14.2 LIMITES E CONTINUIDADE

Revisão técnica: Ricardo Miranda Martins – IMECC – Unicamp

1-22 Determine o limite, se existir, ou mostre que o limite não existe.

1. $\lim\limits_{(x,y)\to(2,3)} (x^2y^2 - 2xy^5 + 3y)$

2. $\lim\limits_{(x,y)\to(-3,4)} (x^3 + 3x^2y^2 - 5y^3 + 1)$

3. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^3 + x^3y^2 - 5}{2 - xy}$

4. $\lim\limits_{(x,y)\to(-2,1)} \dfrac{x^2 + xy + y^2}{x^2 - y^2}$

5. $\lim\limits_{(x,y)\to(\pi,\pi)} x \operatorname{sen}\left(\dfrac{x+y}{4}\right)$

6. $\lim\limits_{(x,y)\to(1,4)} e^{\sqrt{x+2y}}$

7. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x-y}{x^2+y^2}$

8. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{x^2+2y^2}$

9. $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x+y)^2}{x^2+y^2}$

10. $\lim\limits_{(x,y)\to(0,0)} \dfrac{8x^2y^2}{x^4+y^4}$

11. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3+xy^2}{x^2+y^2}$

12. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy+1}{x^2+y^2+1}$

13. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^2+y^2}$

14. $\lim\limits_{(x,y)\to(2,0)} \dfrac{xy-2y}{x^2+y^2-4x+4}$

15. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sqrt{x^2y^2+1}-1}{x^2+y^2}$

16. $\lim\limits_{(x,y)\to(0,1)} \dfrac{xy-x}{x^2+y^2-2y+1}$

17. $\lim\limits_{(x,y)\to(1,-1)} \dfrac{x^2+y^2-2x-2y}{x^2+y^2-2x+2y+2}$

18. $\lim\limits_{(x,y,z)\to(1,2,3)} \dfrac{xz^2-y^2z}{xyz-1}$

19. $\lim\limits_{(x,y,z)\to(2,3,0)} [xe^z + \ln(2x-y)]$

20. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{x^2-y^2-z^2}{x^2+y^2+z^2}$

21. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+zx}{x^2+y^2+z^2}$

22. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{x^2y^2z^2}{x^2+y^2+z^2}$

23-24 Determine $h(x, y) = g(f(x, y))$ e o conjunto no qual h é contínua.

23. $g(t) = e^{-t}\cos t, \quad f(x,y) = x^4 + x^2y^2 + y^4$

24. $g(z) = \operatorname{sen} z, \quad f(x,y) = y\ln x$

25-38 Determine o conjunto de pontos em que a função é contínua.

25. $F(x,y) = \dfrac{x^2+y^2+1}{x^2+y^2-1}$

26. $F(x,y) = \dfrac{x^6+x^3y^3+y^6}{x^3+y^3}$

27. $F(x,y) = \operatorname{tg}(x^4 - y^4)$

28. $G(x,y) = e^{xy}\operatorname{sen}(x+y)$

29. $F(x,y) = \dfrac{1}{x^2-y}$

30. $F(x,y) = \ln(2x+3y)$

31. $G(x,y) = \sqrt{x+y} - \sqrt{x-y}$

32. $f(x,y,z) = \dfrac{xyz}{x^2+y^2-z}$

33. $G(x,y) = 2^{x\operatorname{tg} y}$

34. $f(x,y,z) = x\ln(yz)$

35. $f(x,y,z) = x + y\sqrt{x+z}$

36. $f(x,y) = \begin{cases} \dfrac{2x^2-y^2}{2x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

37. $f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

38. $f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

39. Demonstre, usando a Definição 1, que

(a) $\lim\limits_{(x,y)\to(a,b)} x = a$ (b) $\lim\limits_{(x,y)\to(a,b)} y = b$

(c) $\lim\limits_{(x,y)\to(a,b)} c = c$

40. Utilize coordenadas polares para determinar

$$\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2}$$

[Se (r, θ) são as coordenadas polares do ponto (x, y) com $r \geq o$, observe que $r \to 0^+$ quando $(x,y)\to(0,0)$.]