Symmetry as a Compositional Determinant

Bartok's Music for Strings, Percussion and Celeste

Webern's Variations for Piano, Op 27

Chapter VII. Analysis of Works with Intensive Applications

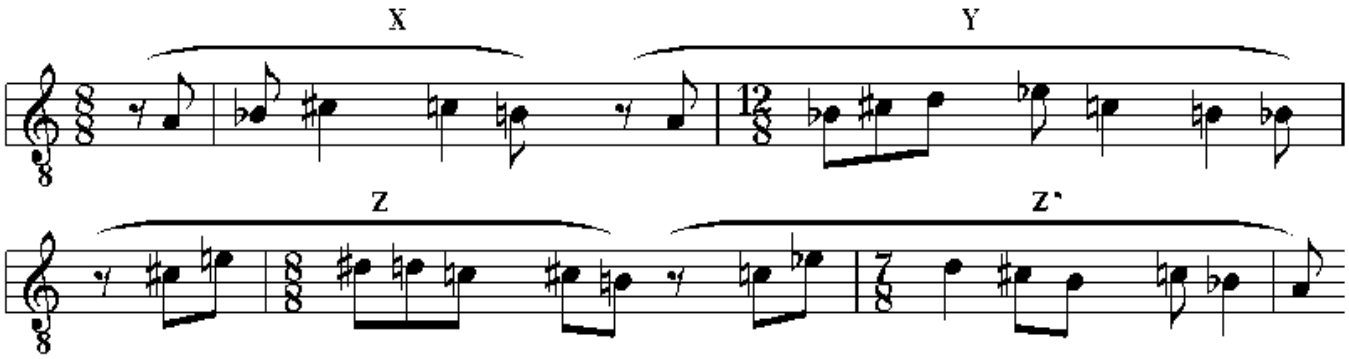
Contrasted with the works of the previous chapter, those analyzed here were known to contain some symmetric properties before their inclusion in this study. These works were analyzed in detail, and the results have shown that symmetry can be and has been used as a powerful constructive principle, tying together several simultaneous levels of a compositional superstructure.

Both of the works chosen are considered to be of major importance as compositions in the first half of this century: Bela Bartok's *Music for String Instruments, Percussion and Celeste* and Anton Webern's *Variations for Piano*, Op. 27. Both have been subject to previous analysis. Notable among these are: the analysis given in the Bartok study score (presumably by the composer), Erno Lendvai's numerological analysis of Bartok's music, and the Webern studies by Leibowitz, Ogdon, and Westergaard.

Bartok's *Music for Strings, Percussion and Celeste*

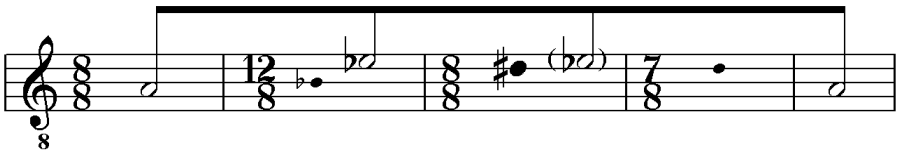
Symmetry is probably the most significant means of unifying this composition. The first movement, almost entirely for strings, has a most remarkable structure, perfect in its symmetrical conception. It is a fugue with the following subject:

Example 79. The fugue subject, **S**, of the first movement of Bartok's *Music for String Instruments, Percussion and Celeste*



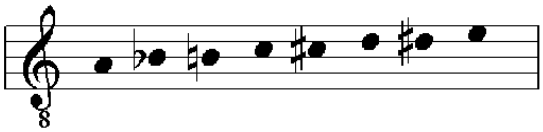
The subject itself (**S**) has a rising and falling contour. It begins on A, climaxes on Eb at the first beat of the 8/8 (see example 79) and the second beat of the 12/8, and then it returns to A. The climax is defined by the contour, the rhythmic-metric stresses, and by pitch height.

Example 80. Generalized motion of the fugue subject.



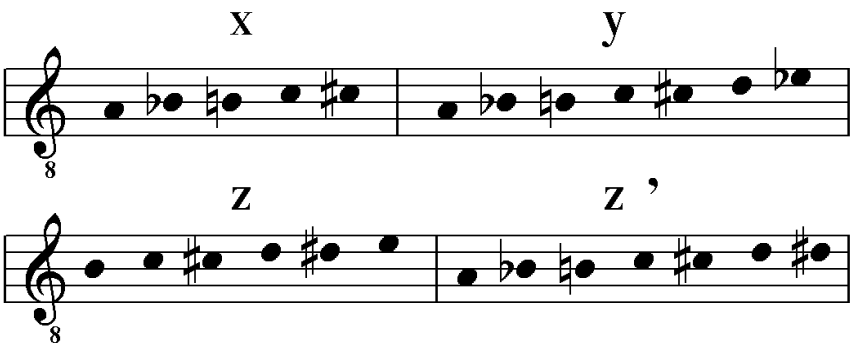
Examination of the pitch content of the subject yields all of the semitones from A to E.

Example 81. The pitch content of the fugue subject.



Each of the four phrases of the subject is also comprised of a complete ambit of semitones which is an automorphic translation of example 81.

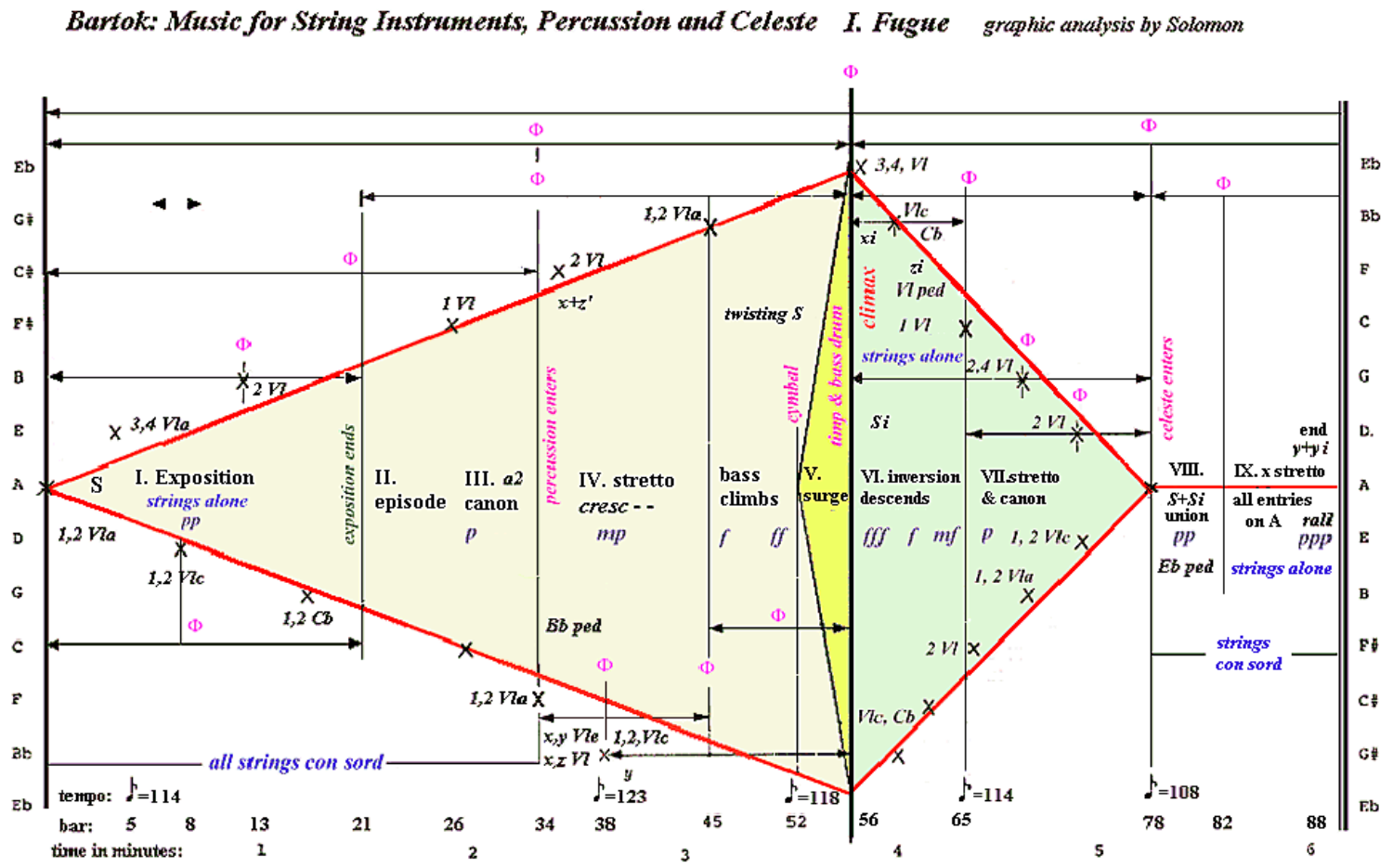
Example 82. The pitch content of each of the four phrases of the fugue subject.



The fugue of the first movement is an arch form having a unique tonal construction centering on A. The subject enters in successive voices passing through the circle of fifths, tonally reflected about this A; the first entry is on A3, the second is a fifth above on E4, the third a fifth below on D3, the fourth on B4 above, the fifth on G3 below, etc.

The five-voiced exposition ends at bar 21, but subject entries continue through the circle of fifths in opposite directions carried through a two-part canon (bar 26), and a stretto (bar 34). At bar 45 there begins a dramatic climb in the bass converging with the upper voices on A4 at bar 52. This marks the beginning of the final surge to the climax, accompanied by cymbal, timpani and bass drum (their only part in the movement is from bar 34 to the climax). From the beginning, adding instruments to the contrapuntal texture results in a gradual crescendo. Mutes are used on the strings at the beginning until they are removed at bar 34 when the percussion first enters. The subject entry pattern continues the logic relentlessly through the cycle of fifths until the climax of the movement is reached on the remote Eb, a tritone from A, at bar 56. Thus, the whole tonal movement of the movement echoes the structure of the subject.

Figure 20. Analytic graph of the first movement of Bartok's *Music for String Instruments, Percussion, and Celeste*

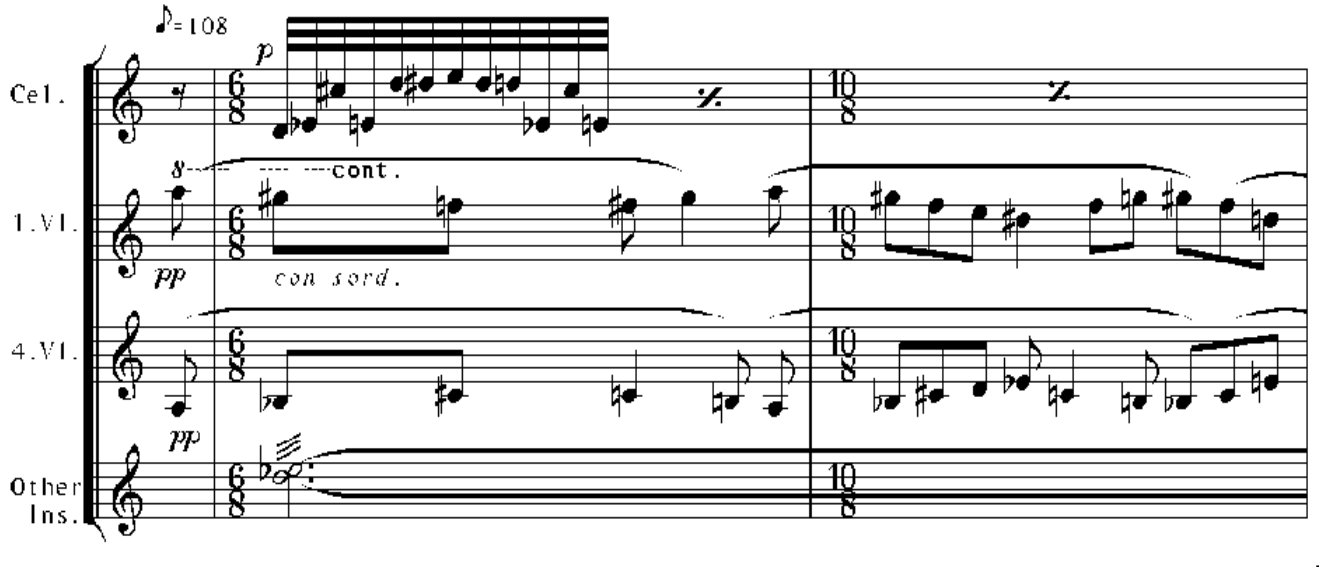


After bar 56, the pattern is rotated 180°, and the subject occurs solely in a reflected form, its inversion, which had not appeared before. Prior to the climax, the subject appears only in direct imitation. Afterwards, its inversion is stated in imitation through the retrograde inversion of the original entry

pattern (backwards through the circle of fifths and inverted in registration), converging at bar 78 on A again. This retrograde-inverted stretto is a kind of automorphic crab of the first 55 measures.

Instruments rapidly drop out after the climax and the texture and dynamics thin. A stretto ensues at bar 65 with entries cascading rapidly down through the strings in the cycle of fifths. At 78 the counterpoint converges on A again. In this measure the complete original upright version of the subject is heard, played simultaneously with its complete inversion. This is the solitary complete union of the whole subject, **S**, and its inversion, **Si**, in the entire movement (designated "**S+Si** union" in figure 20). The celeste enters and plays undulating arpeggios, weaving around the subject and its inversion as the subject unites with its inversion. The two complementary forms begin together on A, climax together on Eb and end together on A, all the while accompanied by an Eb pedal, thus echoing the tonal structure of the subject and the entire movement.

Figure 20b. Measures 78-79. At the completion of the cycle, the fugue subject is combined simultaneously with its inversion.



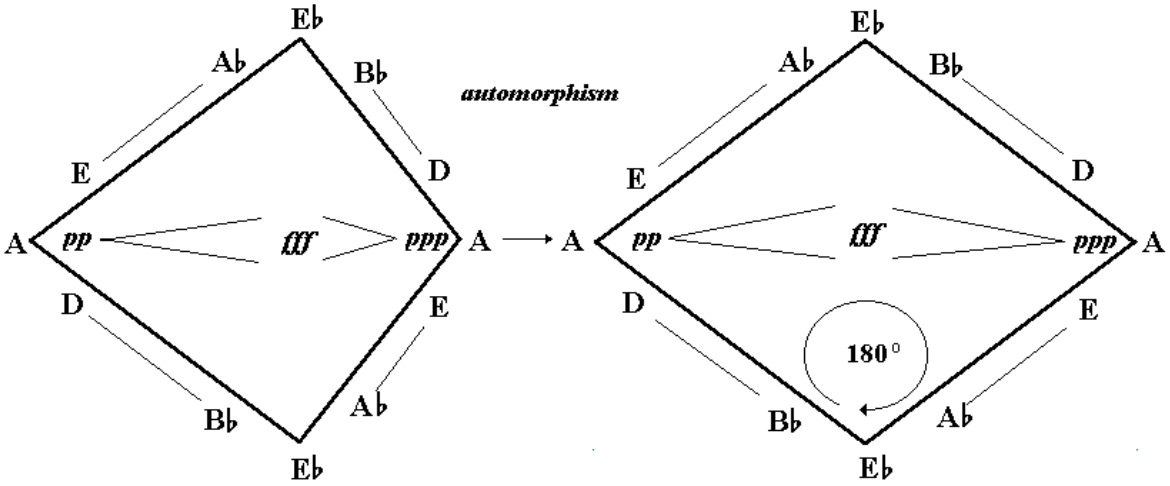
This seems almost programmatic, where Bartok seems to be symbolizing the universal dualities of nature: male-female, positive-negative, north-south, etc.

After this magical union, there begins the final section at 82 with fragments of the subject in stretto with their inversions always stated on A. The events in this sequence are shown horizontally in time in figure 20, a graph of the events in the actual time they occur, as shown by the tempo marks near the bottom of the diagram. The "X's" indicate the order of the subject entries relative to A. Before bar 34 the subject statements are always complete, but thereafter, the subject may be represented by any of its four phrase parts.

The two primary axes are tonal, A, and temporal, bar 56 (the climax). Symmetry is demonstrated by rotating the entire pattern of entry ordering 180 degrees, a retrograde inversion.

The dynamic and textural crescendo-decrescendo may also be reflected about bar 56. In both cases, however, if we wish to show durational as well as ordinal symmetry it is necessary to perform a partial automorphism, operating on the temporal dimensions on one side of the climax.

Figure 21. Operations showing the symmetry in Bartok's *Music for Strings*..., first movement.



The fragmenting of the subject into representative phrases helps to permit the durational contraction after the climax. The entries G# (enharmonically Ab), Bb and F after bar 56 are punctuated by dramatic tonal shifts, unusual pauses, and static harmonies which precede them. The ideal pattern is not fully realized in the music, however. The C# entry after the climax is missing, and the F entry is missing but seems to be substituted as a pedal tone in measures 61-64. The Eb and Bb are treated similarly before the F, but they are accompanied by real entries.

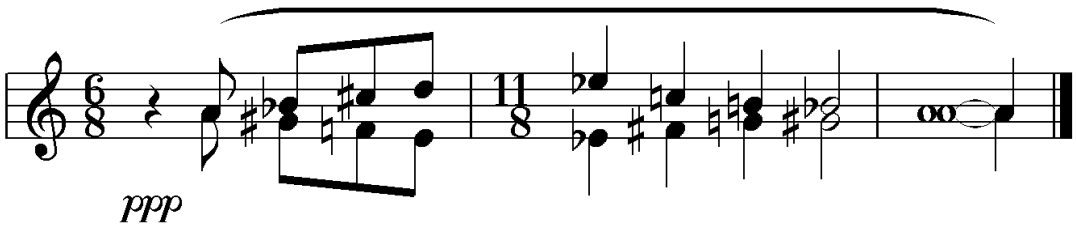
The single surging and subsiding dynamic textural wave corresponds with an increase in pitch range and registrational reflection. The mounting tension of measures 1-55 is echoed by a final surge to the climax at bar 52. The string basses have here reached their highest pitch in the movement, a fortissimo A. From this point the lower strings begin a continuous descent reflected by a simultaneous ascent in the upper strings to the climactic Eb. This motion from A to Eb is a translated statement of the tonal structure of the entire movement, and also of the entire work.

The reflective property of subject-inversion is also treated as structure building in its translation to various levels of the composition. First, we note that the inversion of the subject is not presented until after the climax, the primary axis. After this point the subject's prime form is absent until the tonal cycle is completed at bar 78. Bar 56 may be regarded as the boundary of a metamorphosis, similar to that of the plane of a lens through which all that was upright becomes upside down. This interesting process of metamorphosis began in measure 34 with the fragmenting of the subject, From measure 38 on its shape begins to distort by expanding intervals. From measure 45 a curious twisting begins. A churning of the fragments blurs its identity, until, at the climax, the result appears in focus as the inversion, but not as yet complete.

The stretto sequence at bar 65 hints at the oncoming synthesis, and, with the entry on G before bar 69, we finally hear the complete inversion, followed by a canon of itself in measures 73-77, recalling the canon of section III.

The movement concludes with a stretto of the ***x*** phrase with its inversion and a final statement of ***y***. This is significant as the last statement of the movement. The structure of ***y*** is a translation of the movement's structure. It is here combined simultaneously with its own inversion, symmetrically disposed on A, beginning and ending on this A and climaxing on Eb. It is temporally symmetric to a degree of 60; the simultaneous dyads correspond to the adjacent dyad subject entries of the total movement structure.

Example 83. Reflective structures in the final statement of the first movement fugue



Fibonacci Numbers, Golden Sections, et al

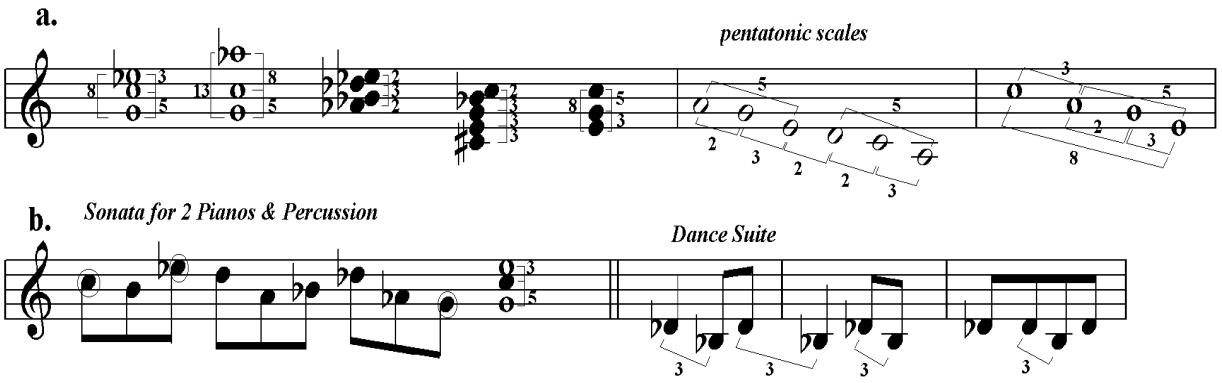
Erno Lendvai claims to have found Fibonacci numbers and Golden Sections in Bartok's *Music for String Instruments, Percussion and Celeste* and other works of this composer. If this is so it would be important to considerations of symmetry since the Fibonacci series is translational by automorphism. It is defined as an arithmetic sequence in which each number is equal to the sum of the previous two. An example is the series:

1,1,2,3,5,8,13,21,34,55,89...

Its derivation from and application to art and nature is somewhat controversial. A general problem lies in the criteria for attribution of significance to

certain numbers. Such is justifiable only if the numbers commonly recur- since the probability of of any number occupying a position is just as great as any other. A well known significant number is pi, the ratio of the circumference of a circle to its diameter. The derivation of pi was a result of its insistent appearance in mathematical equations. When dealing with number series one should not confuse the individual and particular numbers with the law which governs the generation of the series. The recurrence of the minor third is not surprising to us; nor is the major second or perfect fourth, but Lendvai finds Fibonacci numbers in these and other intervals. {Lendvai, 99-137}

Example 84. Intervallic analyses of Bartok's music by Erno Lendvai showing Fibonacci numbers which indicate interval measurement by number of semitones. {Kepes, 174-192}



The problem with this is that other numbers are represented here, yet are ignored. Some add up to 4, 6, 7, 9, and 10. It seems that any number from 1 to 10 can be found here. This looks suspiciously like a selective slant in favor of the most popularly known Fibonacci number series. But, these are not the only Fibonacci numbers. Why, for instance, is the number 1 ignored, and why not include the other possible Fibonacci series, such as:

2,2,4,6,10,16,26...
3,6,9,15,24,39...
3,4,7,11,18,29...

From this, we can see that any number can be considered a Fibonacci number. The intervals 2,3,5 and 8 (in semitones) comprise 30 per cent of all possible intervals in the octave and two out of three of the interval classes. In a random distribution, therefore, these numbers would occur 30 per cent and 67 per cent of the time, respectively. It is well known, in addition, that the semitone and the tritone, intervals 1 and 6, play important roles in Bartok's music, but these are, in fact, ignored by Lendvai.

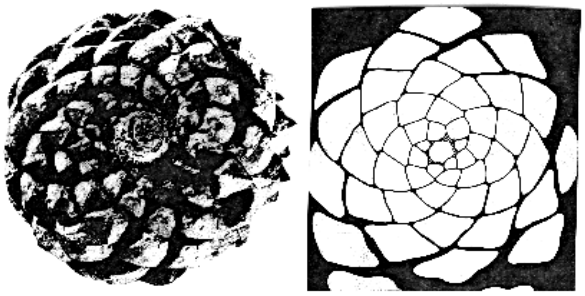
The Fibonacci series is not any fixed group of numbers, although a particular series may represent it. It seems from the following exposition that the ratio which exists as a limit case of the series is the important consideration, the "Golden Section" or ratio, approximately .618:1. As a Fibonacci series gets larger and larger, the ratio between adjacent numbers approaches the Golden Section as a limit. The lower numbers are just an approximation of this ratio. Thus, in the series: 1,1,2,3,5,8,13,21,34,55,89,144,233,377...., the ratio 1:1 is not even close to the Golden Section. Neither is 1:2. The ratio 2:3 is .66667, and 5:8 = .625. The numbers 8:13 = .615, 13:21 = .619, 21:34 = .617647, and 34:55 = .61818. The largest ratio of the numbers shown here is 233:377 = .6180371. Each of these comes closer and closer to the Golden Section, which is an irrational number.

[The Golden Section]... is the name given to an irrational proportion known at least since Euclid which has often been thought to possess some esthetic virtue in itself, some hidden harmonic proportion with the universe. It is defined strictly as a line which is divided in such a way that the smaller part is to the larger part as the larger part is to the whole (line AB cut at C so that CB:AC= AC:AB). In practice it works out at about 8:13 and may be discovered in most works of art....{Murray, P., 89}

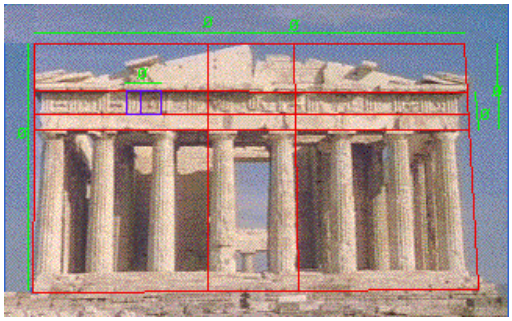
A _____C_____B

The Section has been used consciously by such artists as Le Corbusier, Andre Villon, and Gothic architects, and it is widely used today in architecture. It is found in many works of art, including Dante's *Divine Comedy,* at the end of the sixty-second of the hundred cantos, where Dante parts with Virgil and goes with Beatrice. Sixteenth and seventeenth century instrument makers used it in the design of musical instruments, and a system of tuning has been proposed based upon it. {Barbour, 129} Both the Fibonacci series and the Golden Section appear in nature. The former occurs repeatedly as a function of growth in plants and animals; the ratio occurs in the spirals of the chambered nautilus and the arrangement of seeds in the sunflower and pine cones. {Thompson, XIV}

Example 85. The spiral arrangement of seeds in a pine cone showing the numerical ratio of 8 clockwise spirals and 13 counterclockwise spirals, and a diagram of another cone showing the ratio 5:8. {Sinnott, 154-155}



Example 86. The Greek Parthenon, a building whose facade is enclosed by Golden Rectangles.



If the ratio does occur in Bartok's music, it is significant to symmetry, because the ratio is itself symmetric. Consider the line AC divided at a point B such that AB:BC= BC:AC. If an automorphism enlarges this line to DEF, where DE=BC and AB:BC=DE:EF, then AC=EF.

Figure 22. Symmetry in the Golden Section.

A_____B_____C

D_____E_____F

A_____E_____F

Both automorphic translation and reflection operations are possible demonstrating a second order symmetry. Since AE=EF, the translation is possible without the addition of EF. However, this operation is feasible only by abstracting the dimensions of AB/BC so that they are projected on BC/CA after an automorphism with the multiplier 1/.618.

If a composer wanted to relate two parts of differing size within a whole section or composition so that part A is to part B as part B is to the whole, the ratio of the parts would necessarily be the Golden Section.

Erno Lendvai comes closer to a valid application of the ratio in its projection upon the lengths of the sections of Bartok's music. But, in the *Music for String Instruments, Percussion and Celeste*, first movement, these turn out to be, at best, only rough approximations of the real sectional divisions for the following reasons:

1. The division of the sections are based upon the number of measures. The measure, however, is not of constant length in time due to changes in tempo and frequent change of meter.
2. There are 88 measures in the movement, not 89 as indicated by Lendvai.
3. The climax is in measure 56, not in measure 55.
4. The numbers given do not occur in an ordered sequence as they appear in the Fibonacci series.

A more precise method is needed if we are to determine if the sectional lengths of Bartok's composition correspond to the Golden Section. Bartok has given metronomic marks for all of the tempo changes in the movement. From these it is possible to determine the actual lengths and temporal proportions of sections in the movement.

Table II. Duration of sections in Bartok's *Music for String Instruments, Pereussion and Celeste*, first movement, calculated from the composer's tempo marks.

Tempo	Measures	Bartok's tempi	mean tempo	duration in eighths	duration in minutes	combined sections
1	1-37	116-112	114	300	2.62	3.57
2	38-51	120-126	123	116	0.95	
3	52-64	120-116	118	93	0.79	2.56
4	65-77	116-112	114	98	0.86	

5	78-88	108	108	98	0.91	
total duration					6.13	

The total duration of the movement following the mean tempos would be 6.13 minutes as shown in Table II. Thus, the Golden Section should fall temporally at 3.79 minutes [6.13 x .618 = 3.79]. Bar 52 marks the end of 3.57 minutes. The climax of the movement comes 31 eighths after bar 52 at a tempo of eighth=118, or .26 minutes; 3.57 + .26= 3.83 minutes, which is .04 minutes (2.4 seconds) later than the projected 3.79 Section. This comprises an error of less than 1% for the movement. If this 2.4 seconds is minimized by the only two *rallentandos* in the movement, both occurring after the climax, the error becomes more miniscule.

Until the climactic division of the movement, the vertical dimension of Figure 20 is divided according to ascending and descending fifths from the tonality A, reaching the most remote tonal region, Eb, at the top and bottom of the graph. The arrangement of ascending and descending fifths is reversed after the climax in the graph to correspond with the symmetry of registration. Nine distinct sections are discerned, given by Roman numerals. The criteria for the division of these sections are as follows:

1. Division of the highest order, bar 56: Location of the movement's single wave climax; the only *fff* in the movement, the loudest dynamic; sustaining climactic tone, a high Eb; completion of the tonal climax of the subject entry fifth sequence, A to Eb; end of the use of the subject in proper form until **S+Si** union; begin use of the subject inversion; termination of crescendo and expanding textural density; termination of percussion; begin tonal, textural, and dynamic descent.

2. Division of second order, bar 78: completion of tonal convergence of subject entries upon A, the tonality of the movement and entire work; completion of the tonal and dynamic cycle of the first movement; first entry of the subject proper since the climax; begin union of **S+Si**, the complete subject with its complete inversion simultaneously; begin celeste arpeggio, the sole use in the movement; end of *senza sordina*; tempo change; first pp since beginning.

3. Divisions of the third order:
a. Bar 34; begin *senza sordina*; first percussion entrance; *crescendo* begins; F and C# subject entries in the same measure, unusual; begin statements of subject fragments in lieu of entire statements of subject; begin Bb pedal.
b. Bar 21; end of fugue exposition; begin episode; Bass comes to rest on G.

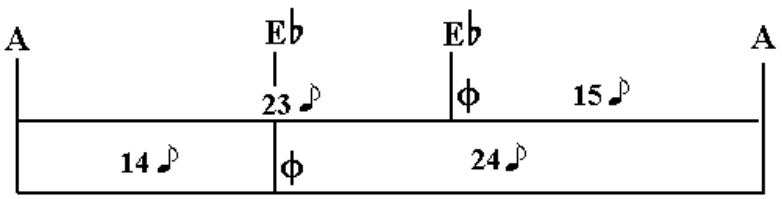
4. Divisions of the fourth order.
a. Bar 65: end of section with rall.; begin stretto section; new tempo; first silence since climax.
b. Bar 82: end of **S+Si** union and celeste arpeggio; begin stretto in contrary motion with entries always on A.
c. Bar 52: cymbal roll climax; first *ff*; new tempo; Bass climaxes on its highest tone in the movement, A; begin bass descent and upper strings ascent in final surge to the climax.

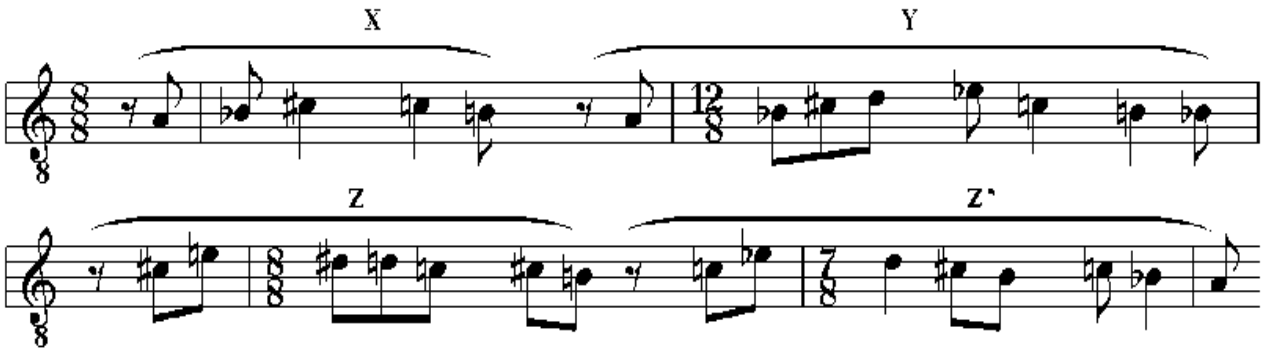
5. Other significant divisions.
a. Bar 45: entry on climactic Eb; begin bass climb; first *f*.
b. Bar 38: new tempo; Bb subject entry; first *mp*.
c. Bar 26: begin canon; end episode; first *p*.
d. Bar 68: canon of inversion; end of stretto; begin *con sordina*

In Figure 20 each *phi* Φ symbol represents a time division by the Golden Section. Three of the largest divisions, aside from the climax, are shown in bold dotted lines on the graph: the end of the exposition (bar 21, which is also the beginning of section II), the beginning of the *senza sordina* and the percussion entrance (bar 34), and the beginning of the **S+Si** union (bar 78). Bar 34 divides the large section from the beginning to the climax into a Golden Section in real time. Bar 21 does likewise for the section from the beginning to bar 34. Bar 78 divides the length from the climax to the end into a Golden Section. Many other smaller but similar divisions are shown on the graph.

The temporal proportions of the fugue subject itself correspond to those of the entire movement. The subject is 38 eighths from its beginning A to its ending A. The second and strongest Eb climaxes in 23 eighths (z, 8/8), and the first Eb comes after 14 or 24 eighths from the end; 38 x .618= 23.5.

Figure 23. Golden Section divisions in the temporal proportions of the fugue subject of the first movement



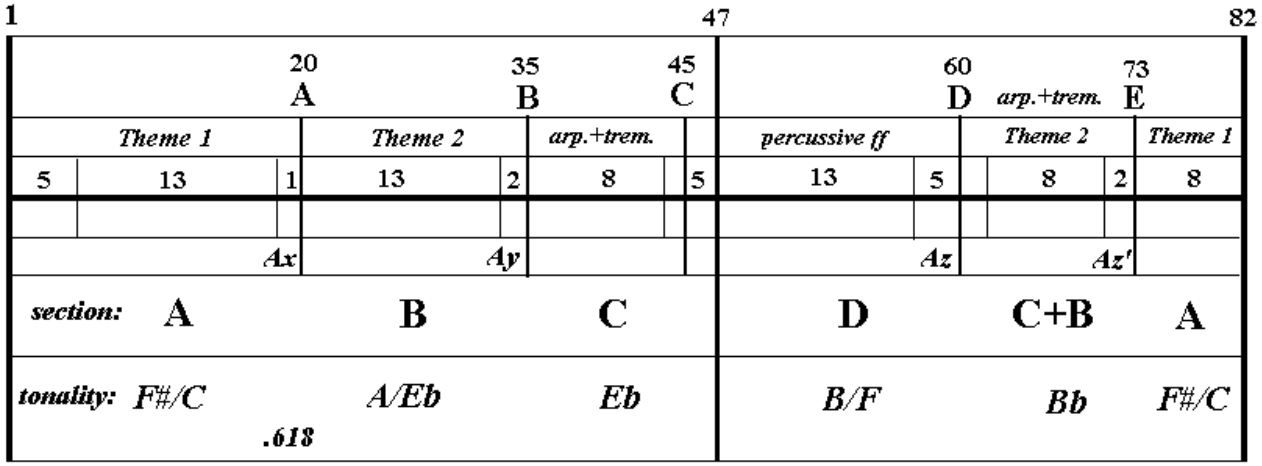


The third movement of the work exhibits a similar symmetrical design to the first movement, shown in Figure 24. It has also been called an *arch structure*: A B C D C B A ; it is temporally reflective about the D section. The tonality is C/F#, a bitonality, which is reflectively related to the tonal center of the fugue movement. There are a total of 82 measures in the third movement. It is divided into six distinct sections corresponding to the parts of the above form. All of these sections are separated by a statement of one of the four parts of the subject of the fugue movement. These occur in their proper order between the six large sections. Adjacent pairs of sections are coupled to produce three larger sections:

| A B | C D | C+B A |
A B A'

Figure 24 shows the formal divisions of the movement in the perspective of time. The complexity of tempo change in the movement makes accurate appraisal of the temporal proportions difficult. However, from Bartok's specified times given in the score at the end of the movement, it is found that the climax of the movement comes at .618 of the total length.

Figure 24. Analytical Diagram of the third movement of Bartok's *Music for String Instruments, Percussion and Celeste.* Note the Fibonacci numbers (center line) indicating the length of sections in measures.



1. Bartok gives the total length of the movement: 6' 35" or 6.58'
6.58 x .618 = 4.06' = 4' 4"

2. Bartok gives the length from the beginning to , letter C as 3' 52". The climax comes about 10" after letter C at his given tempo:

3' 52" + 10" = 4' 2"

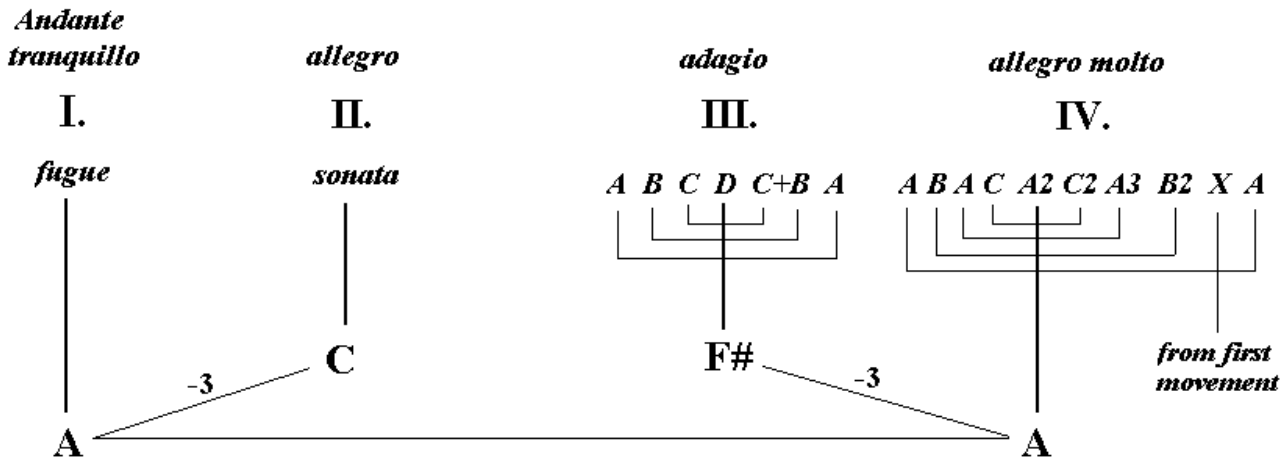
The error of 2" is negligible. Considering a section of the movement from letter B to letter E, according to Bartok's times, the climax lies at a point , again, .618 of the length from letter E.

1. According to Bartok's times: Letter B to C = 55" ; 55"+10" to climax= 65"

2. Letter D to E = 58"; climax to D= 57"-10"= 47"
47" + 58" = 105"; 105" x .618= 65"

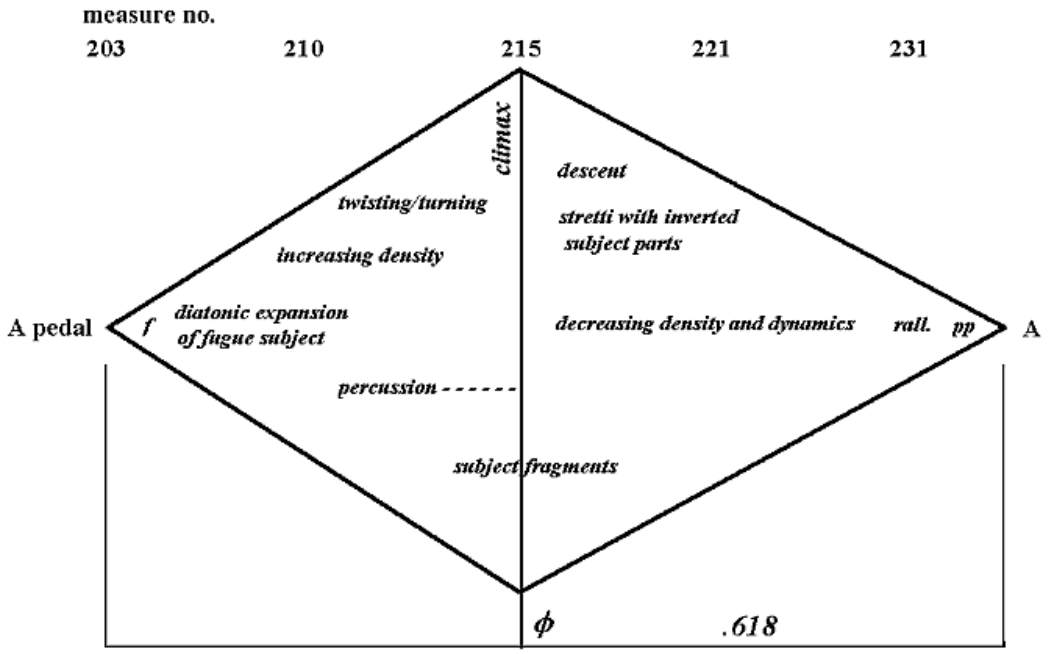
The grand symmetric order of the *Music for String Instruments, Percussion and Celeste* is manifest in its largest scale in the relationships between the four movements. All the movements are either ternary or arch forms and all are temporally symmetric:

Figure 25. Reflective relationships between the movements of Bartok's Music for Strings, Percussion and Celeste



Both the first and the last movements are based upon the tonality A. The last contains a lengthy imitative section labeled X in Figure 25. With strings dominating, this section is built upon the diatonic extension of the fugue subject, a tonal automorphic translation. This section is a restatement, in miniature, of the ideas of the fugue movement as a whole, with imitative treatment, subject inversion, single wave climax, tonal reflection on A, descending *stretti* following the climax, percussion terminating at climax, etc.

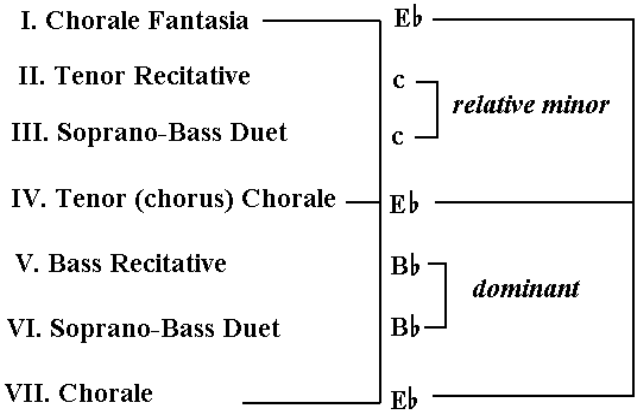
Figure 26. Miniature paraphrase (translation) of the first movement within the last movement of Bartok's *Music for Strings, Percussion and Celeste,* shown as section X on Figure 25. The Golden Section is again calculated by the tempo marks.



The interesting use of automorphic expansion of a subject to tie sister movements together is also found in the *Fourth String Quartet*.

In the *Music for String Instruments, Percussion and Celeste*, as shown in Figure 25, the Allegro is in C, a minor third above the tonic A; of the first and last movements. The Adagio is in F#, reflecting the tonal interval a minor third below the tonic A. This method of tonal centering also occurs in the *Fourth* and *Fifth String Quartets* and other works by Bartok, as well as in works by other composers. Brahm's first symphony in C minor has a second movement in E major and a third movement in Ab major, symmetrically centering on the C minor tonality of the first and last movements. Similar aspects of tonal centering may be found in many of the works of J.S. Bach. The seven movements of Cantata No. 140, *Wachet Auf*, are so related.

Figure 27. Tonal reflection in J.S. Bach's cantata No. 140, *Wachet Auf.*



The *Kreutztab, Christ lag in Todesbanden* cantatas, and the Credo of the B minor Mass are based upon similar schemes.

A final observation in the *Music for String Instruments, Percussion and Celeste* concerns Bartok's plan for the spatial layout of the orchestra. The reflective nature of this plan is most extensively exploited in the antiphonal effects of the second and last movements.

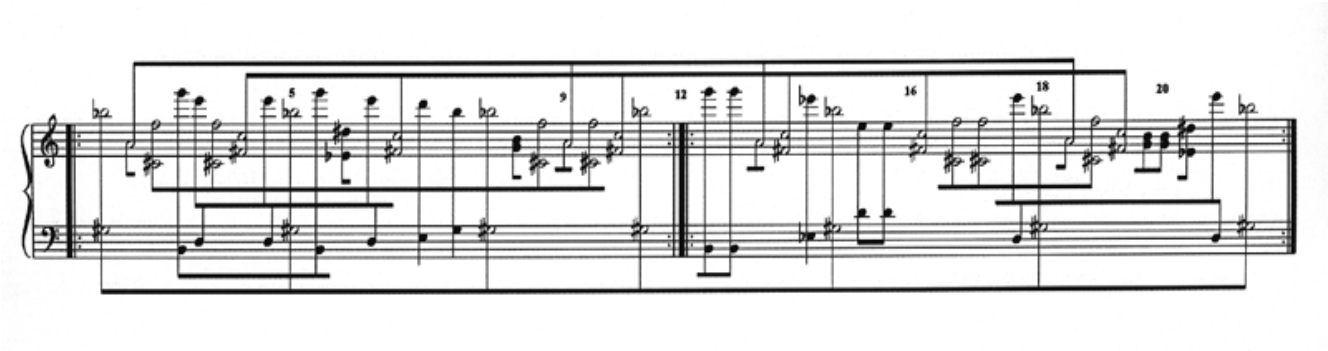
Figure 28. Arrangement of the orchestra for Bartok's composition demonstrating spatial reflection.

Double Bass I		Double Bass II	
Violincello I	Timpani	Bass Drum	Violincello II
Viola I	Side Drums	Cymbals	Viola II
Violin II	Celeste	Xylophone	Violin IV
Violin I	Pianoforte	Harp	Violin III

Webern's Piano Variations, Op. 27

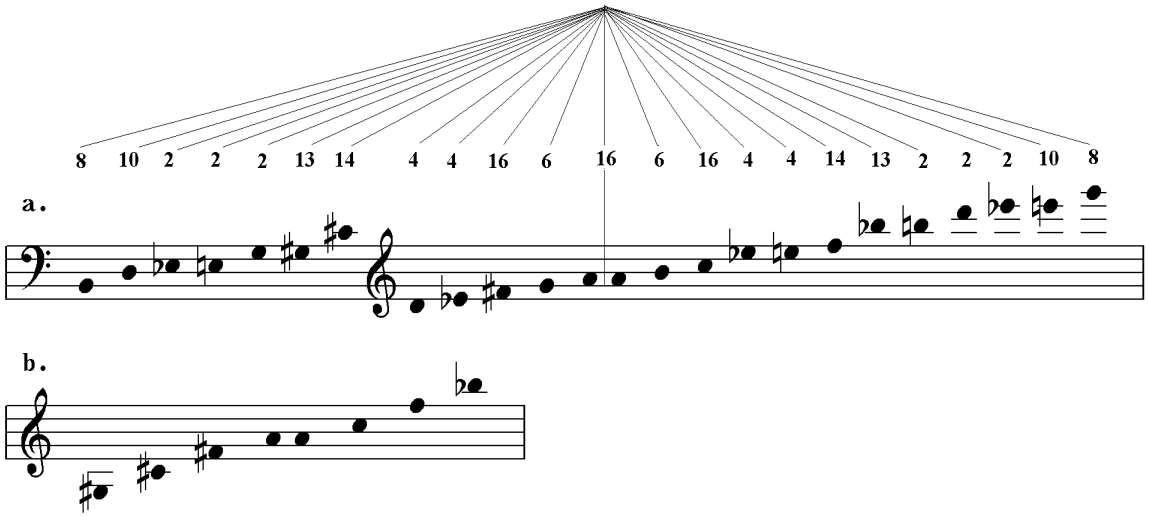
A precise kind of tonal symmetry occurring in the second movement of Webern's *Variations for Piano*, Op. 27, has been pointed out by Wilbur Ogdon. {Ogdon, 133} Example 87 shows all the pitches sounded in this movement in the temporal ordering of their binary groups. It is important to notice that each vertical group is centered symmetrically on the axis tone, without exception. Thus A4 serves, literally, as the tonal center of each binary tone group (or interval) and of the movement as a whole.

Example 87. The binary tone groups in Webern's *Variations for Piano*, second movement. Each interval has a vertical center on A4.



Example 88a gives all the pitches in the movement also, but this time they are given in order of pitch level, from the lowest to highest. Above each note is a number indicating the number of times each is to sound in the movement. The symmetrical centering on A4 is clearly seen here, corresponding, as well, with the number of times each tone is to be sounded about the axis tone.

Example 88. The pitch content of Webern's Op. 27, second movement showing the number of times each pitch is to be sounded: (a) their intervallic and numerical reflections; (b) shows the pitch classes sounded at only one pitch, corresponding to the most numerous tones/of (a), also reflectively centered on A4 From: Wilbur Ogdon,"A Webern Analysis," *Journal of Music Theory*, VI(1962),133.

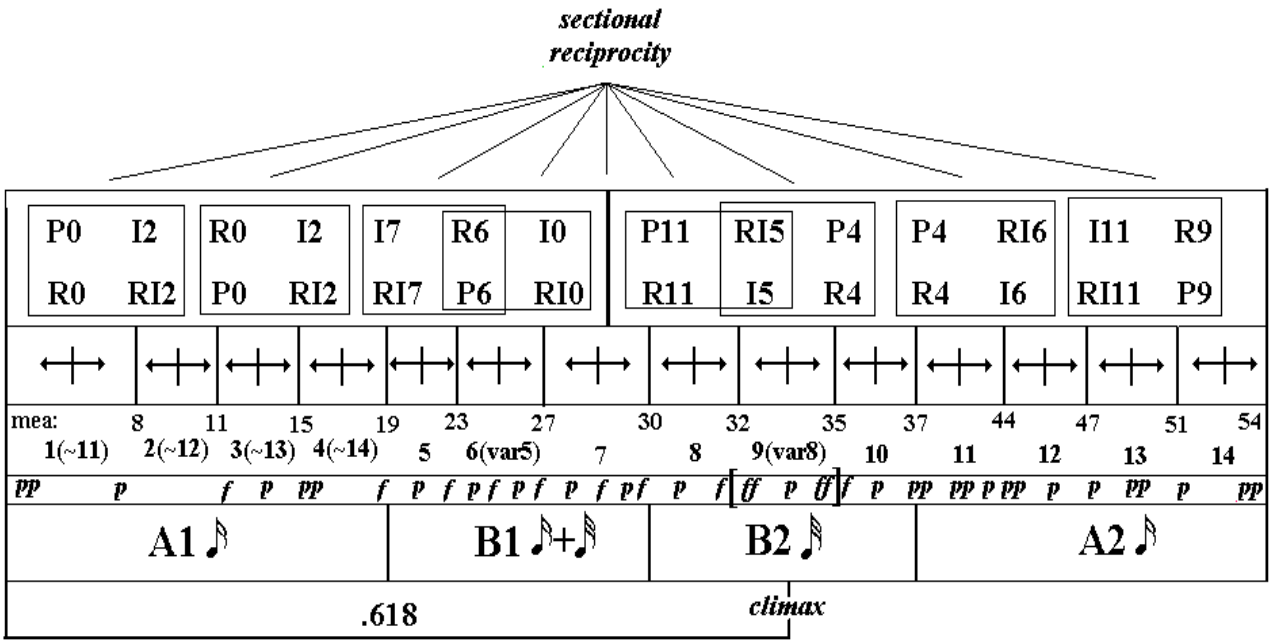


An analytic score of the first two movements can be found at the end of this chapter.

Example 88b shows all the pcs sounded at the same pitch in this movement. They, too, are reflectively disposed on A4. Three of these pitches, as shown in Example 88a, are to be sounded 16 times each, more often than any others, and they are centered on A4.

Leibowitz has described this movement as a canon in contrary motion {Leibowitz, 237}. Just as important is the fact that the larger phrase groups are divided and subdivided into binary groups, and even the smallest groups are composed of pitches reflectively disposed on A4. One is made distinctly aware of this pitch, which is being constantly repeated as well. Therefore, in a work which is often cited as atonal, there is a very defined tonal center! This may seem to contradict Webern's own writings on his treatmeat of tonality, but it is interesting to note his statement in *The Path to the New Music*: "One can also take the view that even with us there is still a tonic present -- I certainly think so..." {Webern, 39}

Figure 27. Analytic diagram of the first movement of Webern's Op. 27



A thorough analysis of the first movement also reveals an astonishing symmetrical design. Figure 27 is an analytic diagram of the first movement showing its many symmetrical features and levels. Consider, first, the forms of the row used in the order of time from the beginning to the end, shown near the top of the diagram,horizontally. Several orders of reflection occur. First is the symmetric placement of unordered row sets. If P+R, vertically, equal A and I+RI=B, then the row order may be represented:

A B A B B A B A B A A B B A

These are divided in the musical context in the following way:

A B A B | B A B || A B A| A B B A

A1 B1 B2 A2

These sections are established through the following criteria:

1. A1 and A2 are set off in the form by their correspondences, i.e., by their sectional content and by their tempos and soft dynamics.

2. B1 and B2 are alike in that they are the only music containing eighth-note motion, by having ternary structures in which their second parts are similarly varied first parts and by their frequent alternation of the more extreme dynamics, *f* and *p* .

3. B1 is the only music having entirely sixteenth and thirty-second note motion and continuous alternation of *ritard* and *a tempo*.

4. The central division is defined further by the bisecting center of the row order, the cessation of B1 and the beginning of B2.

The above form is reflective, first simply by the number of row presentations:

4 3 | 3 4

In figure 27, the upper line of row symbols generally represents forms played in the right hand and the lower line are those played by the left hand. The exception to this occurs in two sections, indicated below the row signs by italic numbers, immediately following the temporal axis, i.e., sections 8 and 9. The upper and lower symbols are here interchanged to clarify the symmetric design. The row forms may be grouped in fours as shown on figure 27 by the boxes; these are all of the possible consecutive groups containing all of the row forms: P, R, I and RI. These groups are also symmetrically arranged about the temporal axis.

All vertical combinations of rows are mutual retrogrades, which result in combinatoriality, hence complementary. Moreover, this is carefully exploited in the structure of the movement by a manifest temporal reflection within each vertical group. For instance, consider the portion of figure 27 indicating the sections of the music. The arrows represent temporal mirrors. If a mirror is placed in the middle of measure 4, it becomes apparent that the order and rhythm of events are in direct point for point correspondence about the mirror. There is a reflection of pitch, register, rhythm, simultaneities, etc. The temporal reflection is perfect and complete.

A similar mirror can be placed in the middle of measure 9, but the symmetry there is not perfect. The diagram shows that the entire movement is comprised of a series of these mirrors, many of which are perfect, all of which are complete. If mutual retrograde forms were not consistently combined as they are, such symmetry would not be possible, but it is noteworthy that the mere use of retrograde rows in combination does not necessitate such symmetry. This feature of "mirroring" is well known in Webern's music. It has been referred to as *Spiegelbild* by Webern analysts and has been cited in Webern's late works by Leibowitz and others. The complexity of this mirroring is not shown in its entirety in figure 27 in order to maintain clarity. As an example, if we consider the mirror in the middle of measure 9, the diagram shows its extent to be from measures 8-11. If we allow some variation in the exact sequence of mirroring and registration, the mirror has a greater span from measures 1-15. Beyond the boundaries of bar 8 and 11 , invertible counterpoint occurs, in retrograde of course.

Sectional correspondences are shown in figure 27 by the symbol ~ , meaning "similar." Section 1 is similar to section 11, 2 is similar to 12, 3 to 13, and 4 to 14. Therefore, the larger section from the beginning to bar 19 is similar to the last large section from bar 37 to the end, making an overall ABA design shown near the bottom of the diagram.

The large B is a binary unit whose division occurs just after bar 30, corresponding to the primary temporal axis of the movement. The first part of B , labelled B1 is comprised of a ternary group corresponding to sections 5,6, and 7, with 5 similar to 7, and 6 a variant of 5. B2 is also constructed in ternary with sections 8,9, and 10. As well as being reflective within themselves, these ternary groups are, of course, reflectively disposed about the primary temporal axis at bar 30, pointing out a symmetry within the binary B. Not only are the individual rows placed reflectively, but their musical realization is as well, both in the context of small scale and large scale divisions.

Do the proportions of the Golden Section occur here as they do in Bartok? The climax of the movement falls in the midst of section 9, in the middle of measure 33. The dynamic mark *piano* occurs here between two *fortissimos*. This *piano* does not make the section less climactic, but to the contrary, calls attention to it as climactic by the dramatic change in dynamics. Since the meter is constant, we can calculate for the Section by measure number, forgoing the problem of *ritards* for a moment. There are 54 measures in the movement with 3 sixteenths per measure= 162 sixteenths.

.618 x 162 = 100 eighths

The climax, falling in the middle of measure 33, comes after 32 measures x 3 sixteenths + 3 sixteenths = 99 sixteenths, comprising an error of less than 1%. There is an approximately equal amount of ritarded time before and after the climax (18 sixteenths before, I7 sixteenths after) which, therefore, does not change the proportion significantly. There is a remarkable correspondence of the climax with the Golden Section.

Along with the large ABA illustrated in figure 27, the principle units of tempo are given for each of the large sections. Tempo change is effected by shifting from sixteenth note motion to thirty-second note motion and vice versa). The first A and the last move solely by sixteenth note motion. Section B1 moves in both sixteenth and thirty-second note motion, and the climactic section, B2, moves solely in thirty-seconds. Since the B sections are the only music containing thirty-second note motion and the A sections are the only music containing solely sixteenths, the tempos are symmetric in the form.

In considering harmonic implications it is essential to note the constant vertical combination of mutual retrograde row forms, the prime with the retrograde and the inversion with the retrograde inversion. The combinations are always of equivalent transpositions for a good harmonic reason. It means that the intervallic-harmonic progression always remains constant with two possible harmonic orders, shown in example 89.

Example 89. The harmonic-intervallic content of the row combinations in the first movement of Webern's Op. 27, where PO= 0 1 9 11 8 10 4 5 6 2 3 7; The numbers indicate the number of semitones, and the minus sign designates an interval complement, or,an interval class with vector designation, or the *directional interval class.*

P/R: -5 2 5 -5 -3 6 || -6 3 5 -5 -2 5

I/RI: 5 -2 -5 5 3 -6 || 6 -3 -5 5 2 -5

The interval class is designated by numbers 1-6, and the interval is equivalent to these numbers, but the minus sign designates an interval complement, e.g., 5= perfect fourth, -5= perfect fifth. The intervals in example 89 always occur in this order in the row combinations of this movement in either the case of P with R or I with RI regardless of transposition. The only difference between the two combinations is that one is the retrograde of the other.

Within each combination the intervals are inversely symmetric about their temporal centers.Therefore, each combination results in a harmonic retrograde inversion in the music, temporally symmetric about an axis following the sixth tone of each combination. This corresponds preciseIy to where the mirrors previously cited occur in the music and conforms to the rhythmic setting.

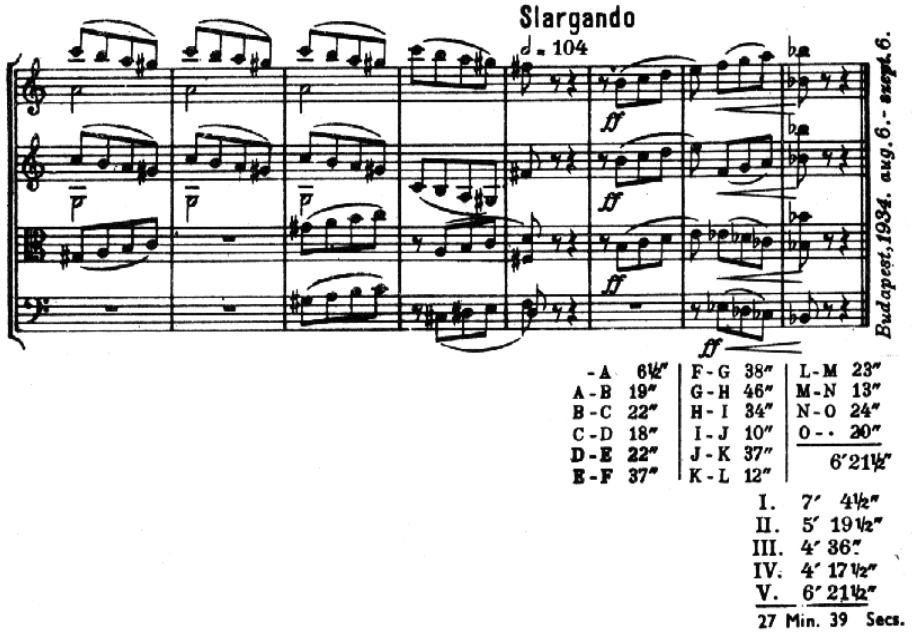
Finally, remembering that every subsequent combination will either be a repeat or its only alternative (P+R following R+I or R+I following P+R), either case will produce an exact harmonic cancrizans, or, at least, a retrograde of interval classes. Since the succession of row combinations is reflective about the primary axis, the entire movement is a harmonic cancrizans.

Some Historic Connections

Bartok's *Music for Strings, Percussion and Celeste* and Webern's *Variations for Piano* were composed in the same year, 1936, notable especially considering their intensive uses of symmetry and their remarkably coincident tone centers (A4). Cubist painters during the period 1910-l940 were intensively involved with symmetrical constructions, Golden Sections, and the like, but there is no apparent referral to such ideas in conversations or writings by Bartok on his own compositions. According to Gerald Abraham, the use of such structures may have been suggested to Bartok by Alfred Lorenz's book *Geheimnis der Form bei Richard Wagner* (1920) in discussions of *Bogenform* {Abraham, 185}.

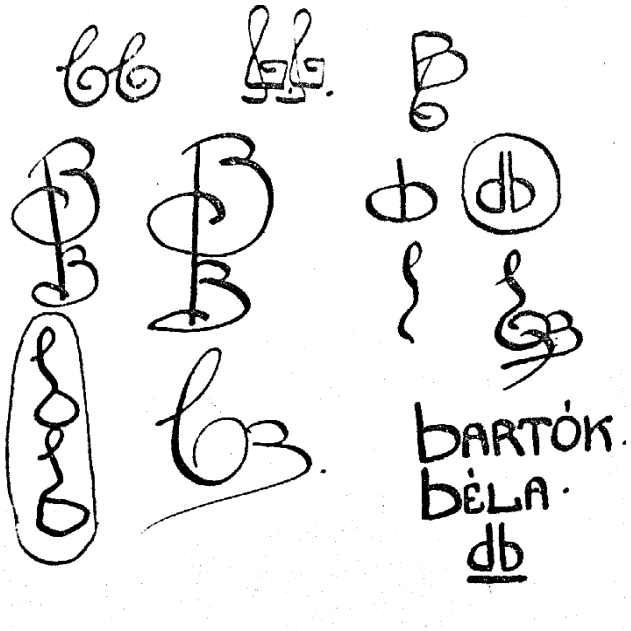
It is well known that both Bartok and Webern were attracted to a scientific knowledge of nature. Both loved plants and marveled at their designs. Bartok was a collector of insects and was very interested in their sense of timing. He even once calculated the number of vibrations per second of a hummingbird wing. Similarly, he was precise about temporal marks in his scores and regarded them as important for performances.

Example 90. Last score of the first movement of Bartok's *Fifth String Quartet* showing the composer's timings for every section of the movement by their letter numbers. The movement lasts a total of 7 minutes 4.5 seconds of which the recapitulation begins after .618 of that length at letter G, with an error of less than 1%. Copyright 1939 by Boosey and Hawkes.



That Bartok would have been unaware of symmetric properties of his music is unlikely, but to what degree symmetry played as an intellectual preconception can never be fully known. That symmetry was important to him on a conscious level is seen in his notes where he experiments with various symmetrical designs of his initials.

Example 91. A page from Bartok's notes showing various experiments with symmetric designs of his initials.



Webern was an amateur botanist and adored the designs of plants. He kept a sketchbook of drawings with plants and geometric designs. Much has been made of Webern's constructivism. Many have accused him of being cold and calculating. Yet how contrived and how conscious were these constructions in his music? It is apparent in reading Webern's letters that at least some of this was unconscious. In a letter to Hildegard Jone dated July 25, 1942, he wrote of the Cantata, Op. 31:

You ask about the shape: at the center are the the words "Because he fell silent on the cross, we must go after him, in all seriousness of bitterness, our breath follows him." What went before is now repeated backwards. Repeated. All shapes are similar and none are the same; thus, the chorus points to the secret law, to a holy riddle.... But the fact that it was just these words that constitute the center of the musical shape came about of its own accord-- indeed it could not have been otherwise. The end was just as difficult, "enmity-coldness": amidst the warmth and friendliness of your words. So here begins in the music (like a breath) "then we turn happily." But that too I only see it Now-- that too came about unintentionally. {Polnauer, 46-48}

Webern was struck by the formal similarities of architecture and music. That he was aware of analogous symmetric superstructures is indicated in a letter to Josef Humplik, dated March 5, 1933:

But, dear Pepo, I also saw the Parthenon Frieze! I stood there for an hour and a half. It's an indescribable miracle. The conception! It is an exact counterpart of out method of composition: always the same thing appearing in a thousand forms. Overwhelming. Comparable too with Bach's "Art of Fugue." {Polnauer, 46-48}

At the same time it is apparent that Webern was quite aware of many aspects of symmetry in his music. At the end of his lecture of 1932, "The Path to Twelve Note Composition," he analyzed his own symphony's second movement as a complex symmetrical structure, and then left his audience with the enigmatic Latin saying, which may be found amidst his sketches in working out a row of like properties for the Concerto, Op. 24:

S A T O R
A R E P O
T E N E T
O P E R A
R O T A S

This is a multi-symmetric magic square which may be translated: "The Sower Arepo Keeps the Works Circling."

Webern's *Variations for Piano, Op. 27*, first and second movements: analytic score. Arrows indicate the placement of mirrors. Pitches with dark numbers trace Prime or Inversion forms of the row; light numbers trace Retrograde or Retrograde-inversion forms. Other symbols explained in text and correlated with Figure 27.

View Analytical Score:
Movement I. page 1, page 2, page 3
Movement II.

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