Hidrodinâmica: Equações Básicas

IFSC, USP

17 de Novembro de 2014

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Princípios Físicos

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Princípios Físicos

- Leis de conservação:

Conceitos Fundamentais

Princípios Físicos

- Leis de conservação:
  - Massa

Conceitos Fundamentais

Princípios Físicos

- Leis de conservação:
  - Massa
  - Energia

Conceitos Fundamentais

Princípios Físicos

- Leis de conservação:
  - Massa
  - Energia
- Segunda lei de Newton

Conceitos Fundamentais

Princípios Físicos

- Leis de conservação:
  - Massa
  - Energia
- Segunda lei de Newton
- Primeira lei da Termodinâmica

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- Volume de controle finito fixo

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- Volume de controle finito fixo
- Volume de controle finito móvel

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Modelos de Fluxo

- Volume de controle finito fixo
- Volume de controle finito móvel
- Volume infinitesimal fixo

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- Volume de controle finito móvel
- Volume infinitesimal fixo
- Volume infinitesimal móvel

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- Volume de controle finito móvel
- Volume infinitesimal fixo
- Volume infinitesimal móvel

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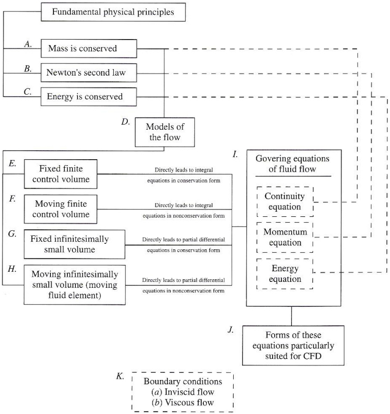
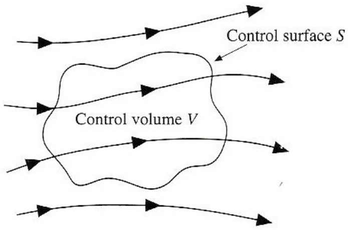
Modelos de Fluxo

- Volume de controle finito fixo
- Volume de controle finito móvel
- Volume infinitesimal fixo
- Volume infinitesimal móvel

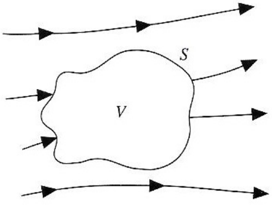


Volume de controle finito



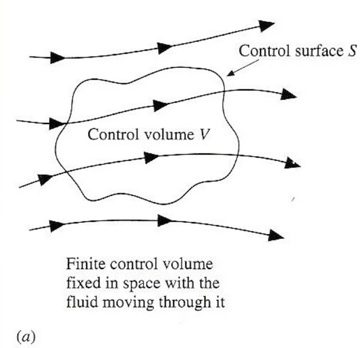
Finite control volume fixed in space with the fluid moving through it

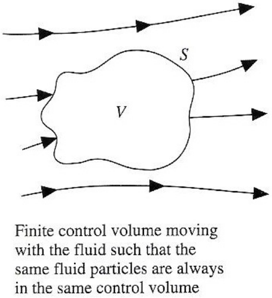
(a)



Finite control volume moving with the fluid such that the same fluid particles are always in the same control volume

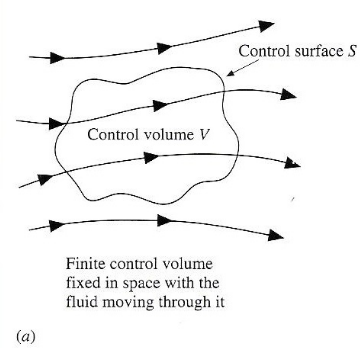
Volume de controle finito

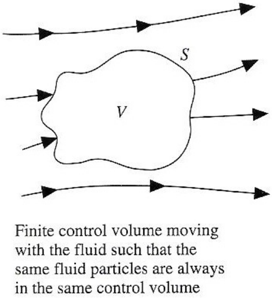




- Forma integral das equações FD

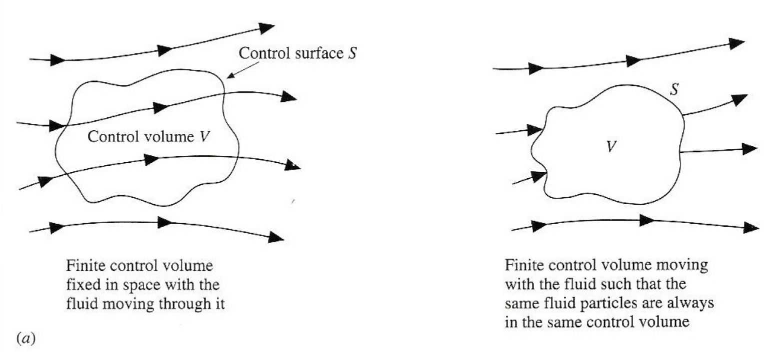
Volume de controle finito





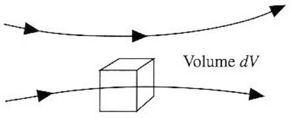
- Forma integral das equações FD
- Forma conservativa: volume fixo

Volume de controle finito



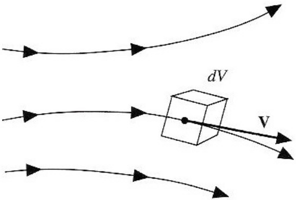
- Forma integral das equações FD
- Forma conservativa: volume fixo
- Forma não-conservativa: volume móvel

Volume de controle infinitesimal



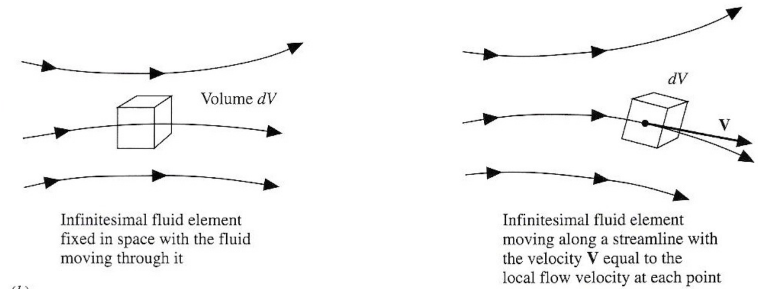
Infinitesimal fluid element fixed in space with the fluid moving through it

(b)



Infinitesimal fluid element moving along a streamline with the velocity **V** equal to the local flow velocity at each point

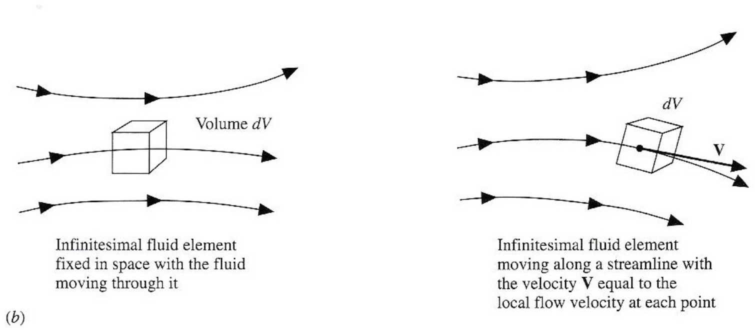
Volume de controle infinitesimal



(b)

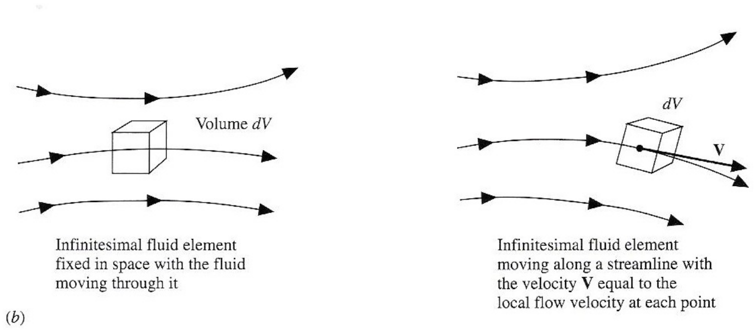
- Forma diferencial das equações FD

Volume de controle infinitesimal



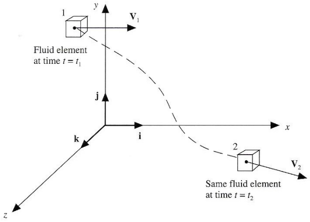
- Forma diferencial das equações FD
- Forma conservativa: volume fixo

Volume de controle infinitesimal



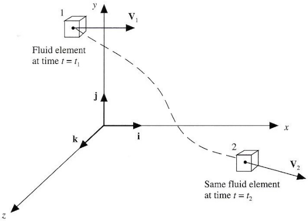
- Forma diferencial das equações FD
- Forma conservativa: volume fixo
- Forma não-conservativa: volume móvel

Derivada total



- $\mathbf{V} = u(x, y, z, t)\mathbf{i} + v(x, y, z, t)\mathbf{j} + w(x, y, z, t)\mathbf{k}$

Derivada total



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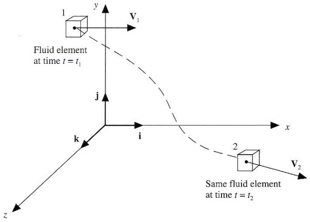
- Densidade no ponto 1: $\rho_1 = \rho_1(x_1, y_1, z_1, t_1)$

Derivada total



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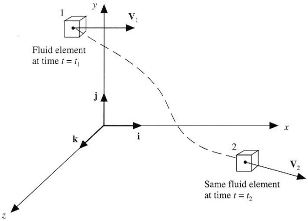
Derivada total



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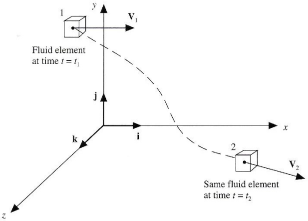
- Taylor:

Derivada total



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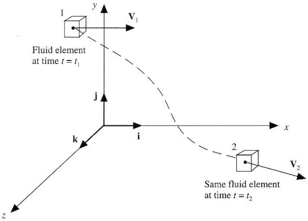
Derivada total



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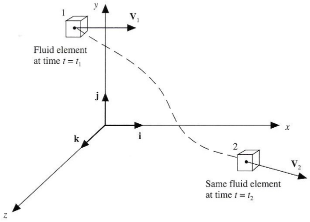
- Derivada total:

$$\lim_{t_2 \to t_1} \frac{\rho_2 - \rho_1}{t_2 - t_1} = u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \frac{\partial \rho}{\partial t} \implies \boxed{\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \mathbf{V} \cdot \nabla \rho}$$

Derivada total



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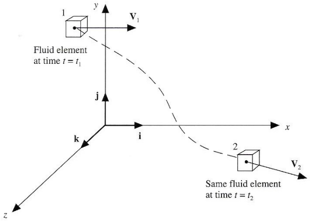
- $\partial / \partial t$ é a derivada local

Derivada total



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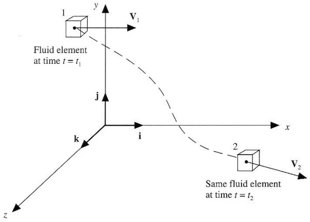
- $\partial / \partial t$ é a derivada local
- $\mathbf{V} \cdot \nabla$ é a derivada convectiva

Derivada total



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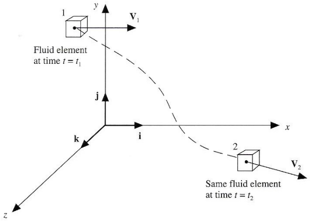
- $\partial/\partial t$ é a derivada local
- $\mathbf{V} \cdot \nabla$ é a derivada convectiva

Derivada total



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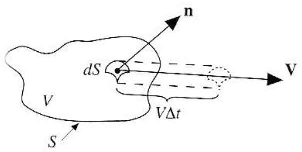
$$\lim_{t_2 \to t_1} \frac{\rho_2 - \rho_1}{t_2 - t_1} = u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z} + \frac{\partial\rho}{\partial t} \Longrightarrow \boxed{\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \mathbf{V} \cdot \nabla\rho}$$

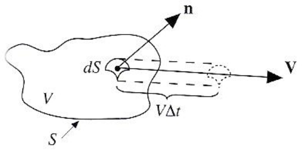
- $\partial/\partial t$ é a derivada local
- $\mathbf{V} \cdot \nabla$ é a derivada convectiva

Divergente da velocidade



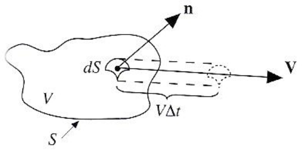
- Volume de controle

Divergente da velocidade



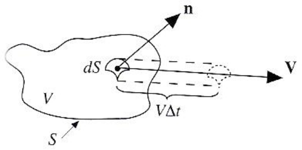
- Volume de controle
- Pequeno: $\Delta\mathcal{V}' = \Delta t\, \mathbf{V} \cdot d\mathbf{S},\ d\mathbf{S} = dS\mathbf{n}$

Divergente da velocidade



- Volume de controle
- Pequeno: $\Delta \mathcal{V}' = \Delta t\, \mathbf{V} \cdot d\mathbf{S}, \; d\mathbf{S} = dS\mathbf{n}$
- Total: $\Delta \mathcal{V} = \oint_S \Delta t\, \mathbf{V} \cdot d\mathbf{S}$

Divergente da velocidade



- Volume de controle
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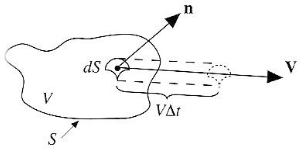
- Total: $\Delta\mathcal{V} = \oint_S \Delta t\, \mathbf{V}\cdot d\mathbf{S}$
- Variação do volume de controle:

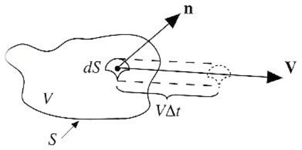
Divergente da velocidade



- Volume de controle
- Pequeno: $\Delta\mathcal{V}' = \Delta t\, \mathbf{V}\cdot d\mathbf{S},\ d\mathbf{S} = dS\mathbf{n}$
- Total: $\Delta\mathcal{V} = \oint_S \Delta t\, \mathbf{V}\cdot d\mathbf{S}$
- Variação do volume de controle:

- $\frac{d\mathcal{V}}{dt} = \lim_{\Delta t\to 0} \frac{\Delta\mathcal{V}}{\Delta t} = \oint_S \mathbf{V}\cdot d\mathbf{S} = \int_{\mathcal{V}} \nabla\cdot\mathbf{V}\, d\mathcal{V}$ (Gauss)

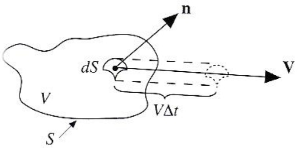
Divergente da velocidade



- Volume de controle
- Pequeno: $\Delta\mathcal{V}' = \Delta t\, \mathbf{V}\cdot d\mathbf{S},\; d\mathbf{S} = dS\mathbf{n}$
- Total: $\Delta\mathcal{V} = \oint_S \Delta t\, \mathbf{V}\cdot d\mathbf{S}$
- Variação do volume de controle:
- $\dfrac{d\mathcal{V}}{dt} = \lim_{\Delta t\to 0} \dfrac{\Delta\mathcal{V}}{\Delta t} = \oint_S \mathbf{V}\cdot d\mathbf{S} = \int_{\mathcal{V}} \nabla\cdot\mathbf{V}\, d\mathcal{V}$ (Gauss)
- Volume de controle infinitesimal: $\mathcal{V} \to \delta\mathcal{V}$

$$\frac{d(\delta\mathcal{V})}{dt} = \nabla\cdot\mathbf{V}\,(\delta\mathcal{V}) \implies \boxed{\nabla\cdot\mathbf{V} = \frac{1}{\delta\mathcal{V}}\frac{d(\delta\mathcal{V})}{dt}}$$

Divergente da velocidade



- Volume de controle
- Pequeno: $\Delta\mathcal{V}' = \Delta t\,\mathbf{V}\cdot d\mathbf{S},\ d\mathbf{S} = dS\mathbf{n}$
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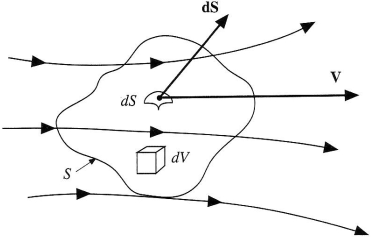
- Variação do volume de controle:
- $\dfrac{d\mathcal{V}}{dt} = \lim_{\Delta t\to 0}\dfrac{\Delta\mathcal{V}}{\Delta t} = \oint_S \mathbf{V}\cdot d\mathbf{S} = \int_{\mathcal{V}} \nabla\cdot\mathbf{V}\,d\mathcal{V}$ (Gauss)
- Volume de controle infinitesimal: $\mathcal{V}\rightarrow\delta\mathcal{V}$

$$\frac{d(\delta\mathcal{V})}{dt} = \nabla\cdot\mathbf{V}\,(\delta\mathcal{V}) \implies \boxed{\nabla\cdot\mathbf{V} = \frac{1}{\delta\mathcal{V}}\frac{d(\delta\mathcal{V})}{dt}}$$

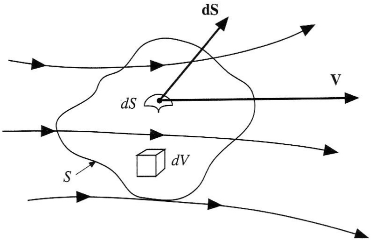
- $\nabla\cdot\mathbf{V}$ é a variação temporal do volume de um fluido em movimento por unidade de volume

Volume finito fixo



- Fluxo de massa através de S = Variação temporal de massa dentro de $\mathcal{V}$ $(A = B)$

Volume finito fixo



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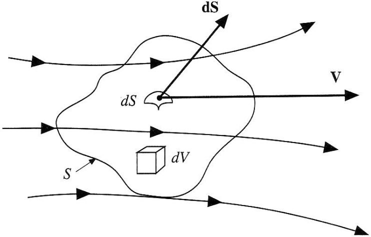

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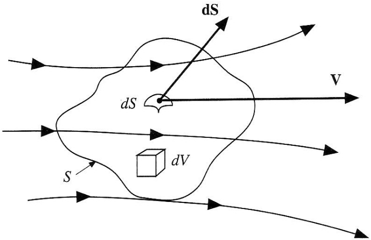
Volume finito fixo



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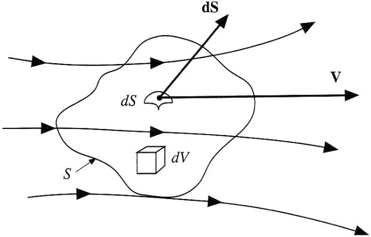

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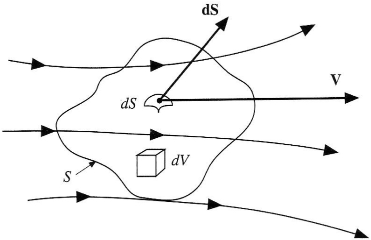

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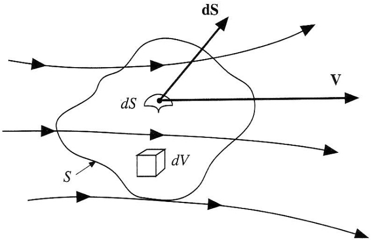
Volume finito fixo



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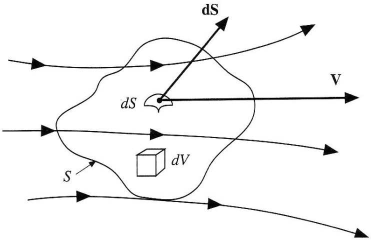

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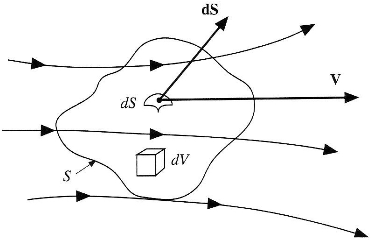

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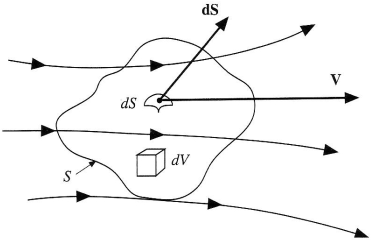

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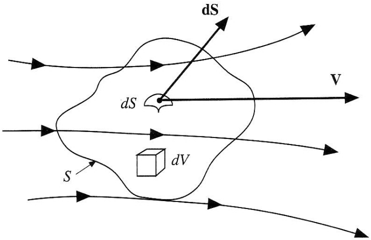

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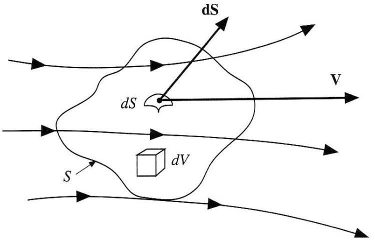
Volume finito fixo



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Aplicação da Eq. da Continuidade

- Forma integral conservativa: $$\oint_s \rho \mathbf{V} \cdot d\mathbf{S} + \frac{\partial}{\partial t} \int_{\mathcal{V}} \rho d\mathcal{V} = 0$$

Aplicação da Eq. da Continuidade

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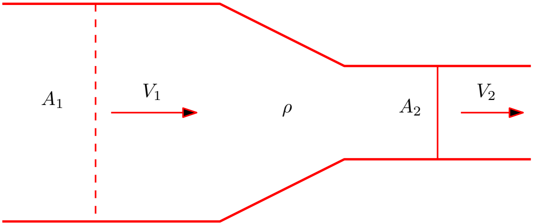
- Tubo com seção reta de área A: $\rho VA = cte. \implies A_1 V_1 = A_2 V_2$



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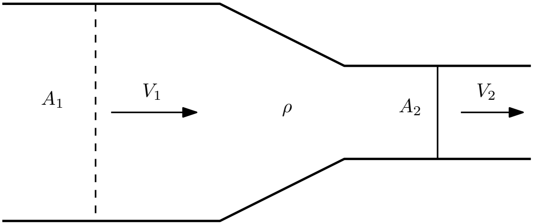

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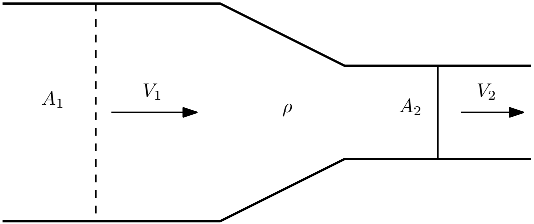
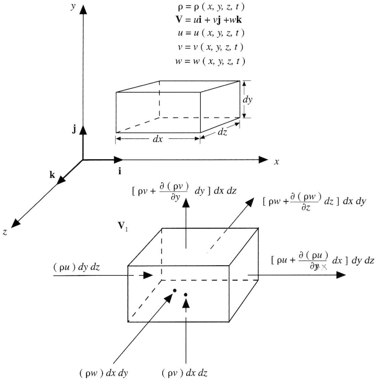


Volume infinitesimal fixo



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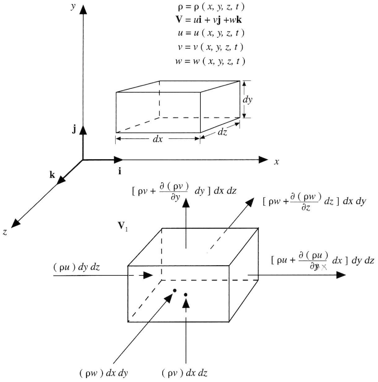

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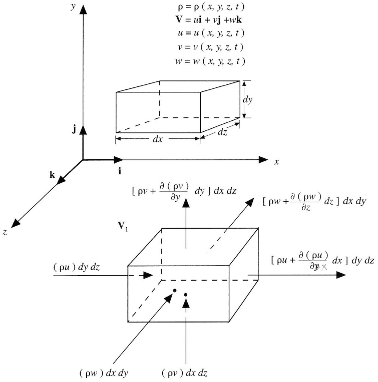
$$\left[\rho u + \frac{\partial(\rho u)}{\partial x}dx\right]dydz - (\rho u)dy\,dz = \frac{\partial(\rho u)}{\partial x}d\mathcal{V}$$

Volume infinitesimal fixo



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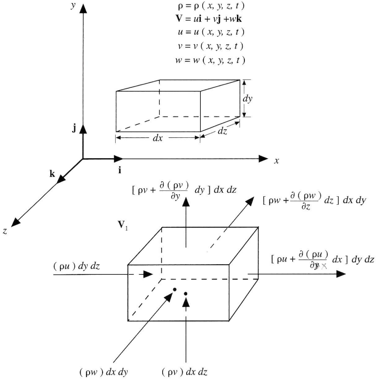

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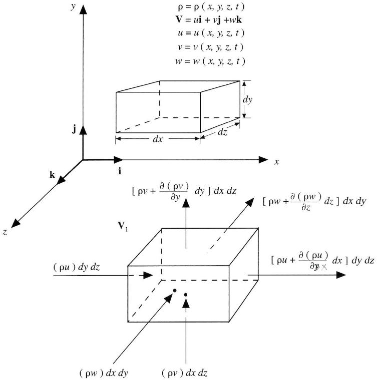

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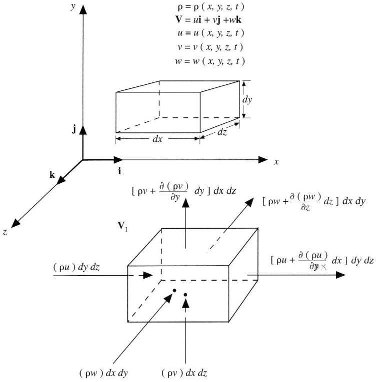

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Volume finito móvel

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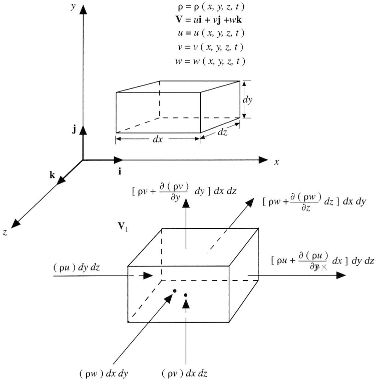

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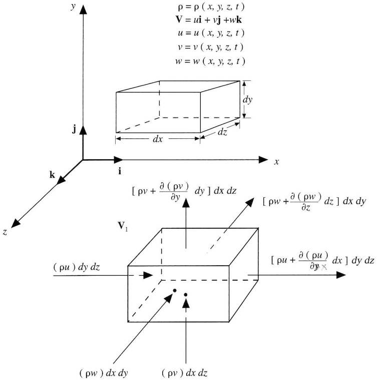

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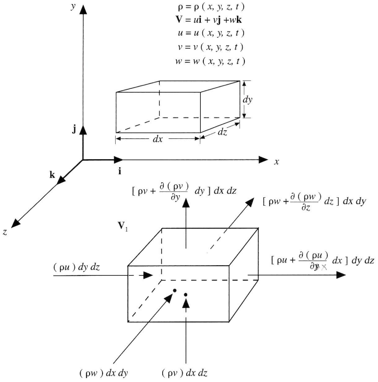

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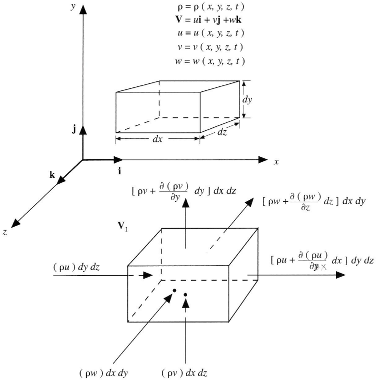
Volume infinitesimal fixo



- Fluxo total de massa = Variação temporal de massa ($A = B$)
- Fluxo em x: ($d\mathcal{V} = dxdydz$)

$$\left[\rho u + \frac{\partial(\rho u)}{\partial x}dx\right]dydz - (\rho u)dy\,dz = \frac{\partial(\rho u)}{\partial x}d\mathcal{V}$$

- $A = \nabla \cdot (\rho \mathbf{V})$ (fluxo total)

  $B = -\frac{\partial}{\partial t}(\rho d\mathcal{V}) = -\frac{\partial \rho}{\partial t}d\mathcal{V}$ (var. massa)
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Volume finito móvel

- Massa $\delta m = \rho \delta \mathcal{V}$ const.: $\frac{d}{dt}\delta m = 0$
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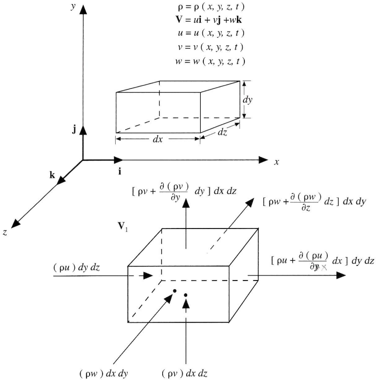

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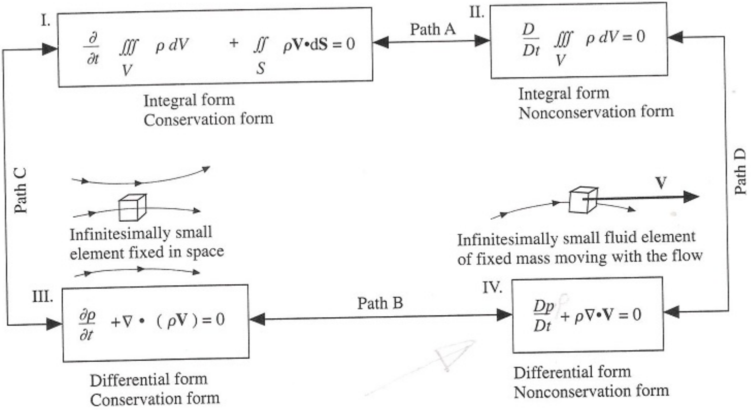
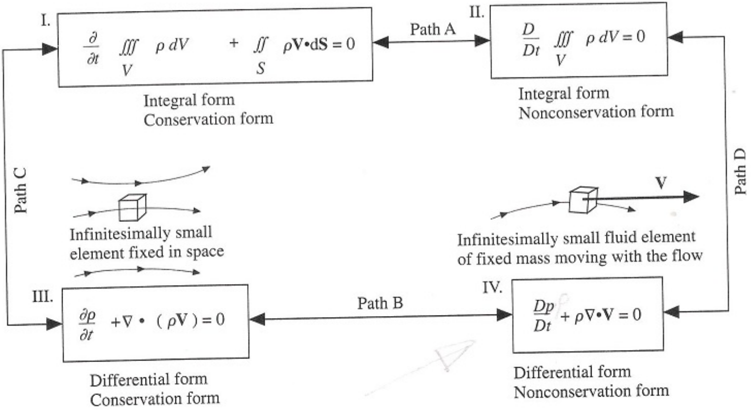

I.
$\frac{\partial}{\partial t}\iiint_V \rho\, dV + \iint_S \rho \mathbf{V}\cdot d\mathbf{S} = 0$
Integral form
Conservation form
Path A
II.
$\frac{D}{Dt}\iiint_V \rho\, dV = 0$
Integral form
Nonconservation form
Path C
Path D
Infinitesimally small element fixed in space
V
Infinitesimally small fluid element of fixed mass moving with the flow
III.
$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \mathbf{V}) = 0$
Differential form
Conservation form
Path B
IV.
$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{V} = 0$
Differential form
Nonconservation form



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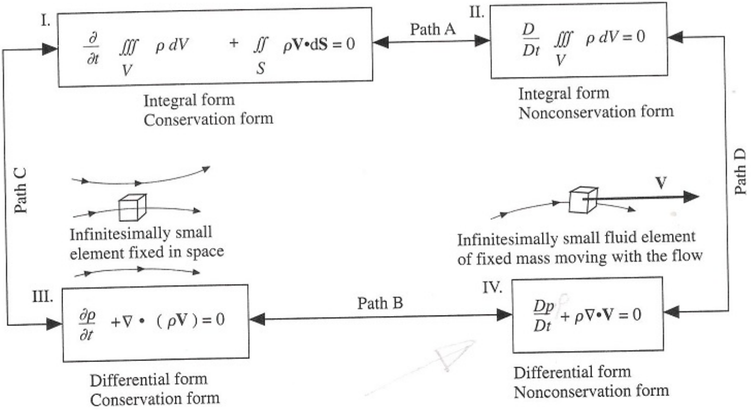
$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{V} = 0$$

- A forma integral permite a presença de descontinuidades dentro do volume de controle (fixo)



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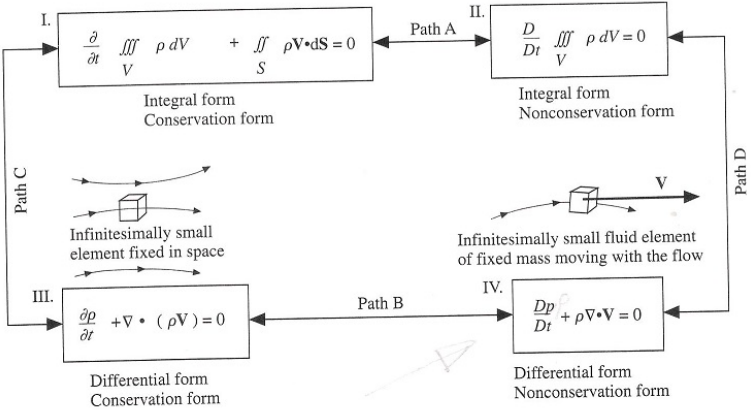
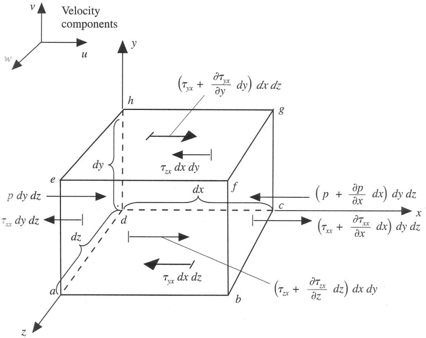
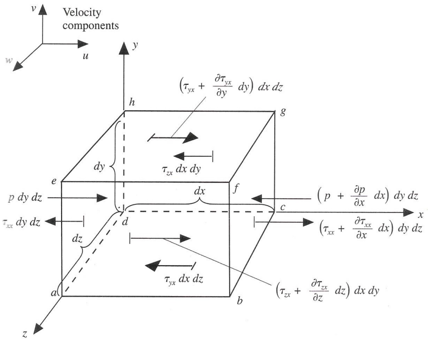


- A forma integral permite a presença de descontinuidades dentro do volume de controle (fixo)
- A forma diferencial assume diferenciabilidade (continuidade)
- A forma integral pode ser considerada mais fundamental



- $\mathbf{F}_b + \mathbf{F}_p + \mathbf{F}_v = m\left(\frac{du}{dt}\mathbf{i} + \frac{dv}{dt}\mathbf{j} + \frac{dw}{dt}\mathbf{k}\right)$



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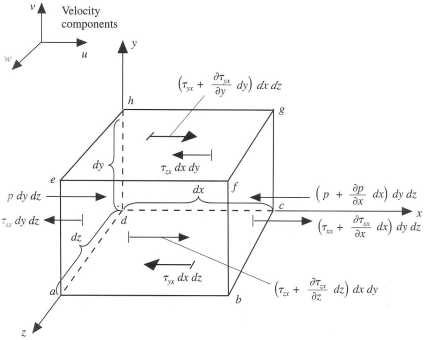

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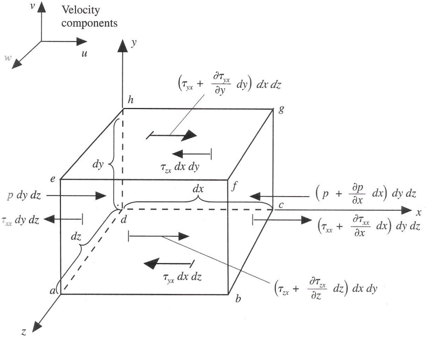

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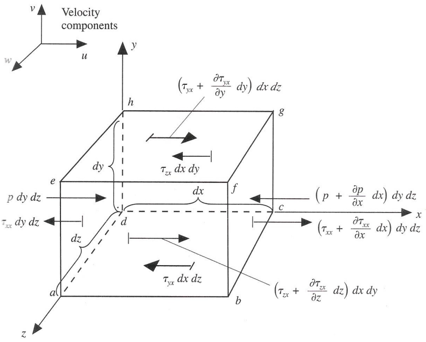
- $\mathbf{F}_v$ = Forças devido a viscosidade: normal (τ_{xx}) e cisalhamento (τ_{yx})



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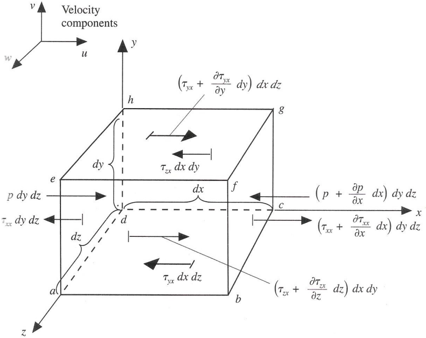

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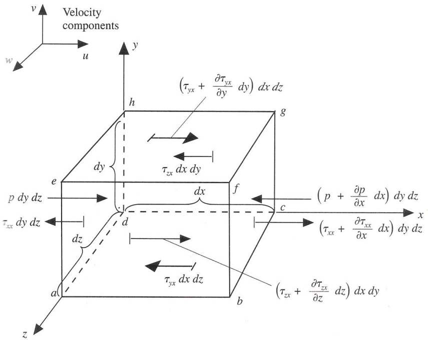

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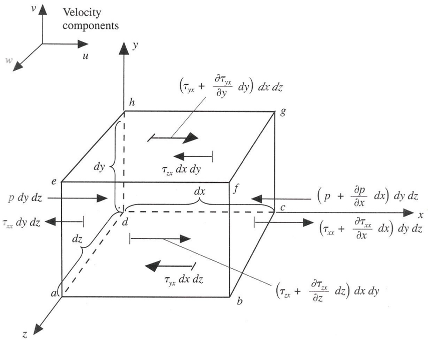

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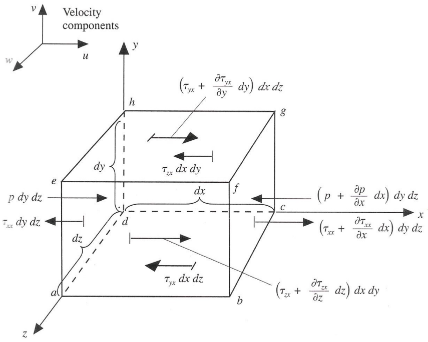

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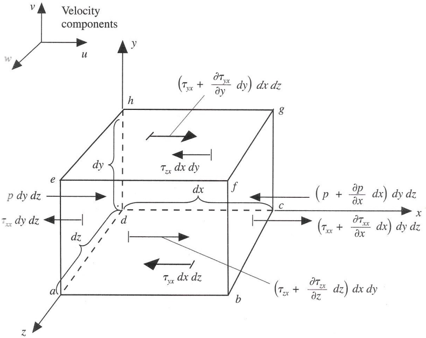

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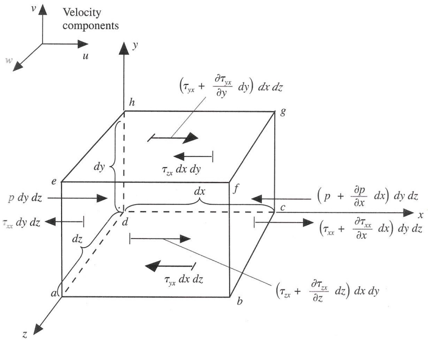
shear stress tensor



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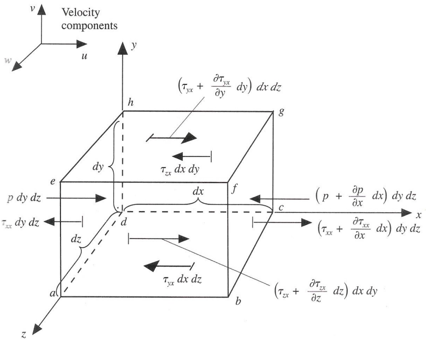
shear stress tensor



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shear stress tensor

Navier-Stokes

- Navier-Stokes (não-conservativa): $\boxed{\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{f} - \nabla p + \nabla \boldsymbol{\tau}}$

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- Fluido Newtoniano (Stokes, 1845):
$\tau_{ii} = \lambda(\nabla \cdot \mathbf{V}) + 2\mu \frac{\partial V_i}{\partial x_i}, \ \tau_{ij} = \mu \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right), \ \tau_{ii} \ll \tau_{ij}, \ 3\lambda = -2\mu (?)$

Navier-Stokes

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Navier-Stokes

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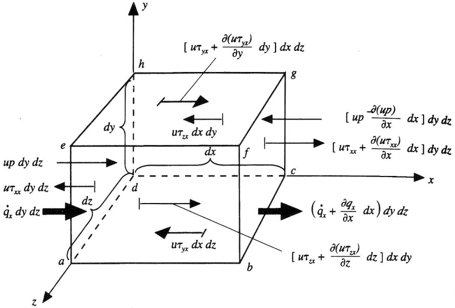

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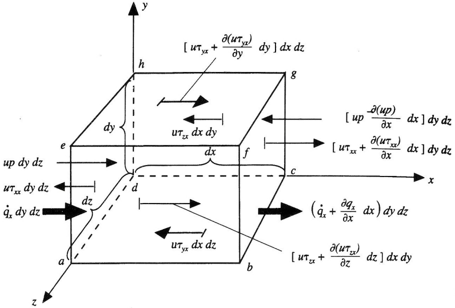

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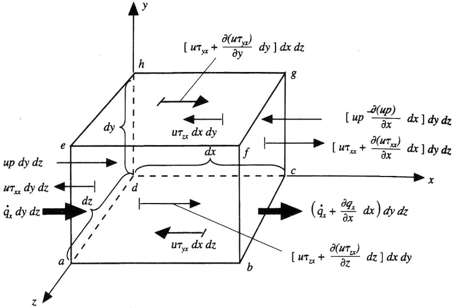

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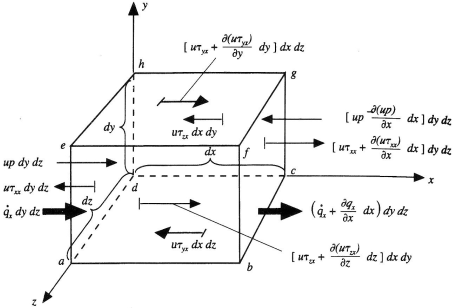

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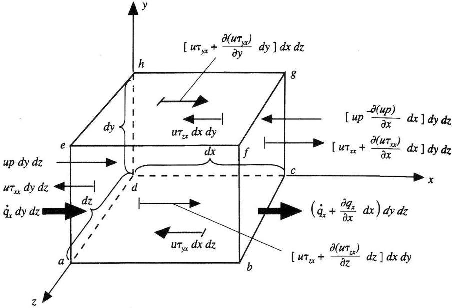

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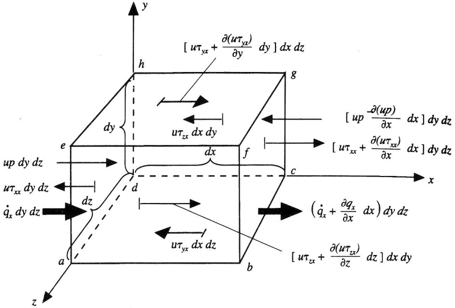

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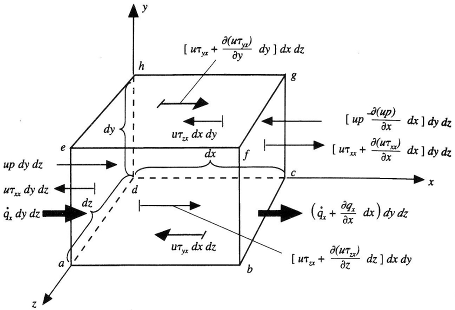

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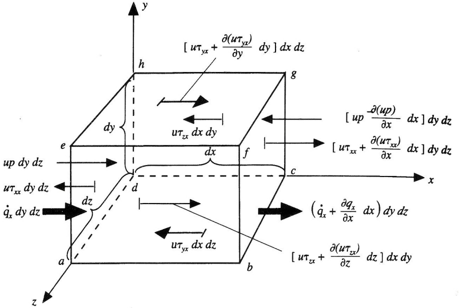

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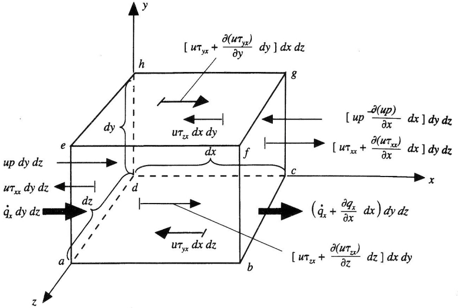

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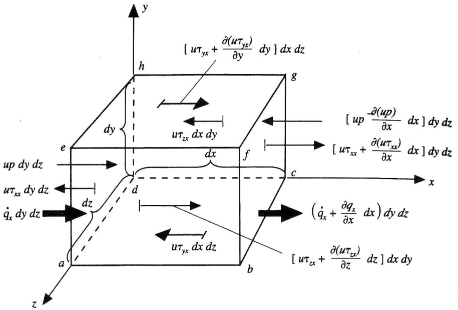

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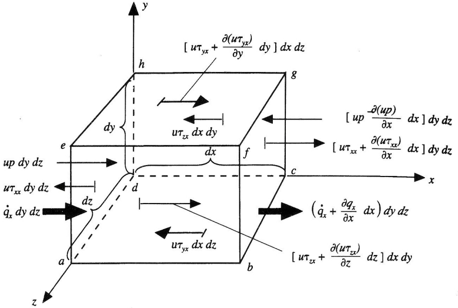

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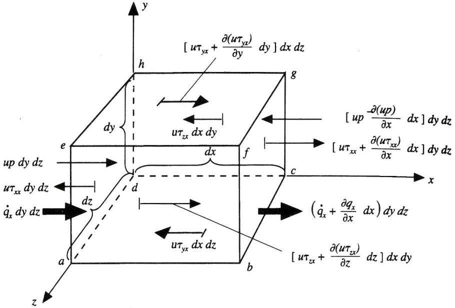

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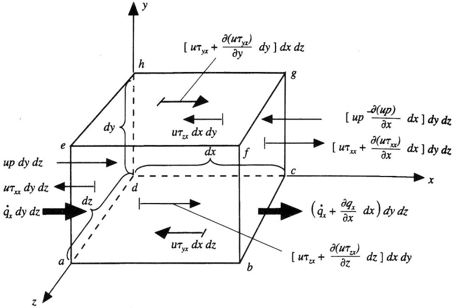

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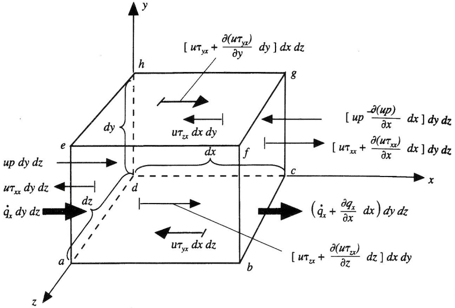

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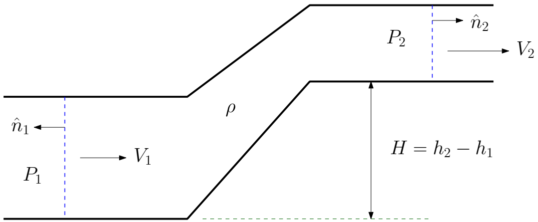

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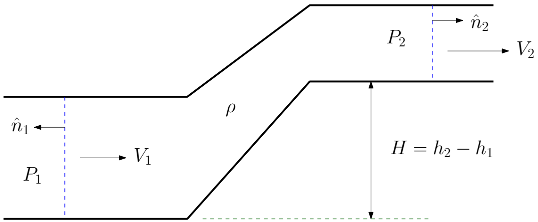

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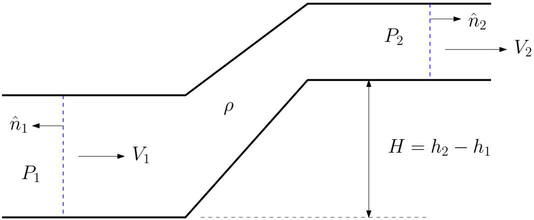

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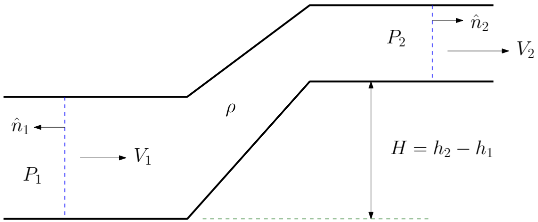

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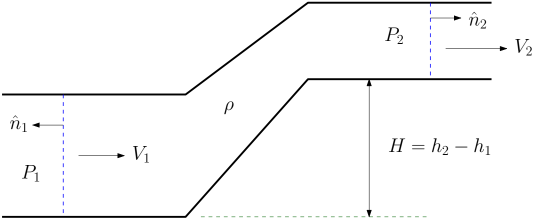

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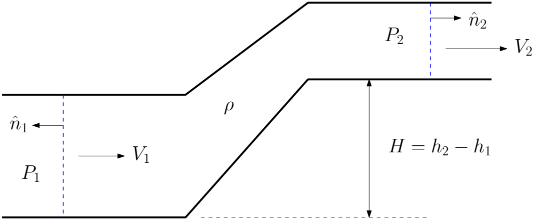

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  $a = r(a,0), \; r = r(a,t)$

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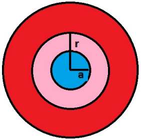

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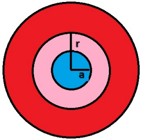

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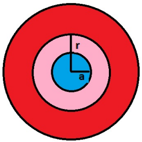

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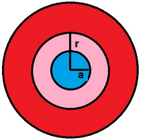

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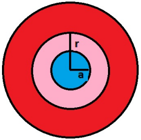

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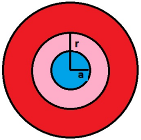

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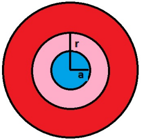

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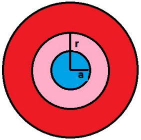

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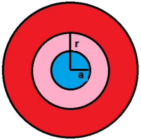
Distribuição esférica



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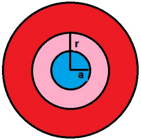

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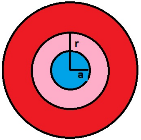

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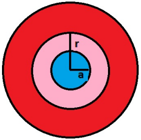

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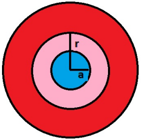

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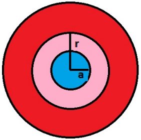

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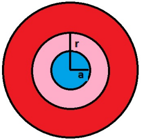

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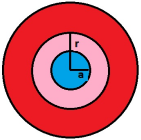

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