Lecture 1-b

Catenary Risers: Global Analysis

Dynamic Problem
equations and time scales

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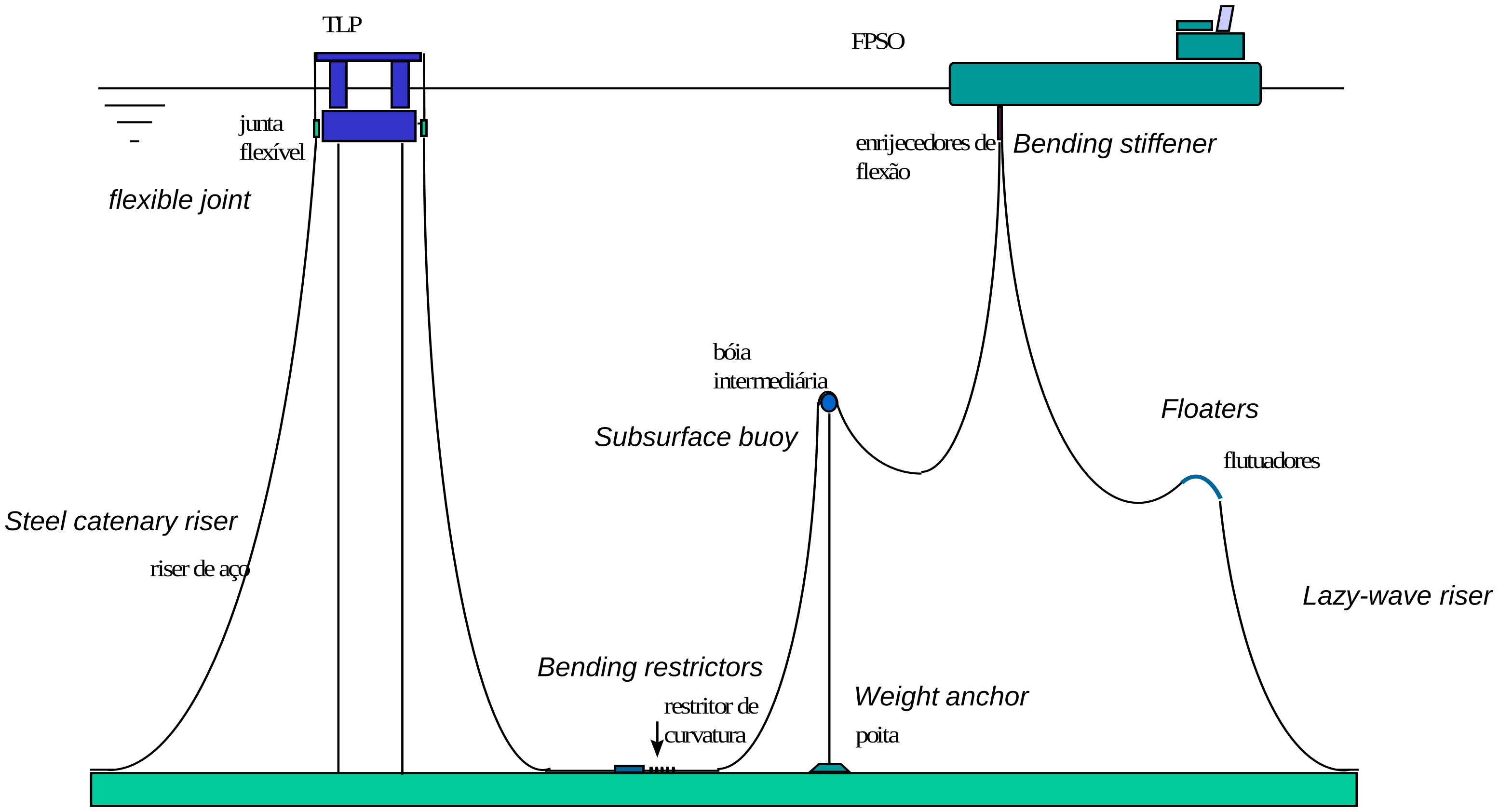
Catenary lines



General Kirshoff-Clebsh-Love Equations (KCL equations)

Curved bars – large displacements, small deformations

$$\frac{\partial \mathrm{T}}{\partial s} - \mathrm{Q}_v \kappa_w + \mathrm{Q}_w \kappa_v + \mathrm{f}_u(t) = m \frac{\partial^2 u}{\partial t^2}$$

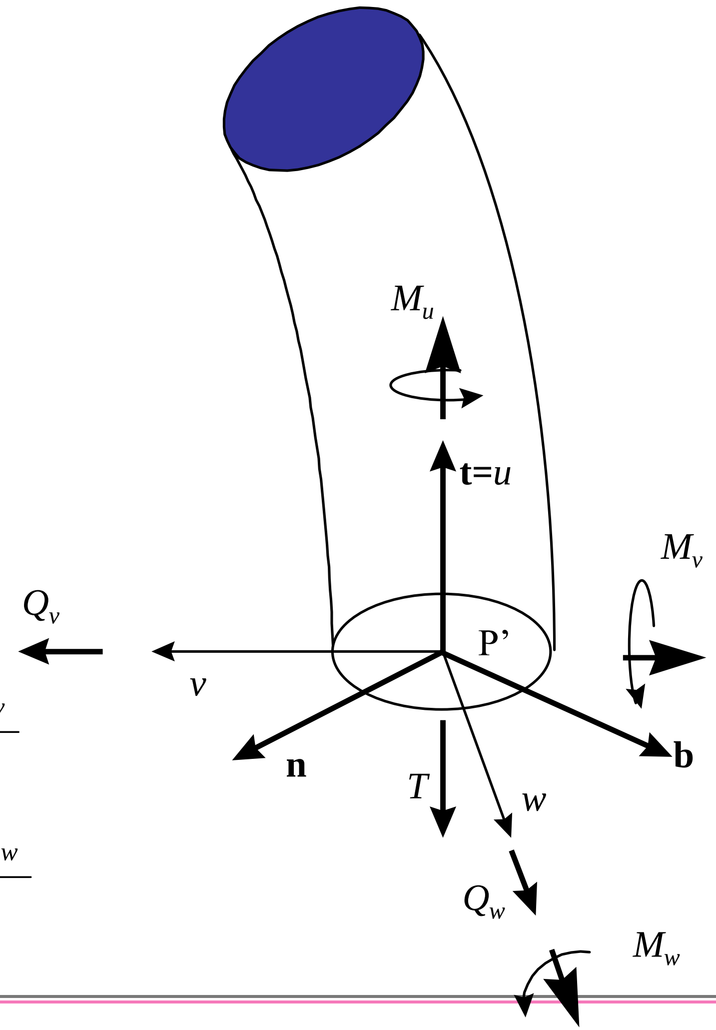
$$\frac{\partial \mathrm{Q}_v}{\partial s} - \mathrm{Q}_w \kappa_u + \mathrm{T} \kappa_w + \mathrm{f}_v(t) = m \frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial \mathrm{Q}_w}{\partial s} - \mathrm{T} \kappa_v + \mathrm{Q}_v \kappa_u + \mathrm{f}_w(t) = m \frac{\partial^2 w}{\partial t^2}$$

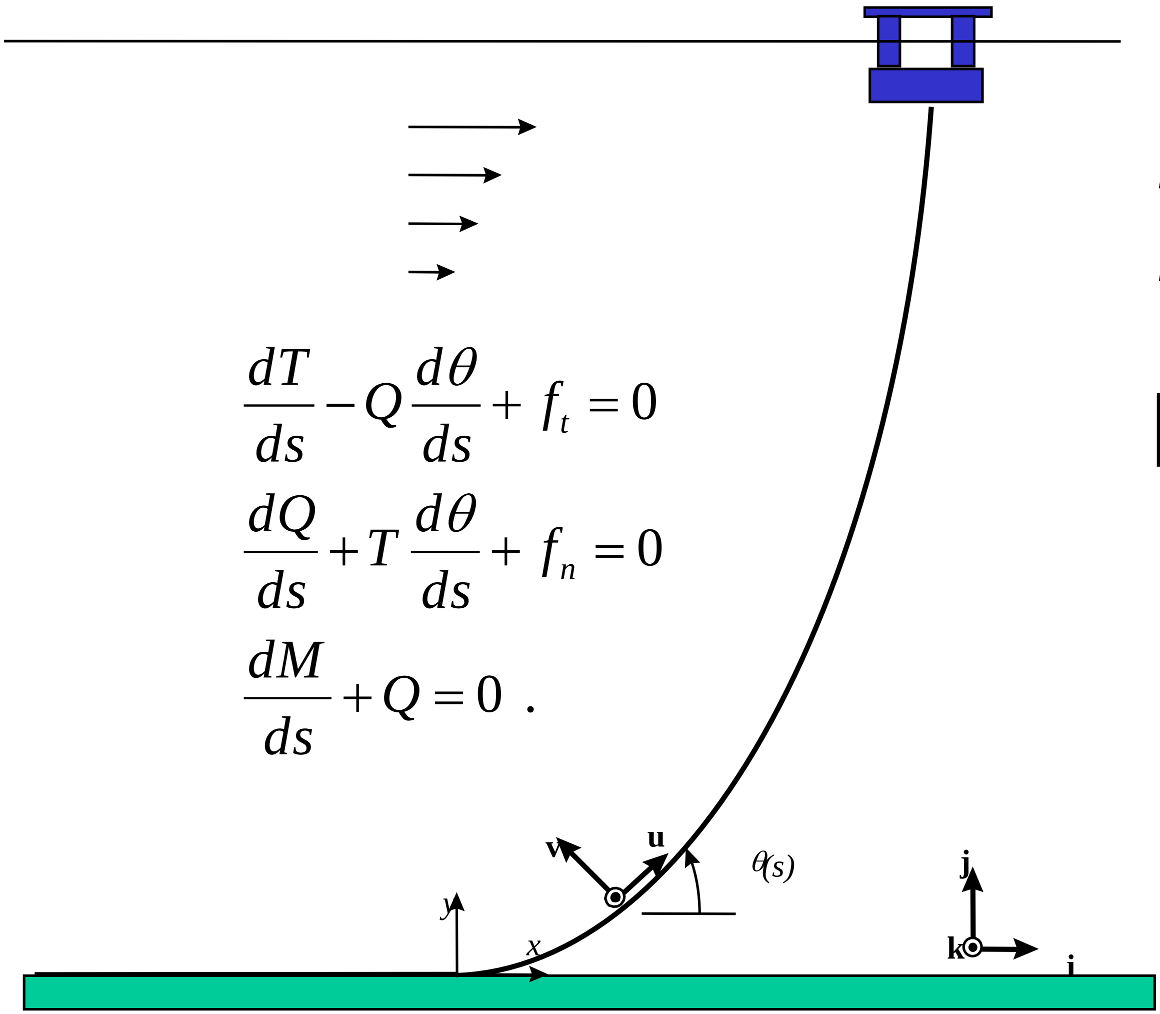
$$\frac{\partial \mathrm{M}_u}{\partial s} - \mathrm{M}_v \kappa_w + \mathrm{M}_w \kappa_v + \mathrm{K}_u(t) = I_{uu} \frac{\partial^2 \varphi_u}{\partial t^2}$$

$$\frac{\partial \mathrm{M}_v}{\partial s} - \mathrm{M}_w \kappa_u + \mathrm{M}_u \kappa_w - \mathrm{Q}_w + \mathrm{K}_v(t) = I_{vv} \frac{\partial^2 \varphi_v}{\partial t^2}$$

$$\frac{\partial \mathrm{M}_w}{\partial s} - \mathrm{M}_u \kappa_v + \mathrm{M}_v \kappa_u + \mathrm{Q}_v + \mathrm{K}_w(t) = I_{ww} \frac{\partial^2 \varphi_w}{\partial t^2}$$



The planar static problem



$$\frac{dT}{ds} - Q\frac{d\theta}{ds} + f_t = 0$$

$$\frac{dQ}{ds} + T\frac{d\theta}{ds} + f_n = 0$$

$$\frac{dM}{ds} + Q = 0 \quad .$$

with:

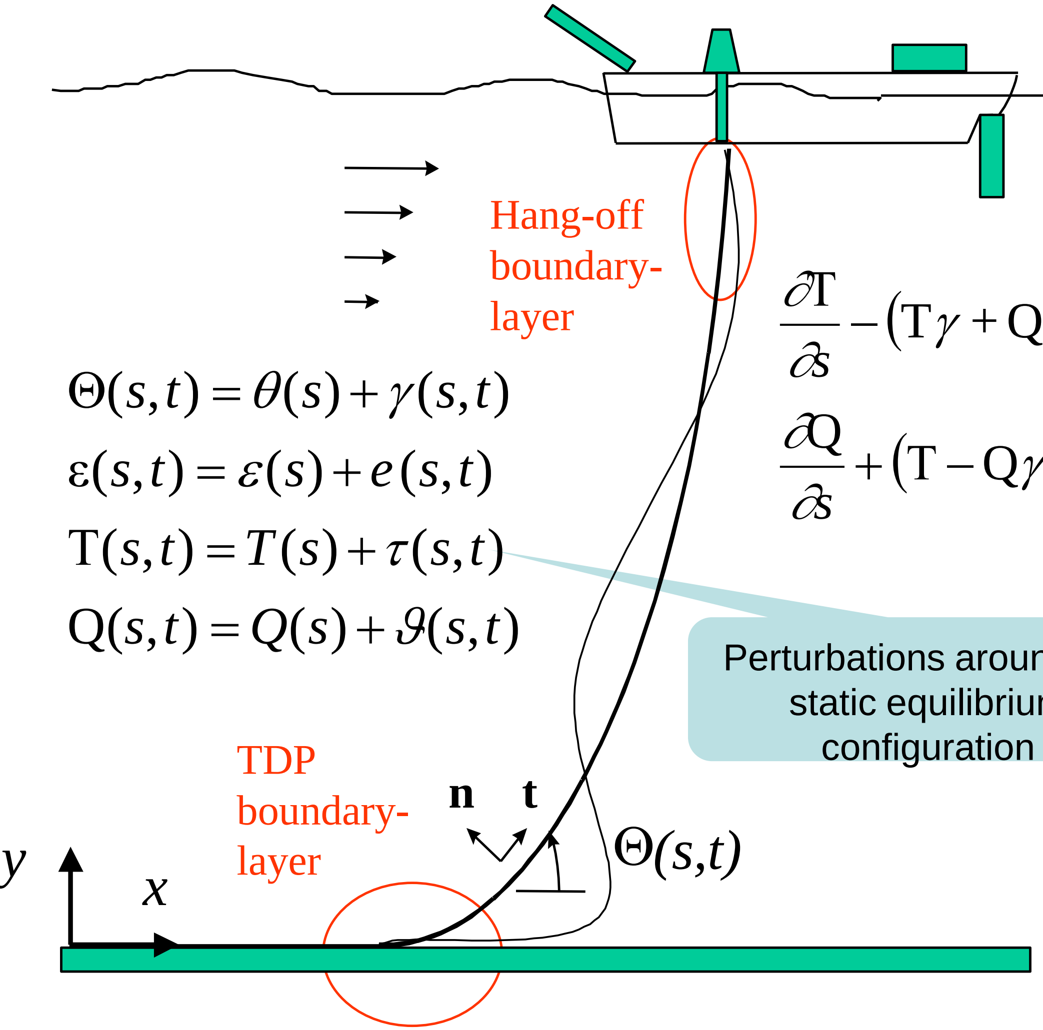
$$f_t = -q\sin\theta + c_t(s)$$

$$f_n = -q\cos\theta + c_n(s)$$

$$\left[c_t(s); c_n(s)\right]$$

Static forces due to current

The planar dynamic problem



$$\Theta(s,t)=\theta(s)+\gamma(s,t)$$
$$\mathrm{\varepsilon}(s,t)=\varepsilon(s)+e(s,t)$$
$$\mathrm{T}(s,t)=T(s)+\tau(s,t)$$
$$\mathrm{Q}(s,t)=Q(s)+\vartheta(s,t)$$

Oscillating hydrodynamic forces

$$\frac{\partial \mathrm{T}}{\partial s}-(\mathrm{T}\gamma+\mathrm{Q})\frac{d\theta}{ds}-\frac{\partial}{\partial s}(\mathrm{Q}\gamma)+c_t+\varpi_t-q\sin\theta=m\frac{\partial^2 u_t}{\partial t^2}$$
$$\frac{\partial \mathrm{Q}}{\partial s}+(\mathrm{T}-\mathrm{Q}\gamma)\frac{d\theta}{ds}+\frac{\partial}{\partial s}(\mathrm{T}\gamma)+c_n+\varpi_n-q\cos\theta=m\frac{\partial^2 u_n}{\partial t^2}$$

Perturbations around the static equilibrium configuration

$$\frac{\partial u_n}{\partial s}+u_t\frac{d\theta}{ds}=(1+\varepsilon(s))\gamma(s,t)$$
$$\frac{\partial u_t}{\partial s}-u_n\frac{d\theta}{ds}=e$$

Kinematic relations

Separating static and dynamic parts:

Static equilibrium, tangential direction

$$\left\{\frac{\partial T}{\partial s} - Q\frac{d\theta}{ds} + c_t - q\sin\theta\right\} + \qquad \longrightarrow \qquad \boxed{\{\bullet\} \equiv 0}$$

$$+\left\{\frac{\partial \tau}{\partial s} - (\mathrm{T}\gamma + \vartheta)\frac{d\theta}{ds} - \frac{\partial}{\partial s}(\mathrm{Q}\gamma) + \varpi_t\right\} = m\frac{\partial^2 u_t}{\partial t^2}$$

Static equilibrium, normal direction

$$\left\{\frac{\partial Q}{\partial s} + T\frac{d\theta}{ds} + c_n - q\cos\theta\right\} + \qquad \longrightarrow \qquad \boxed{\{\bullet\} \equiv 0}$$

$$+\left\{\frac{\partial \vartheta}{\partial s} + (\tau - \mathrm{Q}\gamma)\frac{d\theta}{ds} + \frac{\partial}{\partial s}(\mathrm{T}\gamma) + \varpi_n\right\} = m\frac{\partial^2 u_n}{\partial t^2}$$

Linearized Dynamic Equations

$$\frac{\partial \tau}{\partial s} - \left(\left(T+\tau\right)\gamma + \vartheta\right)\frac{d\theta}{ds} - \frac{\partial}{\partial s}\left(\left(Q+\vartheta\right)\gamma\right) + \varpi_t = m\frac{\partial^2 u_t}{\partial t^2}$$

$$\frac{\partial \vartheta}{\partial s} + \left(\tau - \left(Q+\vartheta\right)\gamma\right)\frac{d\theta}{ds} + \frac{\partial}{\partial s}\left(\left(T+\tau\right)\gamma\right) + \varpi_n = m\frac{\partial^2 u_n}{\partial t^2}$$

Negleting second-order terms

$$\frac{\partial \tau}{\partial s} - \left(T\gamma + \vartheta\right)\frac{d\theta}{ds} - \frac{\partial}{\partial s}\left(Q\gamma\right) + \varpi_t = m\frac{\partial^2 u_t}{\partial t^2}$$

$$\frac{\partial \vartheta}{\partial s} + \left(\tau - Q\gamma\right)\frac{d\theta}{ds} + \frac{\partial}{\partial s}\left(T\gamma\right) + \varpi_n = m\frac{\partial^2 u_n}{\partial t^2}$$

Two dynamic equilibrium equations, linearized around the static equilibrium configurations

Linearized Dynamic Equations: neglecting bending stiffness

Negleting bending stiffness effects:

$$\frac{\partial \tau}{\partial s} - \gamma T \frac{d\theta}{ds} = -\varpi_t + m \frac{\partial^2 u_t}{\partial t^2}$$

$$\tau \frac{d\theta}{ds} + \frac{\partial}{\partial s}(\gamma T) = -\varpi_n + m \frac{\partial^2 u_n}{\partial t^2}$$

Two dynamic equilibrium equations, linearized around the static equilibrium configurations

$$\frac{\partial u_t}{\partial s} - u_n \frac{d\theta}{ds} = e$$

$$\frac{\partial u_n}{\partial s} + u_t \frac{d\theta}{ds} = (1+\varepsilon)\gamma$$

Kinematic relations

$$\tau(s,t) = EAe(s,t) + q_f (u_t \sin\theta + u_n \cos\theta)$$

$$q_f = (\rho_a S_o - \rho_f S_i) g$$

Constitutive equations

Mean current hydrodynamic forces

$$c_t(s) = \frac{1}{2}\rho_a D U(s)\left|U(s)\right| C_T \cos^2\theta(s)$$

$$c_n(s) = -\frac{1}{2}\rho_a D U(s)\left|U(s)\right| C_D \sin^2\theta(s)$$

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Oscillating hydrodynamic forces

Parcelas dinâmicas das forças devido à correnteza e às ondas:

$$\varpi(s,t) = \varpi^{visc}(s,t) + \varpi^{in}(s,t)$$

Viscous parcels

$$\varpi_t^{visc}(s,t) = -\frac{1}{2} C_T \rho_a D \,\text{sinal}(\mathrm{v}_t)\left(-2U(s)\cos\theta\left(\frac{\partial u_t}{\partial t} + U(s)\sin\theta\frac{\partial u_n}{\partial s} - \mathrm{w}_t\right)\right)$$

$$\varpi_n^{visc}(s,t) = -\frac{1}{2} C_D \rho_a D \,\text{sinal}(\mathrm{v}_n)\left(2U(s)\sin\theta\left(\frac{\partial u_n}{\partial t} + U(s)\cos\theta\frac{\partial u_n}{\partial s} - \mathrm{w}_n\right)\right)$$

$$\varpi_t^{in}(s,t) = 0$$

$$\varpi_n^{in}(s,t) = -m_a\left(\frac{\partial^2 u_t}{\partial t^2} + U(s)\cos\theta\frac{\partial \gamma}{\partial t} - \frac{\partial \mathrm{w}_n}{\partial t}\right)$$

Inertial parcels

$$\mathrm{v}_t = \frac{\partial u_t}{\partial t} - U\cos\theta - \mathrm{w}_t$$

$$\mathrm{v}_n = \frac{\partial u_n}{\partial t} + U\sin\theta - \mathrm{w}_n$$

Relative velocity components to the flow

Gravitational waves velocity filed

The bending stiffness effect

From the classic linear constituve equations:

$$\mathrm{M} = EI\chi(s,t) = EI\left(\frac{d\theta}{ds} + c(s,t)\right)$$

e

$$\mathrm{Q} = \frac{\partial \mathrm{M}}{\partial s} = EI\frac{\partial \chi}{\partial s} = Q + \vartheta = EI\frac{d^2\theta}{ds^2} + EI\frac{\partial c}{\partial s}$$

The dynamic equation in the normal direction may be written:

$$\boxed{-EI\frac{\partial^2\chi}{\partial s^2} + \mathrm{T}\chi + \gamma\left(\frac{\partial \mathrm{T}}{\partial s} - EI\frac{d\theta}{ds}\frac{\partial\chi}{\partial s}\right) + c_n + \varpi_n - q\cos\theta = m\frac{\partial^2 u_n}{\partial t^2}}$$

The bending stiffness effect

Defining:

Flexural lenght at TDP

$$\lambda = \sqrt{\frac{EI}{T_0}}$$

Small length parameter

$$\varepsilon = \frac{\lambda}{L}$$

$$\chi_0 = \frac{q}{T_0}$$

Static curvature at TDP

$$\hat{t} = \frac{c_0}{L} t$$

Transversal wave celerity due to geometric stiffness, at TDP

$$c_0 = \sqrt{\frac{T_0}{m + m_a}}$$

Second-order term

The nondimensional dynamic equation in the normal direction reads:

$$-\varepsilon^2 \frac{\partial^2 \hat{\chi}}{\partial \hat{s}^2} + \hat{T}\hat{\chi} + \left(\frac{\partial \hat{T}}{\partial \hat{s}} - \varepsilon^2 \frac{d\theta}{d\hat{s}} \frac{\partial \hat{\chi}}{\partial \hat{s}} \right) \gamma + \hat{c}_n + \hat{\varpi}_n - \hat{\chi}_0 \cos\theta = \frac{\partial^2 \hat{u}_n}{\partial \hat{t}^2}$$

The bending stiffness effect

Rigid risers (Steel): $\varepsilon \approx O(10^{-2})$

Flexible risers: $\varepsilon \approx O(10^{-3})$

Neglecting secon-order terms (bending stiffness):

$$\hat{T}\hat{\chi} + \frac{\partial \hat{T}}{\partial \hat{s}}\gamma + \hat{c}_n + \hat{\varpi}_n - \hat{\chi}_0 \cos\theta = \frac{\partial^2 \hat{u}_n}{\partial \hat{t}^2}(1 + O(\varepsilon^2))$$

Time scales

$$t_1 = \frac{L}{c_g}$$

$$t_2 = \frac{\lambda}{c_g}$$

$$t_3^{(i)} = \frac{L}{c_f^{(i)}}$$

$$t_4^{(i)} = \frac{\lambda}{c_f^{(i)}}$$

$$t_5 = \frac{L}{c_a}$$

$$\bar{c}_g = \sqrt{\frac{\bar{T}}{(m+m_a)}}$$

Wave celerity associated to geometric rigidity

$$\bar{c}_f^{(i)} = \frac{2\pi}{\bar{\lambda}_f^{(i)}} \sqrt{\frac{EI}{(m+m_a)}}$$

Wave celerity associated to bending stiffness

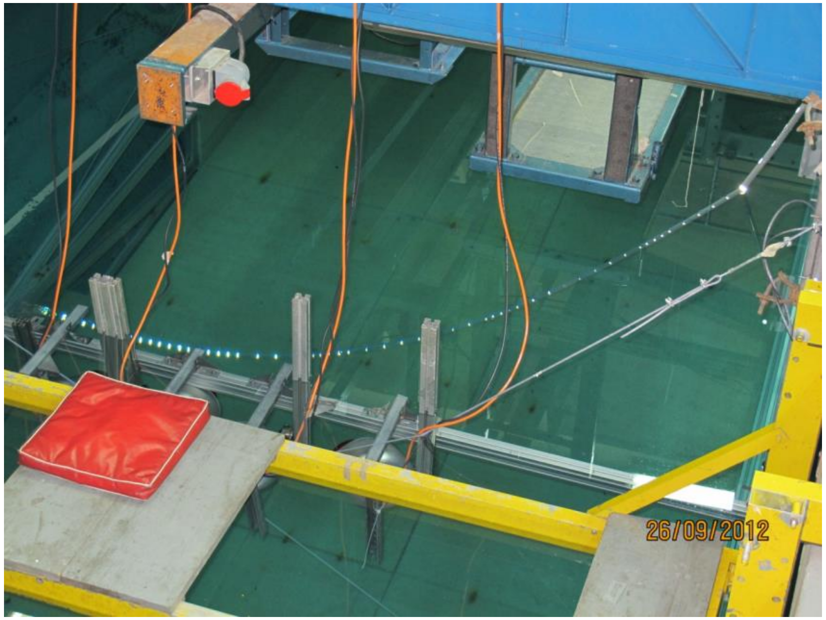
$$c_a = \frac{EA}{m}$$

Wave celerity associated to axial stiffness

$$\lambda(s) = \sqrt{\frac{EI}{T(s)}}$$

Local flexural length

Model similarity and nondimensional group



Number	Symbol and definition	Representation
Froude Number	$F_r = \frac{\omega A}{\sqrt{gL}}$	Dynamic motion in waves
Reynolds Number	$Re = \frac{UD}{\nu}$	Viscous forces
Strouhal Number	$S_t = \frac{f_s D}{U}$	Vortex shedding frequency
Keulegan-Carpenter Number	$KC = \frac{2\pi A}{D}$	Inertial forces vs. drag forces
Structural Damping	$\zeta = \frac{c}{c_c}$	Linear structural damping
Reduced Velocity	$V_r = \frac{U}{f_n D}$	Normalized velocity in VIV
Reduced Shedding Frequency	$f_s^* = \frac{f_s}{f_n} = S_t \frac{U}{f_n D} = S_t V_r$	Vortex shedding normalized frequency
Reduced Mass	$m^* = \frac{m}{m_D}$	Riser mass vs. displaced mass
Added Mass	$a = \frac{m_a}{m}$	Added mass vs. riser mass
Bending Stiffness	$K_f = \frac{\lambda_f}{L} = \frac{1}{L}\sqrt{\frac{EI}{T}}$	Bending vs. geometrical stiffness
Axial Stiffness	$K_a = \frac{EA}{T}$	Axial vs. geometrical stiffness
Soil Stiffness	$K_s = \frac{k_s EI}{T^2}$	Soil vs. bending and geometrical stiffness

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Summary

- The global dynamics is governed by the geometric stiffness or catenary stiffness.
- Bending stiffness effetcs are importante at the extremities and TDP, or for high-order vibration modes for which the mode vibrattion length is of same order of the local flexural length.
- There are several time scales that govern the overall dynamics of a riser.
- A large nondimensional parameters group govern the overall riser dynamics

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