} + Q_x + m_y = 0$$

$$\frac{\partial M_z}{\partial S_i} - M_x \cdot \kappa_{yi} + M_y \cdot \kappa_{xi} + m_z = 0$$

Constitutive Equations for Helical Bars

Hypotheses (de Kirschoff); see, Atanackovic [1], pg. 23 :

- Bar material: homogeneous, isotropic and linear-elastic;
- Constant section;
- Plan sections remain as so (no warping);
- Transversal sections remain orthogonal with respect to the central axis;
- Normal stress components orthogonal to the central axis are neglectable.

Constitutive Equations for Helical Bars

$$\begin{aligned}
 T &= E.A^*. \varepsilon_c - E.S_y^*.[(1+\varepsilon_c).\kappa_{yi} - \kappa_{y1}] + E.S_x^*.[(1+\varepsilon_c).\kappa_{xi} - \kappa_{x1}] \\
 M_y &= -E.S_y^*. \varepsilon_c + E.I_y^*.[(1+\varepsilon_c).\kappa_{yi} - \kappa_{y1}] - E.I_{xy}^*.[(1+\varepsilon_c).\kappa_{xi} - \kappa_{x1}] \\
 M_x &= E.S_x^*. \varepsilon_c - E.I_{xy}^*.[(1+\varepsilon_c).\kappa_{yi} - \kappa_{y1}] + E.I_x^*.[(1+\varepsilon_c).\kappa_{xi} - \kappa_{x1}]
 \end{aligned}$$

Onde:

$$A^* = \iint_A \frac{1}{(1-x_p.\kappa_{y1} + y_p.\kappa_{x1})} dA$$

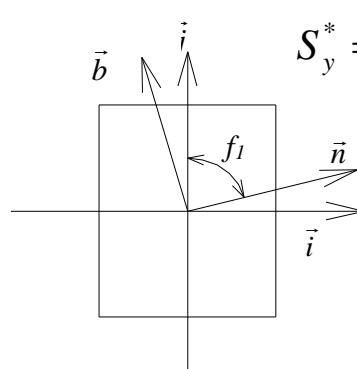
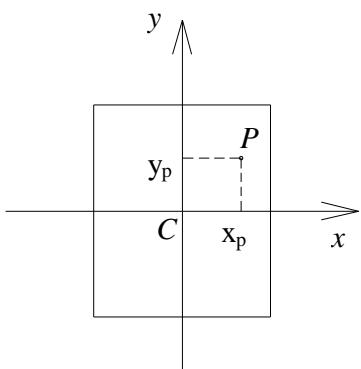
$$I_x^* = \iint_A \frac{y_p^2}{(1-x_p.\kappa_{y1} + y_p.\kappa_{x1})} dA$$

$$S_x^* = \iint_A \frac{y_p}{(1-x_p.\kappa_{y1} + y_p.\kappa_{x1})} dA$$

$$I_y^* = \iint_A \frac{x_p^2}{(1-x_p.\kappa_{y1} + y_p.\kappa_{x1})} dA$$

$$S_y^* = \iint_A \frac{x_p}{(1-x_p.\kappa_{y1} + y_p.\kappa_{x1})} dA$$

$$I_{xy}^* = \iint_A \frac{x_p \cdot y_p}{(1-x_p.\kappa_{y1} + y_p.\kappa_{x1})} dA$$



Note dependency of area and moment of inertia on curvature!!!

Constitutive Equations for Helical Bars:

Case in which principal flexural-torcional directions coincide with Frenet tried

$$\begin{aligned}
 T &= E.A^*.\varepsilon_c - E.S_y^*.[(1+\varepsilon_c).\kappa_{yi} - \chi_1] + E.S_x^*.[(1+\varepsilon_c).\kappa_{xi}] \\
 M_y &= -E.S_y^*.\varepsilon_c + E.I_y^*.[(1+\varepsilon_c).\kappa_{yi} - \chi_1] - E.I_{xy}^*.[(1+\varepsilon_c).\kappa_{xi}] \\
 M_x &= E.S_x^*.\varepsilon_c - E.I_{xy}^*.[(1+\varepsilon_c).\kappa_{yi} - \chi_1] + E.I_x^*.[(1+\varepsilon_c).\kappa_{xi}]
 \end{aligned}$$

Onde:

$$\kappa_{x1} = 0$$

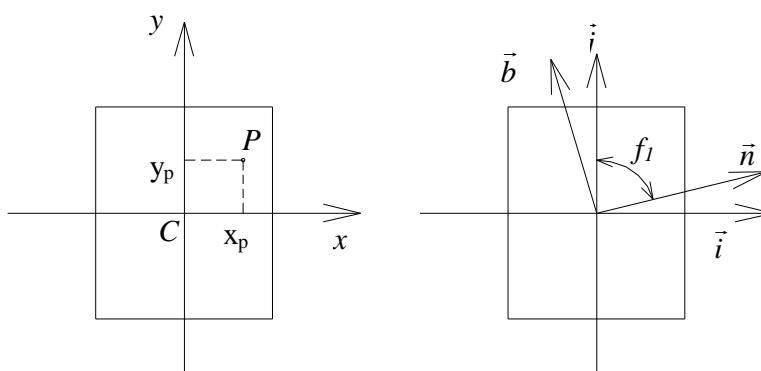
$$\kappa_{y1} = \chi_1$$

$$f_1 = \pi/2$$

$$A^* = \iint_A \frac{1}{(1-x.\chi_1)} dA \quad I_x^* = \iint_A \frac{y^2}{(1-x.\chi_1)} dA$$

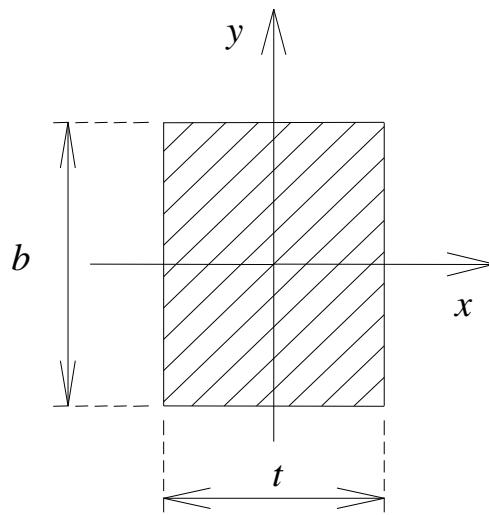
$$S_x^* = \iint_A \frac{y}{(1-x.\chi_1)} dA \quad I_y^* = \iint_A \frac{x^2}{(1-x.\chi_1)} dA$$

$$S_y^* = \iint_A \frac{x}{(1-x.\chi_1)} dA \quad I_{xy}^* = \iint_A \frac{x.y}{(1-x.\chi_1)} dA$$



Dependency of area and moment of inertia on curvature is kept!!!

Constitutive Equations for Helical Bars : Rectangular Sections



$$A^* = \frac{b}{\chi_1} \cdot \ln \left(\frac{2 + t \cdot \chi_1}{2 - t \cdot \chi_1} \right)$$

$$S_x^* = 0$$

$$S_y^* = -\frac{b \cdot t}{\chi_1} + \frac{b}{\chi_1^2} \cdot \ln \left(\frac{2 + t \cdot \chi_1}{2 - t \cdot \chi_1} \right)$$

$$I_x^* = \frac{b^3}{12 \cdot \chi_1} \cdot \ln \left(\frac{2 + t \cdot \chi_1}{2 - t \cdot \chi_1} \right)$$

$$I_y^* = -\frac{b \cdot t}{\chi_1^2} + \frac{b}{\chi_1^3} \cdot \ln \left(\frac{2 + t \cdot \chi_1}{2 - t \cdot \chi_1} \right)$$

$$I_{xy}^* = 0$$

Up to second-order in curvature:

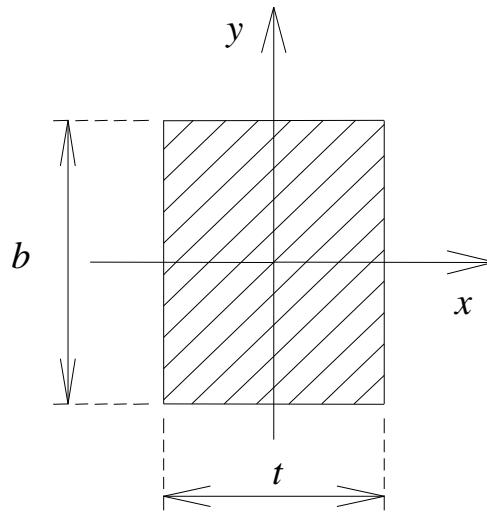
$$A^* = b \cdot t \left[1 + \frac{(t \cdot \chi_1)^2}{12} + O((t \cdot \chi_1)^4) \right]$$

$$S_y^* = \frac{b \cdot t^2}{12} \left[t \cdot \chi_1 + O((t \cdot \chi_1)^3) \right]$$

$$I_x^* = \frac{t \cdot b^3}{12} \left[1 + \frac{(t \cdot \chi_1)^2}{12} + O((t \cdot \chi_1)^4) \right]$$

$$I_y^* = \frac{b \cdot t^3}{12} \left[1 + O((t \cdot \chi_1)^2) \right]$$

Constitutive Equations for Helical Bars : Rectangular Sections



$$T \cong E.\varepsilon_c.(bt) - E.\frac{b.t^2}{12}(t.\chi_1).[\kappa_{yi} - \chi_1]$$

$$M_y \cong -E.\frac{b.t^2}{12}(t.\chi_1).\varepsilon_c + E.\frac{b.t^3}{12}.[\kappa_{yi} - \chi_1]$$

$$M_x \cong E.\frac{t.b^3}{12}.\kappa_{xi}$$

...up to second-order
in curvature.

In first-order:

$$T = EA.\varepsilon_c$$

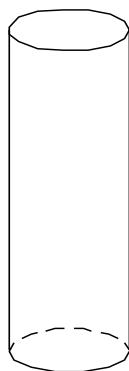
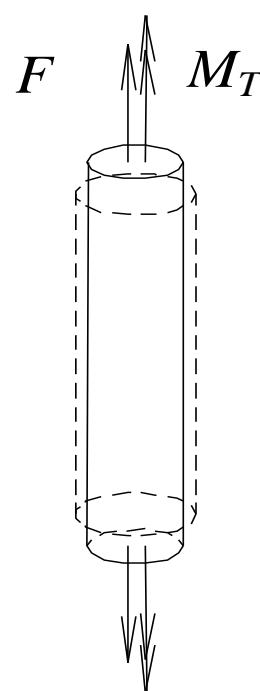
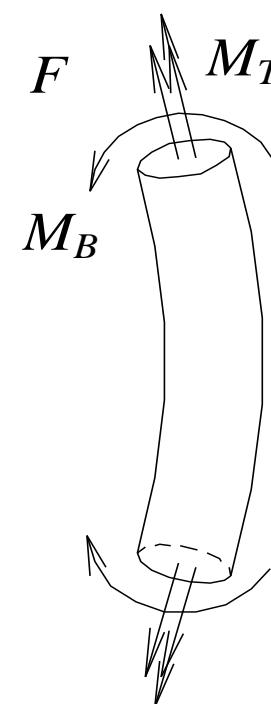
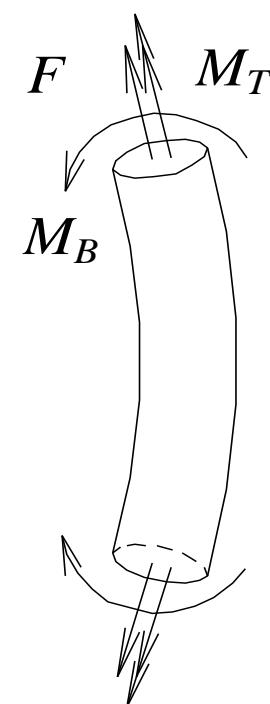
$$M_y = EI_y.(\kappa_{yi} - \kappa_{y1}) = EI_y.(\kappa_{yi} - \chi_1)$$

$$M_x = EI_x.(\kappa_{xi} - \kappa_{x1}) = EI_x.\kappa_{xi}$$

$$A = b.t , \quad I_{xx} = \frac{t.b^3}{12} \quad \text{e} \quad I_{yy} = \frac{b.t^3}{12}$$

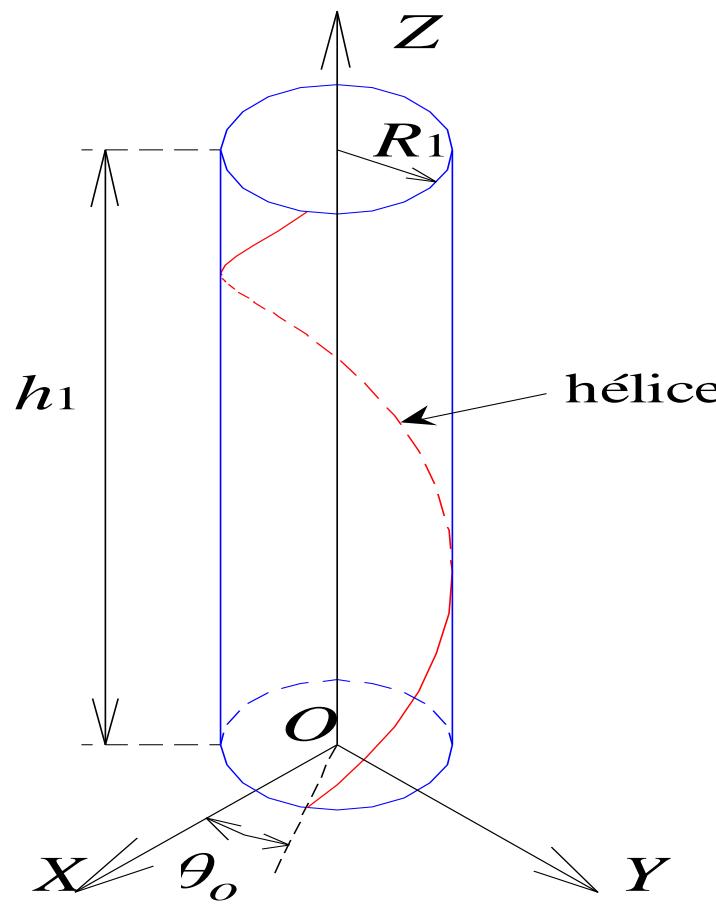
Equations for Helical Components

*Four deformed configurations:
see Ramos Jr, R., 2001 (Appendix C)*

 Σ_1  Σ_2  Σ_3  Σ_4 

Configuration 1: Undeformed supporting cylinder

Ramos Jr., 2001, eqs. (c.1)



$$x_1 = R_1 \cdot \cos \theta_1$$

$$y_1 = R_1 \cdot \sin \theta_1$$

$$z_1 = \frac{h_1 \cdot (\theta_1 - \theta_0)}{2\pi} = \frac{R_1 \cdot (\theta_1 - \theta_0)}{\tan \alpha_1}$$

$$\chi_1 = \left\| \frac{d\vec{t}_1}{dS_1} \right\| = \frac{\sin^2 \alpha_1}{R_1}$$

$$\tau_1 = \left\| \frac{d\vec{b}_1}{dS_1} \right\| = \frac{\sin \alpha_1 \cdot \cos \alpha_1}{R_1}$$

Curvature and torsion in configuration 1

Ramos Jr., 2001, eqs. (c.15-16)

$$\kappa_{x1} = -\chi_1 \cdot \cos f_1$$

$$\kappa_{y1} = \chi_1 \cdot \sin f_1$$

$$f_1 = \frac{\pi}{2} \Rightarrow \frac{df_1}{dS_1} = 0$$

$$\kappa_{t1} = \frac{df_1}{dS_1} + \tau_1$$



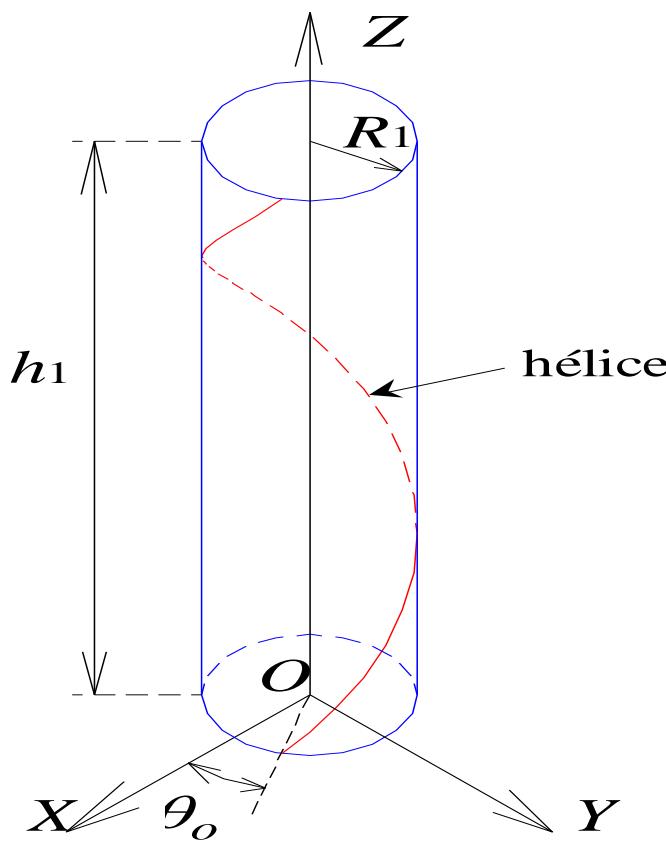
$$\kappa_{x1} = 0$$

$$\kappa_{y1} = \chi_1 = \frac{\sin^2 \alpha_1}{R_1}$$

$$\kappa_{t1} = \tau_1 = \frac{\sin \alpha_1 \cdot \cos \alpha_1}{R_1}$$

Configuration 2: Stretching and torsion without bending

Ramos Jr., 2001, eqs. (c.17)



$$x_2 = R_2 \cdot \cos \theta_2$$

$$y_2 = R_2 \cdot \sin \theta_2$$

$$z_2 = \frac{h_2 \cdot (\theta_2 - \theta_o)}{2\pi} = \frac{R_2 \cdot (\theta_2 - \theta_o)}{\tan \alpha_2}$$

$$\chi_2 = \frac{\sin^2 \alpha_2}{R_2}$$

$$\tau_2 = \frac{\sin \alpha_2 \cdot \cos \alpha_2}{R_2}$$

Strains in configuration 2

Ramos Jr, 2001, eqs. (c.42) e (c.48)

$$\bar{\varepsilon}_{12} = (\varepsilon_r + \varepsilon_\theta) \cdot \sin^2 \alpha_1 + \varepsilon_h \cdot \cos^2 \alpha_1$$

$$\Delta \alpha_{12} = \sin \alpha_1 \cdot \cos \alpha_1 \cdot (\varepsilon_\theta + \varepsilon_r - \varepsilon_h)$$

$$\varepsilon_h = \frac{\Delta h}{h_1} = \frac{\Delta L}{L_1}$$

$$\varepsilon_\theta = \frac{\Delta \phi}{\varphi_1} = \frac{\Delta \phi}{2\pi \cdot N_1} = \frac{\Delta \phi \cdot h_1}{2\pi \cdot L_1}$$

$$\varepsilon_r = \frac{\Delta R}{R_1} = \frac{R_2 - R_1}{R_1}$$

Strains in configuration 2

Ramos Jr, 2001, eqs. (c.43) e (c.49)

$$\bar{\varepsilon}_{12} = (\sin^2 \alpha_1) \cdot \frac{\Delta R}{R_1} + (\sin \alpha_1 \cdot \cos \alpha_1) \cdot R_1 \cdot \frac{\Delta \varphi}{L_1} + (\cos^2 \alpha_1) \cdot \frac{\Delta L}{L_1}$$

$$\Delta \alpha_{12} = \cos^2 \alpha_1 \cdot R_1 \frac{\Delta \varphi}{L_1} + \sin \alpha_1 \cdot \cos \alpha_1 \left(\frac{\Delta R}{R_1} - \frac{\Delta L}{L_1} \right)$$

Curvature and tortuosity variations in configuration 2

Ramos Jr, 2001, eqs. (c.55;56) e (c.58;59)

$$\Delta\chi_{12} = \chi_1 \cdot \left(\frac{2 \cdot \Delta\alpha}{\tan \alpha_1} - \frac{\Delta R}{R_1} \right)$$

ou

$$\Delta\chi_{12} = \chi_1 \cdot \left(2 \cos^2 \alpha_1 \cdot (\varepsilon_\theta + \varepsilon_r - \varepsilon_h) - \varepsilon_r \right)$$

$$\Delta\tau_{12} = \tau_1 \cdot \left[\left(\frac{2}{\tan 2\alpha_1} \right) \cdot \Delta\alpha_{12} - \frac{\Delta R}{R_1} \right]$$

ou

$$\Delta\tau_{12} = \tau_1 \cdot [\cos(2\alpha_1) \cdot (\varepsilon_\theta + \varepsilon_r - \varepsilon_h) - \varepsilon_r]$$

$$\chi_1 = \left\| \frac{d\vec{t}_1}{dS_1} \right\| = \frac{\sin^2 \alpha_1}{R_1}$$

$$\tau_1 = \left\| \frac{d\vec{b}_1}{dS_1} \right\| = \frac{\sin \alpha_1 \cdot \cos \alpha_1}{R_1}$$

Curvature and torsion variations in configuration 2

Ramos Jr, 2001, eqs. (c.60-61)

$$\kappa_{x2} = -\chi_2 \cdot \cos f_2$$

$$\kappa_{y2} = \chi_2 \cdot \sin f_2$$

$$f_2 = \frac{\pi}{2} \Rightarrow \frac{df_2}{dS_2} = 0$$

$$\kappa_{t2} = \frac{df_2}{dS_2} + \tau_2$$



$$\kappa_{x2} = 0$$

$$\kappa_{y2} = \chi_2 = \frac{\sin^2 \alpha_2}{R_2}$$

$$\kappa_{t2} = \tau_2 = \frac{\sin \alpha_2 \cdot \cos \alpha_2}{R_2}$$

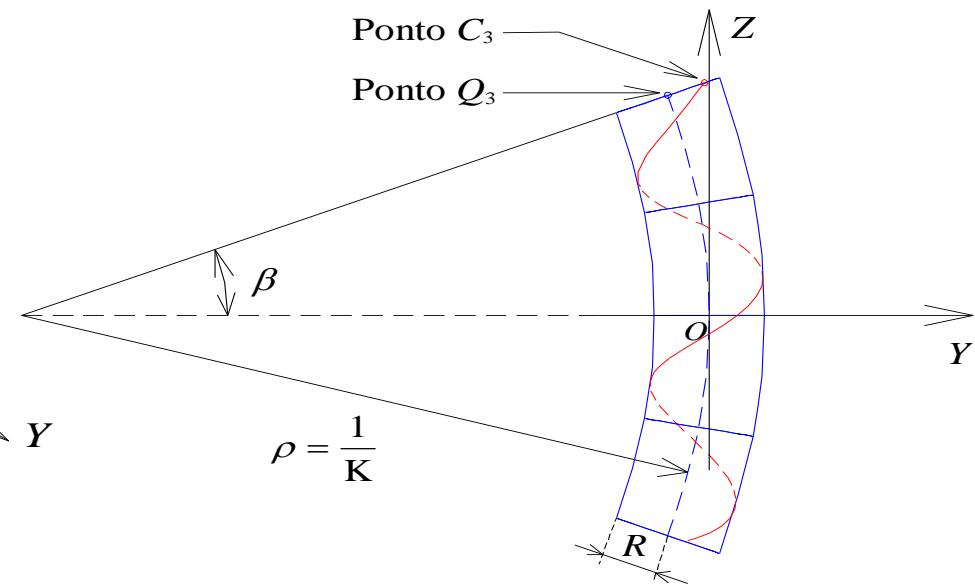
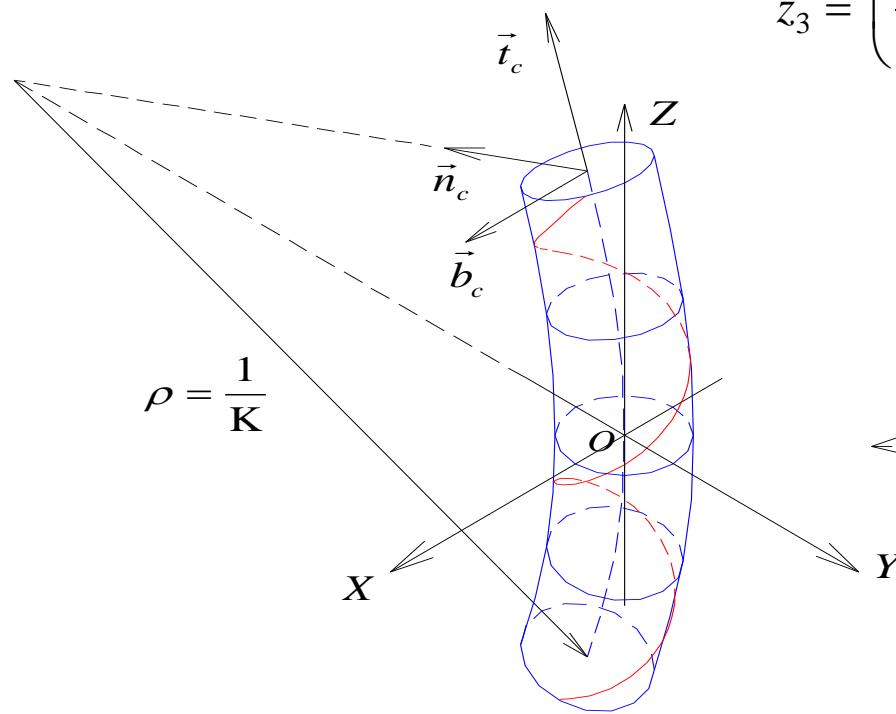
Configuration 3: bending without slipping

Ramos Jr., 2001, eqs. (c.65)

$$x_3 = R_2 \cdot \cos \theta_2$$

$$y_3 = \left(\frac{1}{K} + R_2 \cdot \sin \theta_2 \right) \cdot \cos \left(\frac{K \cdot R_2 \cdot (\theta_2 - \theta_o)}{\tan \alpha_2} \right) - \frac{1}{K}$$

$$z_3 = \left(\frac{1}{K} + R_2 \cdot \sin \theta_2 \right) \cdot \sin \left(\frac{K \cdot R_2 \cdot (\theta_2 - \theta_o)}{\tan \alpha_2} \right)$$

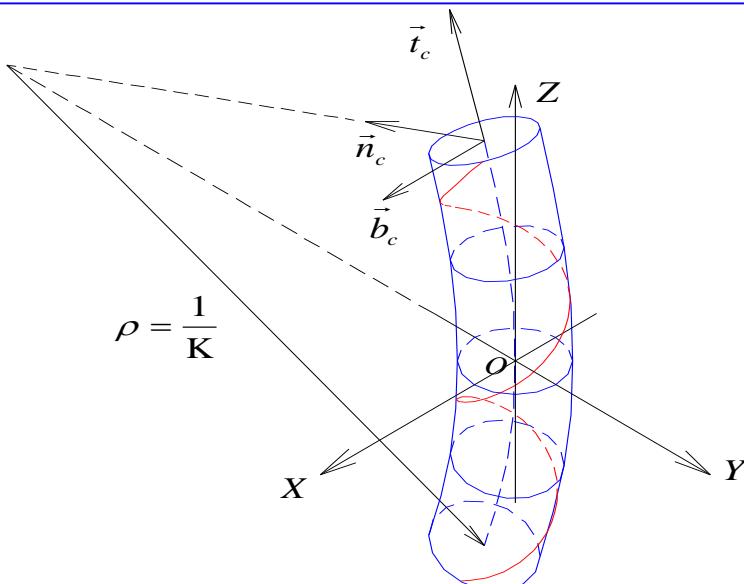


Curvature and Tortuosity in configuration 3

Ramos Jr, 2001, eq. (c.74) e (c.79)

$$\chi_3 = \frac{[f_1(\alpha_2, \eta, \theta_2) + f_2(\alpha_2, \eta, \theta_2)]^{1/2}}{R_2 \cdot \xi^3}$$

$$\tau_3 = \tau_1 \left[1 - \frac{\Delta R}{R_1} + \frac{2}{\tan 2\alpha_1} \cdot \Delta \alpha + \frac{(2\cos^4 \alpha_1 - \cos^2 \alpha_1 - 2) \cdot \sin \theta_1}{\sin^2 \alpha_1} \cdot K R_1 \right]$$



Auxiliary Functions for Curvature and Tortuosity in configuration 3

Ramos Jr, 2001, eq. (c.74)

$$f_1(\alpha_2, \eta, \theta_2) = \eta^2 \cdot \cos^2 \theta_2 \cdot \cos^2 \alpha_2 \cdot (\xi^2 + \sin^2 \alpha_2)^2$$

$$f_2(\alpha_2, \eta, \theta_2) = \xi^2 \cdot (\eta \cdot (1 + \eta \cdot \sin \theta_2) \sin \theta_2 \cdot \cos^2 \alpha_2 + \sin^2 \alpha_2)^2$$

$$\eta = K \cdot R_2$$

$$\xi(\alpha_2, \eta, \theta_2) = [\sin^2 \alpha_2 + \cos^2 \alpha_2 \cdot (1 + \eta \cdot \sin \theta_2)^2]^{1/2}$$

$$\theta_2 = \theta_1 + \frac{\Delta\varphi \cdot R_1 \cdot (\theta_1 - \theta_0)}{L_1 \cdot \tan \alpha_1}$$

Strains in configuration 3

Ramos Jr, 2001, eqs. (c.83)

$$\bar{\varepsilon}_{23} = \xi - 1 = [\sin^2 \alpha_2 + \cos^2 \alpha_2 \cdot (1 + \eta \cdot \sin \theta_2)^2]^{1/2} - 1$$



$$\bar{\varepsilon}_{23} \cong K \cdot R_1 \cdot \cos^2 \alpha_1 \cdot \sin \theta_1$$

$$\bar{\varepsilon}_{13} = (1 + \bar{\varepsilon}_{23}) \cdot (1 + \bar{\varepsilon}_{12}) - 1 \cong \bar{\varepsilon}_{12} + \bar{\varepsilon}_{23}$$

$$\bar{\varepsilon}_{13} = (\sin^2 \alpha_1) \frac{\Delta R}{R_1} + (\sin \alpha_1 \cdot \cos \alpha_1) R_1 \frac{\Delta \varphi}{L_1} + (\cos^2 \alpha_1) \frac{\Delta L}{L_1} + K R_1 \cos^2 \alpha_1 \cdot \sin \theta_1$$

$$\bar{\varepsilon}_{12}$$

Curvature and Torsion in configuration 3

Ramos Jr, 2001, eqs. (c.90-95)

$$\kappa_{x3} = -\chi_3 \cdot \cos f_3$$

$$\kappa_{y3} = \chi_3 \cdot \sin f_3$$

$$\kappa_{t3} = \frac{df_3}{dS_3} + \tau_3$$

$$\frac{df_3}{dS_3} \cong \frac{(2 - \cos^2 \alpha_1) \cdot \sin \theta_1}{\tan \alpha_1} \cdot K$$



$$\kappa_{x3} \cong (-2 + \cos^2 \alpha_1) \cos \alpha_1 \cdot \cos \theta_1 \cdot K$$

$$\kappa_{y3} \cong \chi_1 \left[1 - \frac{\Delta R}{R_1} + \frac{2}{\tan \alpha_1} \Delta \alpha + \frac{\cos(2\alpha_1)}{\tan^2 \alpha_1} \sin \theta_1 \cdot K R_1 \right]$$

$$\kappa_{t3} \cong \tau_1 \cdot \left[1 - \frac{\Delta R}{R_1} + \frac{2}{\tan 2\alpha_1} \Delta \alpha - 2 \cos^2 \alpha_1 \cdot \sin \theta_1 \cdot K R_1 \right]$$

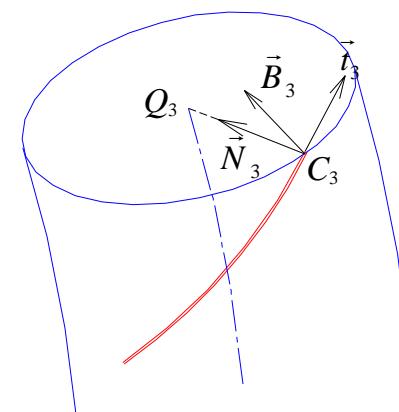
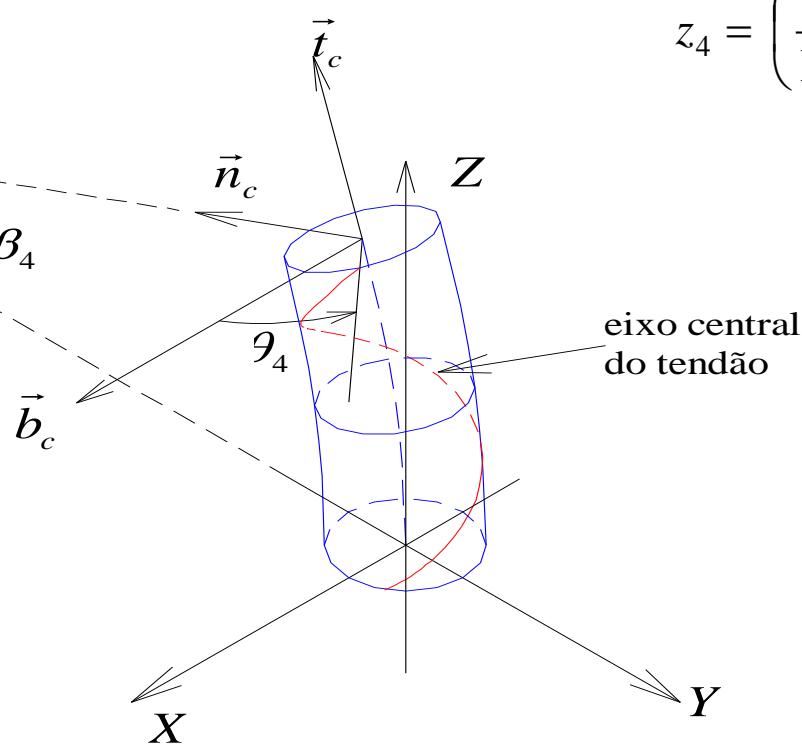
Configuration 4: bending with slipping

Ramos Jr, 2001, eqs. (c.98)

$$x_4 = R_2 \cdot \cos \theta_4$$

$$y_4 = \left(\frac{1}{K} + R_2 \cdot \sin \theta_4 \right) \cdot \cos \beta_4 - \frac{1}{K}$$

$$z_4 = \left(\frac{1}{K} + R_2 \cdot \sin \theta_4 \right) \cdot \sin \beta_4$$



Darboux-Ribaucourt axes

Configuration 4: bending with slipping

Ramos Jr, 2001, eqs. (c.98)

$$\vec{\Delta} \cong \Delta_t \cdot \vec{t}_3 + \Delta_b \cdot \vec{B}_3$$

escorregamento

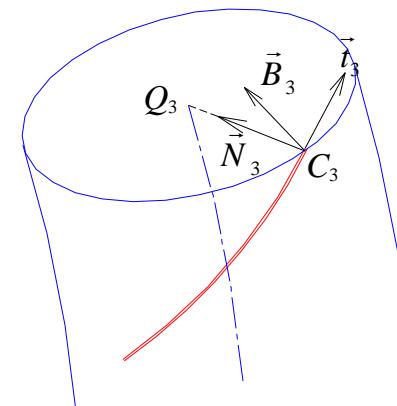
$$\Delta_t = \Delta_t(\theta_2) = \Delta_{tc} \cdot \cos \theta_2 + \Delta_{ts} \cdot \sin \theta_2$$

$$\Delta_b = \Delta_b(\theta_2) = \Delta_{bc} \cdot \cos \theta_2 + \Delta_{bs} \cdot \sin \theta_2$$

$$x_4 = R_2 \cdot \cos \theta_2 + \Delta_t \cdot t_{x3} + \Delta_b \cdot B_{x3}$$

$$y_4 = \left(\frac{1}{K} + R_2 \cdot \sin \theta_2 \right) \cdot \cos \beta - \frac{1}{K} + \Delta_t \cdot t_{y3} + \Delta_b \cdot B_{y3}$$

$$z_4 = \left(\frac{1}{K} + R_2 \cdot \sin \theta_2 \right) \cdot \sin \beta + \Delta_t \cdot t_{z3} + \Delta_b \cdot B_{z3}$$



Darboux-Ribaucourt axes

Configuration 4: slipping

Ramos Jr, 2001, eqs. (c.142-143;147)

$$\vec{\Delta} \cong \Delta_t \cdot \vec{t}_3 + \Delta_b \cdot \vec{B}_3$$

slippage

$$\Delta_{bc} = K \cdot R_1^2 \cdot \cos \alpha_1 \cdot \left(1 + \frac{1}{\sin^2 \alpha_1} \right)$$

$$\Delta_{bs} = 0$$

$$\Delta_{tc} = \frac{K \cdot R_1^2 \cdot \cos^2 \alpha_1}{\sin \alpha_1}$$

$$\Delta_{ts} = 0$$

$$\Delta_t(\theta_2) = \frac{K \cdot R_1^2 \cdot \cos^2 \alpha_1}{\sin \alpha_1} \cdot \cos \theta_2$$

$$\Delta_b(\theta_2) = K \cdot R_1^2 \cdot \cos \alpha_1 \cdot \left(1 + \frac{1}{\sin^2 \alpha_1} \right) \cdot \cos \theta_2$$

Strains in configuration 4

Ramos Jr, 2001, eqs. (c. 112-113)

$$\bar{\varepsilon}_{34} \cong \frac{\sin \alpha_1 \cdot \cos \theta_1}{R_1} \cdot \Delta_{ts} - \frac{\sin \alpha_1 \cdot \sin \theta_1}{R_1} \cdot \Delta_{tc}$$

$$\begin{aligned}\bar{\varepsilon}_{14} &= (1 + \bar{\varepsilon}_{34})(1 + \bar{\varepsilon}_{23})(1 + \bar{\varepsilon}_{12}) - 1 \cong \\ &\cong \bar{\varepsilon}_{12} + \bar{\varepsilon}_{23} + \bar{\varepsilon}_{34}\end{aligned}$$



$$\begin{aligned}\bar{\varepsilon}_{14} \cong & (\sin^2 \alpha_1) \frac{\Delta R}{R_1} + (\sin \alpha_1 \cdot \cos \alpha_1) R_1 \frac{\Delta \varphi}{L_1} + (\cos^2 \alpha_1) \frac{\Delta L}{L_1} + \\ & + K R_1 \cos^2 \alpha_1 \cdot \sin \theta_1 + \frac{\sin \alpha_1 \cdot \cos \theta_1}{R_1} \cdot \Delta_{ts} - \frac{\sin \alpha_1 \cdot \sin \theta_1}{R_1} \cdot \Delta_{tc}\end{aligned}$$

Strains in configuration 4

Ramos Jr, 2001, eqs. (c.112-113)

$$\Delta_{tc} = \frac{K \cdot R_1^2 \cdot \cos^2 \alpha_1}{\sin \alpha_1}$$

$$\Delta_{ts} = 0$$



$$\bar{\varepsilon}_{14} \cong (\sin^2 \alpha_1) \frac{\Delta R}{R_1} + (\sin \alpha_1 \cdot \cos \alpha_1) R_1 \frac{\Delta \phi}{L_1} + (\cos^2 \alpha_1) \frac{\Delta L}{L_1} +$$
$$+ K R_1 \cos^2 \alpha_1 \cdot \sin \theta_1 + \frac{\sin \alpha_1 \cdot \cos \theta_1}{R_1} \cdot (0) - \frac{\sin \alpha_1 \cdot \sin \theta_1}{R_1} \cdot \frac{K_1 R_1^2 \cos^2 \alpha_1}{\sin \alpha_1}$$

Strains in configuration 4

Ramos Jr, 2001, eqs. (c.112-113)

$$\bar{\varepsilon}_{14} \cong (\sin^2 \alpha_1) \frac{\Delta R}{R_1} + (\sin \alpha_1 \cdot \cos \alpha_1) R_1 \frac{\Delta \phi}{L_1} + (\cos^2 \alpha_1) \frac{\Delta L}{L_1}$$

Curvature and Tortuosity in configuration 4

Ramos Jr, 2001, eq. (c.117) e (c.121)

$$\begin{aligned}\chi_4 \cong & \chi_1 \left[1 - \frac{\Delta R}{R_1} + \frac{2}{\tan \alpha_1} \cdot \Delta \alpha_{12} + \frac{\sin \theta_1 \cdot \cos(2\alpha_1)}{\tan^2 \alpha_1} \cdot KR_1 \right] + \\ & + \chi_1 \left[2 \cdot \sin \theta_1 \cdot \cos \alpha_1 \cdot \frac{\Delta_{bc}}{R_1} - 2 \cdot \cos \theta_1 \cdot \cos \alpha_1 \cdot \frac{\Delta_{bs}}{R_1} \right]\end{aligned}$$

$$\begin{aligned}\tau_4 = & \tau_1 \left[1 - \frac{\Delta R}{R_1} + \frac{2}{\tan 2\alpha_1} \cdot \Delta \alpha_{12} + \frac{(2 \cos^4 \alpha_1 - \cos^2 \alpha_1 - 2) \cdot \sin \theta_1}{\sin^2 \alpha_1} \cdot KR_1 \right] + \\ & + \tau_1 \left[2 \cdot \cos \alpha_1 \cdot \sin \theta_1 \cdot \frac{\Delta_{bc}}{R_1} - 2 \cdot \cos \alpha_1 \cdot \cos \theta_1 \cdot \frac{\Delta_{bs}}{R_1} \right]\end{aligned}$$

Curvature and torsion in configuration 4

Ramos Jr, 2001, eqs. (c. 130-134)

$$\kappa_{x4} = -\chi_4 \cdot \cos f_4$$

$$\kappa_{y4} = \chi_4 \cdot \sin f_4$$

$$\kappa_{t4} = \frac{df_4}{dS_4} + \tau_4$$

$$\frac{df_4}{dS_4} \equiv \frac{(2 - \cos^2 \alpha_1) \cdot \sin \theta_1}{\tan \alpha_1} \cdot K +$$

$$-\frac{\sin \alpha_1 \cdot \sin \theta_1}{R_1^2} \cdot \Delta_{bc} + \frac{\sin \alpha_1 \cdot \cos \theta_1}{R_1^2} \cdot \Delta_{bs}$$



$$\kappa_{x4} \equiv (-2 + \cos^2 \alpha_1) \cos \alpha_1 \cdot \cos \theta_1 \cdot K + \chi_1 \left[\cos \theta_1 \cdot \frac{\Delta_{bc}}{R_1} + \sin \theta_1 \cdot \frac{\Delta_{bs}}{R_1} \right]$$

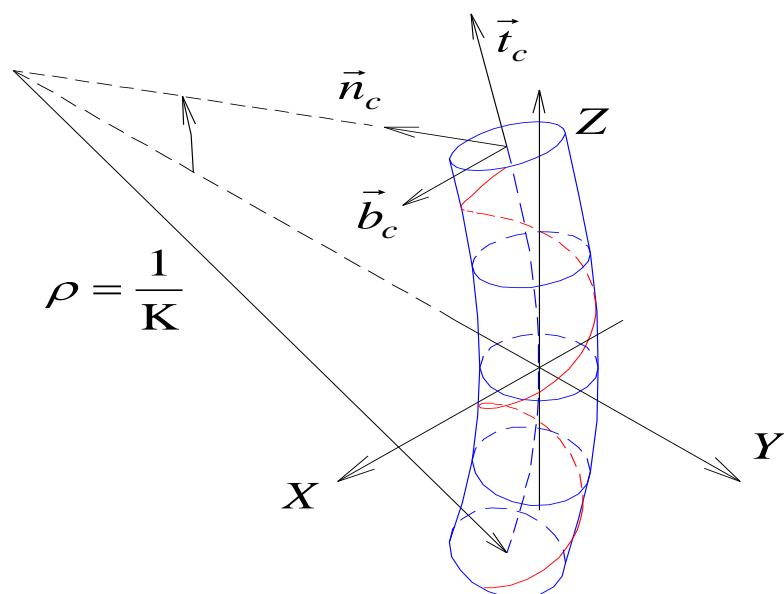
$$\kappa_{y4} \equiv \chi_1 \left[1 - \frac{\Delta R}{R_1} + \frac{2}{\tan \alpha_1} \Delta \alpha + \frac{\cos(2\alpha_1)}{\tan^2 \alpha_1} \sin \theta_1 \cdot K R_1 \right] +$$

$$+ 2 \cdot \cos \alpha_1 \cdot \chi_1 \left[\sin \theta_1 \cdot \frac{\Delta_{bc}}{R_1} - \cos \theta_1 \cdot \frac{\Delta_{bs}}{R_1} \right]$$

$$\kappa_{t4} = \tau_1 \left[1 - \frac{\Delta R}{R_1} + \frac{2}{\tan 2\alpha_1} \cdot \Delta \alpha - 2 \cdot \cos^2 \alpha_1 \cdot \sin \theta_1 \cdot K R_1 \right] +$$

$$+ \tau_1 \cdot \frac{\cos(2\alpha_1)}{\cos \alpha_1} \left[\sin \theta_1 \cdot \frac{\Delta_{bc}}{R_1} - \cos \theta_1 \cdot \frac{\Delta_{bs}}{R_1} \right]$$

Sectional loading in configuration Σ_4



Normal force:

$$F_i$$

Twist moment:

$$M_{t,i}$$

Bending moment:

$$M_{f,i}$$

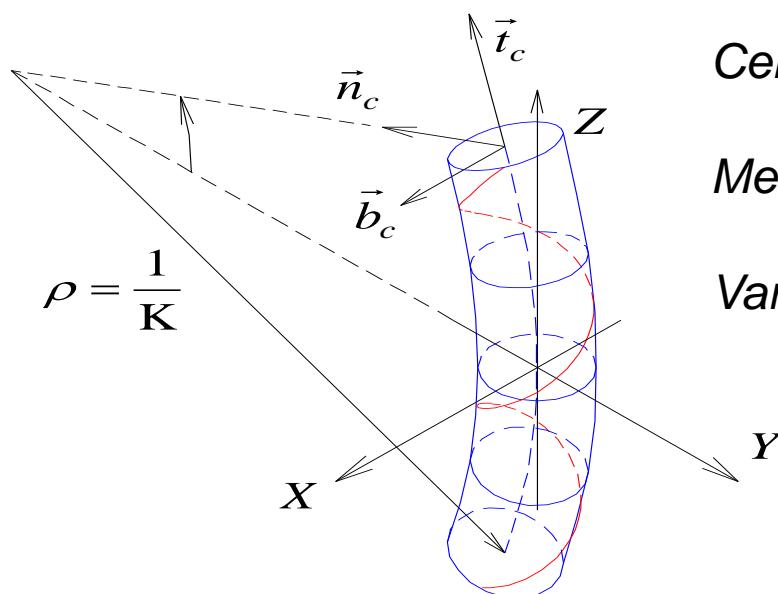
Inter-layer contact pressure:

$$p_{c,i} \text{ e } p_{c,i-1}$$

i : layer

j : helical element

Deformations in the configuration Σ_4



Stretching:

$$\Delta L / L$$

Rotation per unit length:

$$\Delta\varphi / L$$

Central axis curvature:

$$K$$

Mean radius variation:

$$\Delta R_i$$

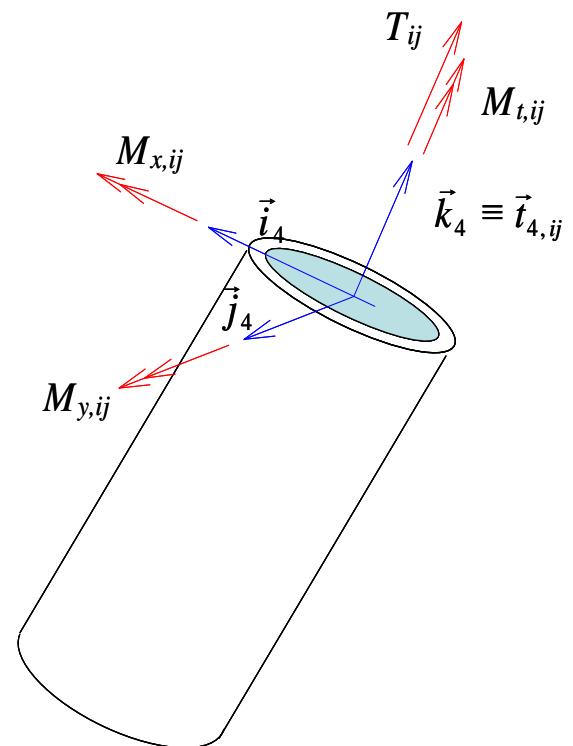
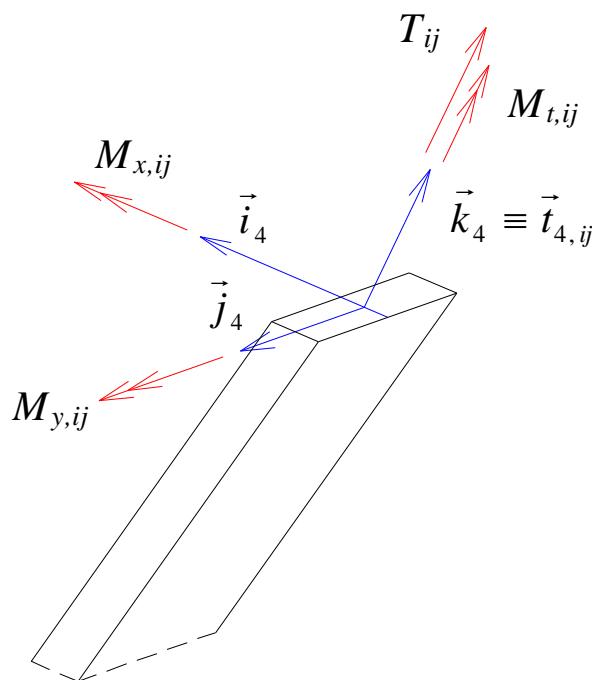
Variation of helical angle:

$$\Delta\alpha_i$$

i : layer

j : helical element

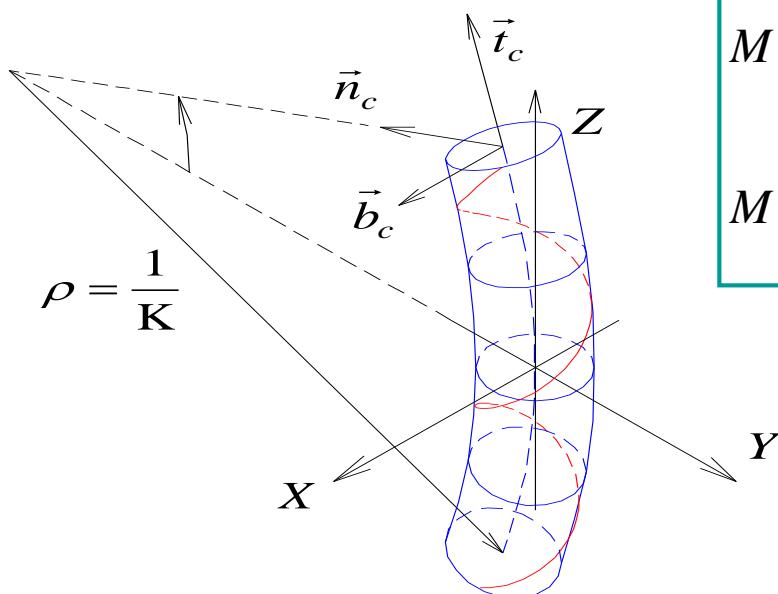
Soliciting efforts on a generic helical tendon section in configuration Σ_4



i : layer

j : helical element

Equilibrium equations, layer i , deformed configuration Σ_4



$$F_i = \sum_{j=1}^{n_i} (T_{ij} \cdot \vec{t}_{ij}) \cdot \vec{t}_c$$

$$M_{t,i} = \sum_{j=1}^{n_i} (M_{x,ij} \cdot \vec{i}_4 + M_{y,ij} \cdot \vec{j}_4 + M_{t,ij} \cdot \vec{k}_4 + T_{ij} \cdot R_{2,i} \cdot \vec{j}_4) \cdot \vec{t}_c$$

$$M_{t,i} = \sum_{j=1}^{n_i} (M_{x,ij} \cdot \vec{i}_4 + M_{y,ij} \cdot \vec{j}_4 + M_{t,ij} \cdot \vec{k}_4 + T_{ij} \cdot R_{2,i} \cdot \vec{j}_4) \cdot \vec{b}_c$$

i : denota a camada

j : denota o elemento

*Equilibrium equations, layer i, deformed configuration Σ_4
no internal pressure*

$$F_i = n_i \cdot (EA)_i \cdot \left[(\cos^3 \alpha_i) \cdot \frac{\Delta L}{L} + (\sin \alpha_i \cdot \cos^2 \alpha_i) \cdot R_i \frac{\Delta \varphi}{L} + (\sin^2 \alpha_i \cdot \cos \alpha_i) \cdot \frac{\Delta R_i}{R_i} \right]$$

$$\begin{aligned} M_{t,i} = & \frac{n_i}{R_i} \cdot \left[(GI_t)_i \cdot \cos \alpha_i \cdot \cos(2\alpha_i) + (EI_y)_i \cdot \sin \alpha_i \cdot \sin(2\alpha_i) \right] \Delta \alpha_i + \\ & + \left[n_i (EA)_i R_i \sin \alpha_i \cos^2 \alpha_i \right] \frac{\Delta L}{L} + \left[n_i (EA)_i R_i \cdot \cos \alpha_i \cdot \sin^2 \alpha_i \right] R_i \frac{\Delta \varphi}{L} + \\ & + n_i \sin \alpha_i \left[(EA)_i R_i \sin^2 \alpha_i - \frac{(GI_t)_i \cos^2 \alpha_i}{R_i} - \frac{(EI_y)_i \sin^2 \alpha_i}{R_i} \right] \cdot \frac{\Delta R_i}{R_i} \end{aligned}$$

$$M_{f,i} = K \cdot n_i \cdot \cos \alpha_i \cdot \left[(GI_t)_i + \frac{3}{2} ((EI_y)_i - (GI_t)_i) \cos^2 \alpha_i \right]$$

Bending uncoupled from
traction and torsion

Equilibrium equations, layer i, deformed configuration Σ_4 under internal pressure

$$F_i = n_i \cdot (EA)_i \left[(\cos^3 \alpha_i) \cdot \frac{\Delta L}{L} + (\sin \alpha_i \cdot \cos^2 \alpha_i) \cdot R_i \frac{\Delta \varphi}{L} + (\sin^2 \alpha_i \cdot \cos \alpha_i) \cdot \frac{\Delta R_i}{R_i} \right] +$$

$$+ n_i \cdot (EA)_i \cdot \cos \alpha_i \cdot \left[\nu_i \frac{r_i}{t_{Ti}} \frac{(p_{in,i} - \bar{p}_{c,i})}{E_i} \right]$$

$\bar{p}_{c,i} = \frac{1}{2}(p_{c,i} + p_{c,i-1})$

$$M_{t,i} = \frac{n_i}{R_i} \cdot [(GI_t)_i \cdot \cos \alpha_i \cdot \cos(2\alpha_i) + (EI)_i \cdot \sin \alpha_i \cdot \sin(2\alpha_i)] \Delta \alpha_i +$$

$$+ n_i (EA)_i R_i \sin \alpha_i \cos^2 \alpha_i \cdot \frac{\Delta L}{L} + [n_i (EA)_i R_i \cdot \cos \alpha_i \cdot \sin^2 \alpha_i] R_i \frac{\Delta \varphi}{L} +$$

$$+ n_i \sin \alpha_i \left[(EA)_i R_i \sin^2 \alpha_i - \frac{(GI_t)_i \cos^2 \alpha_i}{R_i} - \frac{(EI)_i \sin^2 \alpha_i}{R_i} \right] \cdot \frac{\Delta R_i}{R_i} +$$

$$+ n_i (EA)_i R_i \sin \alpha_i \cdot \left[\nu_i \frac{r_i}{t_{Ti}} \frac{(p_{in,i} - \bar{p}_{c,i})}{E_i} \right]$$

Bending uncoupled from traction and torsion

$$M_{f,i} = K \cdot n_i \cdot \cos \alpha_i \cdot (EI)_i \cdot \frac{(2 + 3\nu_i \cdot \cos^2 \alpha_i)}{2 \cdot (1 + \nu_i)}$$

Bending equilibrium equations, layer i , of helical elements in configuration Σ_3 solid or non-pressurized elements

$$M_{f,i} = K n_i \cos \alpha_i \frac{1}{2} \left\{ (EA)_i R_i^2 \cos^3 \alpha_i + 2(GI_t)_i \sin^2 \alpha_i \cos^2 \alpha_i + \right. \\ \left. + [(EI_x)_i (2 - \cos^2 \alpha_i) + (EI_y)_i \cos^2 \alpha_i (2 \cos^2 \alpha_i - 1)] \right\}$$

$$M_{f,i} = K \cdot n_i \cdot \cos \alpha_i \cdot \frac{1}{2} \left\{ (EA)_i R_i^2 \cos^3 \alpha_i + (EI)_i \frac{(2 + 3\nu_i \cdot \cos^2 \alpha_i)}{(1 + \nu_i)} \right\}$$

Solid circular section

Bending uncoupled
from traction and
torsion

Bending equilibrium equations, layer i , of helical elements in configuration Σ_3 tubular, pressurized elements

$$\begin{aligned} M_{f,i} = & K \cdot n_i \cdot \cos \alpha_i \cdot \frac{1}{2} \left\{ (EA)_i \cdot R_i^2 \cos^3 \alpha_i + (EI)_i \frac{(2 + 3\nu_i \cdot \cos^2 \alpha_i)}{(1 + \nu_i)} \right\} + \\ & + (EA)_i R_i \cos \alpha_i \left(\sum_{j=1}^{n_i} \sin \theta_{i,j} \right) \cdot \left(\frac{r_i}{t_{Ti}} \right) \cdot \nu \frac{(p_{in,i} - \bar{p}_{c,i})}{E_i} \end{aligned}$$

Tubular pressurized section

Traction/torsion and bending are now coupled!!

*Helical angle variation, layer i, in configuration Σ_2
tubular, pressurized elements*

$$\Delta\alpha_i = -(\sin \alpha_i \cdot \cos \alpha_i) \frac{\Delta L}{L} + (\cos^2 \alpha_i) R_i \frac{\Delta\varphi}{L} + (\sin \alpha_i \cdot \cos \alpha_i) \frac{\Delta R_i}{R_i} -$$
$$-\tan \alpha_i \cdot v_i \frac{r_i}{t_{T,i}} \frac{(p_{in,i} - \bar{p}_{c,i})}{E_i}$$

Tubular pressurized section

Incognitae

Incognitae	Symbol	Number
Contact pressure or gap between layers i and i+1	p_{ci} or g_i	$n+m-1$
Axial load supported by layer i	F_i	$n+m$
Twist moment supported by layer i	M_{ti}	$n+m$
Bending moment supported by layer i	M_{fi}	$n+m$
Mean radius variation, layer i	ΔR_i	$n+m$
Thickness variation of layer i (*)	Δt_i	$n+m$
Laying angle variation (helical layer i)	α_i	n
Axial elongation	$\Delta L/L$	1
Twist per unit length	$\Delta \varphi/L$	1
Bending curvature	K	1
Total		$7n+6m+2$

(*) Except if the first layer is a umbilical central core.

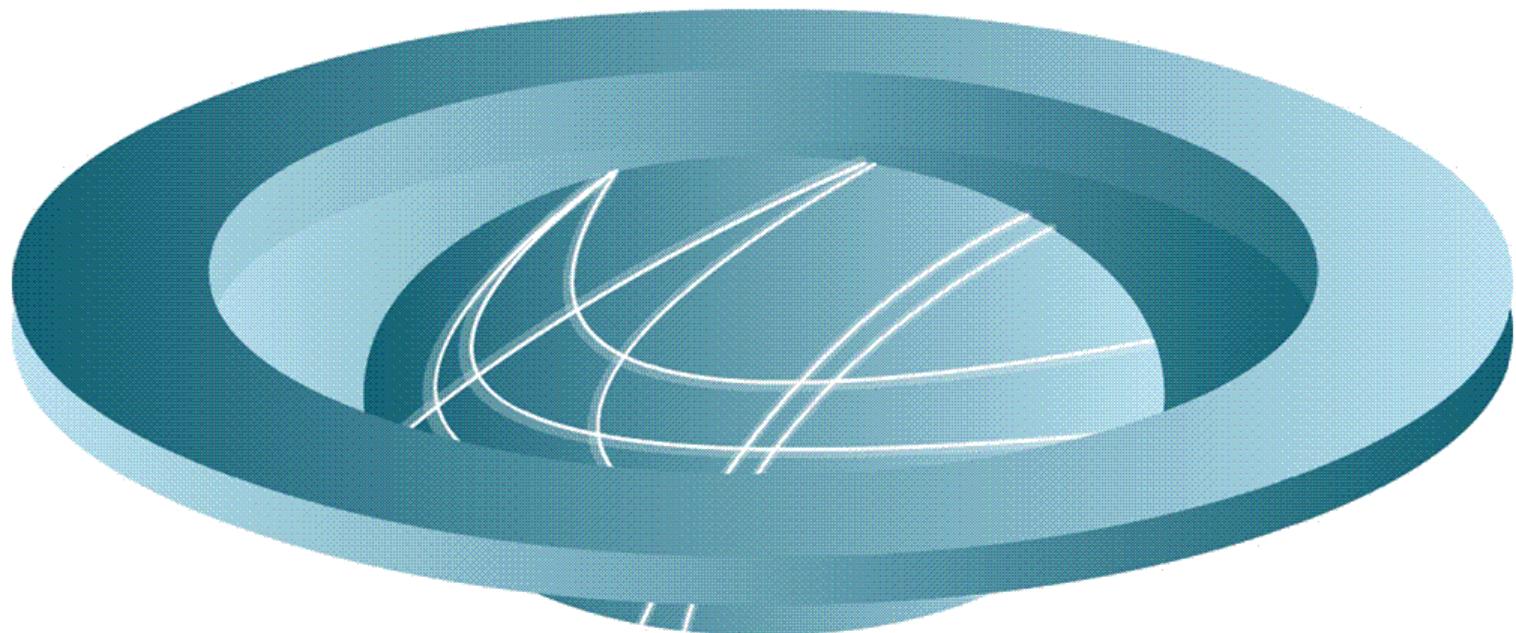
Summary

- Kirschoff-Clebsh hypotheses lead to equilibrium equations for helical elements.
- Small strains and curvature hypotheses lead to classic constitutive equations, relating soliciting efforts to stretching, and curvature and torsion variation on helical elements.
- Differential geometry and variational calculus lead to equations relating strains and soliciting efforts in the helical elements sections to stretching, curvature and torsion imposed to the cable or pipe.
- The effect of internal pressurization of a particular tubular helical element can be taken into account.



Acknowledgements





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