Lucky Numbers

ONE DAY at Princeton I was sitting in the lounge and overheard some mathematicians talking about the series for e^x, which is $1 + x + x^2/2! + x^3/3!$ Each term you get by multiplying the preceding term by x and dividing by the next number. For example, to get the next term after $x^4/4!$ you multiply that term by x and divide by 5. It's very simple.

When I was a kid I was excited by series, and had played with this thing. I had computed e using that series, and had seen how quickly the new terms became very small.

I mumbled something about how it was easy to calculate e to any power using that series (you just substitute the power for x).

"Oh yeah?" they said, "Well, then, what's e to the 3.3?" said some joker—I think it was Tukey.

I say, "That's easy. It's 27.11."

Tukey knows it isn't so easy to compute all that in your head. "Hey! How'd you do that?"

Another guy says, "You know Feynman, he's just faking it. It's not really right."

They go to get a table, and while they're doing that, I put on a few more figures: "27.1126," I say.

They find it in the table. "It's right! But how'd you *do* it!"

"I just summed the series."

"Nobody can sum the series that fast. You must just happen to know that one. How about e to the 3?"

"Look," I say. "It's hard work! Only one a day!"

"Hah! It's a fake!" they say, happily.

"All right," I say, "It's 20.085."

They look in the book as I put a few

more figures on. They're all excited now, because I got another one right.

Here are these great mathematicians of the day, puzzled at how I can compute e to any power! One of them says, "He just *can't* be substituting and summing—it's too hard. There's some trick. You couldn't do just any old number like e to the 1.4."

I say, "It's hard work, but for you, OK. It's 4.05."

As they're looking it up, I put on a few more digits and say, "And that's the last one for the day!" and walk out.

What happened was this: I happened to know three numbers—the logarithm of 10 to the base e (needed to convert numbers from base 10 to base e), which is 2.3026 (so I knew that e to the 2.3 is very close to 10), and because of radioactivity (mean-life and half-life), I knew the log of 2 to the base e, which is .69315 (so I also knew that e to the .7 is nearly equal to 2). I also knew e (to the 1), which is 2.71828.

The first number they gave me was e to the 3.3, which is e to the 2.3—ten—times e, or 27.18. While they were sweating about how I was doing it, I was correcting for the extra .0026—2.3026 is a little high.

I knew I couldn't do another one; that was sheer luck. But then the guy said e to the 3: that's e to the 2.3 times e to the .7, or ten times two. So I knew it was 20.something, and while they were worrying how I did it, I adjusted for the .693.

Now I was *sure* I couldn't do another one, because the last one was again by sheer luck. But the guy said e to the 1.4, which is e to the .7 times itself. So all I had to do is fix up 4 a little bit!

They never did figure out how I did it.

When I was at Los Alamos I found out that Hans Bethe was absolutely topnotch at calculating. For example, one time we were putting some numbers into a formula, and got to 48 squared. I reach for the Marchant calculator, and he says, "That's 2300." I begin to push the buttons, and he says, "If you want it exactly, it's 2304."

The machine says 2304. "Gee! That's pretty remarkable!" I say.

"Don't you know how to square numbers near 50?" he says. "You square 50—that's 2500—and subtract 100 times the difference of your number from 50 (in this case it's 2), so you have

2300. If you want the correction, square the difference and add it on. That makes 2304."

A few minutes later we need to take the cube root of 2 1/2. Now to take cube roots on the Marchant you had to use a table for the first approximation. I open the drawer to get the table—it takes a little longer this time—and he says, "It's about 1.35."

I try it out on the Marchant and it's right. "How did you do that one?" I ask. "Do you have a secret for taking cube roots of numbers?"

"Oh," he says, "the log of 2 1/2 is so-and-so. Now one-third of that log is between the logs of 1.3, which is this, and 1.4, which is that, so I interpolated."

So I found out something: first, he knows the log tables; second, the amount of arithmetic he did to make the interpolation alone would have taken me longer to do than reach for the table and punch the buttons on the calculator. I was very impressed.

After that, I tried to do those things. I memorized a few logs, and began to notice things. For instance, if somebody says, "What is 28 squared?" you notice that the square root of 2 is 1.4, and 28 is 20 times 1.4, so the square of 28 must be around 400 times 2, or 800.

If somebody comes along and wants to divide 1 by 1.73, you can tell them immediately that it's .577, because you notice that 1.73 is nearly the square root of 3, so 1/1.73 must be one-third of the square root of 3. And if it's 1/1.75, that's equal to the inverse of 7/4, and you've memorized the repeating decimals for sevenths: .571428 . . .

I had a lot of fun trying to do arithmetic fast, by tricks, with Hans. It was very rare that I'd see something he didn't see and beat him to the answer, and he'd laugh his hearty laugh when I'd get one. He was nearly always able to get the answer to any problem within a percent. It was easy for him—every number was near something he knew.

One day I was feeling my oats. It was lunch time in the technical area, and I don't know how I got the idea, but I announced, "I can work out in sixty seconds the answer to any problem that anybody can state in ten seconds, to 10 percent!"

People started giving me problems they thought were difficult, such as integrating a function like $1/(1+x^4)$, which hardly changed over the range they gave me. The hardest one somebody gave me was the binomial coefficient of x^{10} in $(1+x)^{20}$; I got that just in time.

They were all giving me problems and I was feeling great, when Paul Olum walked by in the hall. Paul had worked with me for a while at Princeton before coming out to Los Alamos, and he was always cleverer than I was. For instance, one day I was absent-mindedly playing with one of those measuring tapes that snap back into your hand when you push a button. The tape would always slap over and hit my hand, and it hurt a little bit. "Geez!" I exclaimed. "What a *dope* I am. I keep playing with this thing, and it hurts me every time."

He said, "You don't hold it right," and took the damn thing, pulled out the tape, pushed the button, and it came right back. No hurt.

"Wow! How do you *do* that?" I exclaimed.

"Figure it out!"

For the next two weeks I'm walking all around Princeton, snapping this tape back until my hand is absolutely raw. Finally I can't take it any longer. "Paul! I give up! How the hell do you hold it so it doesn't hurt?"

"Who says it doesn't hurt? It hurts me too!"

I felt so stupid. He had gotten me to go around and hurt my hand for two weeks!

So Paul is walking past the lunch place and these guys are all excited. "Hey, Paul!" they call out. "Feynman's terrific! We give him a problem that can be stated in ten seconds, and in a minute he gets the answer to 10 percent. Why don't you give him one?"

Without hardly stopping, he says, "The tangent of 10 to the 100th."

I was sunk: you have to divide by pi to 100 decimal places! It was hopeless.

One time I boasted, "I can do by other methods any integral anybody else needs contour integration to do."

So Paul puts up this tre*men*dous damn integral he had obtained by starting out with a complex function that he knew the answer to, taking out the real part of it and leaving only the complex part. He had unwrapped it so it was *only* possible by contour integration! He was always deflating me like that. He was a very smart fellow.

The first time I was in Brazil I was eating a noon meal at I don't know what time—I was always in the restaurants at the wrong time—and I was the only customer in the place. I was eating rice with steak (which I loved), and there were about four waiters standing around.

A Japanese man came into the restaurant. I had seen him before, wandering around; he was trying to sell abacuses. He started to talk to the waiters, and challenged them: He said he could add numbers faster than any of them could do.

The waiters didn't want to lose face, so they said, "Yeah, yeah. Why don't you go over and challenge the customer over there?"

The man came over. I protested, "But I don't speak Portuguese well!"

The waiters laughed. "The numbers are easy," they said.

They brought me a pencil and paper.

The man asked a waiter to call out some numbers to add. He beat me hollow, because while I was writing the numbers down, he was already adding them as he went along.

I suggested that the waiter write down two identical lists of numbers and hand them to us at the same time. It didn't make much difference. He still beat me by quite a bit.

However, the man got a little bit excited: he wanted to prove himself some more. "*Multiplicação!*" he said.

Somebody wrote down a problem. He beat me again, but not by much, because I'm pretty good at products.

The man then made a mistake: he proposed we go on to division. What he didn't realize was, the harder the problem, the better chance I had.

We both did a long division problem. It was a tie.

This bothered the hell out of the Japanese man, because he was apparently very well trained on the abacus, and here he was almost beaten by this customer in a restaurant.

"*Raios cubicos!*" he says, with a vengeance. Cube roots! He wants to do cube roots by arithmetic! It's hard to find a more difficult fundamental problem in arithmetic. It must have been his topnotch exercise in abacus-land.

He writes a number on some paper—any old number—and I

still remember it: 1729.03. He starts working on it, mumbling and grumbling: "*Mmmmmmagmmmmbrrr*"—he's working like a demon! He's poring away, doing this cube root.

Meanwhile I'm just *sitting* there.

One of the waiters says, "What are you doing?"

I point to my head. "Thinking!" I say. I write down 12 on the paper. After a little while I've got 12.002.

The man with the abacus wipes the sweat off his forehead: "Twelve!" he says.

"Oh, no!" I say. "More digits! More digits!" I know that in taking a cube root by arithmetic, each new digit is even more work than the one before. It's a hard job.

He buries himself again, grunting "*Rrrrgrrrrmmmmmm . . .,*" while I add on two more digits. He finally lifts his head to say, "12.0!"

The waiters are all excited and happy. They tell the man, "Look! He does it only by thinking, and you need an abacus! He's got more digits!"

He was completely washed out, and left, humiliated. The waiters congratulated each other.

How did the customer beat the abacus? The number was 1729.03. I happened to know that a cubic foot contains 1728 cubic inches, so the answer is a tiny bit more than 12. The excess, 1.03, is only one part in nearly 2000, and I had learned in calculus that for small fractions, the cube root's excess is one-third of the number's excess. So all I had to do is find the fraction 1/1728, and multiply by 4 (divide by 3 and multiply by 12). So I was able to pull out a whole lot of digits that way.

A few weeks later the man came into the cocktail lounge of the hotel I was staying at. He recognized me and came over. "Tell me," he said, "how were you able to do that cube-root problem so fast?"

I started to explain that it was an approximate method, and had to do with the percentage of error. "Suppose you had given me 28. Now, the cube root of 27 is 3 . . ."

He picks up his abacus: *zzzzzzzzzzzzzzz*— "Oh yes," he says.

I realized something: he doesn't *know* numbers. With the abacus, you don't have to memorize a lot of arithmetic combinations; all you have to do is learn how to push the little beads up and down. You don't have to memorize 9 + 7 = 16; you just know that when you add 9 you push a ten's bead up and pull a one's bead down. So we're slower at basic arithmetic, but we know numbers.

Furthermore, the whole idea of an approximate method was beyond him, even though a cube root often cannot be computed exactly by any method. So I never could teach him how I did cube roots or explain how lucky I was that he happened to choose 1729.03.

O Americano, Outra Vez!

ONE TIME I picked up a hitchhiker who told me how interesting South America was, and that I ought to go there. I complained that the language is different, but he said just go ahead and learn it—it's no big problem. So I thought, that's a good idea: I'll go to South America.

Cornell had some foreign language classes which followed a method used during the war, in which small groups of about ten students and one native speaker speak only the foreign language—nothing else. Since I was a rather young-looking professor there at Cornell, I decided to take the class as if I were a regular student. And since I didn't know yet where I was going to end up in South America, I decided to take Spanish, because the great majority of the countries there speak Spanish.

So when it was time to register for the class, we were standing outside, ready to go into the classroom, when this pneumatic blonde came along. You know how once in a while you get this feeling, WOW? She looked terrific. I said to myself, "Maybe she's going to be in the Spanish class—that'll be *great!*" But no, she walked into the Portuguese class. So I figured, What the hell—I might as well learn Portuguese.

I started walking right after her when this Anglo-Saxon attitude that I have said, "No, that's not a good reason to decide which language to speak." So I went back and signed up for the Spanish class, to my utter regret.

Some time later I was at a Physics Society meeting in New York, and I found myself sitting next to Jaime Tiomno, from Bra-