Lecture 2

Catenary Risers: Local Analysis

Hot Spots

Bending Stiffness and Soil Effects at TDZ

Bending and tensioning at TOP

Celso P. Pesce

Professor of Mechanical Sciences

PhD in Ocean Engineering, MSc Marine Hydrodynamics, Naval Architect

ceppesce@usp.br

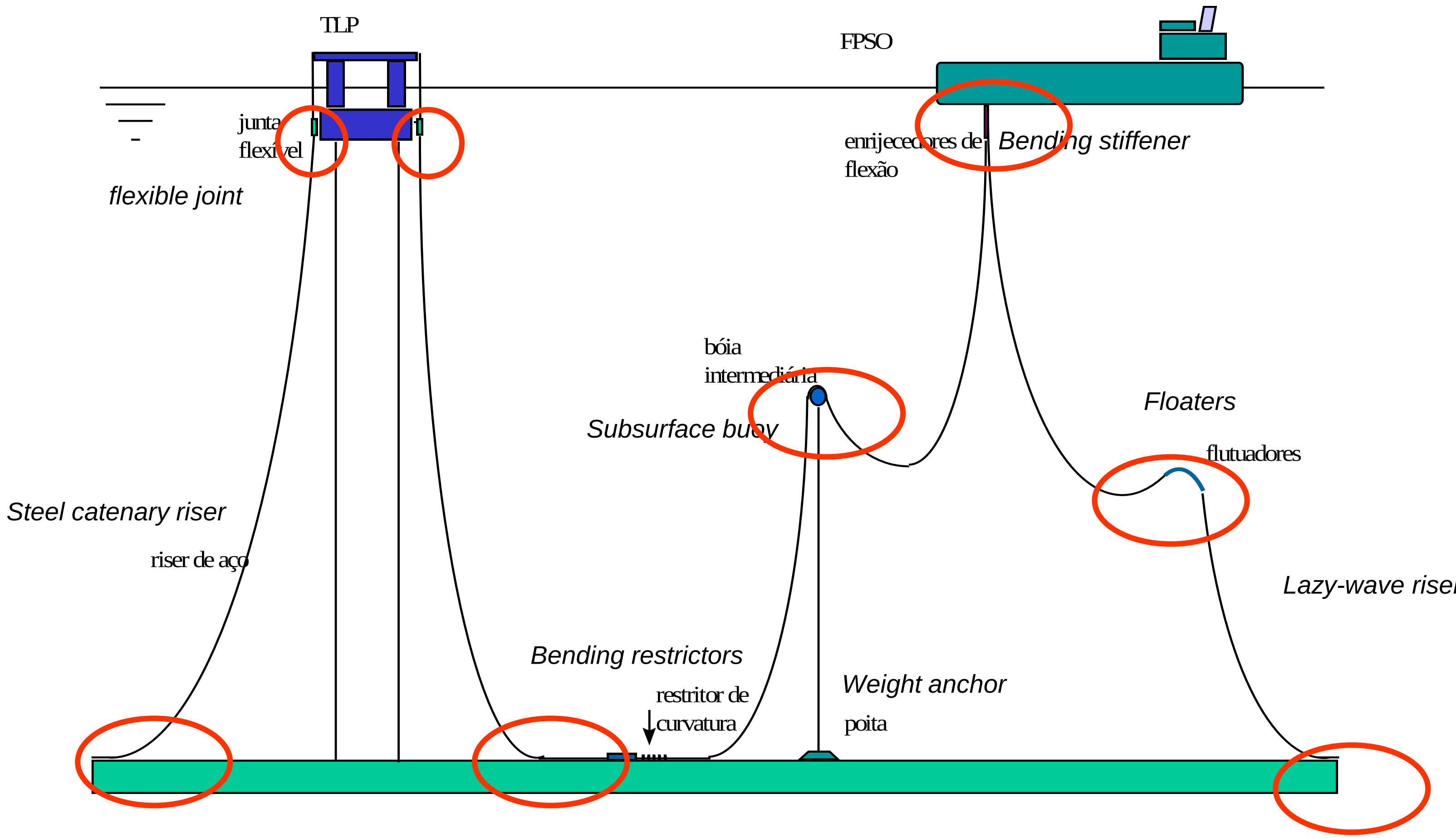
LMO - Offshore Mechanics Laboratory

Escola Politécnica

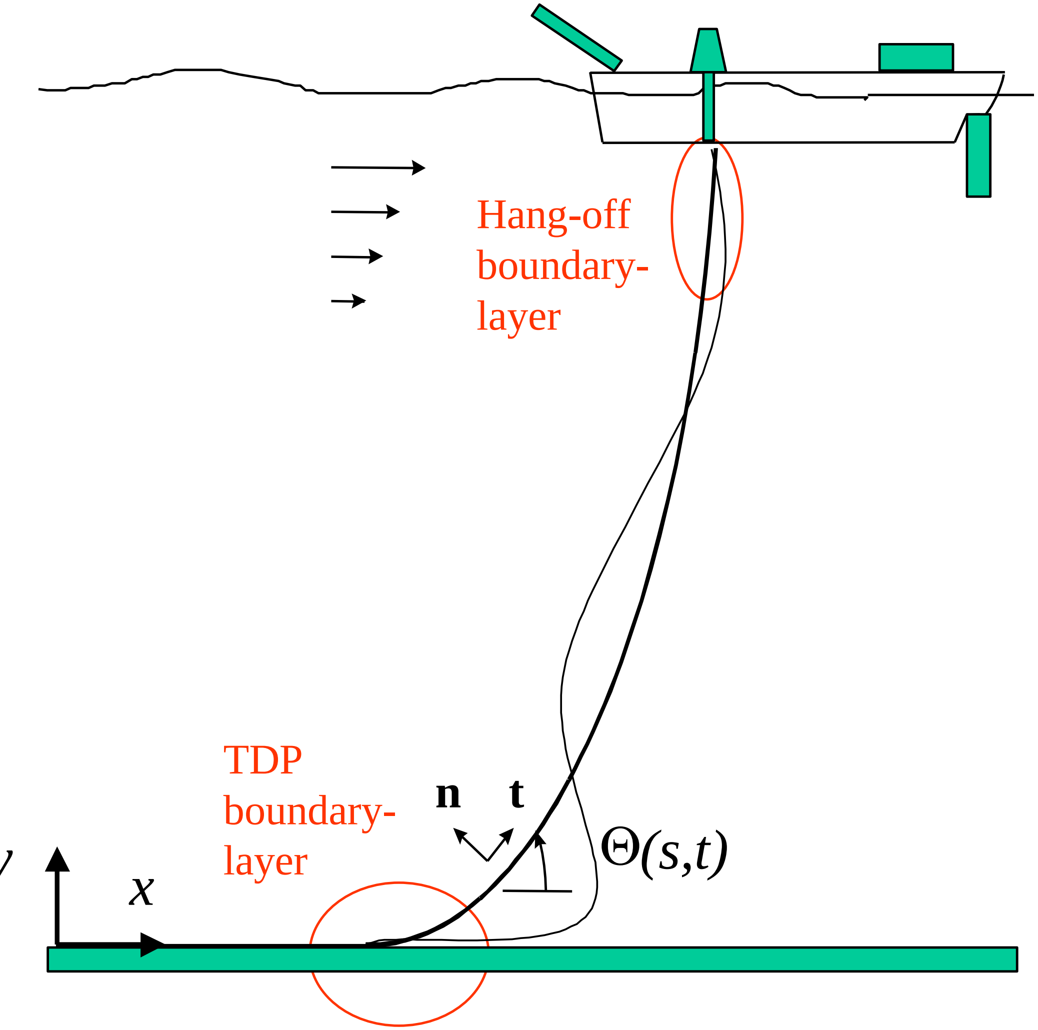
University of São Paulo

Brazil

Catenary lines



The dynamic problem



- Global dynamics governed by catenary rigidity.
- Bending stiffness effetcs are importante at the extremities and TDP, or for high-order vibration modes for which the mode vibrattion length is of same order of the local flexural length.
- There are several time scales that govern the overall dynamics of a riser.

Classic dynamic analysis

- Extensible cable static equilibrium solution under current.
- Linearized dynamic solution in frequency domain or in time domain – extensible cable.
- Vibration modes may be determined numerically or assessed analytically.
- **Bending stiffness effects can be accounted for *a posteriori*, through boundary layer techniques at Top and TDZ.**

The Bending Stiffness Effect

Total curvature

From the classic relations:

Static curvature

$$M = EI\chi(s,t) = EI\left(\frac{d\theta}{ds} + \kappa(s,t)\right)$$

Dynamic curvature

e

$$Q = -\frac{\partial M}{\partial s} = -EI\frac{\partial \chi}{\partial s} = Q + \vartheta = -EI\frac{d^2\theta}{ds^2} - EI\frac{\partial \kappa}{\partial s}$$

Mean hydrodynamic force

The dynamic equilibrium equation in the normal direction reads:

$$-EI\frac{\partial^2 \chi}{\partial s^2} + T\chi + \gamma\left(\frac{\partial T}{\partial s} - EI\frac{d\theta}{ds}\frac{\partial \chi}{\partial s}\right) + c_n + \varpi_n - q\cos\theta = m\frac{\partial^2 u_n}{\partial t^2}$$

Oscillatory hydrodynamic force

The Bending Stiffness Effect

Defining:

$$\lambda = \sqrt{\frac{EI}{T_0}}$$

Flexural length at TDP

$$\varepsilon = \frac{\lambda}{L}$$

$$\chi_0 = \frac{q}{T_0}$$

Static curvature at TDP

$$\hat{t} = \frac{c_0}{L} t$$

Transversal wave celerity dominated by catenary rigidity at TDP

$$c_0 = \sqrt{\frac{T_0}{m + m_a}}$$

The dynamic equation in the normal direction reads:

$$-\varepsilon^2 \frac{\partial^2 \hat{\chi}}{\partial \hat{s}^2} + \hat{T}\hat{\chi} + \left(\frac{\partial \hat{T}}{\partial \hat{s}} - \varepsilon^2 \frac{d\theta}{d\hat{s}} \frac{\partial \hat{\chi}}{\partial \hat{s}} \right) \gamma + \hat{c}_n + \hat{\varpi}_n - \hat{\chi}_0 \cos\theta = \frac{\partial^2 \hat{u}_n}{\partial \hat{t}^2}$$

Time scales

$$t_1 = \frac{L}{c_g}$$

$$t_2 = \frac{\lambda}{c_g}$$

$$t_3^{(i)} = \frac{L}{c_f^{(i)}}$$

$$t_4^{(i)} = \frac{\lambda}{c_f^{(i)}}$$

$$t_5 = \frac{L}{c_a}$$

$$\bar{c}_g = \sqrt{\frac{\bar{T}}{(m+m_a)}}$$

Transversal waves celerity associated to geometric rigidity

$$\bar{c}_f^{(i)} = \frac{2\pi}{\bar{\lambda}_f^{(i)}} \sqrt{\frac{EI}{(m+m_a)}}$$

Transversal waves celerity associated to bdending rigidity

$$c_a = \sqrt{\frac{EA}{m}}$$

Axial waves celerity associated to axial rigidity

$$\lambda(s) = \sqrt{\frac{EI}{T(s)}}$$

Local flexural length

A measure of the length of the bending stiffness influence

The bending stiffness effect

Rigid risers (Steel): $\varepsilon \approx O(10^{-2})$

Flexible risers: $\varepsilon \approx O(10^{-3})$

Neglecting secon-order terms (bending stiffness) the dynamic equation is dominated by tension:

$$\hat{T}\hat{\chi} + \frac{\partial \hat{T}}{\partial \hat{s}}\gamma + \hat{c}_n + \hat{\varpi}_n - \hat{\chi}_0 \cos\theta = \frac{\partial^2 \hat{u}_n}{\partial \hat{t}^2}(1 + O(\varepsilon^2))$$

Global Dynamic Analysis Procedure

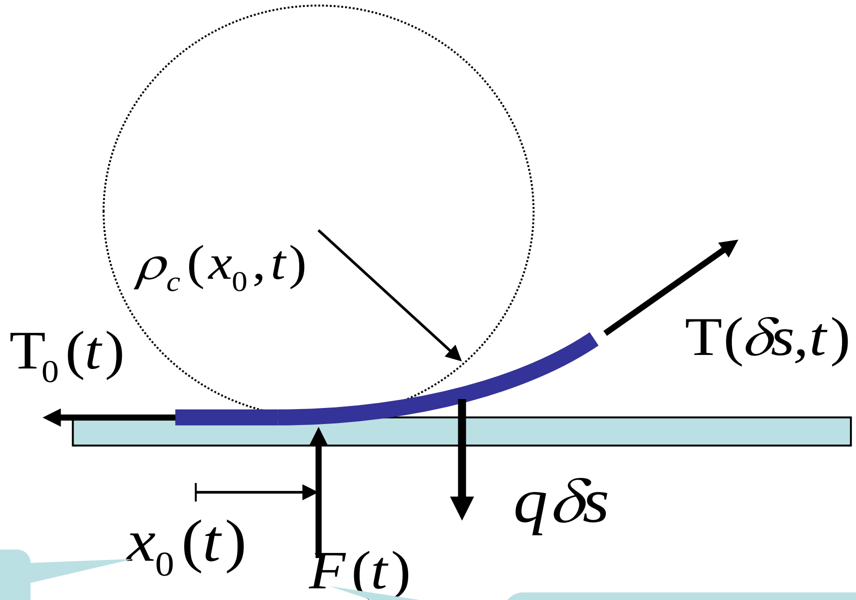
- Static solution under current.
- Dynamic solution around the static configuration in time domain with nonlinear contac boundary condition at TDP:
    - *or*
- Dynamic solution around the static configuration in frequency domain withe the following boundary conditions:
    - Pinned at TDP and TOP;
    - Pin at TDP linked to na equivalent linear spring representing the the effectively extensible length on the bottom.
    - Rotational spring at top representing the *bending-stiffner.*
    - Correction on the curvature, angle and elastic line near the extremities, by incorporating the bending stiffness effects – a posteriori – thorugh the 'boundary-layer' asymptotic technique.

Shock condition against the soil at TDP

Catenary:

$$\tan\theta_c(s) = \frac{qs}{T_0} = \chi_0 s$$

$$T(s) = T_0 \sec\theta_c$$



In first-order:

Instantaneous TDP position

Possible impacting force

$$T(\delta s,t) = T(\delta s) + \tau(\delta s,t) \cong T_0 + \tau_0(t) + \left.\frac{\partial T}{\partial s}\right|_{s=0} \delta s =$$

$$= T_0\left(1 + \frac{\tau_0(t)}{T_0}\right) + O(\chi_0 \delta s)^2 \approx T_0; \quad \text{se} \quad \frac{\tau_0(t)}{T_0} << 1$$

Shock condition at TDP...

Local kinematics:

$$y_c(x_0(t),t) \equiv 0$$

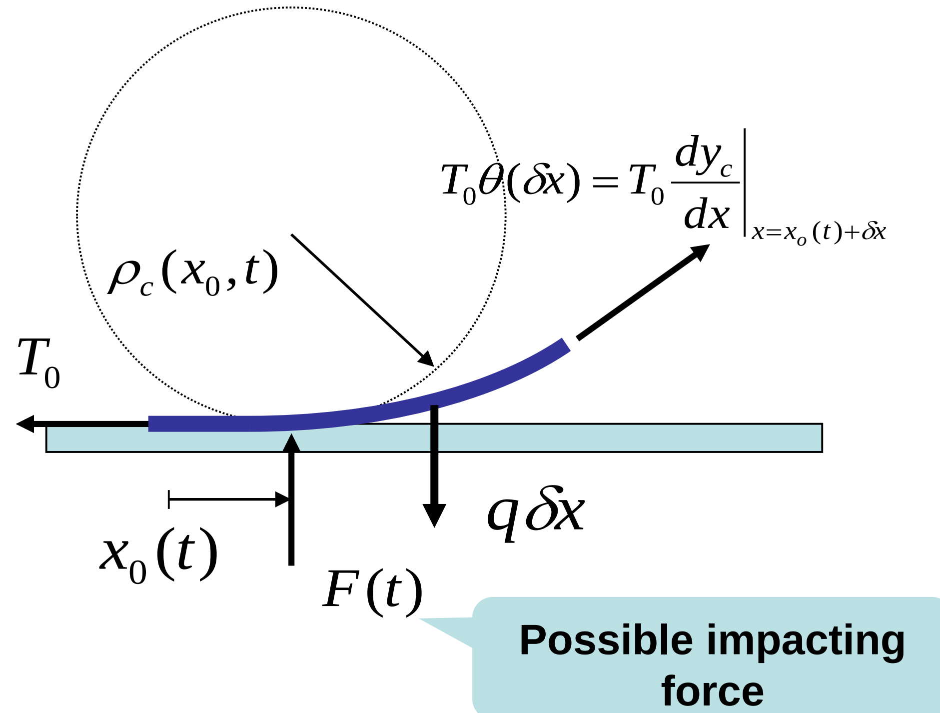
$$\downarrow$$

$$\frac{d}{dt}\big(y_c(x_0(t),t)\big) =$$

$$\frac{\partial}{\partial t} y_c(x_0(t),t) + \dot{x}_0(t)\frac{\partial}{\partial x}\big(y_c(x_0(t),t)\big) \equiv 0$$

Como:

$$\frac{d\delta x}{dt} = -\dot{x}_0(t)$$



Local dynamic equilibrium:

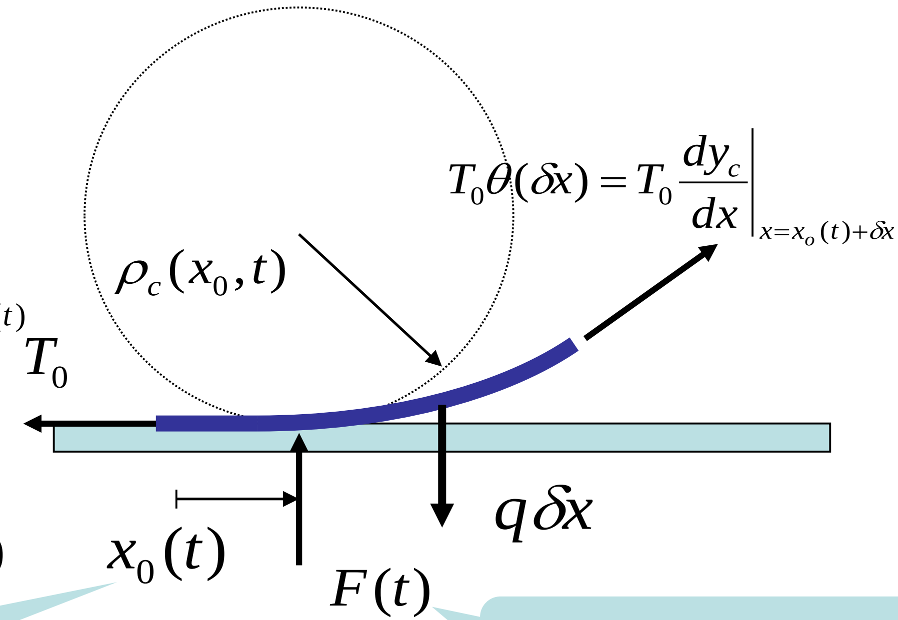
$$\frac{d}{dt}\left[(m+m_a)\delta x(t)\left.\frac{\partial y_c}{\partial t}\right|_{x_0+\delta x}\right] = F(t) + T_0\left.\frac{\partial y_c}{\partial x}\right|_{x_0+\delta x} - q\delta x(t)$$

Shock condition at TDP...

In the limit:

$$\delta x \to 0 \quad \Rightarrow$$

$$-(m+m_a)\dot{x}_0(t)\left.\frac{\partial y_c}{\partial t}\right|_{x_0(t)} = F(t) + T_0 \left.\frac{\partial y_c}{\partial x}\right|_{x_0(t)}$$



E como:

$$\frac{\partial}{\partial t} y_c(x_0(t),t) + \dot{x}_0(t)\frac{\partial}{\partial x}\big(y_c(x_0(t),t)\big) \equiv 0$$

Instantaneous TDP position

Possible impacting force

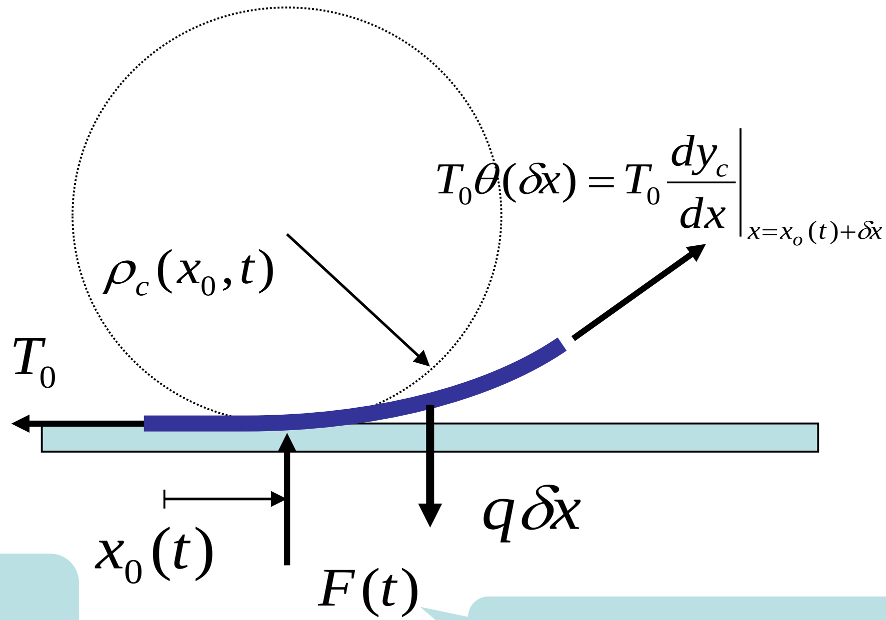
The dynamic equilibrium reads:

$$(m+m_a)\dot{x}_0{}^2(t)\left.\frac{\partial y_c}{\partial x}\right|_{x_0(t)} = F(t) + T_0 \left.\frac{\partial y_c}{\partial x}\right|_{x_0(t)}$$

Shock condition at TDP...

The local dynamic equilibrium equation rearranged :

$$\left(\dot{x}_0^{\,2}(t) - c_0^{\,2}\right)\left.\frac{\partial y_c}{\partial x}\right|_{x_0(t)} = \frac{F(t)}{(m+m_a)}$$



where:

$$c_0 = \sqrt{\frac{T_0}{(m+m_a)}}$$

Local 'cable' transversal wave celerity

Possible impacting force

Rewriting:

$$\left(\mathrm{M}^2 - 1\right)\left.\frac{\partial y_c}{\partial x}\right|_{x_0(t)} = \frac{F(t)}{T_0}$$

$$\mathrm{M} = \dot{x}_0 / c_0$$

A local 'Mach' number

Shock condition at TDP...

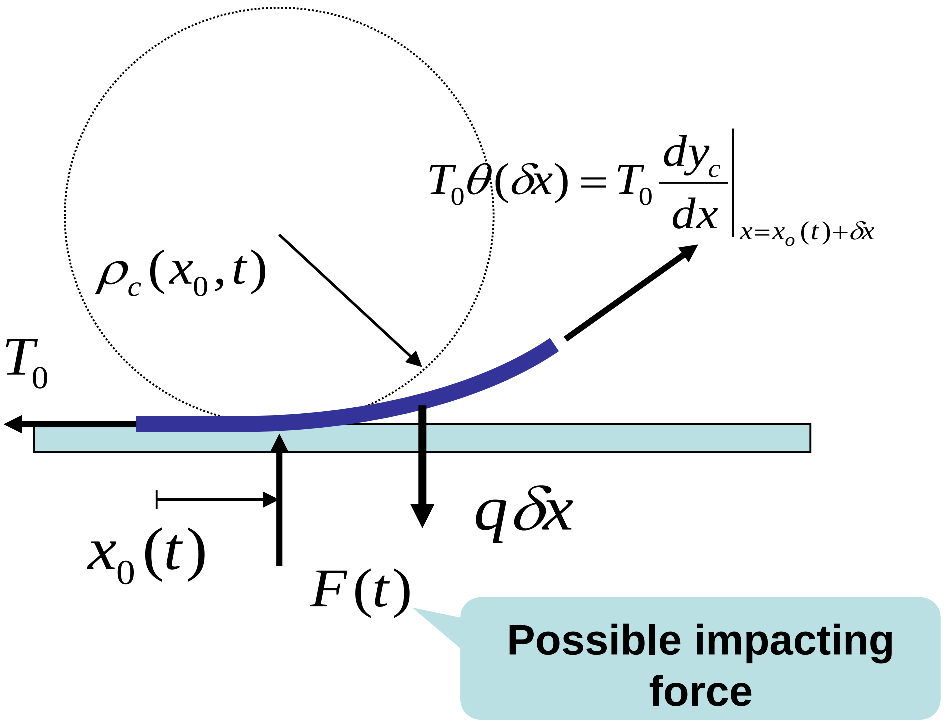
Therefore:

$$\left(\mathrm{M}^2 - 1\right)\left.\frac{\partial y_c}{\partial x}\right|_{x_0(t)} = \frac{F(t)}{T_0}$$

with: $\mathrm{M} = \dot{x}_0 / c_0$

However:

$$\left.\frac{\partial y_c}{\partial x}\right|_{x_0(t)} \geq 0$$



Impact exists if and only if: $\mathrm{M} > 1$ → i.e., if: $\dot{x}_0 > c_0$

No-shock condition

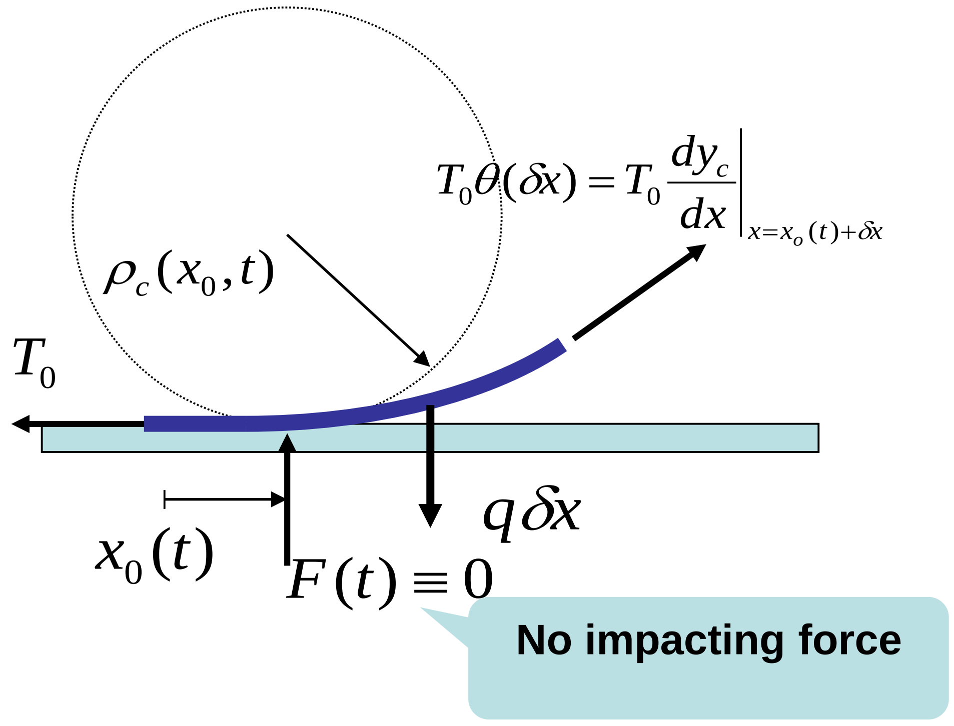
In this case:

$$\left(\mathrm{M}^2-1\right)\frac{\partial y_c}{\partial x}\bigg|_{x_0(t)}=\frac{F(t)}{T_0}=0$$

What necessarily implies:

$$\frac{\partial}{\partial x}\left(y_c(x_0(t),t)\right)\equiv 0$$

$$\rightarrow \quad \boxed{\frac{\partial y_c}{\partial t}\bigg|_{x_0(t)}\equiv 0;\ \ \mathrm{M}<1}$$



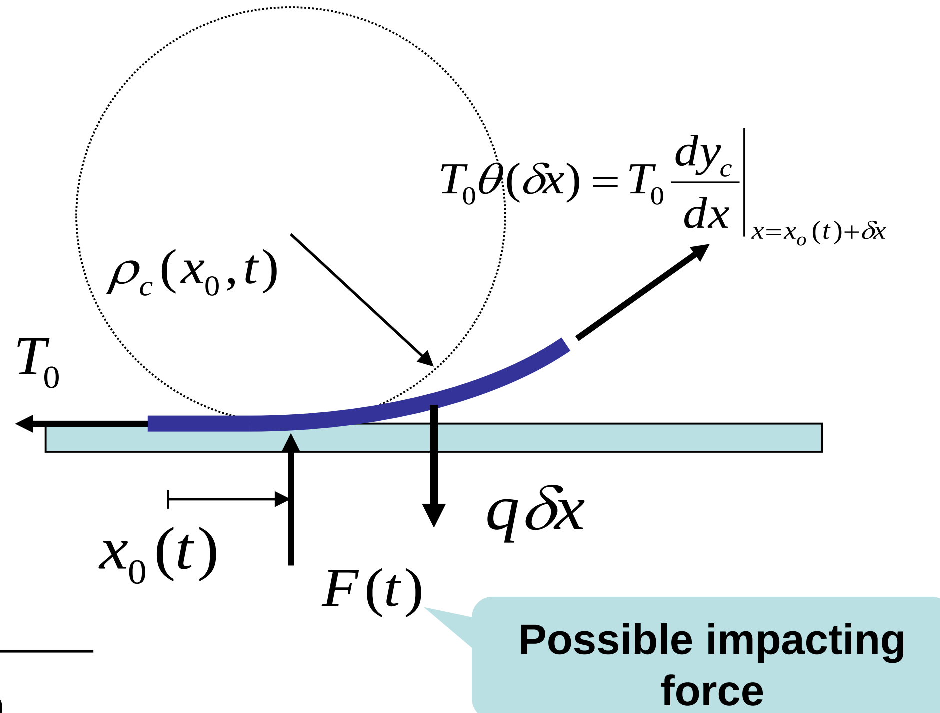
There is no impact if: $\mathrm{M}<1$ → i.e., if: $\dot{x}_0<c_0$

Shock condition of a cable against the soil

Physical interpretation:

When the TDP excursion speed is lower than the natural transversal wave propagation speed, there is enough time for the cable to adjust its geometry as to prevent shock agaisnt the soil.

Otherwise, impact is expected to occur.



In other words, since: $c_0 = \sqrt{\frac{T_0}{(m+m_a)}}$

The smaller is the static tension at TDP, the larger is the'Mach' number for a given TDP excursion speed!

Local subcritical kinematics at TDP

There is no shock.

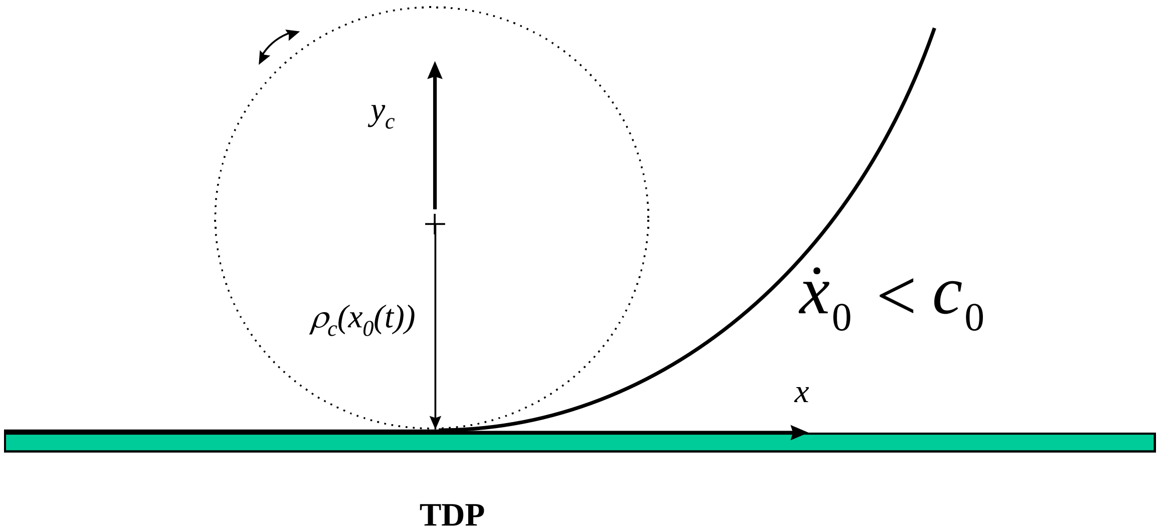
The cable 'rolls' smoothly on the soil.

Analogously to a 'disc' rolling without slipping on a horizontal plane.

In this case, the 'disc' radius varies according to the varying curvature radius of the cable at the instantaneous TDP position.

$$\left.\frac{\partial y_c}{\partial t}\right|_{x_0(t)} \equiv 0;\ \mathrm{M} < 1$$

$$\frac{\partial}{\partial x}\big(y_c(x_0(t),t)\big) \equiv 0$$



The vertical TDP acceleration is:

$$\frac{\partial^2}{\partial t^2} y_c(x_0(t),t) = \frac{\dot{x}_0^{\ 2}(t)}{\rho_c(x_0(t))}$$

Local subcritical kinematics at TDP

In fact, since:

$$\frac{d}{dt} y_c(x_0(t),t) = \frac{\partial y_c(x_0(t),t)}{\partial t} + \dot{x}_0(t)\frac{\partial y_c(x_0(t),t)}{\partial x} \equiv 0$$

then:

$$\left(\frac{\partial^2 y_c}{\partial x \partial t} + \dot{x}_0(t)\frac{\partial^2 y_c}{\partial x^2}\right)_{x=x_0(t)} \equiv 0$$

e

$$\left(\frac{\partial^2 y_c}{\partial t^2} + \ddot{x}_0\frac{\partial y_c}{\partial x} + \dot{x}_0(t)\frac{\partial^2 y_c}{\partial t \partial x}\right)_{x=x_0(t)} \equiv 0$$

0

$$\frac{\partial^2}{\partial t^2} y_c(x_0(t),t) = \frac{\dot{x}_0{}^2(t)}{\rho_c(x_0(t))}$$

Compare with the instantaneous center of rotation of a disc rolling on a plane without slipping

Local subcritical kinematics at TDP

Analogously, the vertical coordinate variation at x=0 may be evaluated, from the following Taylor series expansions:

$$\frac{y_c(x_0(t),t)}{\rho_{0c}} = \frac{1}{\rho_{0c}}\left(y_c(0,t) + x_0(t)\frac{\partial y_c}{\partial x}(0,t) \right) + O(\frac{x_o{}^2(t)}{\rho_{0c}{}^2}) \equiv 0$$

and

$$\frac{1}{\rho_{0c}}\frac{\partial y_c}{\partial x}(x_0(t),t) = \frac{1}{\rho_{0c}}\left(\frac{\partial y_c}{\partial x}(0,t) + x_0(t)\frac{\partial^2 y_c}{\partial x^2}(0,t) \right) + O(\frac{x_o{}^2(t)}{\rho_{0c}{}^2}) \equiv 0$$

Leading to:

$$\frac{y_c(0,t)}{\rho_{0c}} \cong \frac{x_0{}^2(t)}{\rho_{0c}}\frac{\partial^2 y_c}{\partial x^2}(0,t) \cong O\left(\frac{a_0}{\rho_{0c}} \right)^2$$

TDP excursion amplitude

Static curvature radius at TDP

Local subcritical kinematics at TDP

Summary

- There is no impact if the excursion TDP speed is smaller than the local tnasversal wave propagation speed.

- In such conditions the kinematics and dynamics are said *subcritical.*

- TDP may then be treated, in first-order, as a pin.

The bending stiffness effect near the TDP: the *Boundary-layer technique*

- If the kinematics is subcritical (no shock), it can be shown that hydrodynamic and inertial forces are, *locally*, of order O(M^2)<<*1*, if compared to the tension and shear forces.
- The equation governing the curvature is, therefore, *locally quasi-static*, when obeserved from the global dynamics time scale point of view:

Dynamic tension atTDP

$$-\lambda^2 \frac{d^2\chi}{ds^2} + \left(1 + \frac{\tau(t)}{T_0}\right)\chi \cong \chi_0\left(1 + O(\mathrm{M}^2)\right)$$

Static curvature at TDP

‘Flexural length’ at TDP

$$\lambda = \sqrt{\frac{EI}{T_0}} \qquad c_0 = \sqrt{\frac{T_0}{m + m_a}} \qquad \hat{t} = \frac{c_0}{L} t$$

The bending stiffness effect near the TDP: the *Boundary-layer technique*

- Locally this is a linear second order O.D.E. in the curvilinear coordinate *s*.
- The solution is exponential, resulting:

$$\frac{\chi(s,t;\lambda)}{\chi_0} = \begin{cases} \dfrac{1}{1+\tau(t)/T_0}\left[1-\exp\left(-\beta(s,t;\lambda)\right)\right]; & \beta(s,t;\lambda) > 0 \\ 0; & \beta(s,t;\lambda) \le 0 \end{cases}$$

onde

$$\beta(s,t;\lambda) = \frac{\sqrt{1+\tau(t)/T_0}}{\lambda}\left(s - s_f(t)\right)$$

Instantaneous 'real' TDP position, bending stiffness effect included

$$s_f(t) = x_0(t) - \lambda \Big/ \sqrt{1+\tau(t)/T_0}$$

Instantaneous cable TDP position

$$x_0(t) = -\gamma(0,t)\rho_0$$

$$\bar{s}_f = -\lambda$$

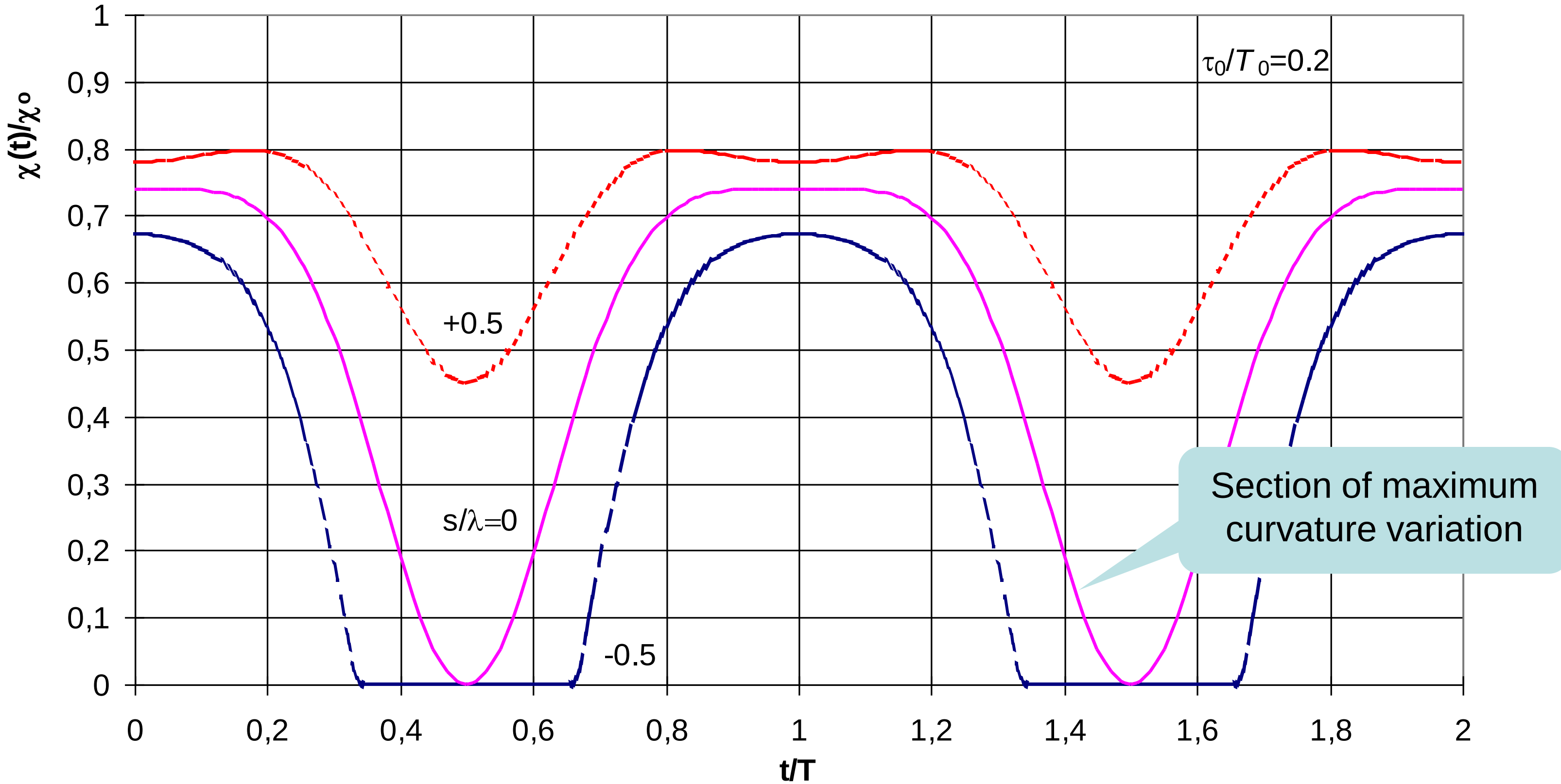
Average real TDP position, displaced to the left as a result of bending stiffness effect

Recalling:
Global Dynamic Analysis Procedure

- Static solution under current.
- Dynamic solution around the static configuration in time domain with nonlinear contac boundary condition at TDP:
  - *or*
- Dynamic solution around the static configuration in frequency domain withe the following boundary conditions:
  - Pinned at TDP and TOP;
  - Pin at TDP linked to na equivalent linear spring representing the the effectively extensible length on the bottom.
  - Rotational spring at top representing the *bending-stiffner.*
  - Correction on the curvature, angle and elastic line near the extremities, by incorporating the bending stiffness effects – a posteriori – via the 'boundary-layer' asymptotic technique.

The asymptotic analytical solution at TDZ

$$\frac{x_0(t)}{\lambda} = \frac{a_0}{\lambda}\cos\omega t \qquad \frac{a_0}{\lambda} = 1$$



Nondimensional curvature vs. time

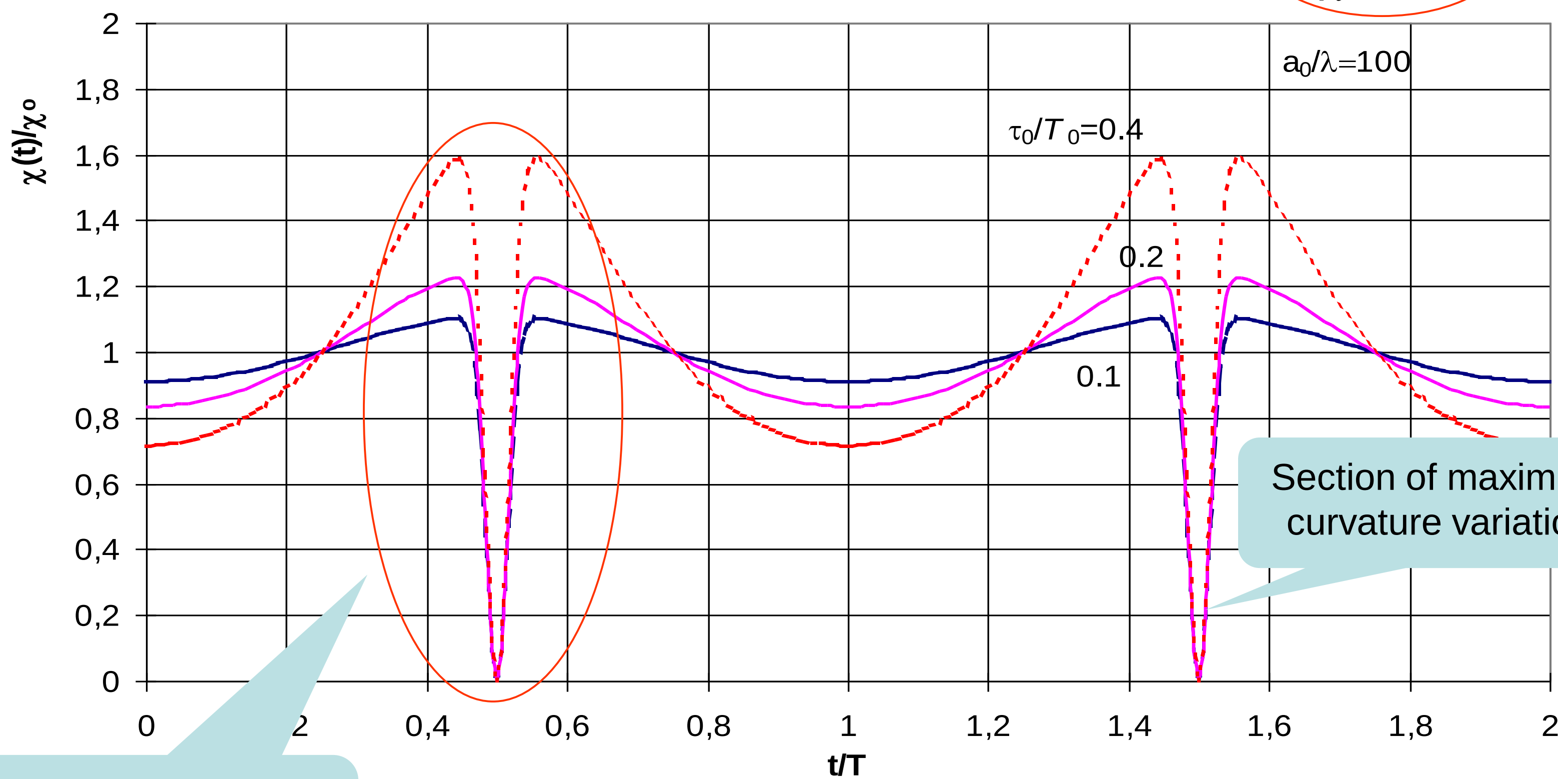
for three distinct sections

The asymptotic analytical solution at TDP

$$\frac{x_0(t)}{\lambda} = \frac{a_0}{\lambda}\cos\omega t$$

Very small bending stiffness

$$\frac{a_0}{\lambda} = 100$$



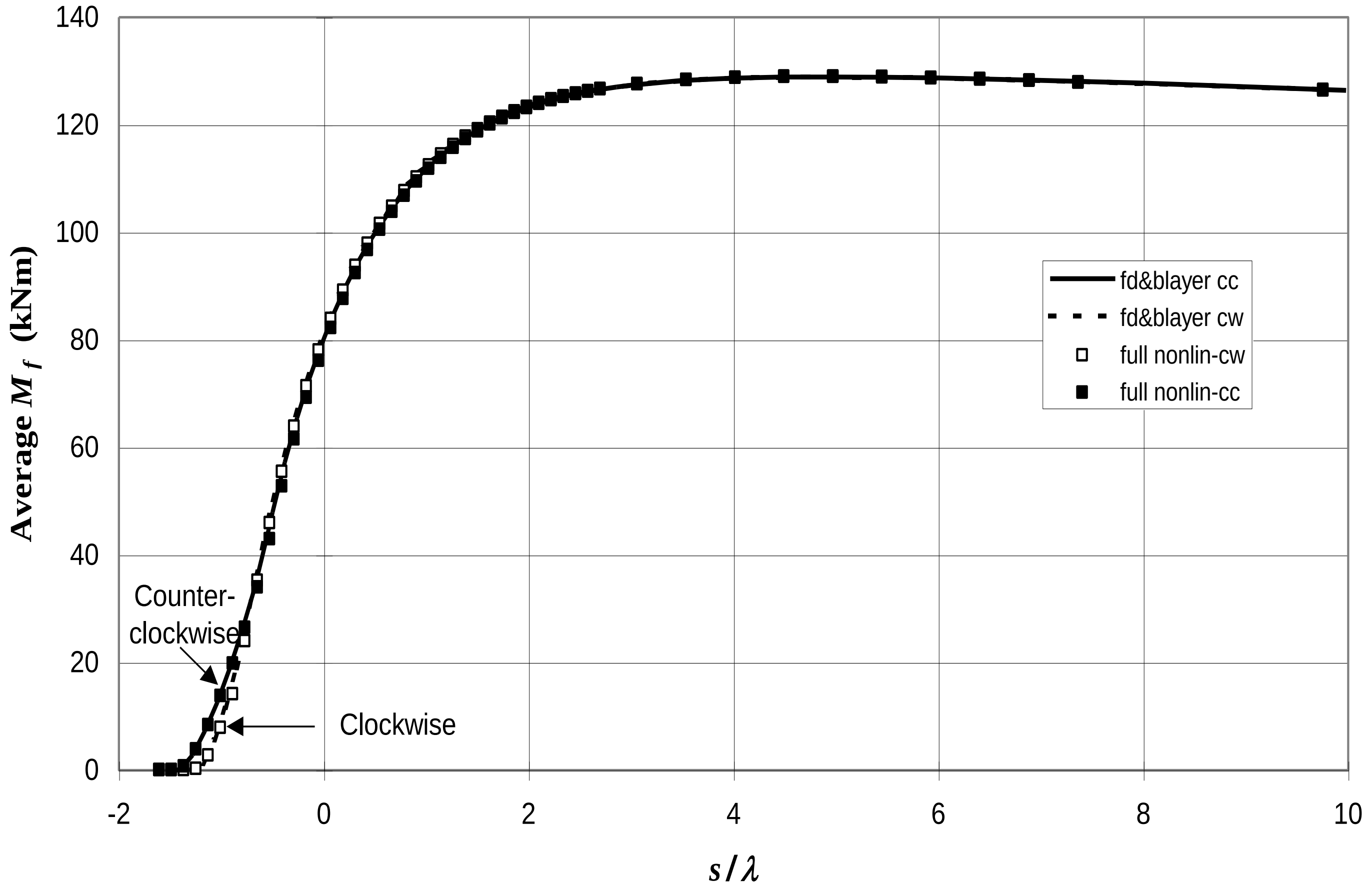
Section of maximum curvature variation

Sudden variations might be confused with numerical instabilities

Nondimensional curvature vs. time

for three distinct dynamic tension amplitudes

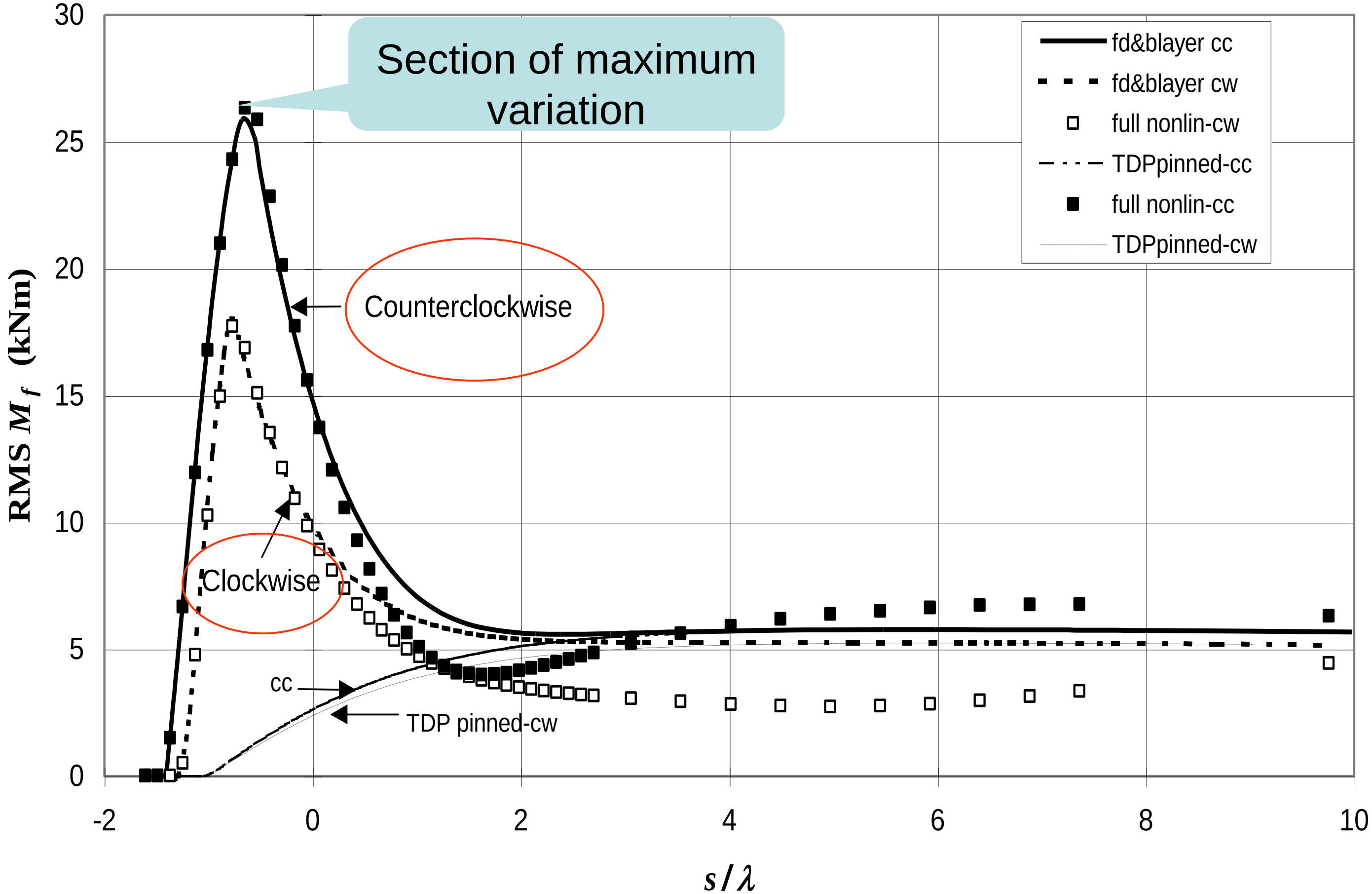
The asymptotic analytical solution at TDZ



Time Average Bending Moment at TDZ

SCR (16”; 575m); compared with ORCAFLEX

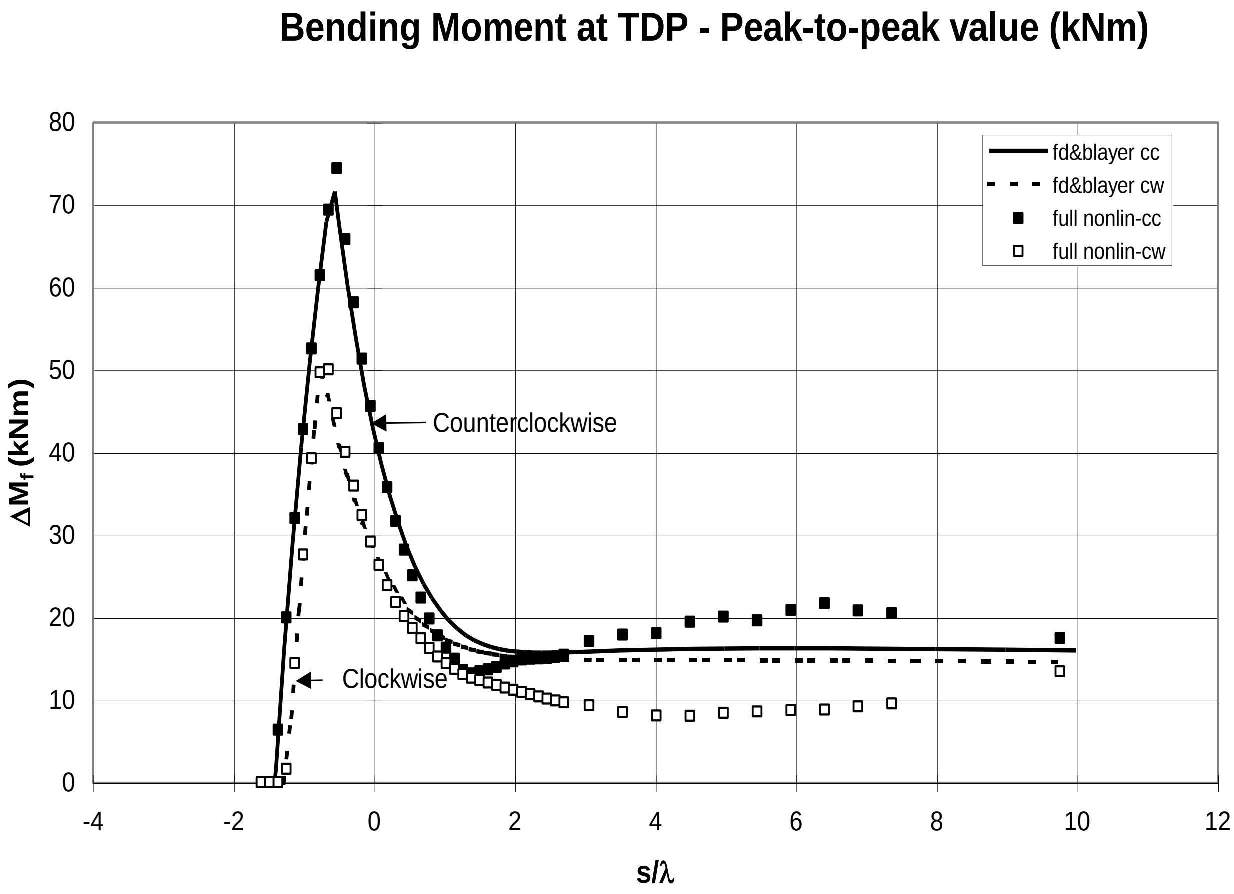
The asymptotic analytical solution at TDZ



RMS Bending Moment at TDZ

SCR (16”; 575m); compared with ORCAFLEX

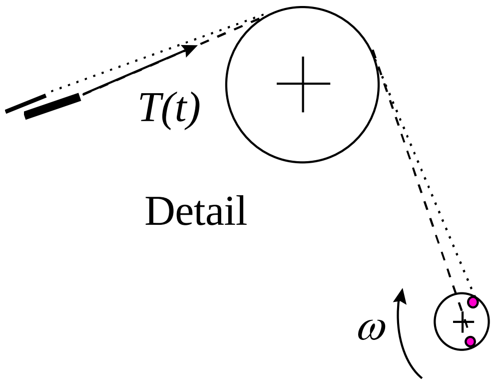
The asymptotic analytical solution at TDZ



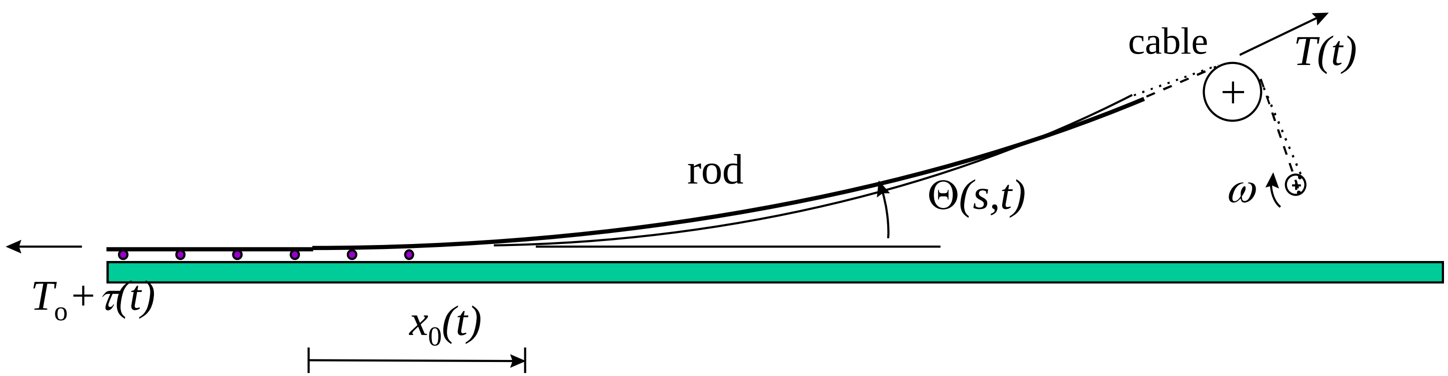
Peak-to-peak Bending Moment, at TDZ

SCR (16”; 575m) ; comparação com ORCAFLEX

The asymptotic analytical solution at TDZ: experimental validation



	Rod	Riser
External diameter(m)	0.0127	0.2731
Internal diameter (m)	-	0.2320
Linear weight (*q*) (N/m)	9.807	1250.4
Young modulus (*E*) (GPa)	206	206
Bending stiffness (*B*) (kNm2)	0.226	2.12E06



Experimental sketch – POLI/IPT; 1997.

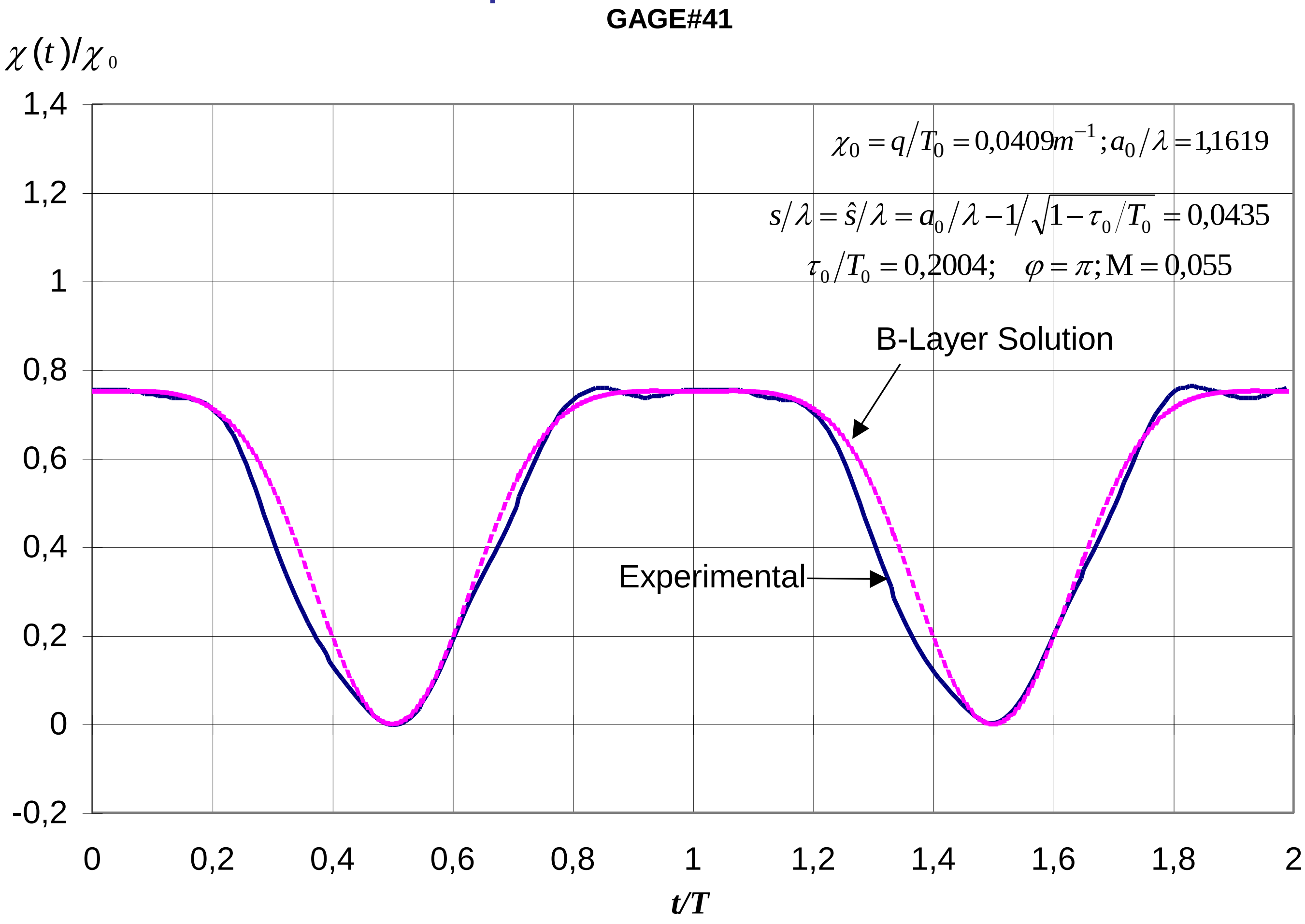
The asymptotic analytical solution at TDZ: experimental validation

T_0	0.2396 kN
χ_0	0.0409 m^{-1}
λ	0.9640 m
$\overline{T}_{top}$	0.2727 m
$\overline{T}_{TDP}$	0.2514 kN
a_0	1.125 m
φ	180 ^{o}
ω	2.3621 rad/s
$v_0 = \omega a_0$	2.6456 m/s
c_0	48.4672 m/s
τ_0 / T_0	0.2004
$\tau_0 / \overline{T}_{TDP}$	0.1910
a_0 / λ	1.1619
$M_0 = v_0 / c_0$	0.0546
$M_0^{\ 2}$	0.0030

	a_0/λ	τ_0/T_0	$M_0 = \omega a_0 / c_0$
16” - Sea 3	0,23	0,06	0,045
16”- Sea 4	0,55	0,08	0,099
8” - Sea 3	0,22	0,07	0,040
8” - Sea 4	0,39	0,10	0,064
Experiment	1,20	0,20	0,055

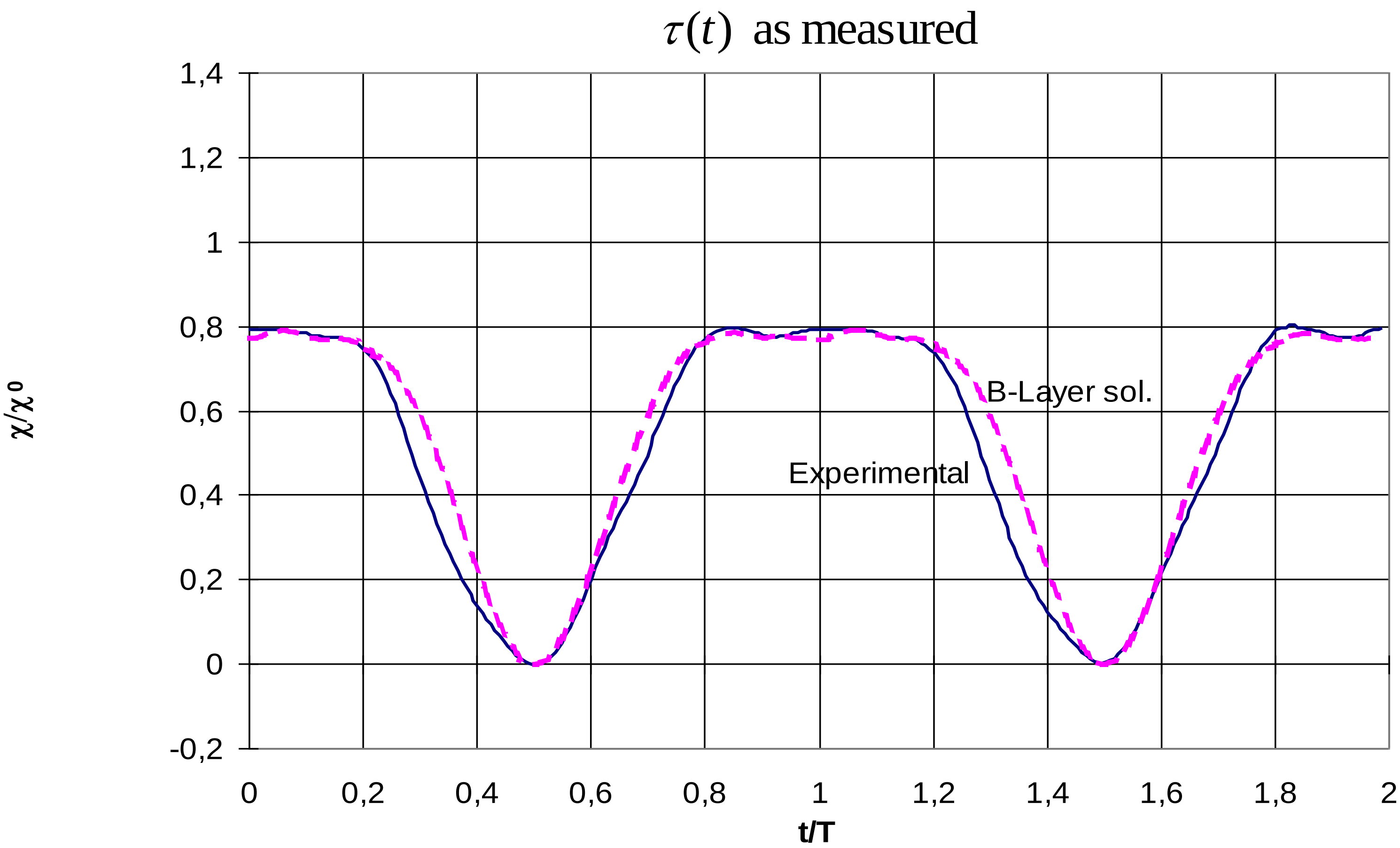
POLI/IPT; 1997: typical and experimental conditions.

The asymptotic analytical solution at TDZ: experimental validation



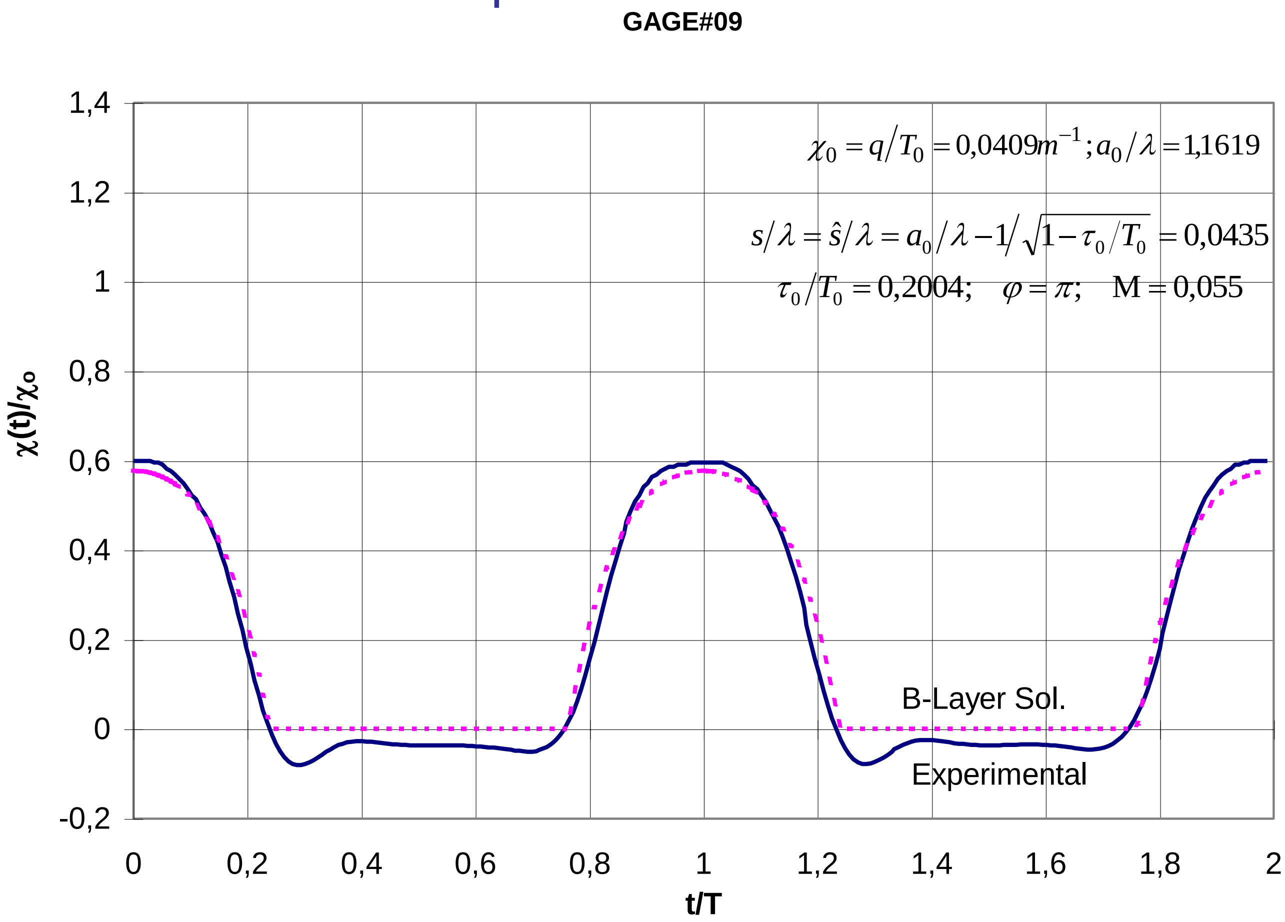
Experimental validation – POLI/IPT; 1997.

The asymptotic analytical solution at TDZ: experimental validation



Experimental validation – POLI/IPT; 1997.

The asymptotic analytical solution at TDZ: experimental validation



Experimental validation – POLI/IPT; 1997.

Soil Elasticity Effect

Hypotheses

- No shock against the soil $\mathrm{M} = \left|\dot{x}_0 / c_0\right| < 1$
- The dynamics of the suspended part governs the local dynamics

$$\hat{t} = \mathrm{t}\, c_0 / \lambda$$

Local nondimensional time-scale (rapid)

$$c_0 = \sqrt{T_0 / (m + m_a)}$$

Cable transversal wave celerity

$$t = \mathrm{t}\, c_0 / L = \varepsilon \hat{t}$$

Global nondimensional time-scale (slow)

$$\varepsilon = \lambda / L$$

Small parameter

$$\frac{\partial^4 \eta}{\partial \xi^4} - b^2(t) \frac{\partial^2 \eta}{\partial \xi^2} + K\eta = -\varepsilon^2 \frac{\partial^2 \eta}{\partial t^2}$$

Beam equation under dynamic tension on elastic bed

Universidade de São Paulo

LIFE&MO

Solution via boundary-layer technique

Local Dynamics on Linearly Elastic Soil

Elastica at TDZ

$$\eta(\xi,t)=\frac{X_0}{b(t)}\frac{\sqrt{2}}{K^{1/2}}\frac{1}{K^{1/4}+\sqrt{2}b(t)}\exp\left(\frac{K^{1/4}}{\sqrt{2}}(\xi-\xi_K(t))\right)\times$$

$$\times\sin\left(\frac{K^{1/4}}{\sqrt{2}}(\xi-\xi_K(t))\right)$$

valid for $(\xi-\xi_K(t))\leq 0$

and

$$\eta(\xi,t)=\frac{X_0}{b^2(t)}\left\{\frac{\xi^2-\xi_K^2(t)}{2}-\xi_0(t)(\xi-\xi_K(t))\right\}+$$

$$+\frac{X_0}{b^2(t)}\left\{\frac{1}{b^2(t)}\frac{K^{1/4}}{K^{1/4}+\sqrt{2}b(t)}\left[1-\exp\left[-b(t)(\xi-\xi_K(t))\right]\right]\right\}$$

valid for $(\xi-\xi_K(t))\geq 0$

$$K=\frac{kEJ}{T_0^{\ 2}}$$

nondimensional soil rigidity parameter

$$X_0=\chi_0\lambda$$

nondimensional static curvature

$$b^2(t)=1+\tau_0(t)/T_0$$

nondimensional total tension function

Instantaneous TDP position

$$\xi_K(t)=\xi_0(t)-\frac{K^{1/4}}{K^{1/4}+\sqrt{2}b(t)}\left(\frac{1}{b(t)}-\frac{b(t)}{K^{1/2}}\right)$$

Local Dynamics on Linearly Elastic Soil

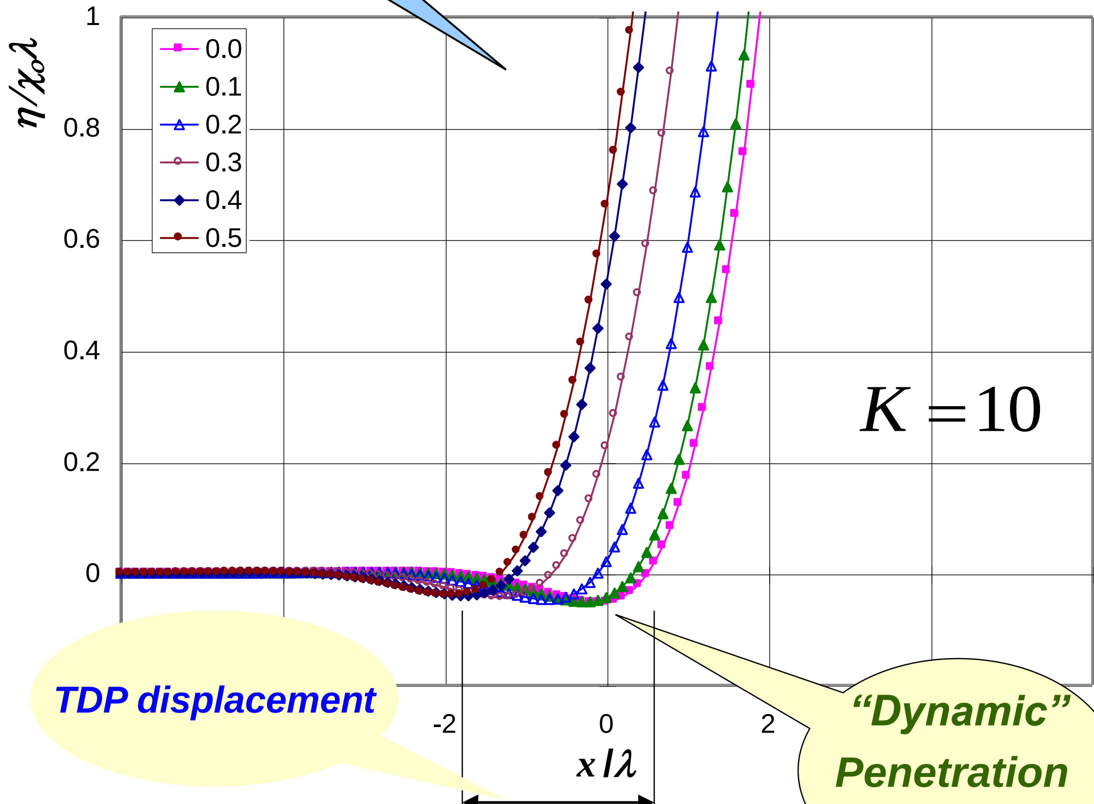
Elastica at TDZ

Half cycle of snapshots at

$t/\mathrm{T} = 0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ \mathrm{T} = 2\pi\omega^{-1}$

$\xi_0(t) = x_0(t)/\lambda = a_0 \cos \omega t$

Cable TDP displacement



$b^2(t) = 1 - \tau_0(t)/T_0 = 1 - \beta_0 \cos \omega t$

nondimensional dynamic tension

TDP displacement

"Dynamic" Penetration into the soil

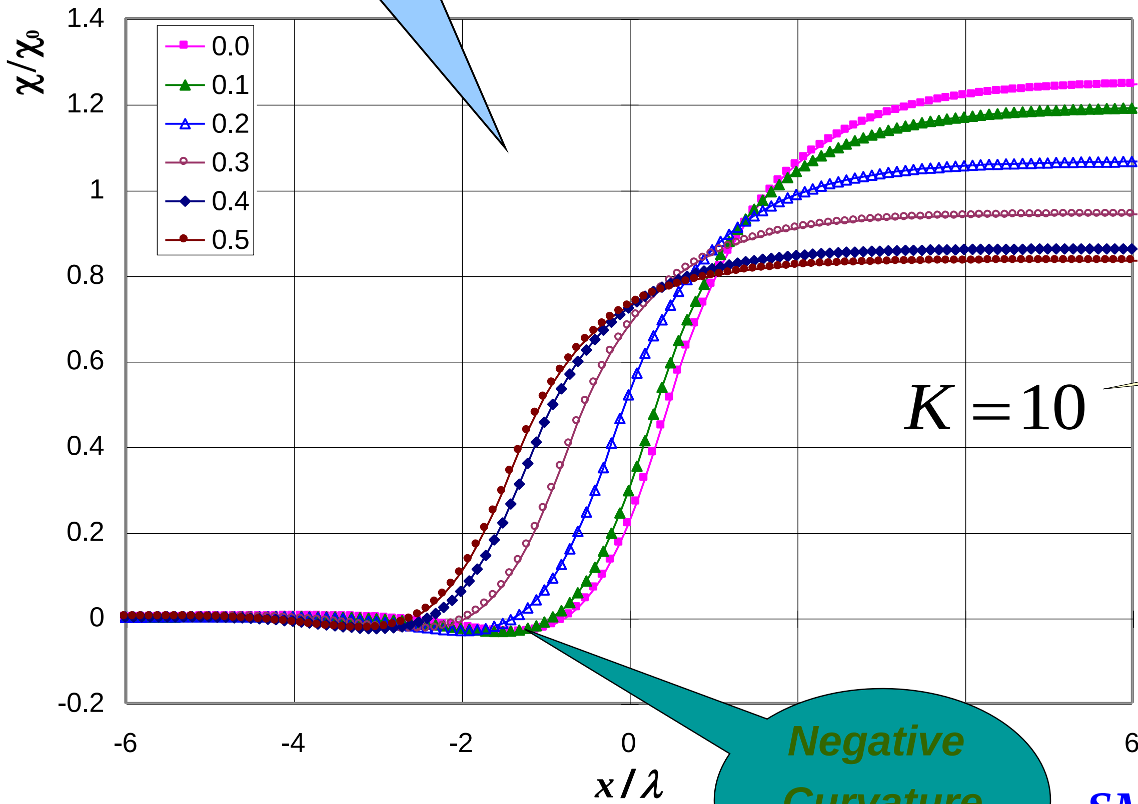
$a_0 = 1; \quad \beta_0 = 0.2$

Local Dynamics on Linearly Elastic Soil

Curvature at TDZ

Half cycle of snapshots at

$$t/\mathrm{T} = 0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ \mathrm{T} = 2\pi\omega^{-1}$$



Even for "SOFT" SOIL

Negative Curvature on the soil

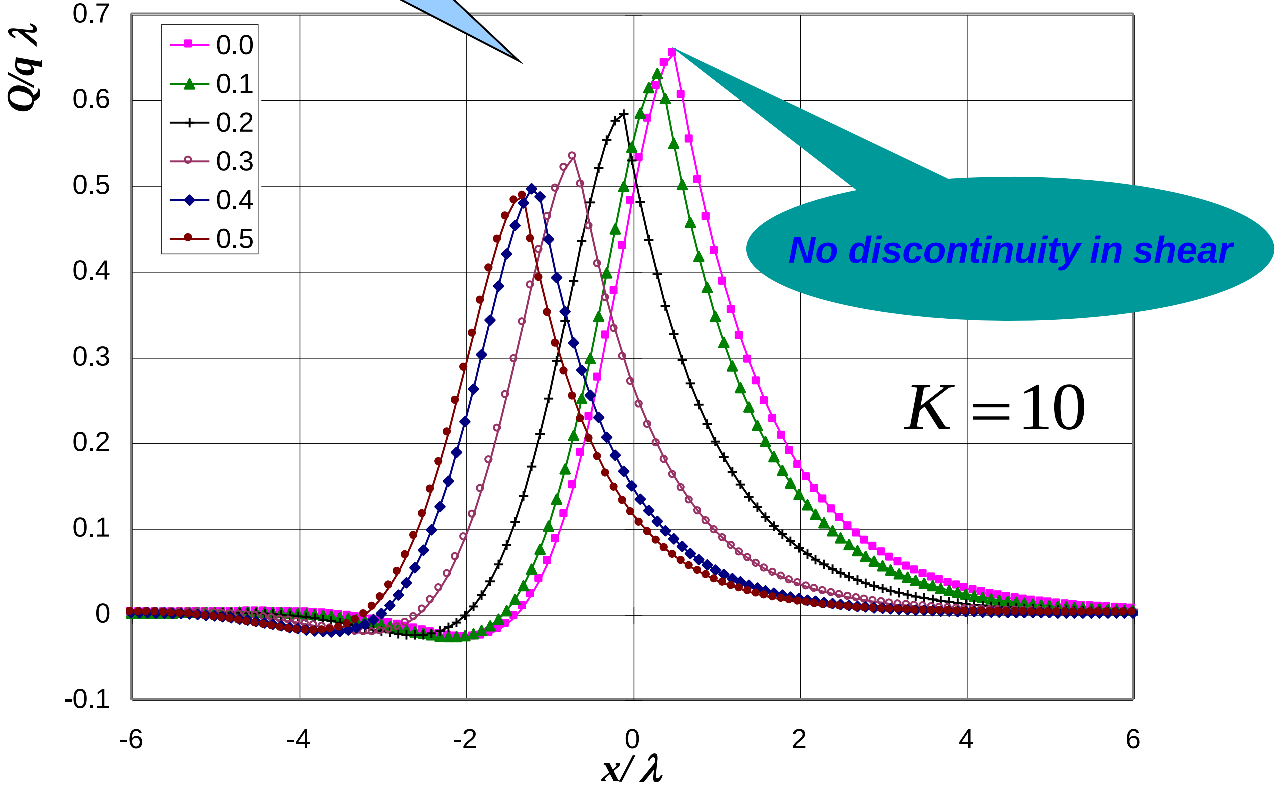
SMALL VALUES

Local Dynamics on Linearly Elastic Soil

Shear at TDZ

Half cycle of snapshots at

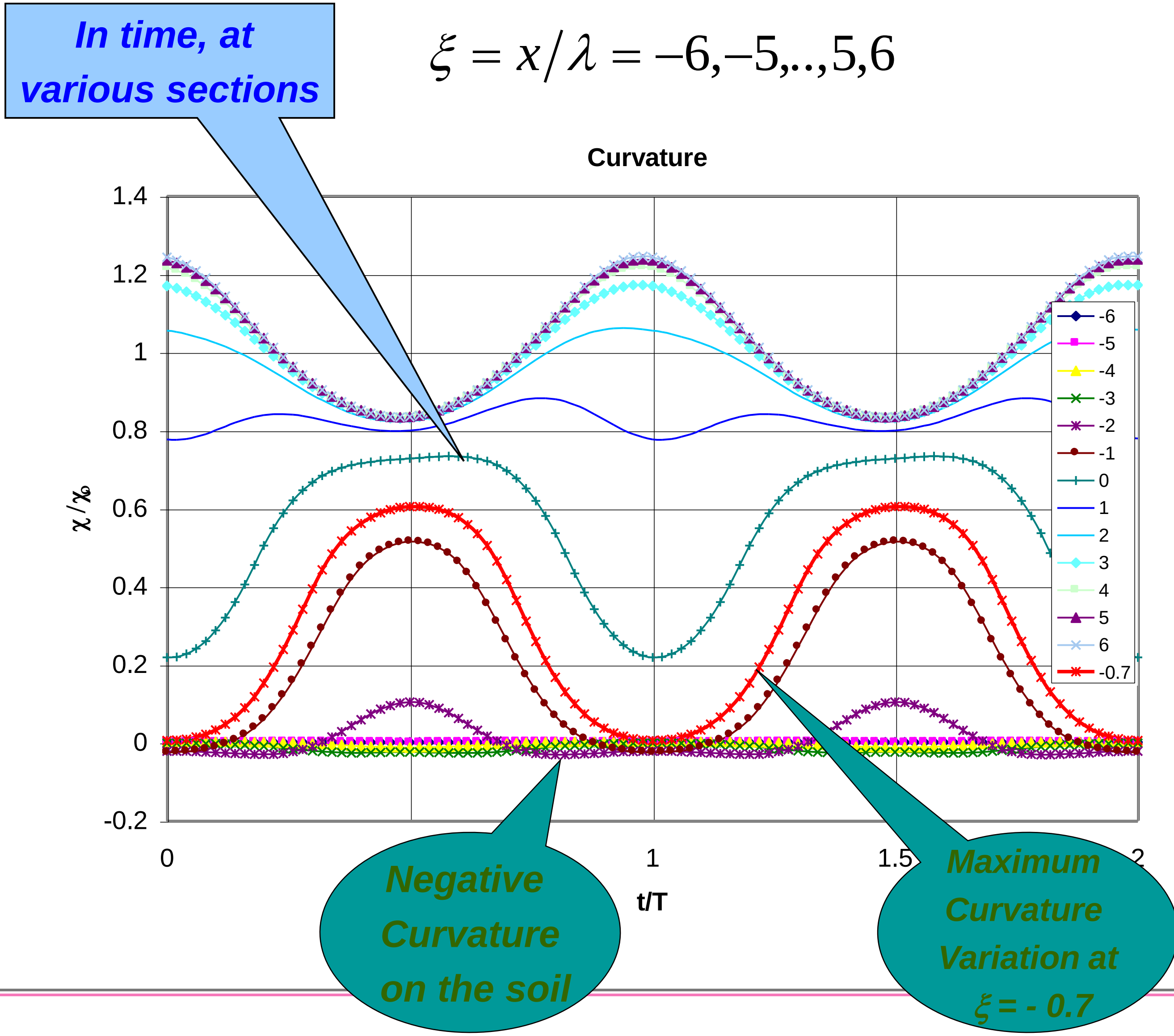
$t/\mathrm{T} = 0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5;\ \mathrm{T} = 2\pi\omega^{-1}$



Local Dynamics on Linearly Elastic Soil

Curvature at TDZ

$$\xi = x/\lambda = -6,-5,..,5,6$$



"SOFT" SOIL

$$K = 10$$

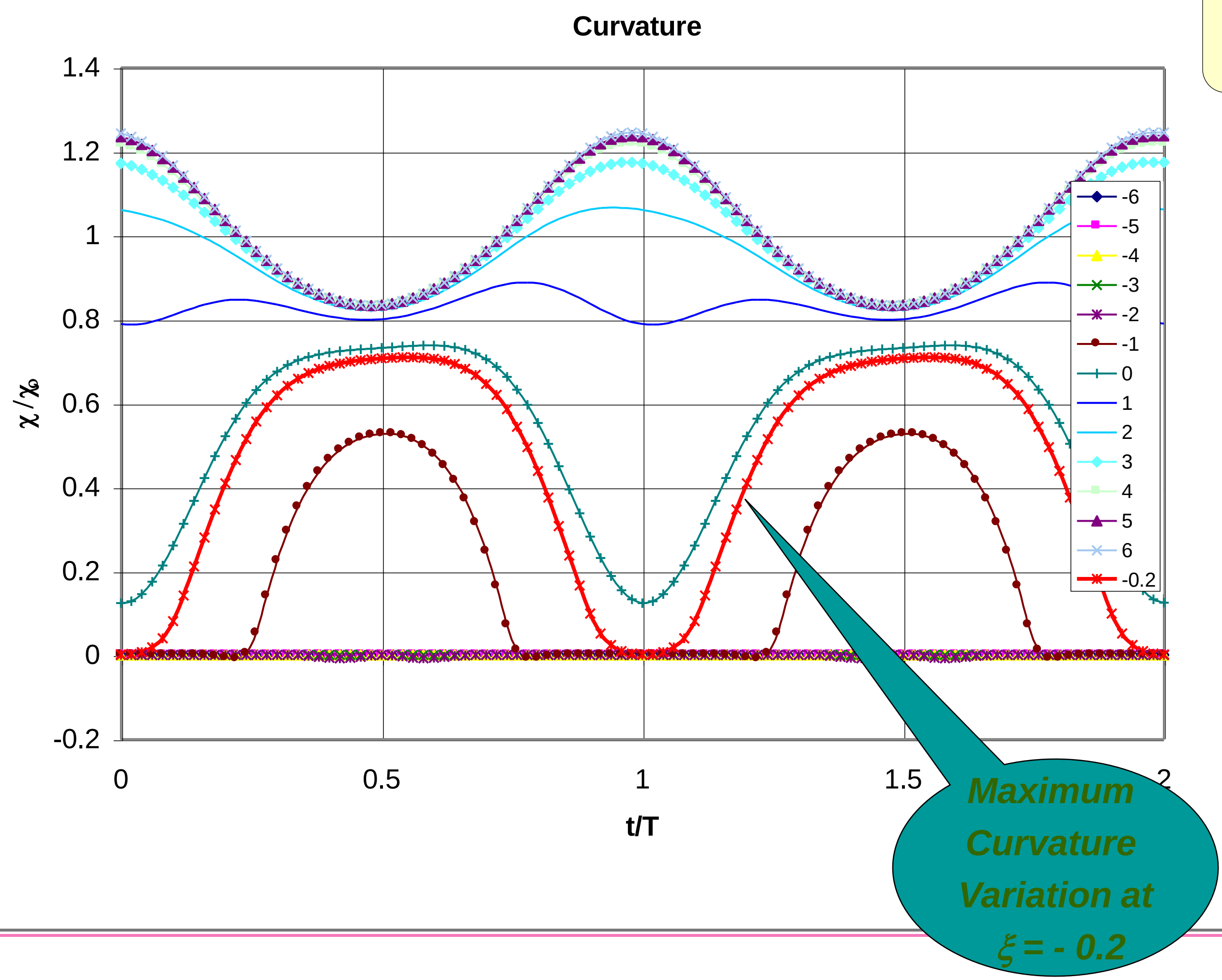
$$a_0 = 1$$

$$\beta_0 = 0.2$$

Local Dynamics on Linearly Elastic Soil

Curvature at TDZ

$$\xi = x/\lambda = -6,-5,..,5,6$$



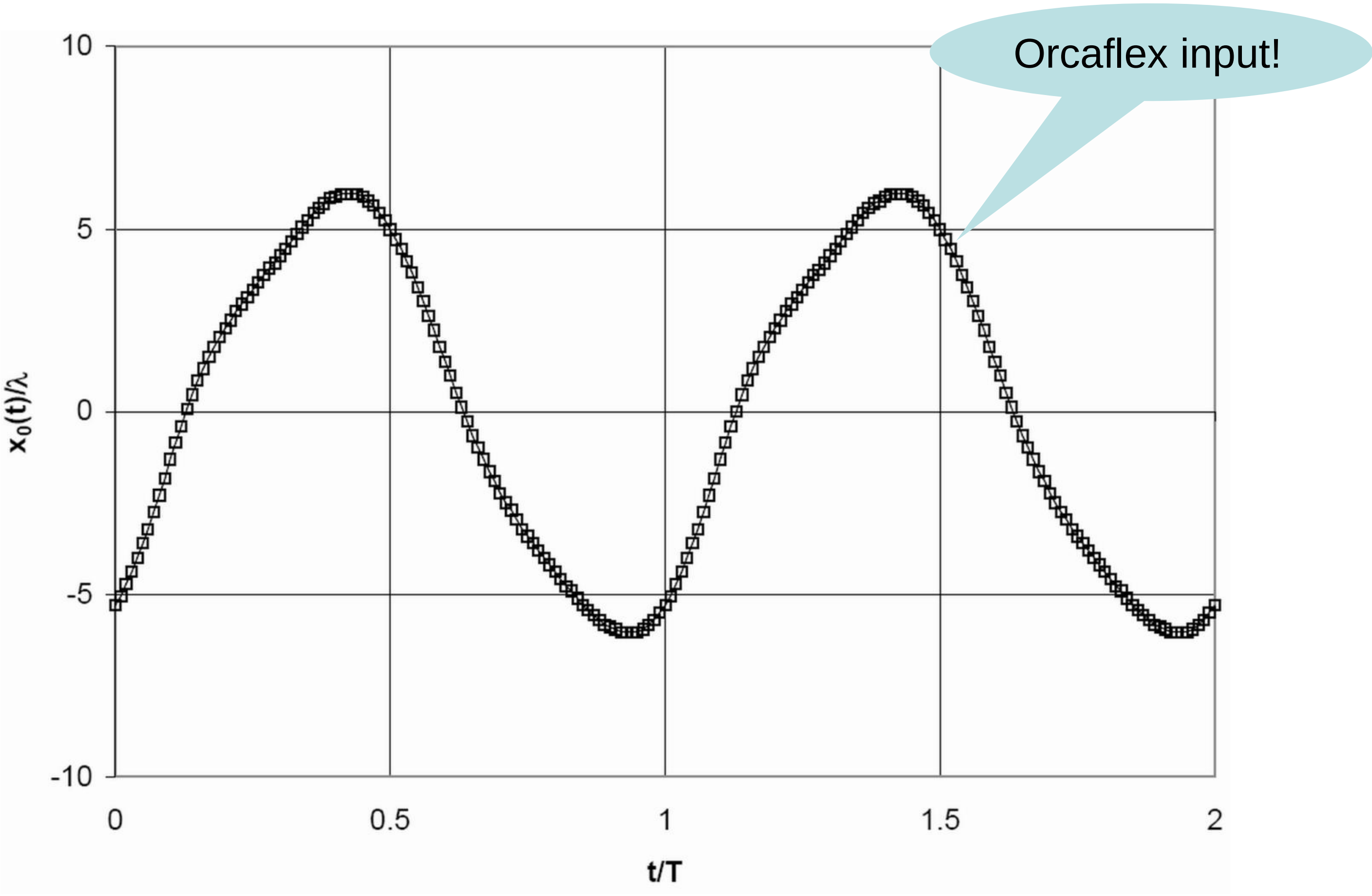
Very "STIFF" SOIL

$$K = 10^4$$

$$a_0 = 1$$

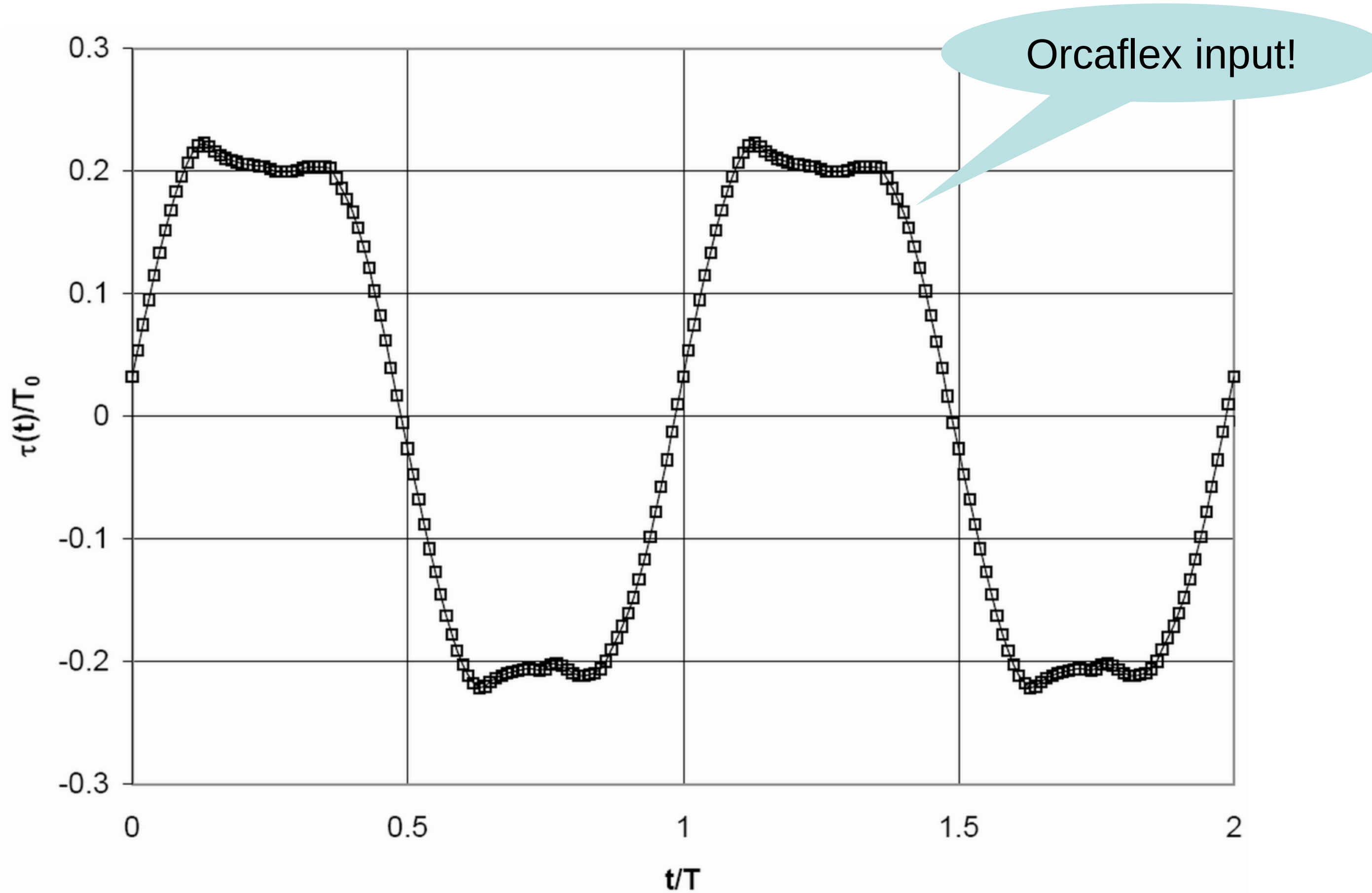
$$\beta_0 = 0.2$$

Asymptotic solution at TDZ with a linear soil-structure interaction: comparing with ORCAFLEX



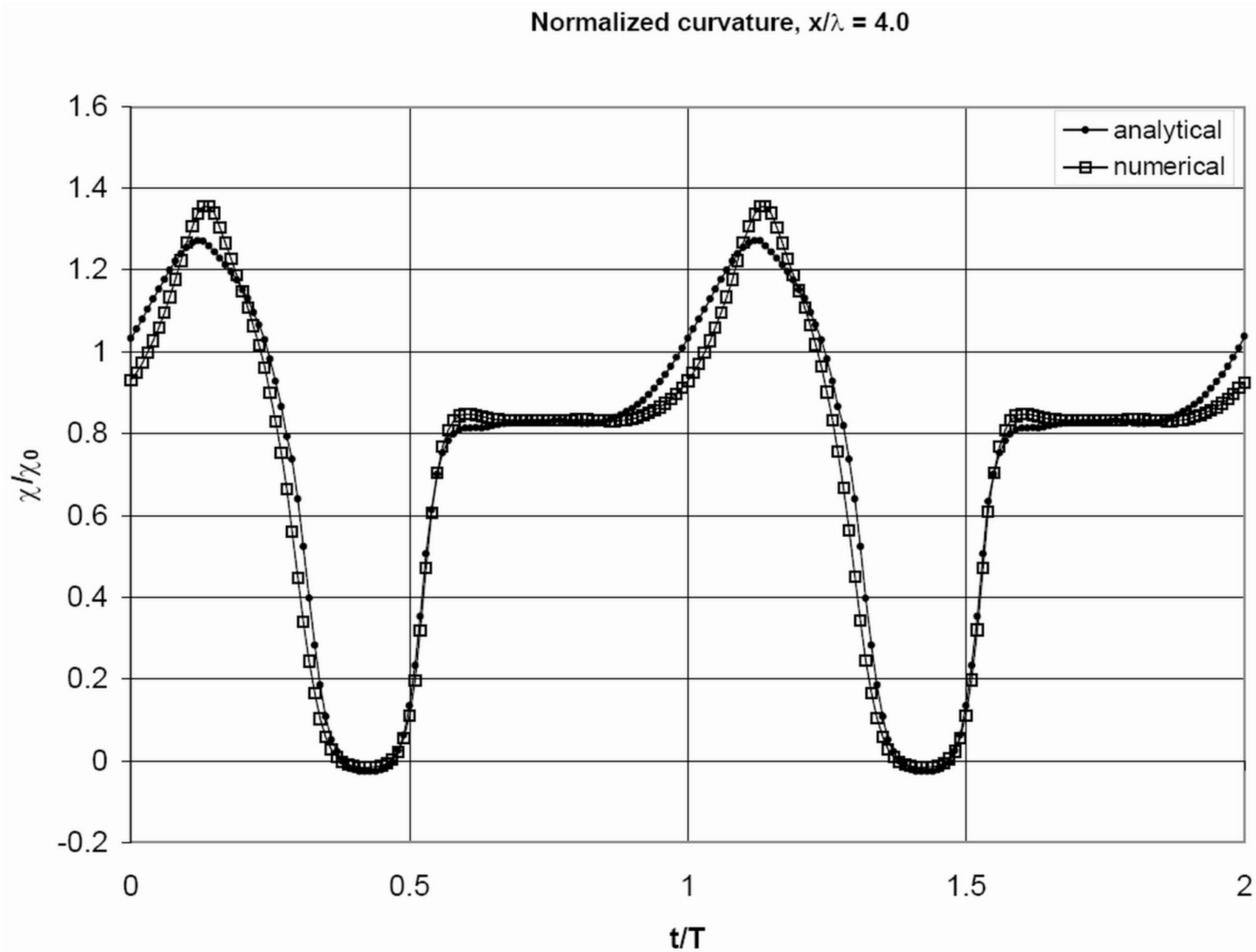
TDP excursion

Asymptotic solution at TDZ with a linear soil-structure interaction : comparing with ORCAFLEX



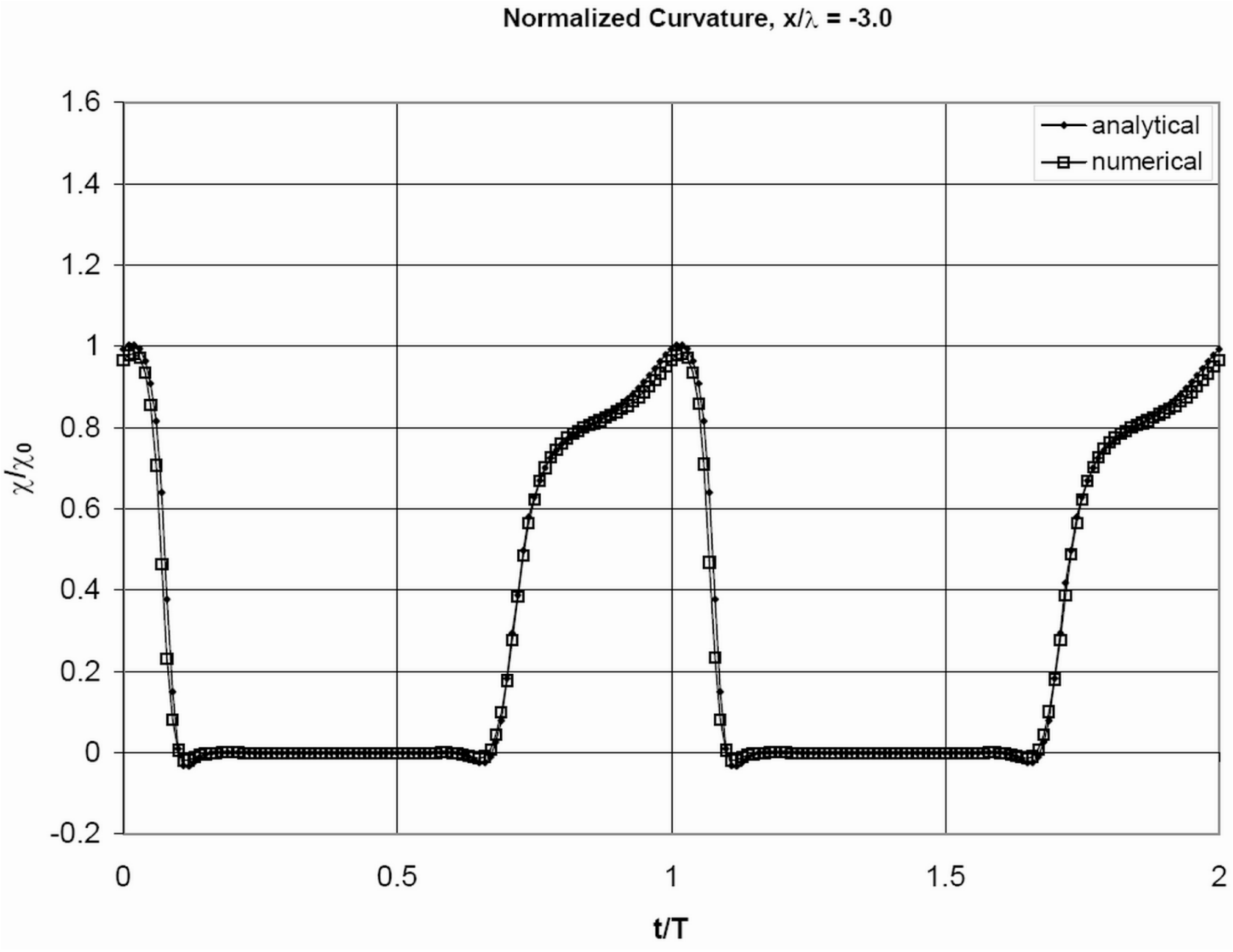
Tension at TDP

Asymptotic solution at TDZ with a linear soil-structure interaction : comparing with ORCAFLEX



Curvature at TDZ: $x/\lambda=4$

Asymptotic solution at TDZ with a linear soil-structure interaction : comparing with ORCAFLEX



Curvature at TDZ at x/λ=-3

Universidade de São Paulo

LIFE&MO

SCR
A first-order approximation for VIV

Shedding frequency

$$\Omega_s = 2\pi S_t \frac{U}{D}$$

Strouhal Number

$$U^* = \frac{U}{f_n D} = \frac{2\pi U}{\Omega_n D}$$

Reduced Velocity

$$\alpha = A/\lambda$$

Modal Amplitude

$$\Omega_n = \frac{2\pi}{U^*}\frac{U}{D} \longrightarrow \frac{\Omega_s}{\Omega_n} = S_t U^*$$

$$S_t \approx 0.18 - 0.2$$

VIV peaks at $\frac{\Omega_s}{\Omega_n} \cong 1 \longrightarrow U^* \approx 5.5$

In $U = 0.5\,\mathrm{m/s} \Rightarrow \Omega_{25} \cong 2.9\,\mathrm{rad/s}$

$$\omega_{25} = (L/c_0)\times\Omega_{25} = 37.038\times\Omega_{25} \cong 107.4$$

Universidade de São Paulo

LIFE&MO

SCR

A first-order approximation for VIV

Dynamics at TDA

$$\xi_0(t) \cong \varepsilon\omega X_0^{-1}\alpha\cos\omega t$$

$$b^2(t) \cong 1-\varepsilon\omega X_0\alpha\cos\omega t$$

$$\mathrm{M} = \varepsilon^2\omega^2 X_0^{-1}\alpha < 1$$

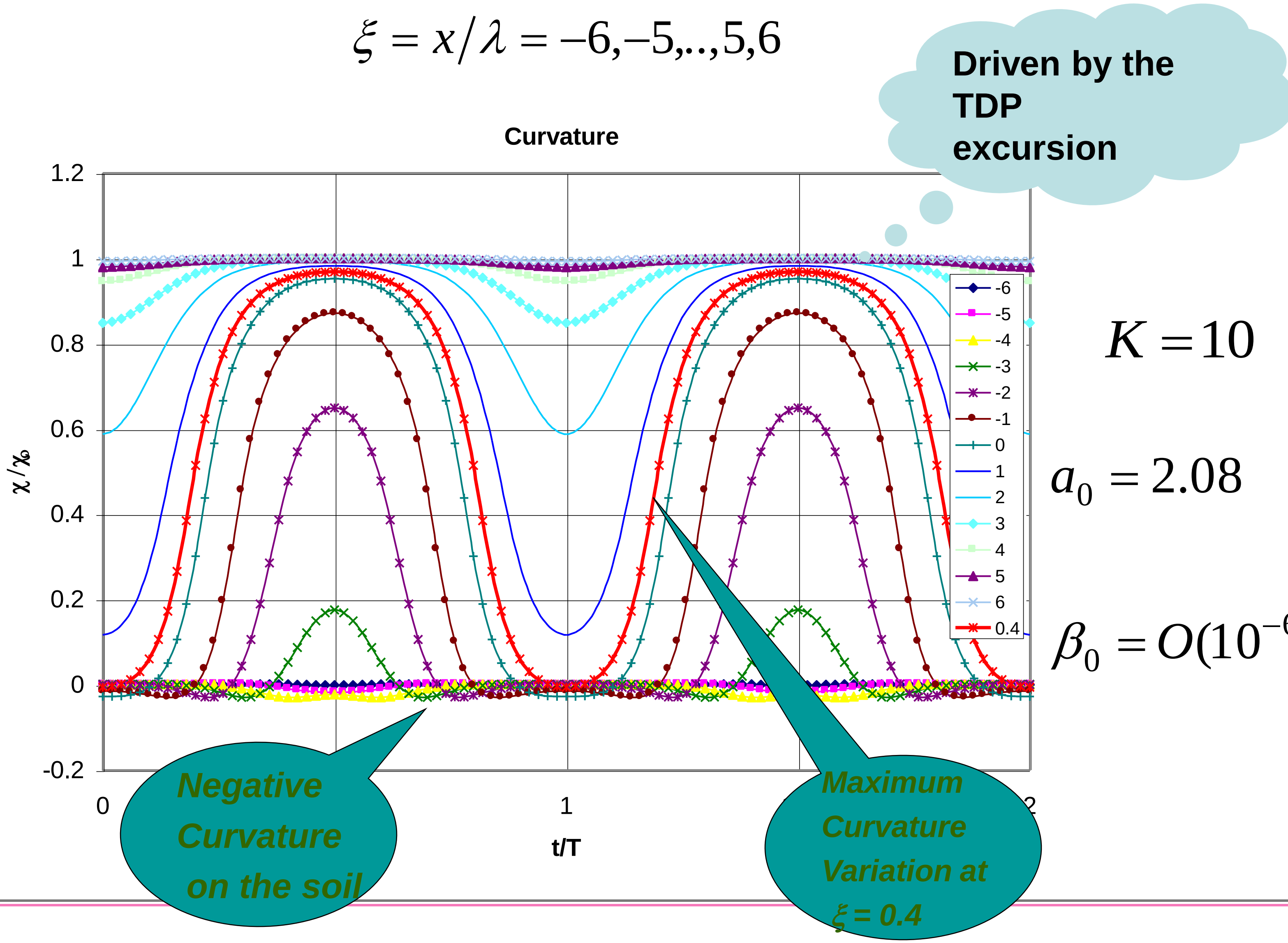
$$\alpha = A/\lambda$$

$A = 1.0\times D \Rightarrow$ $\mathrm{M} = 0.33 \Rightarrow$ ***No shocking***

Local Dynamics on Linearly Elastic Soil

Corrected Curvature at TDA

$$\xi = x/\lambda = -6,-5,..,5,6$$



$$K = 10$$

$$a_0 = 2.08$$

$$\beta_0 = O(10^{-6})$$

Discussion

- *Inside the boundary-layer:* **curvature variation attains a maximum of order χ_0 at a section that slightly penetrates the soil.**

- *Outside the boundary-layer(from WKB solution):* **curvature variation attains a maximum of order (Pesce et al., 2006):**

$$\Delta\chi_{WKB_{\max}} \approx O(2M\chi_0) = 0.66\chi_0$$

at a suspended section

$$(\xi^* = \varepsilon^{-1}\breve{\xi}^* \approx \pi/(2\varepsilon\omega)) \approx 10$$

TYPICAL P-52 SCR DATA : RONCADOR FIELD, CAMPOS BASIN

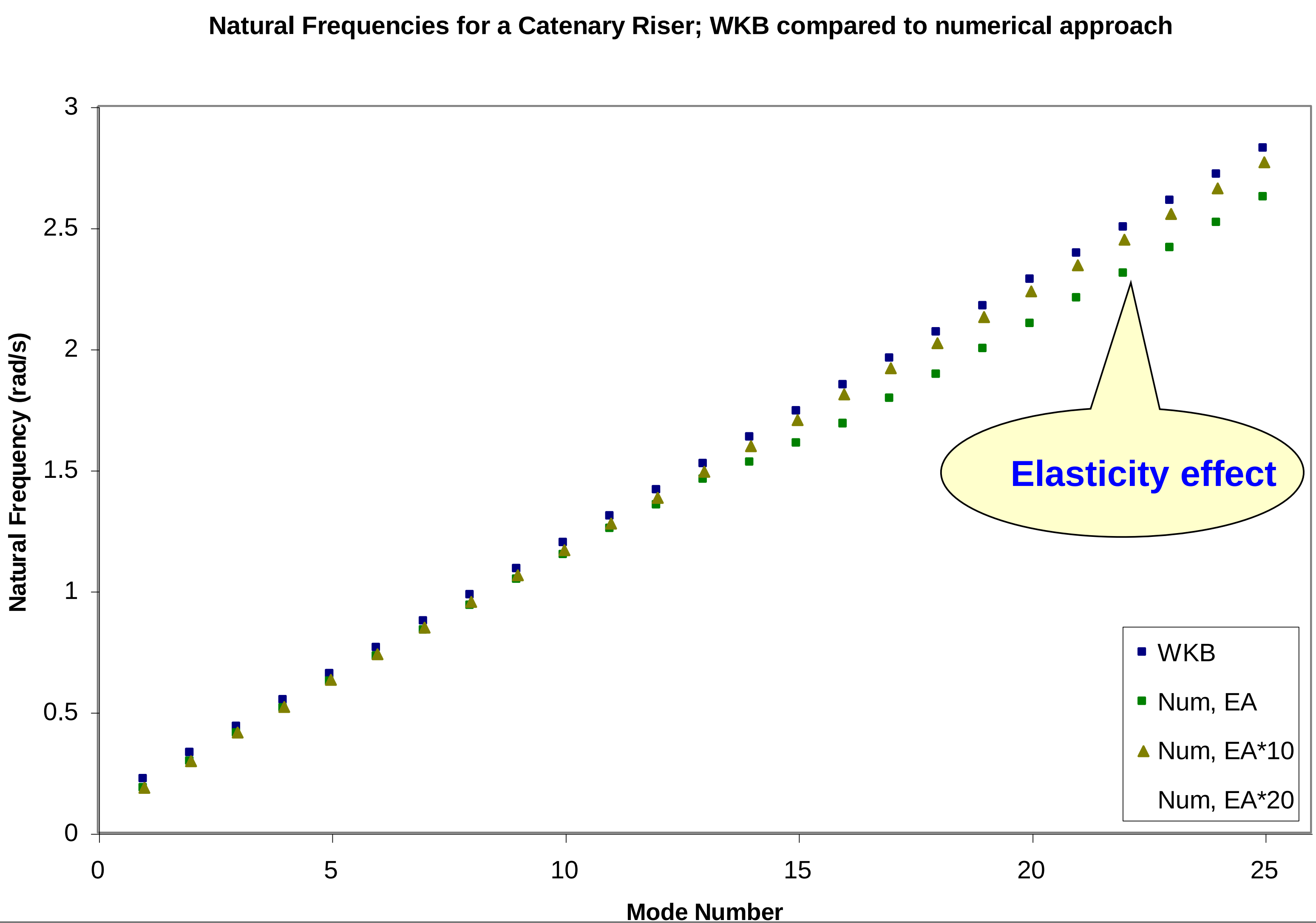
Axial Rigidity, *EA* (kN)	2.314 x10^6
Bending Stiffness, *EI* (kNm^2)	9915
Immersed weight, *q* (kN/m)	0.727
m (kg/m) (filled with water)	108.0
External diameter, *D* (m)	0.2032
Thickness (mm)	19.05
Depth *H* (m)	1800
Total length (m)	5047
Angle at top, θ_L (^{o}) (no current)	70 (w.r.t. horizontal)
Soil Rigidity, *k* (kN/m/m)	466.37
Suspended length, *L* (m)	2571
Static tension at TDP, T_0 (kN)	680.55
Flexural length, λ(m)	3.82
Curvature at TDP, χ_0 (m^{-1})	1.077E-03
Nondimensional curvature at TDP, $X_0 = \chi_0 \lambda$	4.114E-03
Local scale, $\varepsilon = \lambda / L$	1.486E-03
Nondimensional soil rigidity parameter, $K = kEI / T_0^2$	10

L/D~O(10^4)

43 SCRs designed for the Semi-submersible P52 in 1800 m waterdepth

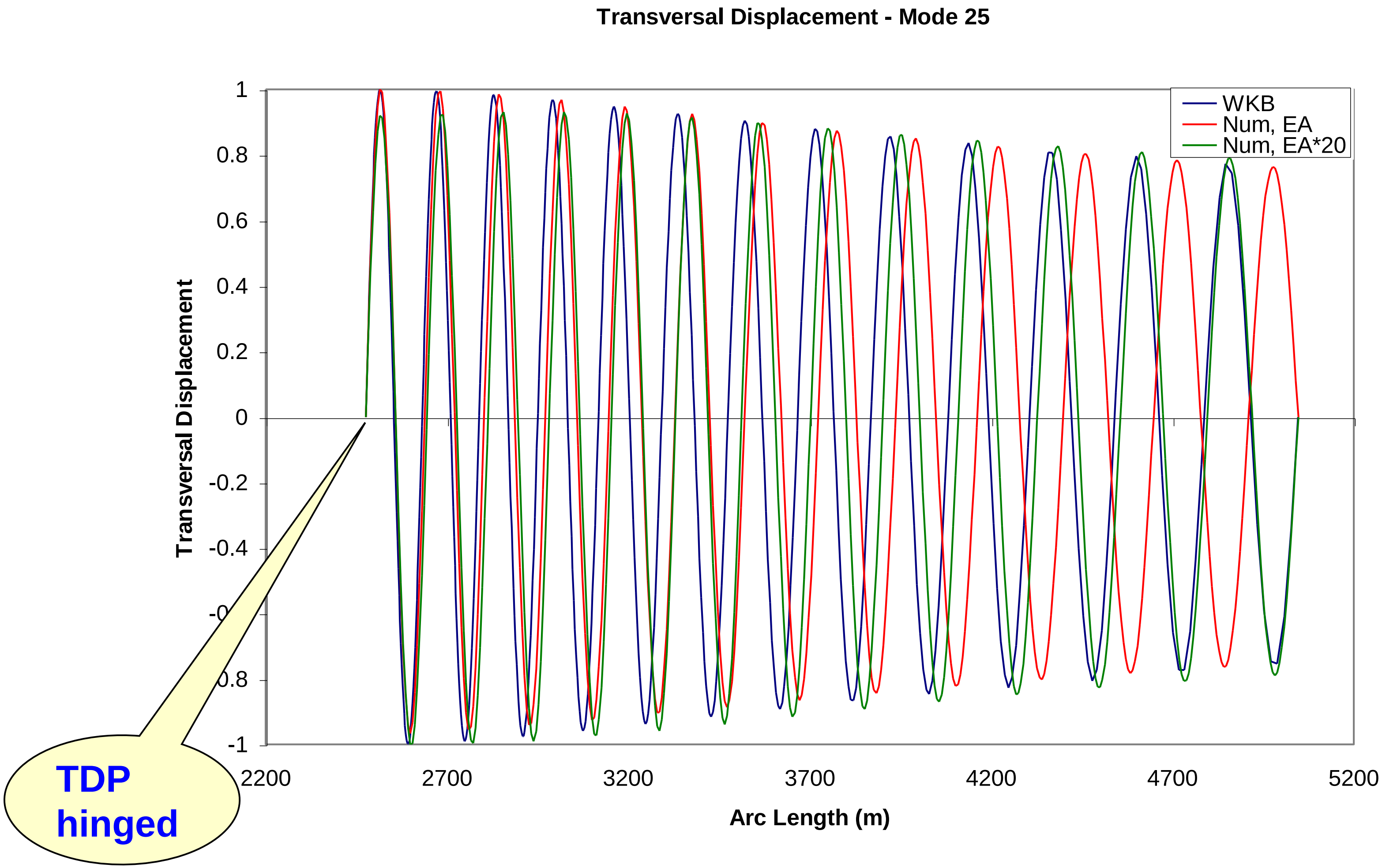
WKB vs. POLIFLEX

SCR



WKB vs. POLIFLEX

SCR: Mode 25



WKB Asymptotic Technique gives, for SCR eigenfrequecies:

$$\Omega_n \cong \Lambda_n \sqrt{\frac{(1-\cos\theta_L)}{\cos\theta_L}} \sqrt{\frac{(m^*-1)}{(m^*+1)}} \sqrt{\frac{g}{H}} \, ,$$

$$\varphi_n(\theta;\theta_L) \cong A_n (\cos\theta)^{-1/4} \sin\left\{ \Lambda_n \int_0^{\theta} \frac{d\theta}{(\cos\theta)^{3/2}} \right\}$$

$$\Lambda_n = \Lambda_n(\theta_L) \cong \frac{n\pi}{\int_0^{\theta_L} \frac{d\theta}{(\cos\theta)^{3/2}}} \, .$$

For θ_L=70 degrees w.r.t horizontal (20 degrees w.r.t. vertical):

$$\Lambda_n \sqrt{\frac{1-\cos\theta_L}{\cos\theta_L}} \cong 2n \;\Rightarrow \Omega_n \cong 2n \sqrt{\frac{(m^*-1)}{(m^*+1)}} \sqrt{\frac{g}{H}}$$

Defining a 'buoyancy' Froude number:

$$Fr^* = \frac{U}{\sqrt{(m^*-1)gD}}$$

$$\Rightarrow \quad \frac{\Omega_s}{\Omega_n} \cong \frac{\pi S_t F_r^*}{n} \sqrt{\frac{H}{D}} \sqrt{(m^*+1)}$$

$$m^* = \frac{m}{m_d} \cong \frac{m}{m_a} = a^{-1}$$

The typical SCR:
D=0.203 m; H=1800m; m*=3.26
Primary resonance expected when:

$$n \cong \pi S_t F_r * \sqrt{\frac{H}{D}} \sqrt{(m^*+1)}$$

At U=0.1 m/s, eigenmode 5 is excited, with a frequency just inside the typical first-order wave-frequency range:

$$\Omega \cong 0.5 \text{ rad/s}$$

U (m/s)	Fr*	n	Wn (rad/s)	Tn (s)
0,1	0,047	5	0,537	11,70
0,2	0,093	11	1,182	5,32
0,3	0,140	17	1,826	3,44
0,4	0,187	22	2,364	2,66
0,5	0,234	28	3,008	2,09
0,7	0,327	39	4,190	1,50
0,8	0,374	45	4,835	1,30
0,9	0,421	51	5,479	1,15
1	0,467	57	6,124	1,03

This would even enhance the possibility of sub-harmonic and parametric resonance induced by platform motion.

Summary

- The riser-soil interaction problem was addressed.
- A quasi-static analytical solution was derived for the local dynamics of a catenary riser at touchdown region.
- The present solution is quasi-static in the sense that the local dynamics is governed by the relatively slow dynamics of the suspended part.
- This is valid if soil rigidity is sufficiently large and if the motion is not so fast as to provoke impact of the riser against the soil.
- Even being the soil modeled as linear elastic, the main features of the nonlinear nature of the contact was preserved, by properly considering the oscillatory excursion of the TDP and the dynamic tension.
- An estimate for curvature variation at TDZ due to VIV has been addressed.

The asymptotic solution near the TOP (upper end)

Local equation:

$$\frac{\partial^2 \chi}{\partial s^2} - \frac{1}{\lambda_L^{\ 2}}\left(1+\frac{\tau}{T_L}\right)\chi = -\frac{1}{\lambda_L^{\ 2}}\left(1+\frac{\tau}{T_L}\right)\chi^c(s,t)\big((1+O(\delta)\big)$$

$$\lambda_L = \frac{EI}{T_L}$$

Flexural length at TOP

Curvatura local:

$$\chi(s,t) = \chi^c(s,t) -$$

$$-\frac{1}{\lambda_L}\left(\chi_L^c(t)\lambda_L + \frac{k_F\lambda_L}{EI}\big(\Theta_L(t)-\Phi(t)\big)\right)\exp\left(\sqrt{1+\frac{\tau(t)}{T_L}}\,\frac{s-L}{\lambda_L}\right)$$

Bending stiffness rigidity

Cable curvature

Cable angle at Top

Imposed angle at the connection

The asymptotic solution near the TOP (upper end)

Angle along the local boundary-layer:

Cable angle

Imposed angle to the connection

$$\Theta(s,t)=\Theta^{c}(s,t)-\frac{1}{\sqrt{1+\frac{\tau(t)}{T_L}}}\left(\chi_L^c(t)\lambda_L+\frac{k_F\lambda_L}{EI}\left(\Theta_L(t)-\Phi(t)\right)\right)\exp\left(\sqrt{1+\frac{\tau(t)}{T_L}}\,\frac{s-L}{\lambda_L}\right)$$

Cable angle at TOP

Cable angle at TOP:

$$\Theta_L(t)=\frac{\Theta_L^c(t)+\frac{1}{\sqrt{1+\tau(t)/T_L}}\left(\frac{k_F\lambda_L}{EI}\Phi(t)-\chi_L^c(t)\lambda_L\right)}{1+\frac{k_F\lambda_L}{EI}\frac{1}{\sqrt{1+\tau(t)/T_L}}}$$

The asymptotic solution near the TOP (upper end)

Static curvature along the local boundary layer:

$$\chi(s) \cong \chi^{c}(s) - \left(\chi_L^c + \frac{k_F}{EI}(\theta_L - \Phi) \right) e^{(s-L)/\lambda_L}$$

Static angle:

$$\theta(s) = \theta^{c}(s) - \left(\chi_L^c \lambda_L + \frac{k_F \lambda_L}{EI}(\theta_L - \Phi) \right) e^{(s-L)/\lambda_L}$$

Angle at TOP:

$$\theta_L = \frac{\theta_L^c + \dfrac{k_F \lambda_L}{EI}\Phi - \chi_L^c \lambda_L}{1 + \dfrac{k_F \lambda_L}{EI}} = \frac{\theta_L^c + \dfrac{k_F}{\lambda_L T_L}\Phi - \chi_L^c \lambda_L}{1 + \dfrac{k_F}{\lambda_L T_L}}$$

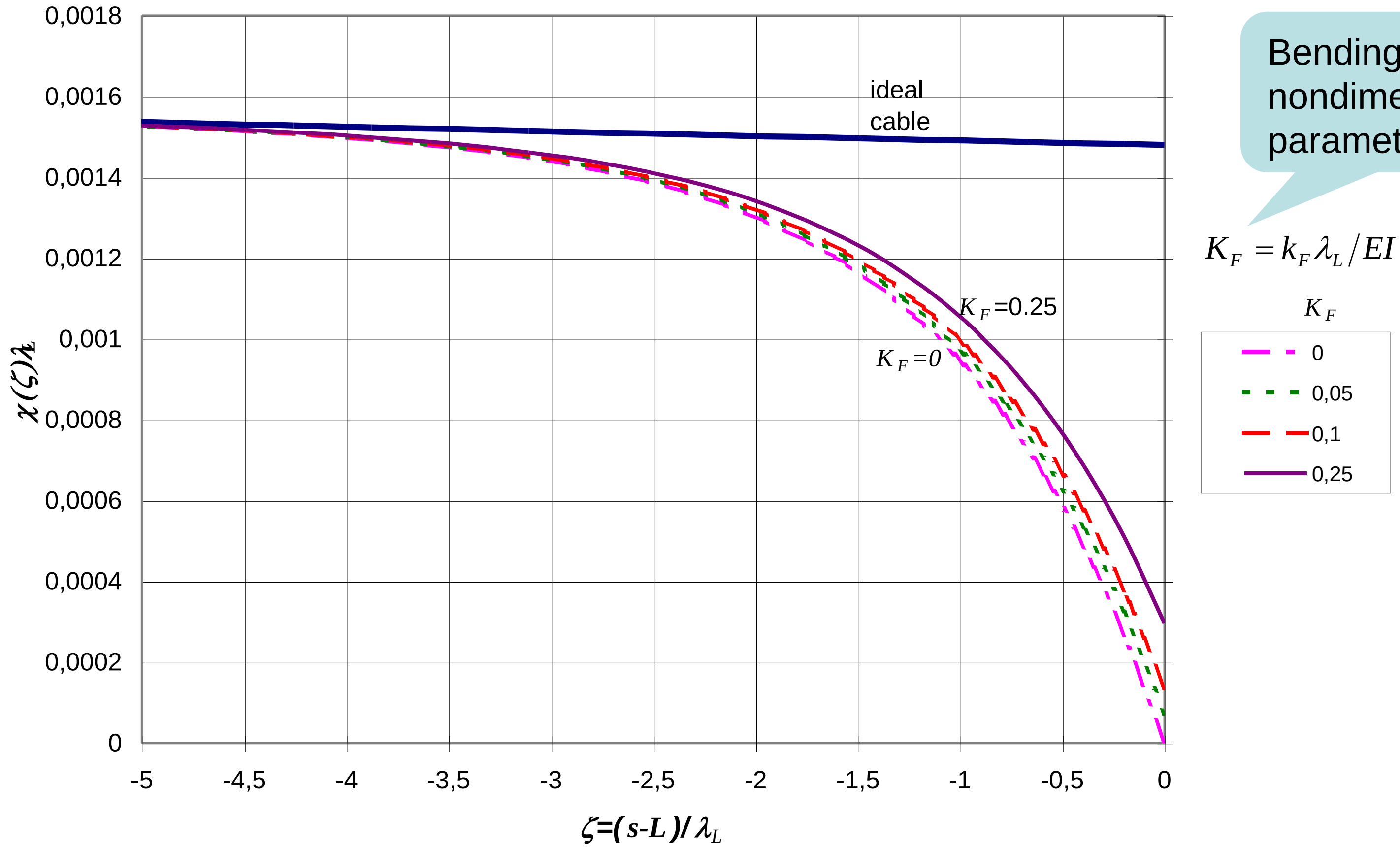
The asymptotic solution near the TOP (upper end)

Ideal design condition:

$$\Phi^* = \theta_L^c - \chi_L^c \lambda_L \qquad \chi(s) \cong \chi^c(s) - \chi_L^c e^{(s-L)/\lambda_L}$$

Such that: $\chi(L) = \chi_L \equiv 0$

The asymptotic solution near the TOP (upper end)



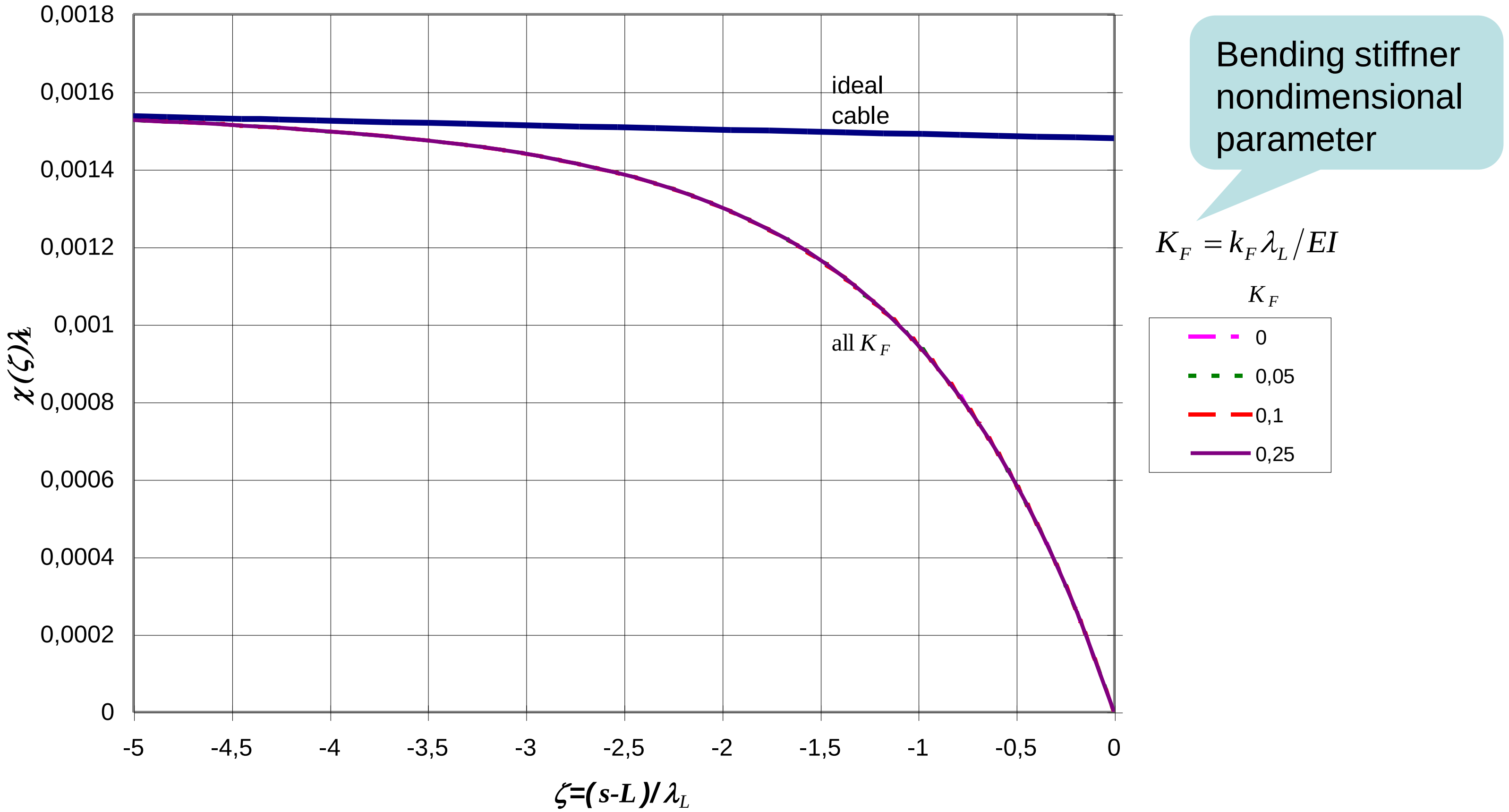
Bending stiffner nondimensional parameter

$K_F = k_F \lambda_L / EI$

Nondimensional static curvature

$\theta_L^c = 68.4^o; \Phi = \theta_L^c = 68.4^o$

The asymptotic solution near the TOP (upper end)

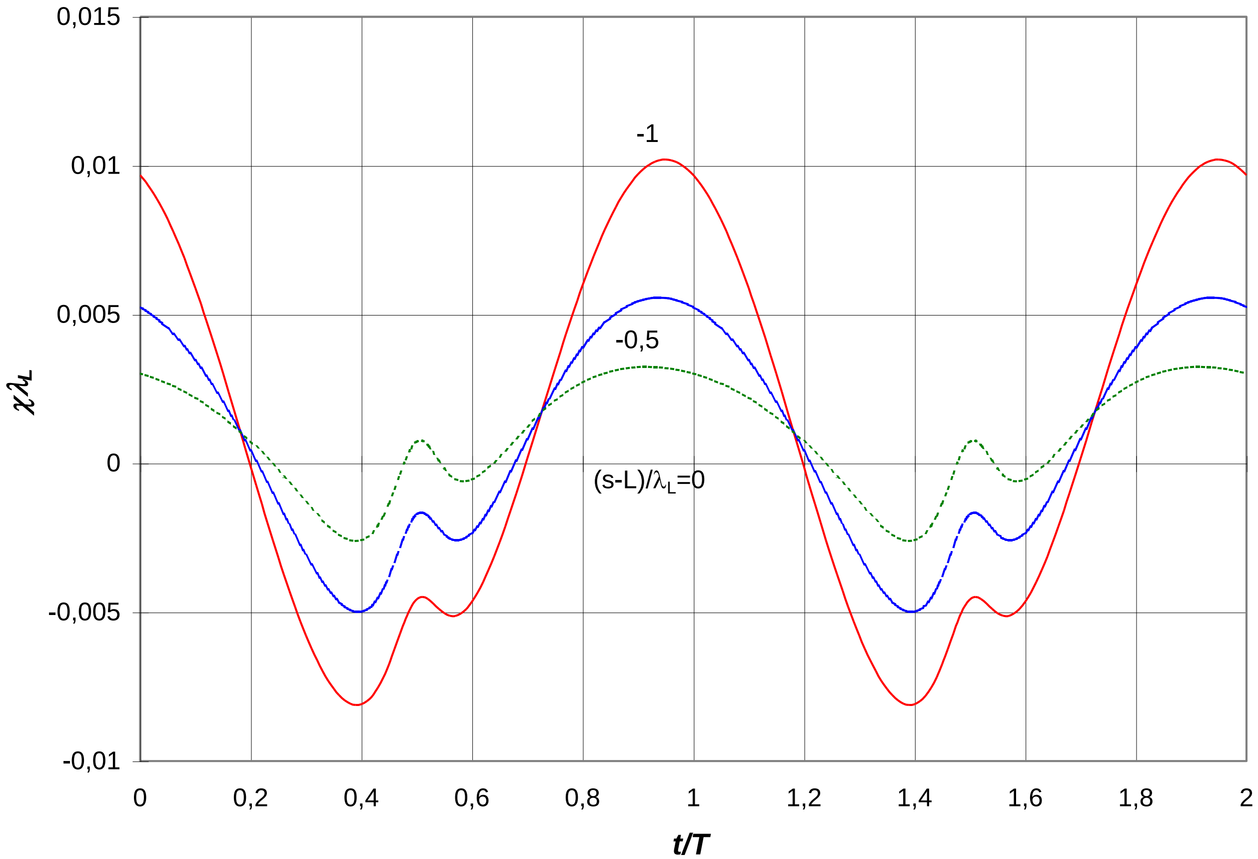


Nondimensional static curvature. Ideal design condition.

$\theta_L^c = 68.4^o; \Phi = \theta_L^c = 68.4^o$

The asymptotic solution near the TOP (upper end)

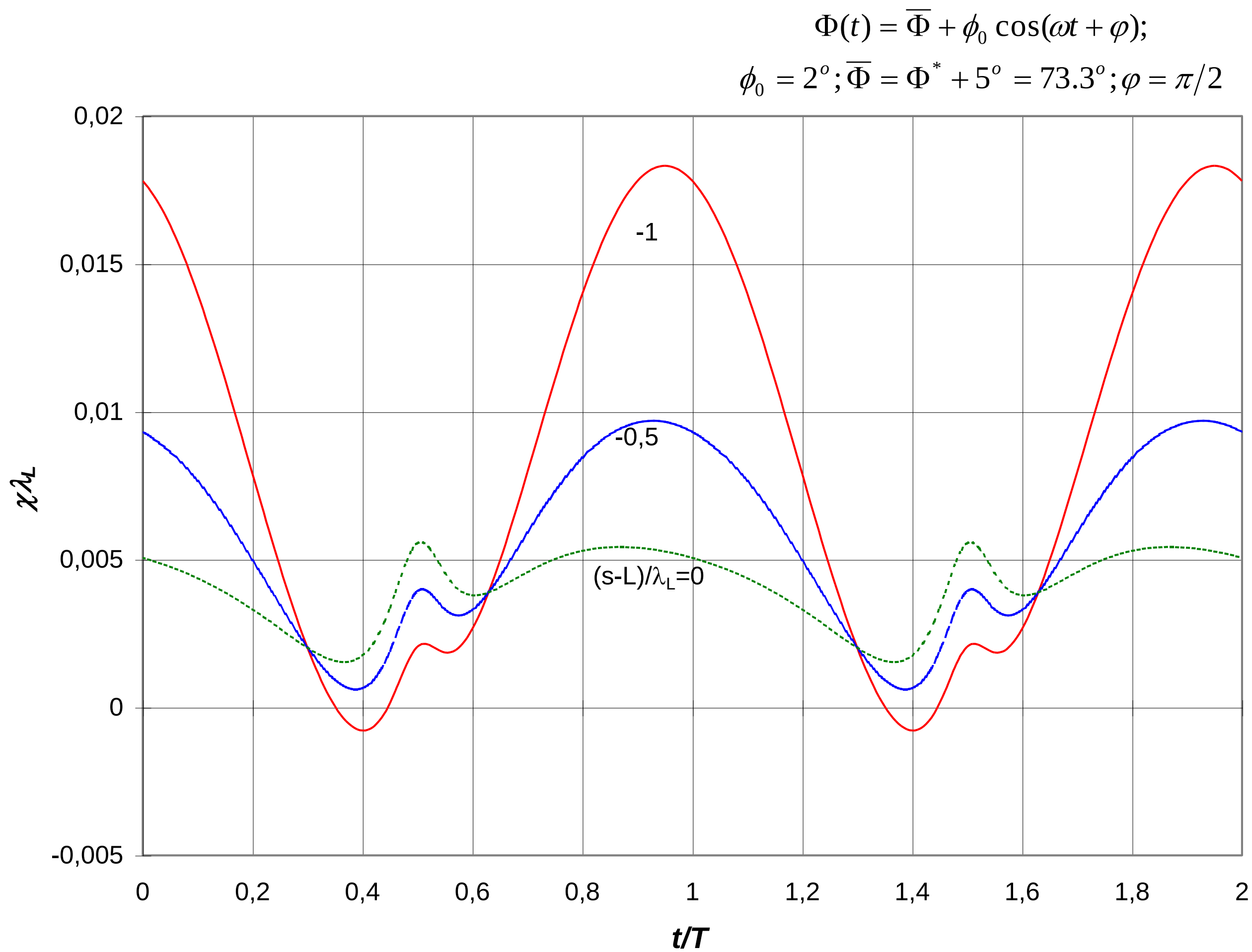
$$\Phi(t) = \Phi^* + \phi_0 \cos(\omega t + \varphi); \phi_0 = 2^o; \varphi = \pi/2$$



Nondimensional dynamic curvature. Ideal design condition.

$\Phi^* = \theta_L - \chi_L^c \lambda_L = 1.192\text{rad} = 68.3^o; \tau_0/T_L = 0.9$ $\theta_L^c = 68.4^o$

The asymptotic solution near the TOP (upper end)



Nondimensional dynamic curvature. Survival heeling condition: 5 degrees.

$\Phi^* = \theta_L - \chi_L^c \lambda_L = 1.192\text{rad} = 68.3^o; \tau_0 / T_L = 0.9$ $\theta_L^c = 68.4^o$

USP
Universidade de São Paulo

LIFE&MO

Acknowledgements

FAPESP 50 ANOS

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

FINEP
FINANCIADORA DE ESTUDOS E PROJETOS
MINISTÉRIO DA CIÊNCIA E TECNOLOGIA

PRYSMIAN
CABLES & SYSTEMS

Draka

BR
PETROBRAS

prh Programa de Recursos Humanos

anp Agência Nacional do Petróleo

TPN

ipt INSTITUTO DE PESQUISAS TECNOLÓGICAS

NDF

Universidade de São Paulo

LIFE&MO

LIFE&MO

FLUID-STRUCTURE INTERACTION AND OFFSHORE MECHANICS LABORATORY