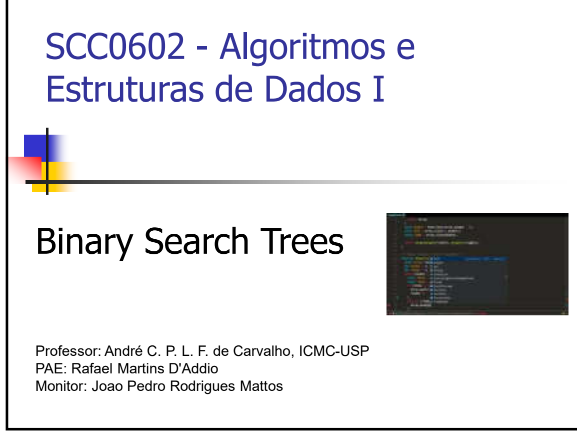

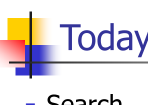

Today

- Search
- Linear search
- Binary Search Trees
  - Tree traversals (using divide-and-conquer)
  - Searching
  - Insertion
  - Deletion

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Introduction

- Search is frequently in several applications
  - Games
    - Best players in checker and chess are search algorithms
  - Minimum path
    - Traveller salesman problem are solved by search algorithms
  - Search engines
    - Search algorithms can find the most relevant sites
    - Dictionaries

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Chess

- In 1997, the current world champion Gary Kasparov played 6 games against Deep Blue, a program written by IBM researchers
  - Deep Blue won 3, lost 2, tied 1
    - Searched 126.000.000 nodes per sec
    - Generated 30 billion positions per reaching depth 14 routinely

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Dictionaries

- *Dictionary* Abstract data type (ADT)
- Dynamic set with methods:
  - **Search(S, k)** – *a query method that returns a pointer x to an element, where x.key = k*
  - **Insert(S, x)** – *a modifier method that adds the element pointed to by x to S*
  - **Delete(S, x)** – *a modifier method that removes the element pointed to by x from S*
- An element has a *key* part and a *satellite data* part

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Ordered Dictionaries

- Besides the previous functions, it should also support the priority-queue-type operations
  - **Min(S)**
  - **Max(S)**
- It would be useful to support
  - **Predecessor(S, k)**
  - **Successor(S, k)**
- These operations require the keys to be comparable

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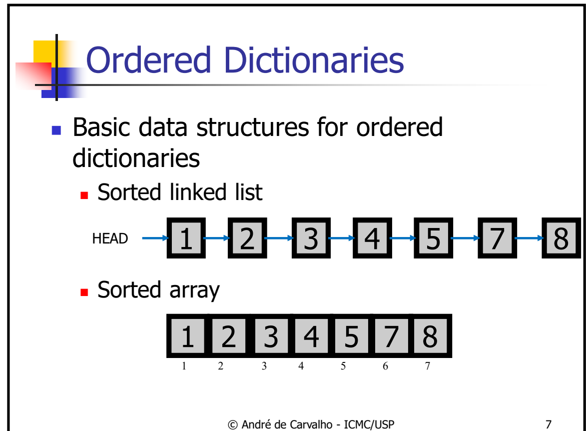
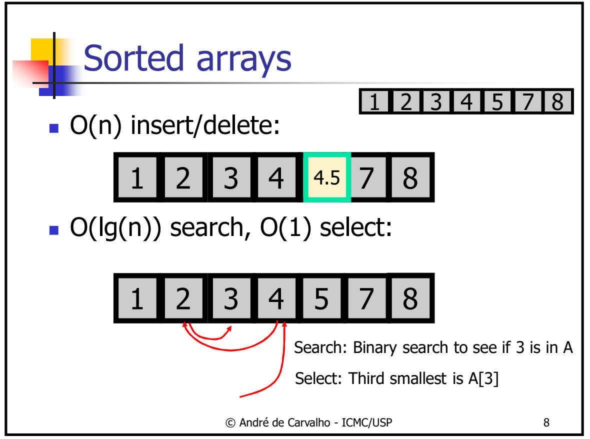
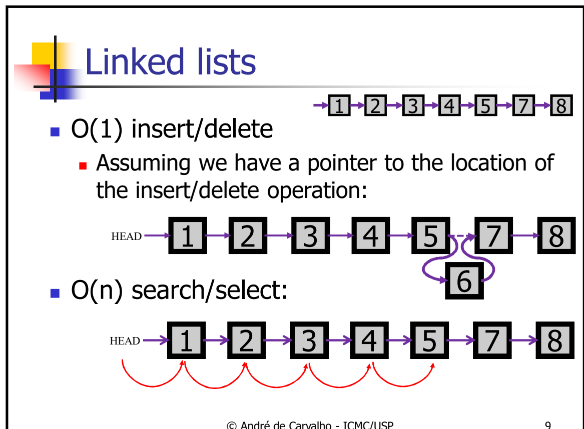
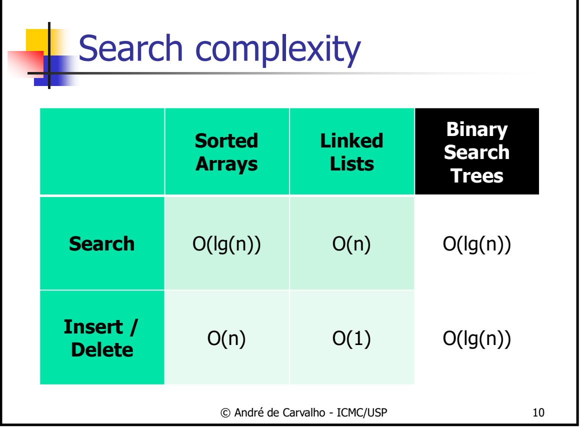
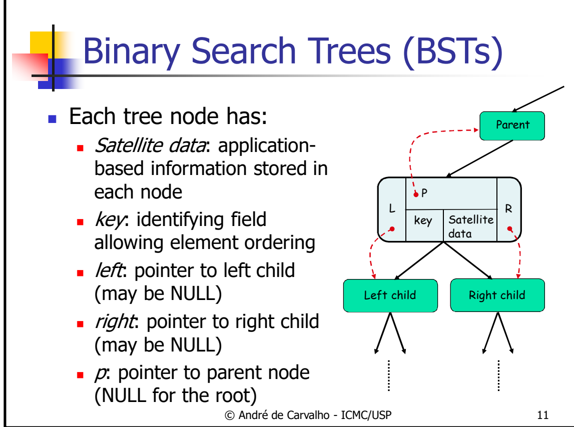


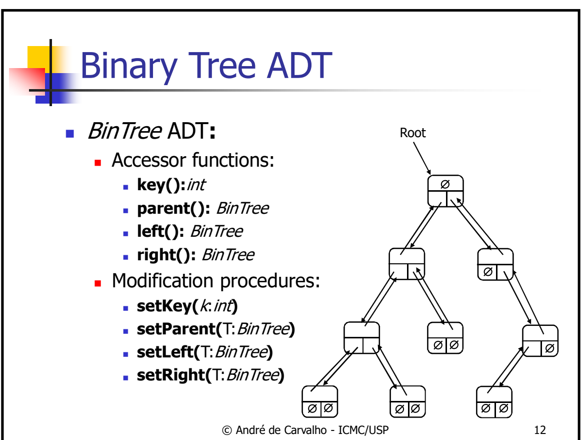


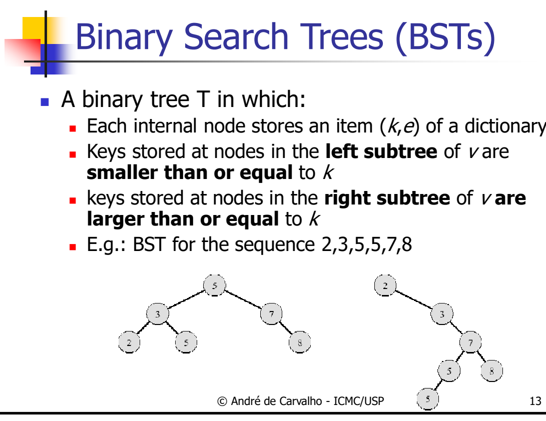




	Sorted Arrays	Linked Lists	Binary Search Trees
Search	O(lg(n))	O(n)	O(lg(n))
Insert / Delete	O(n)	O(1)	O(lg(n))






Binary Search Trees (BSTs)
A binary tree T in which:
Each internal node stores an item (k,e) of a dictionary
Keys stored at nodes in the left subtree of v are smaller than or equal to k
keys stored at nodes in the right subtree of v are larger than or equal to k
E.g.: BST for the sequence 2,3,5,5,7,8
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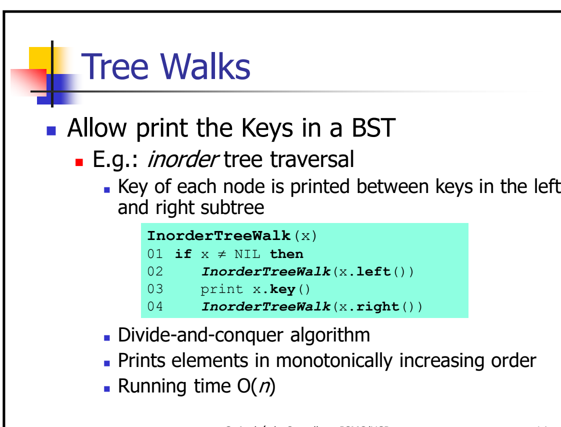
Tree Walks
Allow print the Keys in a BST
E.g.: inorder tree traversal
Key of each node is printed between keys in the left and right subtree
InorderTreeWalk(x)
01 if x ≠ NIL then
02 InorderTreeWalk(x.left())
03 print x.key()
04 InorderTreeWalk(x.right())
Divide-and-conquer algorithm
Prints elements in monotonically increasing order
Running time O(n)
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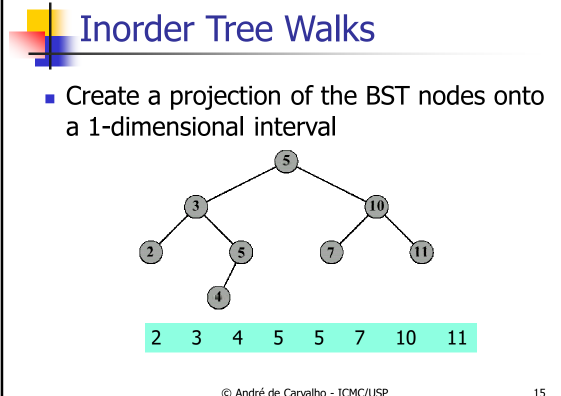
Inorder Tree Walks
Create a projection of the BST nodes onto a 1-dimensional interval
5
3
10
2
5
7
11
4
2 3 4 5 5 7 10 11
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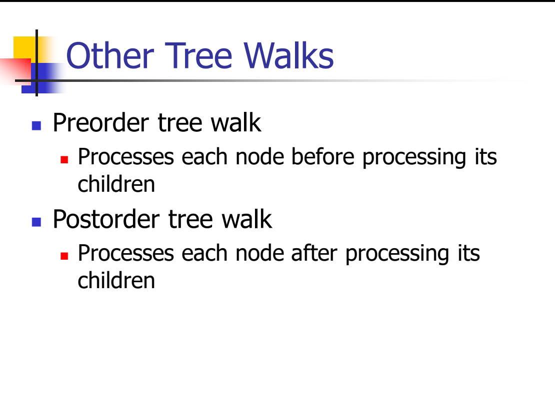
Other Tree Walks
Preorder tree walk
Processes each node before processing its children
Postorder tree walk
Processes each node after processing its children
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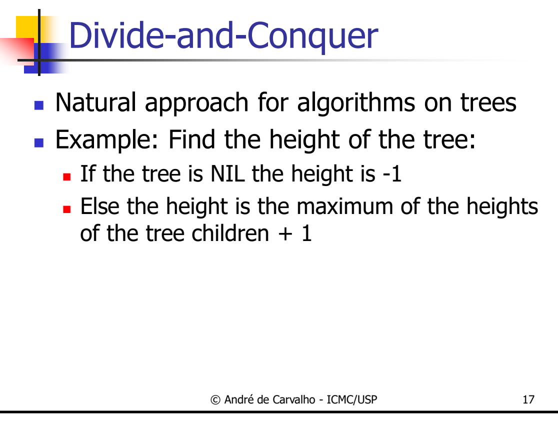
Divide-and-Conquer
Natural approach for algorithms on trees
Example: Find the height of the tree:
If the tree is NIL the height is -1
Else the height is the maximum of the heights of the tree children + 1
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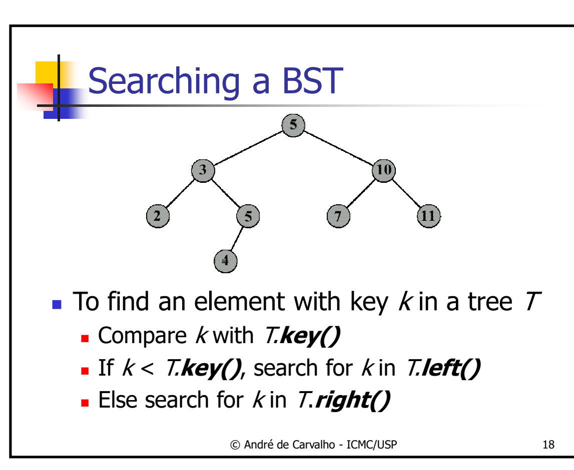
Searching a BST
5
3
10
2
5
7
11
4
To find an element with key k in a tree T
Compare k with T.key()
If k < T.key(), search for k in T.left()
Else search for k in T.right()
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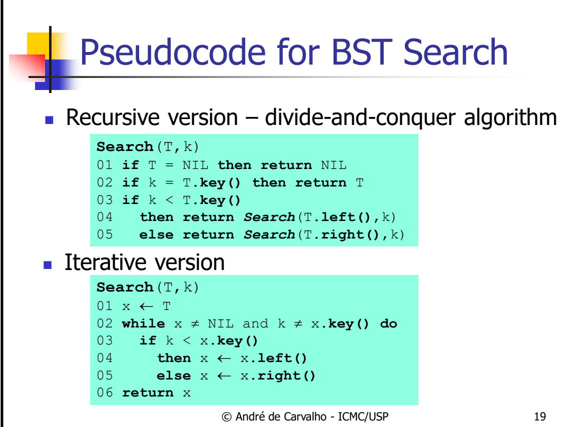
Pseudocode for BST Search
Recursive version – divide-and-conquer algorithm
Search(T,k)
01 if T = NIL then return NIL
02 if k = T.key() then return T
03 if k < T.key()
04 then return Search(T.left(),k)
05 else return Search(T.right(),k)
Iterative version
Search(T,k)
01 x ← T
02 while x ≠ NIL and k ≠ x.key() do
03 if k < x.key()
04 then x ← x.left()
05 else x ← x.right()
06 return x
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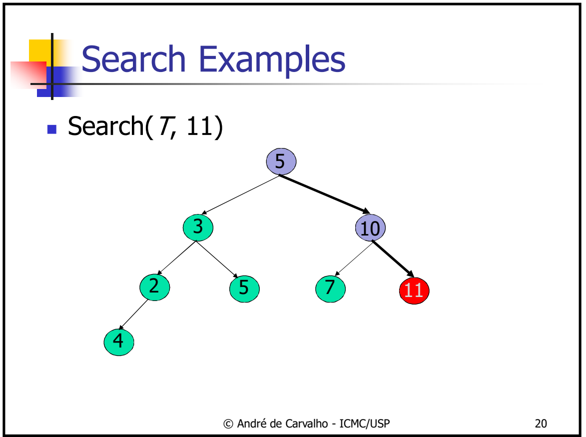
Search Examples
Search(T, 11)
5
3
10
2
5
7
11
4
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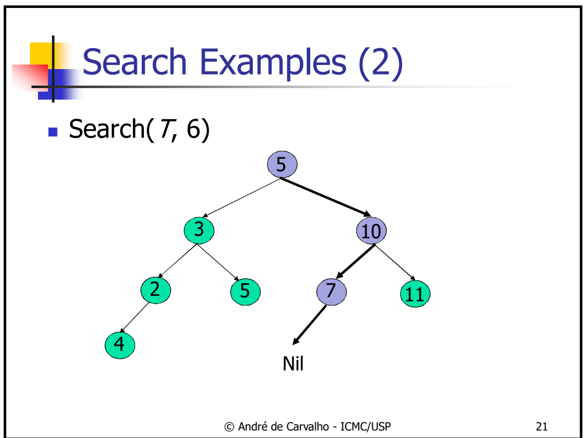
Search Examples (2)
Search(T, 6)
5
3
10
2
5
7
11
4
Nil
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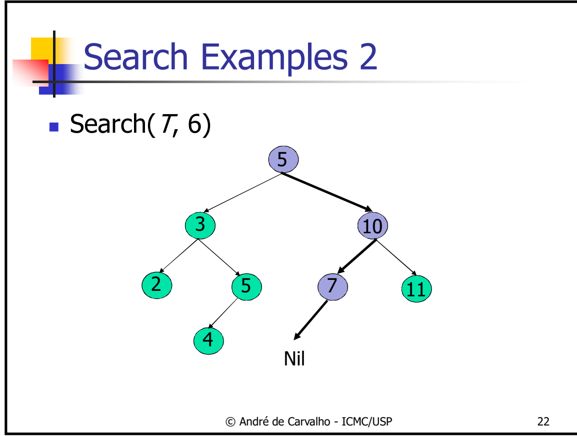
Search Examples 2
Search(T, 6)
5
3
10
2
5
7
11
4
Nil
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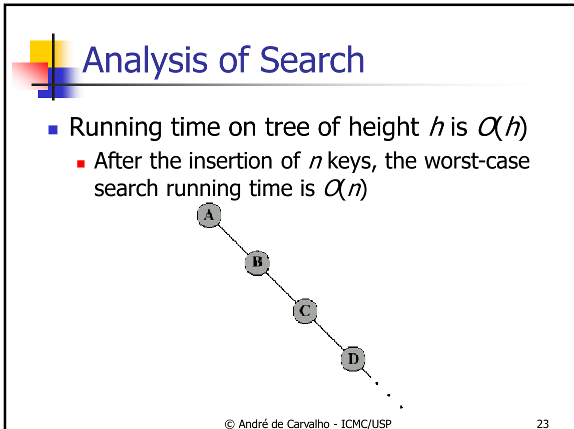
Analysis of Search
Running time on tree of height h is O(h)
After the insertion of n keys, the worst-case search running time is O(n)
A
B
C
D
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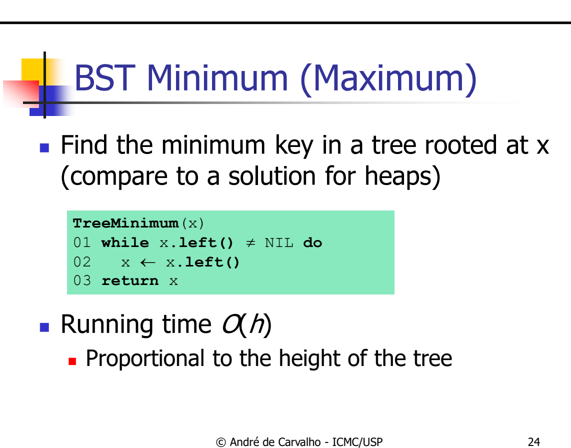
BST Minimum (Maximum)
Find the minimum key in a tree rooted at x (compare to a solution for heaps)
TreeMinimum(x)
01 while x.left() ≠ NIL do
02 x ← x.left()
03 return x
Running time O(h)
Proportional to the height of the tree
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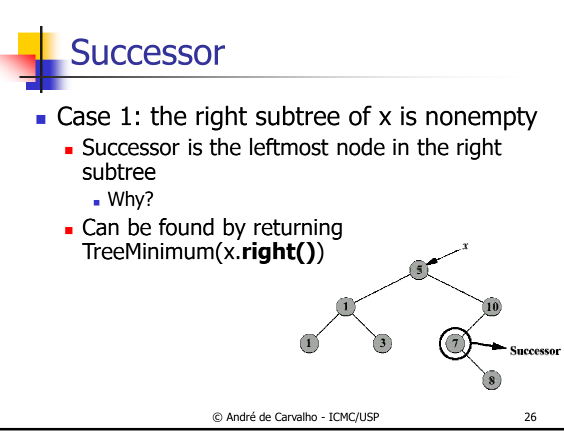
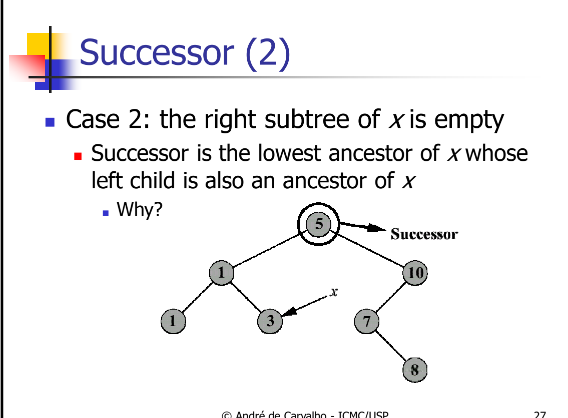
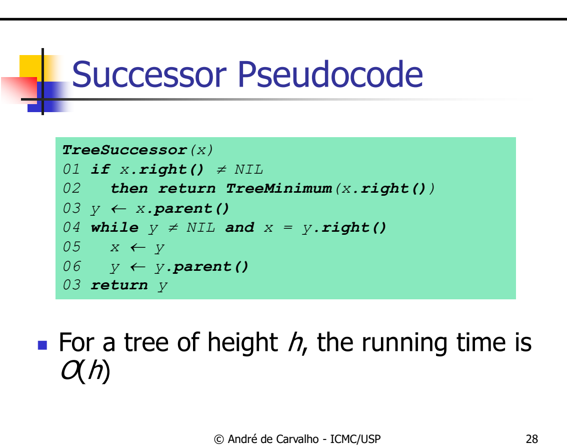
Successor

- Given *x*, find the node with the smallest key that is larger than *x*.**key()**
- There are two possible cases, depending on the right subtree of *x*
  - Case 1: the right subtree of x is nonempty
  - Case 2: the right subtree of *x* is empty





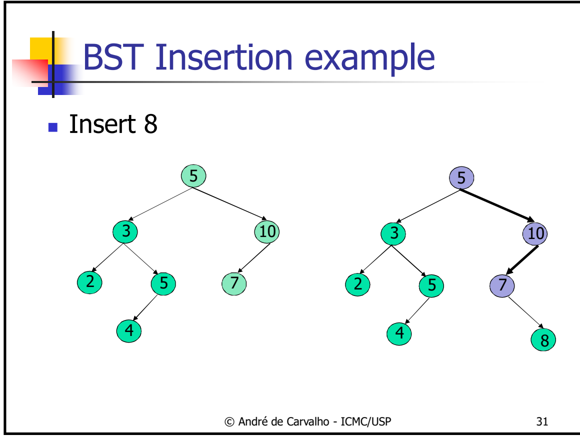


BST Insertion

- The basic idea is similar to searching
  - Take an element (tree) *z* (whose left and right children are NIL) and insert it into *T*
  - Find place in *T* where *z* belongs, as if searching for *z.key()*, and add *z* there
- The running on a tree of height *h* is $O(h)$

BST Insertion Pseudo Code

```
TreeInsert(T,z)
01 y ← NIL
02 x ← T
03 while x ≠ NIL
04   y ← x
05   if z.key() < x.key()
06     then x ← x.left()
07     else x ← x.right()
08 z.setParent(y)
09 if y ≠ NIL
10   then if z.key() < z.key()
11          then y.setLeft(z)
12          else y.setRight(z)
13   else T ← z
```


BST Insertion example
Insert 8
5
3
10
2
5
7
4
5
3
10
2
5
7
4
8
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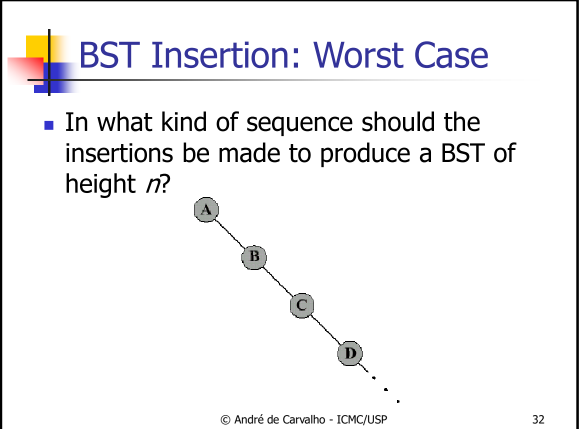
BST Insertion: Worst Case
In what kind of sequence should the insertions be made to produce a BST of height n?
A
B
C
D
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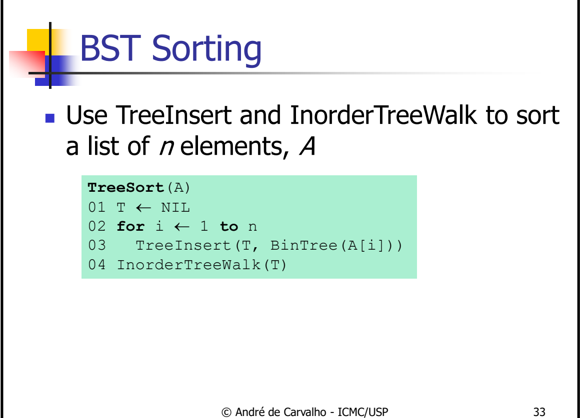
BST Sorting
Use TreeInsert and InorderTreeWalk to sort a list of n elements, A
TreeSort(A)
01 T ← NIL
02 for i ← 1 to n
03   TreeInsert(T, BinTree(A[i]))
04 InorderTreeWalk(T)
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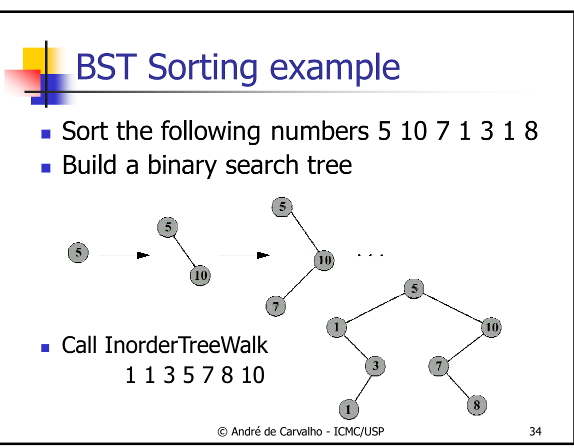
BST Sorting example
Sort the following numbers 5 10 7 1 3 1 8
Build a binary search tree
Call InorderTreeWalk
1 1 3 5 7 8 10
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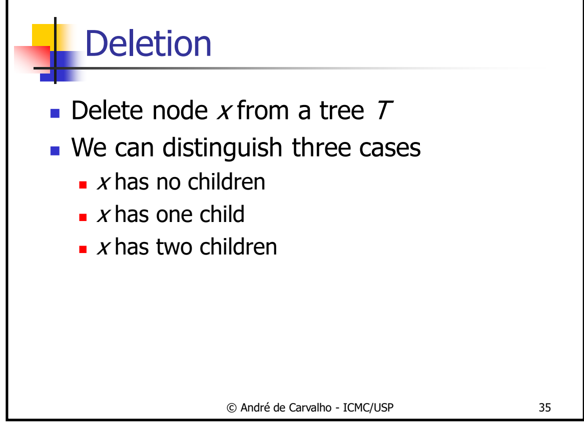
Deletion
Delete node x from a tree T
We can distinguish three cases
x has no children
x has one child
x has two children
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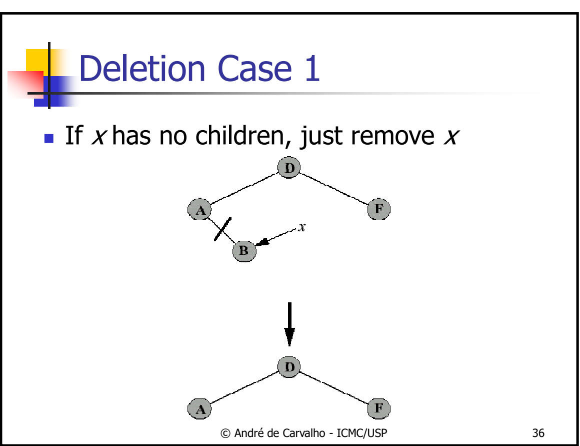
Deletion Case 1
If x has no children, just remove x
D
A
F
B
x
D
A
F
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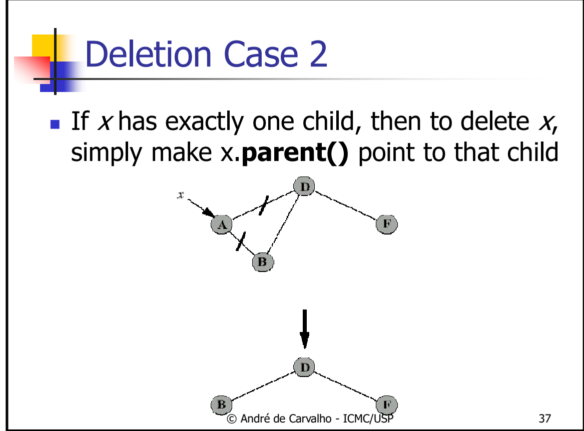
Deletion Case 2
If x has exactly one child, then to delete x, simply make x.parent() point to that child
x
A
B
D
F
B
D
F
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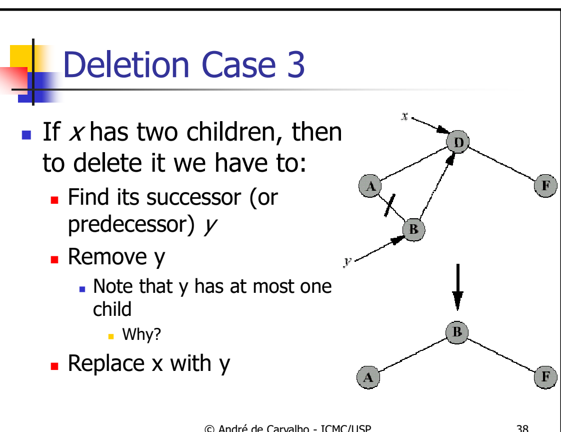
Deletion Case 3
If x has two children, then to delete it we have to:
Find its successor (or predecessor) y
Remove y
Note that y has at most one child
Why?
Replace x with y
x
D
A
F
B
y
B
A
F
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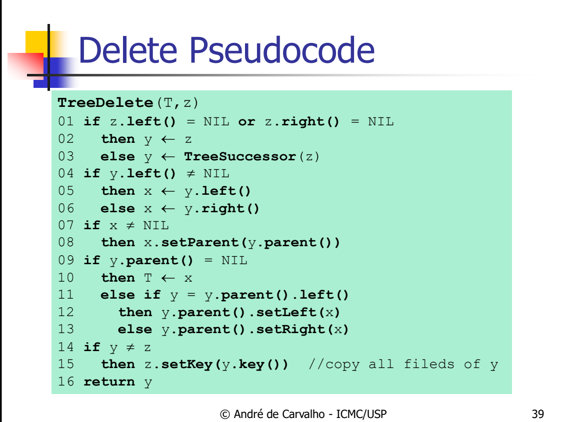
Delete Pseudocode
TreeDelete(T,z)
01 if z.left() = NIL or z.right() = NIL
02 then y ← z
03 else y ← TreeSuccessor(z)
04 if y.left() ≠ NIL
05 then x ← y.left()
06 else x ← y.right()
07 if x ≠ NIL
08 then x.setParent(y.parent())
09 if y.parent() = NIL
10 then T ← x
11 else if y = y.parent().left()
12 then y.parent().setLeft(x)
13 else y.parent().setRight(x)
14 if y ≠ z
15 then z.setKey(y.key()) //copy all fileds of y
16 return y
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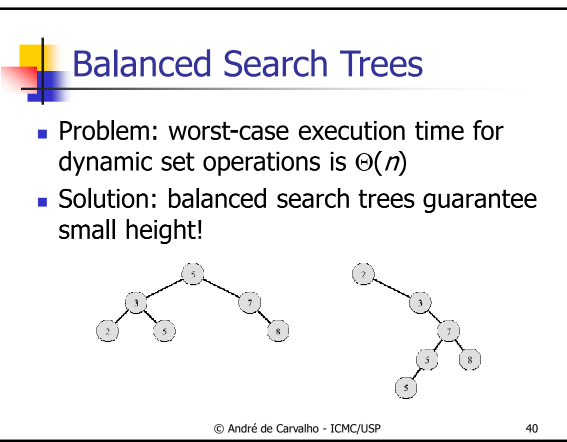
Balanced Search Trees
Problem: worst-case execution time for dynamic set operations is Θ(n)
Solution: balanced search trees guarantee small height!
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Next Lecture
Hashing
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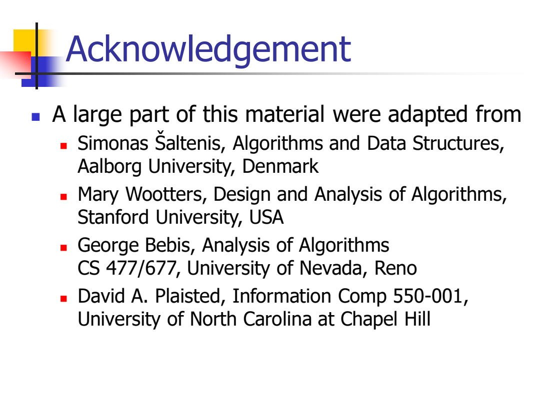
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George Bebis, Analysis of Algorithms CS 477/677, University of Nevada, Reno
David A. Plaisted, Information Comp 550-001, University of North Carolina at Chapel Hill
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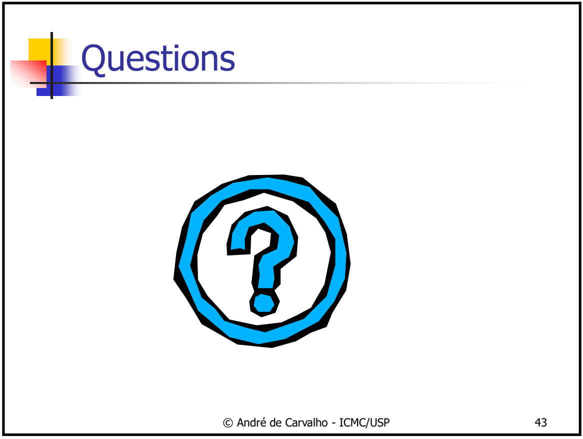
Questions
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