Álgebra de Chaveamento

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Álgebra de Chaveamento

- Ao final do estudo deste tópico você saberá:
  - Os conceitos da Algebra Booleana e da Álgebra de Chaveamento
  - Os axiomas e Teoremas da Álgebra de Chaveamento
  - Demonstração de Teoremas por Indução Finita
  - Os Teoremas de DeMorgan
  - As portas lógicas Inversora, AND e OR
  - O Diagrama Lógico
  - O Princípio da Dualidade
  - A Tabela Verdade
  - Os conceitos de Literal, Termo Produto, Soma de Produtos, Termo Soma, Produto de Somas, Termo Normal, Mintermo e Maxtermo
  - A Soma e o Produto Canônico

Teoremas de 1 variável

(T1)	$X + 0 = X$	(T1′)	$X \cdot 1 = X$	(Identities)
(T2)	$X + 1 = 1$	(T2′)	$X \cdot 0 = 0$	(Null elements)
(T3)	$X + X = X$	(T3′)	$X \cdot X = X$	(Idempotency)
(T4)	$(X')' = X$			(Involution)
(T5)	$X + X' = 1$	(T5′)	$X \cdot X' = 0$	(Complements)

Fonte das figuras: Wakerly - Digital Design

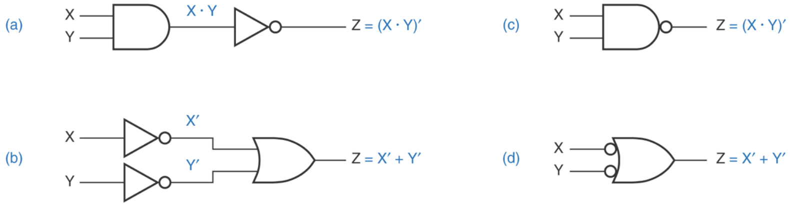
Teoremas de 2 ou 3 variáveis

(T6)	$X + Y = Y + X$	(T6′)	$X \cdot Y = Y \cdot X$	(Commutativity)
(T7)	$(X + Y) + Z = X + (Y + Z)$	(T7′)	$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$	(Associativity)
(T8)	$X \cdot Y + X \cdot Z = X \cdot (Y + Z)$	(T8′)	$(X + Y) \cdot (X + Z) = X + Y \cdot Z$	(Distributivity)
(T9)	$X + X \cdot Y = X$	(T9′)	$X \cdot (X + Y) = X$	(Covering)
(T10)	$X \cdot Y + X \cdot Y' = X$	(T10′)	$(X + Y) \cdot (X + Y') = X$	(Combining)
(T11)	$X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$			(Consensus)
(T11′)	$(X + Y) \cdot (X' + Z) \cdot (Y + Z) = (X + Y) \cdot (X' + Z)$			

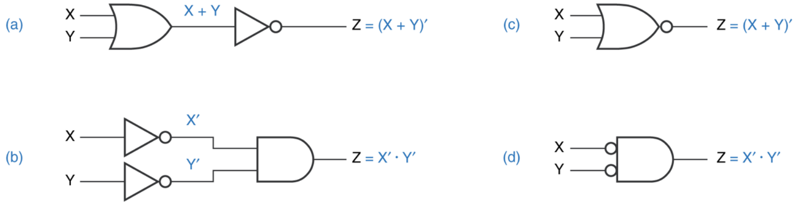
Teoremas de n variáveis

(T12)	$X + X + \cdots + X = X$	(Generalized idempotency)
(T12′)	$X \cdot X \cdot \cdots \cdot X = X$	
(T13)	$(X_1 \cdot X_2 \cdot \cdots \cdot X_n)' = X_1' + X_2' + \cdots + X_n'$	(DeMorgan's theorems)
(T13′)	$(X_1 + X_2 + \cdots + X_n)' = X_1' \cdot X_2' \cdot \cdots \cdot X_n'$	
(T14)	$[F(X_1, X_2, \ldots, X_n, +, \cdot)]' = F(X_1', X_2', \ldots, X_n', \cdot, +)$	(Generalized DeMorgan's theorem)
(T15)	$F(X_1, X_2, \ldots, X_n) = X_1 \cdot F(1, X_2, \ldots, X_n) + X_1' \cdot F(0, X_2, \ldots, X_n)$	(Shannon's expansion theorems)
(T15′)	$F(X_1, X_2, \ldots, X_n) = [X_1 + F(0, X_2, \ldots, X_n)] \cdot [X_1' + F(1, X_2, \ldots, X_n)]$	

DeMorgan: Teorema T13



DeMorgan: Teorema T13'



Circuito Lógico "Tipo 1"

(a)

X	Y	Z
LOW	LOW	LOW
LOW	HIGH	LOW
HIGH	LOW	LOW
HIGH	HIGH	HIGH

(b)

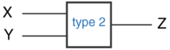
X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

(c)

X	Y	Z
1	1	1
1	0	1
0	1	1
0	0	0

Circuito Lógico "Tipo 2"

(a)



X	Y	Z
LOW	LOW	LOW
LOW	HIGH	HIGH
HIGH	LOW	HIGH
HIGH	HIGH	HIGH

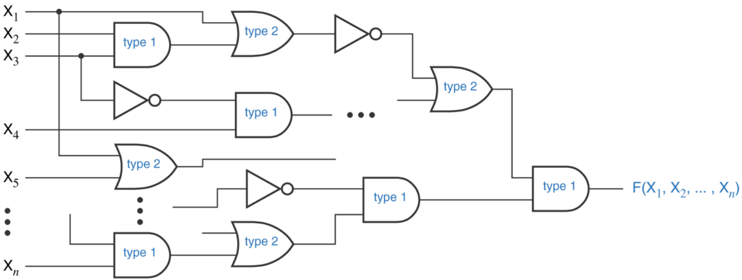
(b)

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

(c)

X	Y	Z
1	1	1
1	0	0
0	1	0
0	0	0

Circuito de Lógica Positiva



Circuito de Lógica Negativa

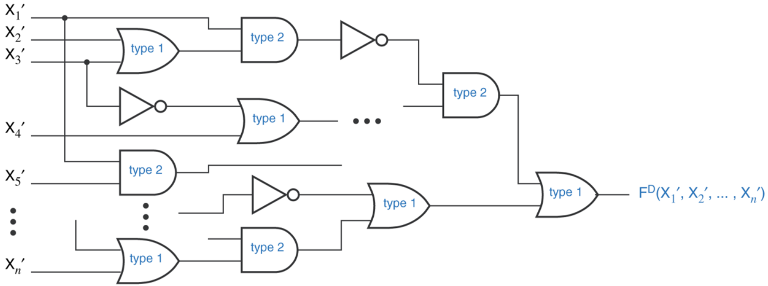


Tabela Verdade

Row	X	Y	Z	F	*Minterm*	*Maxterm*
0	0	0	0	F(0,0,0)	$X' \cdot Y' \cdot Z'$	$X + Y + Z$
1	0	0	1	F(0,0,1)	$X' \cdot Y' \cdot Z$	$X + Y + Z'$
2	0	1	0	F(0,1,0)	$X' \cdot Y \cdot Z'$	$X + Y' + Z$
3	0	1	1	F(0,1,1)	$X' \cdot Y \cdot Z$	$X + Y' + Z'$
4	1	0	0	F(1,0,0)	$X \cdot Y' \cdot Z'$	$X' + Y + Z$
5	1	0	1	F(1,0,1)	$X \cdot Y' \cdot Z$	$X' + Y + Z'$
6	1	1	0	F(1,1,0)	$X \cdot Y \cdot Z'$	$X' + Y' + Z$
7	1	1	1	F(1,1,1)	$X \cdot Y \cdot Z$	$X' + Y' + Z'$