11.10 SOLUÇÕES

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1.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$(1+x)^{-2}$	1
1	$-2(1+x)^{-3}$	-2
2	$2\cdot 3(1+x)^{-4}$	$2\cdot 3$
3	$-2\cdot 3\cdot 4(1+x)^{-5}$	$-2\cdot 3\cdot 4$
4	$2\cdot 3\cdot 4\cdot 5(1+x)^{-6}$	$2\cdot 3\cdot 4\cdot 5$
...	...	...

Logo $f^{(n)}(0) = (-1)^n\,(n+1)!$ e

$$\frac{1}{(1+x)^2} = \sum_{n=0}^{\infty} \frac{(-1)^n\,(n+1)!}{n!} x^n = \sum_{n=0}^{\infty} (-1)^n\,(n+1)\,x^n$$

Se $a_n = (-1)^n\,(n+1)\,x^n$, então $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = |x|$, logo $R = 1$.

2.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$x/\,(1-x)$	0
1	$(1-x)^{-2}$	1
2	$2(1-x)^{-3}$	2
3	$3\cdot 2(1-x)^{-4}$	$3\cdot 2$
4	$4\cdot 3\cdot 2(1-x)^{-5}$	$4\cdot 3\cdot 2$
...	...	...

$f^{(n)}(0) = n!$ exceto quando $n = 0$, então

$$\frac{x}{1-x} = \sum_{n=1}^{\infty} \frac{n!}{n!} x^n = \sum_{n=1}^{\infty} x^n. \quad \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = |x| < 1$$

para convergência, então $R = 1$.

3.

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	x^{-1}	1
1	$-x^{-2}$	-1
2	$2x^{-3}$	2
3	$-3\cdot 2x^{-4}$	$-3\cdot 2$
4	$4\cdot 3\cdot 2x^{-5}$	$4\cdot 3\cdot 2$
...	...	...

Então $f^{(n)}(1) = (-1)^n\,n!$, e

$$\frac{1}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n\,n!}{n!} (x-1)^n = \sum_{n=0}^{\infty} (-1)^n\,(x-1)^n.$$ Se

$a_n = (-1)^n (x-1)^n$, então $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = |x-1| < 1$ para convergência, logo $0 < x < 2$ e $R = 1$.

4.

n	$f^{(n)}(x)$	$f^{(n)}(4)$
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	2^{-2}
2	$-\frac{1}{4}x^{-3/2}$	-2^{-5}
3	$\frac{3}{8}x^{-5/2}$	$3\cdot 2^{-8}$
4	$-\frac{15}{16}x^{-7/2}$	$-15\cdot 2^{-11}$
...	...	...

$f^{(n)}(4) = \dfrac{(-1)^{n-1}\,1\cdot 3\cdot 5\cdot\cdots\cdot(2n-3)}{2^{3n-1}}$ para $n \geq 2$, então

$$\sqrt{x} = 2 + \frac{x-4}{4} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1}\,1\cdot 3\cdot 5\cdot\cdots\cdot(2n-3)}{2^{3n-1}\,n!}(x-4)^n$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-4|}{8} \lim_{n\to\infty} \left(\frac{2n-1}{n+1}\right) = \frac{|x-4|}{4} < 1$$

para convergência, então $|x-4| < 4 \quad\Rightarrow\quad R = 4$.

5.

n	$f^{(n)}(x)$	$f^{(n)}\left(\frac{\pi}{4}\right)$
0	$\operatorname{sen} x$	$\sqrt{2}/2$
1	$\cos x$	$\sqrt{2}/2$
2	$-\operatorname{sen} x$	$-\sqrt{2}/2$
3	$-\cos x$	$-\sqrt{2}/2$
4	$\operatorname{sen} x$	$\sqrt{2}/2$
...	...	...

$$\begin{aligned}
\operatorname{sen} x &= f\left(\tfrac{\pi}{4}\right) + f'\left(\tfrac{\pi}{4}\right)\left(x-\tfrac{\pi}{4}\right) + \frac{f''\left(\frac{\pi}{4}\right)}{2!}\left(x-\tfrac{\pi}{4}\right)^2 \\
&\quad + \frac{f^{(3)}\left(\frac{\pi}{4}\right)}{3!}\left(x-\tfrac{\pi}{4}\right)^3 + \frac{f^{(4)}\left(\frac{\pi}{4}\right)}{4!}\left(x-\tfrac{\pi}{4}\right)^4 + \cdots \\
&= \tfrac{\sqrt{2}}{2}\Big[1 + \left(x-\tfrac{\pi}{4}\right) - \tfrac{1}{2!}\left(x-\tfrac{\pi}{4}\right)^2 \\
&\qquad - \tfrac{1}{3!}\left(x-\tfrac{\pi}{4}\right)^3 + \tfrac{1}{4!}\left(x-\tfrac{\pi}{4}\right)^4 + \cdots\Big] \\
&= \tfrac{\sqrt{2}}{2}\left[1 - \tfrac{1}{2!}\left(x-\tfrac{\pi}{4}\right)^2 + \tfrac{1}{4!}\left(x-\tfrac{\pi}{4}\right)^4 - \cdots\right] \\
&\qquad + \tfrac{\sqrt{2}}{2}\left[\left(x-\tfrac{\pi}{4}\right) - \tfrac{1}{3!}\left(x-\tfrac{\pi}{4}\right)^3 + \cdots\right] \\
&= \tfrac{\sqrt{2}}{2}\sum_{n=0}^{\infty}(-1)^n\Big[\tfrac{1}{(2n)!}\left(x-\tfrac{\pi}{4}\right)^{2n} \\
&\qquad + \tfrac{1}{(2n+1)!}\left(x-\tfrac{\pi}{4}\right)^{2n+1}\Big]
\end{aligned}$$

As séries também podem ser escritas em uma forma mais elegante:

$$\operatorname{sen} x = \frac{\sqrt{2}}{2}\sum_{n=0}^{\infty}\frac{(-1)^{n(n-1)/2}\left(x-\frac{\pi}{4}\right)^n}{n!}.$$ Se

$a_n = \dfrac{(-1)^{n(n-1)/2}\left(x-\frac{\pi}{4}\right)^n}{n!}$, então

$\lim_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\dfrac{\left|x-\frac{\pi}{4}\right|}{n+1} = 0 < 1$ para todo x, então $R = \infty$.

6.

n	$f^{(n)}(x)$	$f^{(n)}\left(-\frac{\pi}{4}\right)$
0	$\cos x$	$\frac{\sqrt{2}}{2}$
1	$-\operatorname{sen} x$	$\frac{\sqrt{2}}{2}$
2	$-\cos x$	$-\frac{\sqrt{2}}{2}$
3	$\operatorname{sen} x$	$-\frac{\sqrt{2}}{2}$
4	$\cos x$	$\frac{\sqrt{2}}{2}$
...	...	...

$f^{(n)}\left(-\frac{\pi}{4}\right) = (-1)^{n(n-1)/2}\,\frac{\sqrt{2}}{2}$, então

$$\cos x = \sum_{n=0}^{\infty} \frac{f^{(n)}\left(-\frac{\pi}{4}\right)}{n!}\left(x+\frac{\pi}{4}\right)^n = \frac{\sqrt{2}}{2}\sum_{n=0}^{\infty}\frac{(-1)^{n(n-1)/2}\left(x+\frac{\pi}{4}\right)^n}{n!}$$

com $R = \infty$ pelo Teste da Razão (como no Problema 5).

7. $e^{3x} = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!} = \sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$, com $R = \infty$.

8. $\operatorname{sen} 2x = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$,

$R = \infty$

9. $x^2 \cos x = x^2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n)!}$,

$R = \infty$

10. $\cos\left(x^3\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(x^3\right)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$, $R = \infty$.

11.
$$x \operatorname{sen}\left(\frac{x}{2}\right) = x \sum_{n=0}^{\infty} \frac{(-1)^n (x/2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+1)!\,2^{2n+1}}$$ com $R = \infty$.

12.
$$xe^{-x} = x\sum_{n=0}^{\infty}\frac{(-x)^n}{n!} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n!} = \sum_{n=1}^{\infty}\frac{(-1)^{n-1} x^n}{(n-1)!},\ R = \infty.$$

13.
$$\frac{1-\cos x}{x^2} = x^{-2}\left[1 - \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}\right] = x^{-2}\left[-\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}\right] = \sum_{n=1}^{\infty}\frac{(-1)^{n+1} x^{2n-2}}{(2n)!} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+2)!}$$

uma vez que a série é igual a $\frac{1}{2}$ quando $x = 0$; $R = \infty$.

14.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$(1+2x)^{-1/2}$	1
1	$-\frac{1}{2}(1+2x)^{-3/2}(2)$	-1
2	$\frac{3}{2}(1+2x)^{-5/2}(2)$	3
3	$-3\cdot\frac{5}{2}(1+2x)^{-7/2}(2)$	$-3\cdot 5$
...	...	...

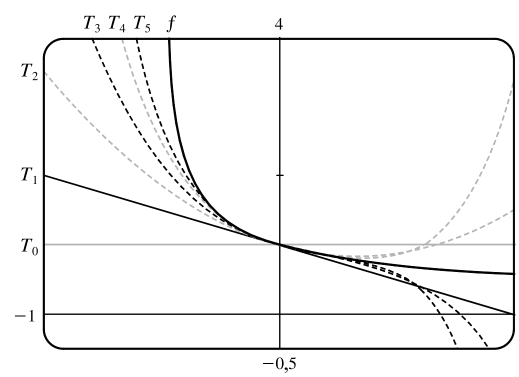
$f^{(n)}(0) = (-1)^n\, 1\cdot 3\cdot 5\cdot 7\cdot\cdots\cdot(2n-1)$, então

$$(1+2x)^{-1/2} = \sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = \sum_{n=0}^{\infty}\frac{(-1)^n\, 1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{n!}x^n$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{2n+1}{n+1}|x| = 2|x| < 1$$

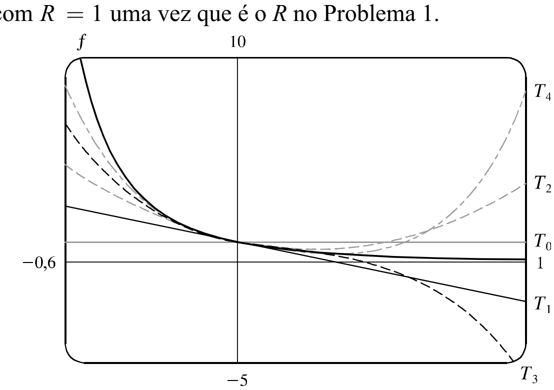
para convergência, logo $R = \frac{1}{2}$.

Outro método: Utilize a série binominal.



15.
$$f(x) = (1+x)^{-3} = -\frac{1}{2}\frac{d}{dx}\left[\frac{1}{(1+x)^2}\right] = -\frac{1}{2}\frac{d}{dx}\left[\sum_{n=0}^{\infty}(-1)^n(n+1)x^n\right] \quad \left[\begin{array}{l}\text{do}\\ \text{Problema 1}\end{array}\right]$$
$$= -\frac{1}{2}\sum_{n=1}^{\infty}(-1)^n\, n(n+1)x^{n-1} = \sum_{n=0}^{\infty}\frac{(-1)^n(n+1)(n+2)x^n}{2}$$

com $R = 1$ uma vez que é o R no Problema 1.



16. $\ln(1+x) = \displaystyle\int \frac{dx}{1+x} = \int \sum_{n=0}^{\infty} (-1)^n x^n \, dx$

$$= C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

com $C = 0$ e $R = 1$, então $\ln(1{,}1) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(0{,}1)^n}{n}$.

Esta é uma série alternada com

$b_5 = \dfrac{(0{,}1)^5}{5} = 0{,}000002$, logo, até cinco casas decimais,

$\ln(1{,}1) \approx \displaystyle\sum_{n=1}^{4} \frac{(-1)^{n-1}(0{,}1)^n}{n} \approx 0{,}09531$.

17. $\displaystyle\int \operatorname{sen}(x^2)\,dx = \int \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{(2n+1)!}\,dx$

$$= \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}\,dx$$

$$= C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$$

18. $\displaystyle\int e^{x^3}\,dx = \int \sum_{n=0}^{\infty} \frac{(x^3)^n}{n!}\,dx = C + \sum_{n=0}^{\infty} \frac{x^{3n+1}}{(3n+1)\,n!}$ com $R = \infty$.

19. Usando a série do Problema 17, obtemos

$$\int_0^1 \operatorname{sen}(x^2)\,dx = \sum_{n=0}^{\infty} \left[\frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}\right]_0^1$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+3)(2n+1)!}$$

e $|c_3| = \dfrac{1}{75\,600} < 0{,}000014$, logo, pelo Teorema da Estimativa da Série Alternada, temos

$\displaystyle\sum_{n=0}^{2} \frac{(-1)^n}{(4n+3)(2n+1)!} = \frac{1}{3} - \frac{1}{42} + \frac{1}{1320} \approx 0{,}310$

(correta até três casas decimais).

20. $\cos(x^2) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n}}{(2n)!}$, logo

$$\int_0^{0{,}5} \cos(x^2)\,dx = \int_0^{0{,}5} \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}\,dx$$

$$= \sum_{n=0}^{\infty} \left[\frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}\right]_0^{0{,}5}$$

$$= 0{,}5 - \frac{(0{,}5)^5}{5\cdot 2!} + \frac{(0{,}5)^9}{9\cdot 4!} - \cdots$$

mas $\dfrac{(0{,}5)^9}{9\cdot 4!} \approx 0{,}000009$, logo, pelo Teorema da Estimativa da Série Alternada, temos

$\int_0^{0{,}5} \cos(x^2)\,dx \approx 0{,}5 - \dfrac{(0{,}5)^5}{5\cdot 2!} \approx 0{,}497$ (correta até três casas decimais).

21.

$$\begin{array}{r|l}
 & -x + \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 + \cdots \\ \hline
1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 + \cdots & -x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 - \cdots \\
 & -x - \phantom{\tfrac{1}{2}}x^2 - \tfrac{1}{2}x^3 - \cdots \\ \hline
 & \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 - \cdots \\
 & \tfrac{1}{2}x^2 + \tfrac{1}{2}x^3 + \cdots \\ \hline
 & \phantom{-x - \tfrac{1}{2}x^2} -\tfrac{1}{3}x^3 + \cdots \\
 & \phantom{-x - \tfrac{1}{2}x^2} -\tfrac{1}{3}x^3 + \cdots \\ \hline
 & \phantom{-x - \tfrac{1}{2}x^2 -\tfrac{1}{3}x^3} \cdots
\end{array}$$

A partir do Exemplo 6 na Seção 11.9, temos

$\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \cdots$, $|x| < 1$. Portanto,

$y = \dfrac{\ln(1-x)}{e^x} = \dfrac{-x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \cdots}{1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots}$. Então, pela

divisão acima, $\dfrac{\ln(1-x)}{e^x} = -x + \dfrac{x^2}{2} - \dfrac{x^3}{3} + \cdots$,

$|x| < 1$.

22. $\displaystyle\sum_{n=2}^{\infty} \frac{x^{3n+1}}{n!} = x \sum_{n=2}^{\infty} \frac{(x^3)^n}{n!} = x\left[\sum_{n=0}^{\infty} \frac{(x^3)^n}{n!} - 1 - x^3\right]$

$$= x\left(e^{x^3} - 1 - x^3\right) \text{ por (11)}$$

23. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!} = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$$= \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) - 1$$

$$= e^x - 1 \text{ por (11)}$$

24. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n\,(n+1)!} = \sum_{n=0}^{\infty} \frac{(x/2)^n}{(n+1)!} = \frac{2}{x} \sum_{n=0}^{\infty} \frac{(x/2)^n}{(n+1)!}$

$$= \frac{2}{x}\left[(x/2) + \frac{(x/2)^2}{2!} + \frac{(x/2)^3}{3!} + \cdots\right]$$

$$= \frac{2}{x}\left(e^{x/2} - 1\right)$$

25. Por (11), $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots$, mas para $x > 0$, todos os termos após dos dois primeiros no RHS são positivos, logo $e^x > 1 + x$ para $x > 0$.

26. Para os Exercícios 12 e 24 no texto, $\cosh x = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^6 + \cdots \geq 1 + \frac{1}{2}x^2$ para todo x uma vez que existem somente potências pares de x no RHS, logo todos os termos remanescentes da expansão são positivos.

27. $(1+x^2)^{1/3} = \displaystyle\sum_{n=0}^{\infty} \binom{\frac{1}{3}}{n} x^{2n}$

$$= 1 + \frac{x^2}{3} + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}x^4 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}x^6 + \cdots$$

$$= 1 + \frac{x^2}{3} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1}\cdot 2\cdot 5\cdot 8\cdot\cdots\cdot(3n-4)\,x^{2n}}{3^n\,n!}$$

com $R = 1$.

28. $[1+(-x)]^{-1/2} = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} (-x)^n$

$$= 1 + \left(-\tfrac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x)^2 + \cdots$$

$$= 1 + \frac{x}{2} + \frac{1 \cdot 3}{2^2 2!}x^2 + \frac{1 \cdot 3 \cdot 5}{2^3 3!}x^3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2^4 4!}x^4 + \cdots$$

$$= 1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n n!} x^n$$

logo $\frac{x}{\sqrt{1-x}} = x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n n!} x^{n+1}$ com $R = 1$.

29. $(2+x)^{-1/2} = \frac{1}{\sqrt{2}}\left(1 + \frac{x}{2}\right)^{-1/2}$

$$= \frac{\sqrt{2}}{2} \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} \left(\frac{x}{2}\right)^n$$

$$= \frac{\sqrt{2}}{2}\left[1 + \left(-\tfrac{1}{2}\right)\left(\frac{x}{2}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(\frac{x}{2}\right)^2 + \cdots\right]$$

$$= \frac{\sqrt{2}}{2}\left[1 + \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)\, x^n}{2^{2n} \cdot n!}\right]$$

com $|x/2| < 1$, então $|x| < 2$ e $R = 2$.

30. $\left[1 + (-x^3)\right]^{-1/2} = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} (-x^3)^n$

$$= 1 + \left(-\tfrac{1}{2}\right)(-x^3) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x^3)^2 + \cdots$$

$$= 1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)\, x^{3n}}{2^n \cdot n!}$$

então $\frac{x^2}{\sqrt{1-x^3}} = x^2 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)\, x^{3n+2}}{2^n \cdot n!}$

com $R = 1$.

31. $(1-x)^{-5} = 1 + (-5)(-x) + \frac{(-5)(-6)}{2!}(-x)^2 + \frac{(-5)(-6)(-7)}{3!}(-x)^3 + \cdots$

$$= 1 + \sum_{n=1}^{\infty} \frac{5 \cdot 6 \cdot 7 \cdots (n+4)}{n!} x^n = \sum_{n=0}^{\infty} \frac{(n+4)!}{4! \cdot n!} x^n$$

$\Rightarrow \quad \frac{x^5}{(1-x)^5} = \sum_{n=0}^{\infty} \frac{(n+4)!}{4! \cdot n!} x^{n+5}$ ou

$\sum_{n=0}^{\infty} \frac{(n+1)(n+2)(n+3)(n+4)}{24} x^{n+5}$, com $R = 1$.

32. $\sqrt[5]{x-1} = -[1+(-x)]^{1/5} = -\sum_{n=0}^{\infty} \binom{\frac{1}{5}}{n} (-x)^n$

$$= -\left[1 + \tfrac{1}{5}(-x) + \frac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)}{2!}(-x)^2 + \frac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)\left(-\frac{9}{5}\right)}{3!}(-x)^3 + \cdots\right]$$

$$= -1 + \frac{x}{5} + \sum_{n=2}^{\infty} \frac{4 \cdot 9 \cdots (5n-6)\, x^n}{5^n \cdot n!}$$ com $R = 1$.

33. (a) $(1+x)^{-1/2} = 1 + \left(-\tfrac{1}{2}\right)x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}x^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}x^3 + \cdots$

$$= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n\, 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n \cdot n!} x^n$$

(b) Tome $x = 0{,}1$ nas séries acima.
$\frac{1 \cdot 3 \cdot 5 \cdot 7}{2^4 4!}(0{,}1)^4 < 0{,}00003$, logo
$\frac{1}{\sqrt{1{,}1}} \approx 1 - \frac{0{,}1}{2} + \frac{1 \cdot 3}{2^2 \cdot 2!}(0{,}1)^2 - \frac{1 \cdot 3 \cdot 5}{2^3 \cdot 3!}(0{,}1)^3 \approx 0{,}953$

34. (a) $(8+x)^{1/3} = 2\left(1 + \frac{x}{8}\right)^{1/3} = 2\sum_{n=0}^{\infty} \binom{\frac{1}{3}}{n} \left(\frac{x}{8}\right)^n$

$$= 2\left[1 + \frac{1}{3}\left(\frac{x}{8}\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\frac{x}{8}\right)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}\left(\frac{x}{8}\right)^3 + \cdots\right]$$

$$= 2\left[1 + \frac{x}{24} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1} \cdot 2 \cdot 5 \cdot \cdots \cdot (3n-4)\, x^n}{24^n \cdot n!}\right]$$

(b) $(8+0{,}2)^{1/3} = 2\left[1 + \frac{0{,}2}{24} - \frac{(0{,}2)^2}{24^2} + \frac{2 \cdot 5(0{,}2)^3}{24^3 \cdot 3!} - \cdots\right]$

$$\approx 2\left[1 + \frac{0{,}2}{24} - \frac{(0{,}2)^2}{24^2}\right]$$

uma vez que $2 \cdot \frac{2 \cdot 5\,(0{,}2)^3}{24^3 \cdot 3!} \approx 0{,}000002$, logo $\sqrt[3]{8{,}2} \approx 2{,}0165$.