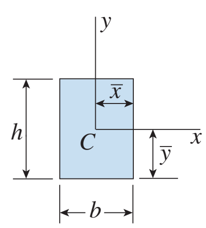
Apêndice A

Propriedades das figuras planas

Notação: A = área
$\bar{x}, \bar{y}$ = distâncias ao centroide C
I_x, I_y = momentos de inércia em relação aos eixos x e y, respectivamente
I_{xy} = produto de inércia em relação aos eixos x e y
$I_P = I_x + I_y$ = momento de inércia polar em relação à origem dos eixos x e y
I_{BB} = momento de inércia em relação ao eixo B-B

1



Retângulo (Origem dos eixos no centroide)

$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = 0 \qquad I_P = \frac{bh}{12}(h^2 + b^2)$$

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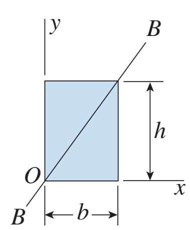
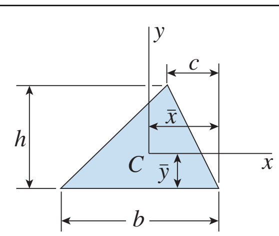


Retângulo (Origem dos eixos no canto)

$$I_x = \frac{bh^3}{3} \qquad I_y = \frac{hb^3}{3} \qquad I_{xy} = \frac{b^2h^2}{4} \qquad I_P \frac{bh}{3}(h^2 + b^2)$$

$$I_{BB} = \frac{b^3h^3}{6(b^2 + h^2)}$$

3

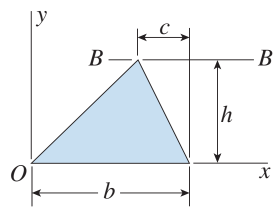


Triângulo retângulo (Origem dos eixos no centroide)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b + c}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{bh}{36}(b^2 - bc + c^2)$$

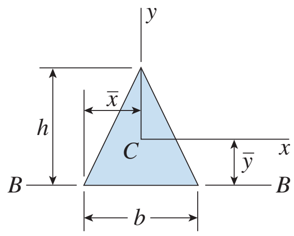
$$I_{xy} = \frac{bh^2}{72}(b - 2c) \qquad I_P = \frac{bh}{36}(h^2 + b^2 - bc + c^2)$$

4 

Triângulo (Origem dos eixos no vértice)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{bh}{12}(3b^2 - 3bc + c^2)$$

$$I_{xy} = \frac{bh^2}{24}(3b - 2c) \qquad I_{BB} = \frac{bh^3}{4}$$

5 

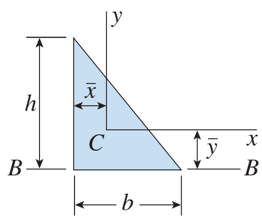
Triângulo isósceles (Origem dos eixos no centroide)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48} \qquad I_{xy} = 0$$

$$I_P = \frac{bh}{144}(4h^2 + 3b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

(*Note:* For an equilateral triangle, $h = \sqrt{3}\, b/2$.)

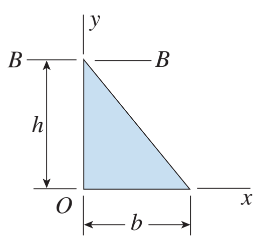
6 

Triângulo retângulo (Origem dos eixos no centroide)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36} \qquad I_{xy} = -\frac{b^2h^2}{72}$$

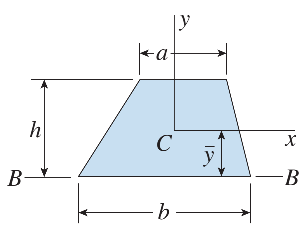
$$I_P = \frac{bh}{36}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

7 

Triângulo retângulo (Origem dos eixos no vértice

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = \frac{b^2h^2}{24}$$

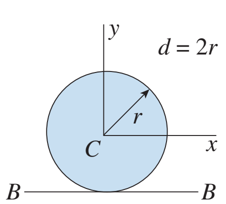
$$I_P = \frac{bh}{12}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{4}$$

8 

Trapezoide (Origem dos eixos no centroide)

$$A = \frac{h(a + b)}{2} \qquad \bar{y} = \frac{h(2a + b)}{3(a + b)}$$

$$I_x = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)} \qquad I_{BB} = \frac{h^3(3a + b)}{12}$$

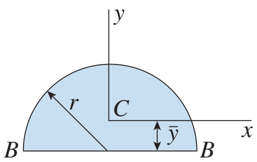
9 

Círculo (Origem dos eixos no centro)

$$A = \pi r^2 = \frac{\pi d^2}{4} \qquad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{xy} = 0 \qquad I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \qquad I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

10

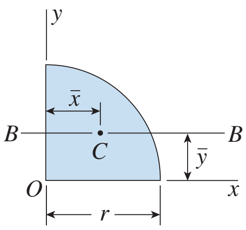


Semicírculo (Origem dos eixos no centroide)

$$A = \frac{\pi r^2}{2} \qquad \bar{y} = \frac{4r}{3\pi}$$

$$I_x = \frac{(9\pi^2 - 64)r^4}{72\pi} \approx 0{,}1098 r^4 \qquad I_y = \frac{\pi r^4}{8} \qquad I_{xy} = 0 \qquad I_{BB} = \frac{\pi r^4}{8}$$

11

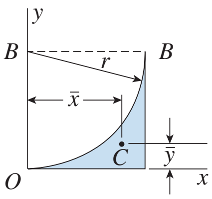


Quarto de círculo (Origem dos eixos no centro do círculo)

$$A = \frac{\pi r^2}{4} \qquad \bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = I_y = \frac{\pi r^4}{16} \qquad I_{xy} = \frac{r^4}{8} \qquad I_{BB} = \frac{(9\pi^2 - 64)r^4}{144\pi} \approx 0{,}05488 r^4$$

12

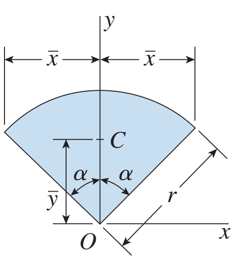


Arco de quarto de círculo (Origem dos eixos no ponto de tangência)

$$A = \left(1 - \frac{\pi}{4}\right)r^2 \qquad \bar{x} = \frac{2r}{3(4 - \pi)} \approx 0{,}7766r \qquad \bar{y} = \frac{(10 - 3\pi)r}{3(4 - \pi)} \approx 0{,}2234r$$

$$I_x = \left(1 - \frac{5\pi}{16}\right)r^4 \approx 0{,}01825 r^4 \qquad I_y = I_{BB} = \left(\frac{1}{3} - \frac{\pi}{16}\right)r^4 \approx 0{,}1370 r^4$$

13



Setor circular (Origem dos eixos no centro do círculo)

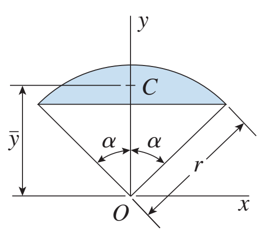
α = ângulos em radianos $(\alpha \leq \pi/2)$

$$A = \alpha r^2 \qquad \bar{x} = r \operatorname{sen} \alpha \qquad \bar{y} = \frac{2r \operatorname{sen} \alpha}{3\alpha}$$

$$I_x = \frac{r^4}{4}(\alpha + \operatorname{sen} \alpha \cos \alpha) \qquad I_y = \frac{r^4}{4}(\alpha - \operatorname{sen} \alpha \cos \alpha)$$

$$I_{xy} = 0 \qquad I_P = \frac{\alpha r^4}{2}$$

14



Segmento circular (Origem dos eixos no centro do círculo)

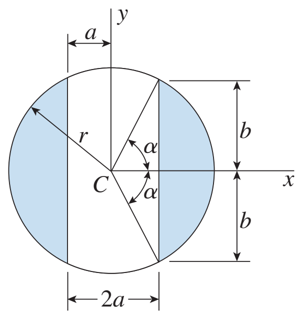
α = ângulos em radianos $(\alpha \leq \pi/2)$

$$A = r^2(\alpha - \operatorname{sen} \alpha \cos \alpha) \qquad \bar{y} = \frac{2r}{3}\left(\frac{\operatorname{sen}^3 \alpha}{\alpha - \operatorname{sen} \alpha \cos \alpha}\right)$$

$$I_x = \frac{r^4}{4}(\alpha - \operatorname{sen} \alpha \cos \alpha + 2 \operatorname{sen}^3 \alpha \cos \alpha) \qquad I_{xy} = 0$$

$$I_y = \frac{r^4}{12}(3\alpha - 3 \operatorname{sen} \alpha \cos \alpha - 2 \operatorname{sen}^3 \alpha \cos \alpha)$$

15



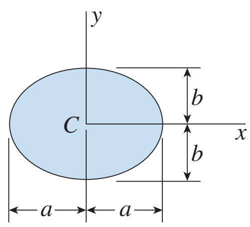
Círculo com o núcleo removido (Origem dos eixos no centro do círculo)

α = ângulos em radianos $\quad (\alpha \leq \pi/2)$

$$\alpha = \arccos\frac{a}{r} \qquad b = \sqrt{r^2 - a^2} \qquad A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$$

$$I_x = \frac{r^4}{6}\left(3\alpha - \frac{3ab}{r^2} - \frac{2ab^3}{r^4}\right) \qquad I_y = \frac{r^4}{2}\left(\alpha - \frac{ab}{r^2} + \frac{2ab^3}{r^4}\right) \qquad I_{xy} = 0$$

16



Elipse (Origem dos eixos no centroide)

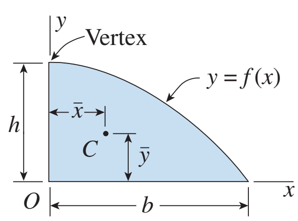
$$A = \pi ab \qquad I_x = \frac{\pi ab^3}{4} \qquad I_y = \frac{\pi ba^3}{4}$$

$$I_{xy} = 0 \qquad I_P = \frac{\pi ab}{4}(b^2 + a^2)$$

$$\text{Circunferência} \approx \pi[1{,}5(a + b) - \sqrt{ab}] \qquad (a/3 \leq b \leq a)$$

$$\approx 4{,}17b^2/a + 4a \qquad (0 \leq b \leq a/3)$$

17



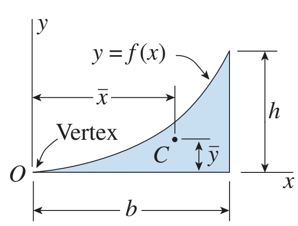
Semissegmento parabólico (Origem dos eixos no canto)

$$y = f(x) = h\left(1 - \frac{x^2}{b^2}\right)$$

$$A = \frac{2bh}{3} \qquad \bar{x} = \frac{3b}{8} \qquad \bar{y} = \frac{2h}{5}$$

$$I_x = \frac{16bh^3}{105} \qquad I_y = \frac{2hb^3}{15} \qquad I_{xy} = \frac{b^2h^2}{12}$$

18



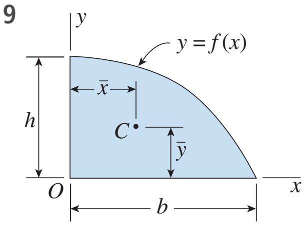
Arco parabólico (Origem dos eixos no vértice)

$$y = f(x) = \frac{hx^2}{b^2}$$

$$A = \frac{bh}{3} \qquad \bar{x} = \frac{3b}{4} \qquad \bar{y} = \frac{3h}{10}$$

$$I_x = \frac{bh^3}{21} \qquad I_y = \frac{hb^3}{5} \qquad I_{xy} = \frac{b^2h^2}{12}$$

19



Semissegmento de grau *n*-ésimo (Origem dos eixos no canto)

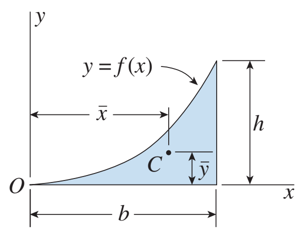
$$y = f(x) = h\left(1 - \frac{x^n}{b^n}\right) \quad (n > 0)$$

$$A = bh\left(\frac{n}{n+1}\right) \quad \bar{x} = \frac{b(n+1)}{2(n+2)} \quad \bar{y} = \frac{hn}{2n+1}$$

$$I_x = \frac{2bh^3n^3}{(n+1)(2n+1)(3n+1)} \quad I_y = \frac{hb^3n}{3(n+3)}$$

$$I_{xy} = \frac{b^2h^2n^2}{4(n+1)(n+2)}$$

20



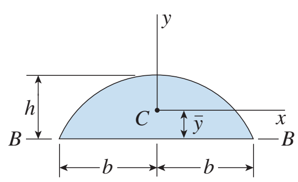
Arco de grau *n*-ésimo (Origem dos eixos no ponto de tangência)

$$y = f(x) = \frac{hx^n}{b^n} \quad (n > 0)$$

$$A = \frac{bh}{n+1} \quad \bar{x} = \frac{b(n+1)}{n+2} \quad \bar{y} = \frac{h(n+1)}{2(2n+1)}$$

$$I_x = \frac{bh^3}{3(3n+1)} \quad I_y = \frac{hb^3}{n+3} \quad I_{xy} = \frac{b^2h^2}{4(n+1)}$$

21



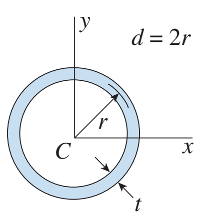
Onda senoidal (Origem dos eixos no centroide)

$$A = \frac{4bh}{\pi} \quad \bar{y} = \frac{\pi h}{8}$$

$$I_x = \left(\frac{8}{9\pi} - \frac{\pi}{16}\right)bh^3 \approx 0{,}08659bh^3 \quad I_y = \left(\frac{4}{\pi} - \frac{32}{\pi^3}\right)hb^3 \approx 0{,}2412hb^3$$

$$I_{xy} = 0 \quad I_{BB} = \frac{8bh^3}{9\pi}$$

22

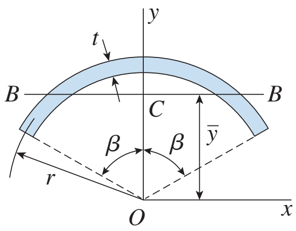


Anel circular fino (Origem dos eixos no centro). Fórmulas aproximadas para o caso em que t é pequeno

$$A = 2\pi rt = \pi dt \quad I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$

$$I_{xy} = 0 \quad I_P = 2\pi r^3 t = \frac{\pi d^3 t}{4}$$

23



Arco circular fino (Origem dos eixos no centro do círculo). Fórmulas aproximadas para o caso em que t é pequeno

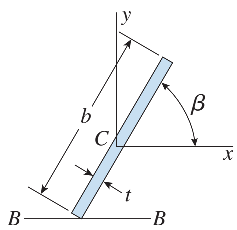
β = ângulo em radianos (*Observação*: Para arco semicircular, $\beta = \pi/2$.)

$$A = 2\beta rt \qquad \bar{y} = \frac{r \operatorname{sen} \beta}{\beta}$$

$$I_x = r^3 t(\beta + \operatorname{sen} \beta \cos \beta) \qquad I_y = r^3 t(\beta - \operatorname{sen} \beta \cos \beta)$$

$$I_{xy} = 0 \qquad I_{BB} = r^3 t\left(\frac{2\beta + \operatorname{sen} 2\beta}{2} - \frac{1 - \cos 2\beta}{\beta}\right)$$

24

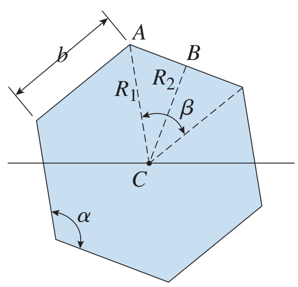


Retângulo fino (Origem dos eixos no centroide). Fórmulas aproximadas para o caso em que t é pequeno

$$A = bt$$

$$I_x = \frac{tb^3}{12} \operatorname{sen}^2 \beta \qquad I_y = \frac{tb^3}{12} \cos^2 \beta \qquad I_{BB} = \frac{tb^3}{3} \operatorname{sen}^2 \beta$$

25



Polígono regular com *n* lados (Origem dos eixos no centroide)

C = centroide (no centro do polígono)

n = número de lados ($n \geqslant 3$) $\qquad$ b = comprimento de um lado

β = ângulo central para um lado $\qquad$ α = ângulo interior (ou ângulo do vértice)

$$\beta = \frac{360°}{n} \qquad \alpha = \left(\frac{n-2}{n}\right)180° \qquad \alpha + \beta = 180°$$

R_1 = raio do círculo circunscrito (linha CA)

R_2 = raio do círculo inscrito (linha CB)

$$R_1 = \frac{b}{2} \operatorname{cossec} \frac{\beta}{2} \qquad R_2 = \frac{b}{2} \operatorname{cotg} \frac{\beta}{2} \qquad A = \frac{nb^2}{4} \operatorname{cotg} \frac{\beta}{2}$$

I_c = momento de inércia ao redor de qualquer eixo através de C (o centroide C é um ponto principal e cada eixo através de C é um eixo principal)

$$I_c = \frac{nb^4}{192}\left(\operatorname{cotg} \frac{\beta}{2}\right)\left(3 \operatorname{cotg}^2 \frac{\beta}{2} + 1\right) \qquad I_P = 2I_c$$