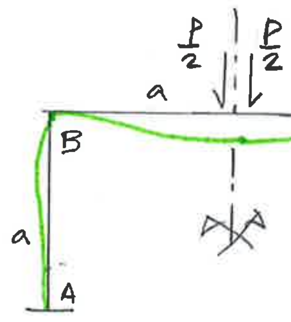
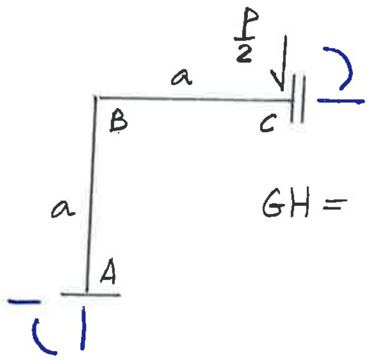


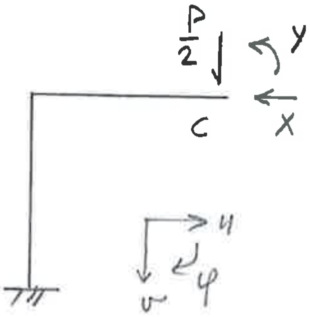



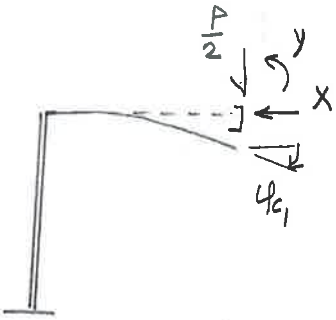

GH = 5 - 3 = 2

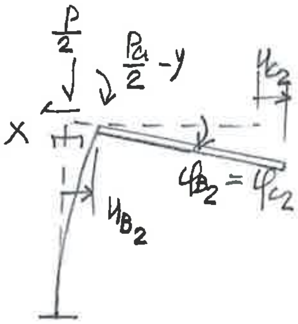
## EIF

$u_c = 0$

$\varphi_c = 0$




$$u_{c_1} = 0$$

$$\varphi_{c_1} = \frac{(P/2)a^2}{2EI} - \frac{Ya}{EI}$$

$$u_{c_2} = u_{B_2} = \frac{\left(\frac{Pa}{2} - Y\right)a^2}{2EI} - \frac{Xa^3}{3EI} = -\frac{Xa^3}{3EI} - \frac{Ya^2}{2EI} + \frac{Pa^3}{4EI}$$

$$\varphi_{c_2} = \varphi_{B_2} = \frac{\left(\frac{Pa}{2} - Y\right)a}{EI} - \frac{Xa^2}{2EI} = -\frac{Xa^2}{2EI} - \frac{Ya}{EI} + \frac{Pa^2}{2EI}$$

Eqs. de compatibili//

$$u_c = 0 \Rightarrow \frac{Xa}{3} + \frac{Y}{2} = \frac{Pa}{4}$$

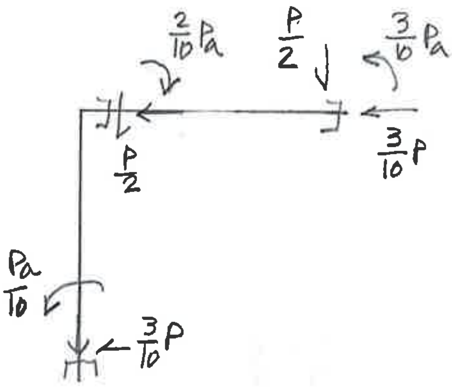
$$\varphi_c = 0 \Rightarrow \frac{Pa}{4} - Y - \frac{Xa}{2} - Y + \frac{Pa}{2} = 0 \Rightarrow \frac{Xa}{2} + 2Y = \frac{3Pa}{4}$$

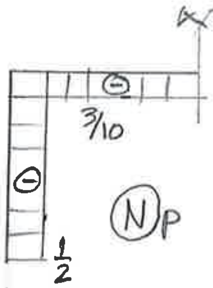
$$\begin{bmatrix} 1/3 & 1/2 \\ 1/2 & 2 \end{bmatrix} \begin{bmatrix} Xa \\ Y \end{bmatrix} = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix} Pa$$

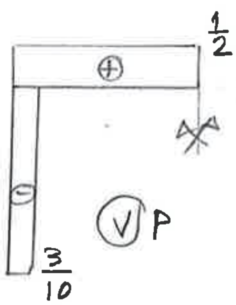
$$Xa = \frac{\begin{vmatrix} 1/4 & 1/2 \\ 3/4 & 2 \end{vmatrix} Pa}{\begin{vmatrix} 1/3 & 1/2 \\ 1/2 & 2 \end{vmatrix}} = \frac{\frac{1}{2} - \frac{3}{8}}{\frac{2}{3} - \frac{1}{4}} Pa = \frac{3}{10} Pa \Rightarrow X = \frac{3}{10} P$$

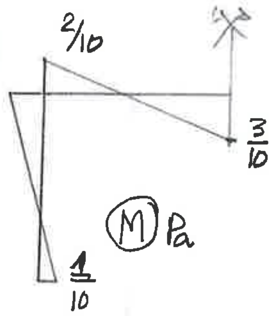
$$\frac{3}{10} - \frac{1}{2} = \frac{3-5}{10} = \frac{2}{10}$$

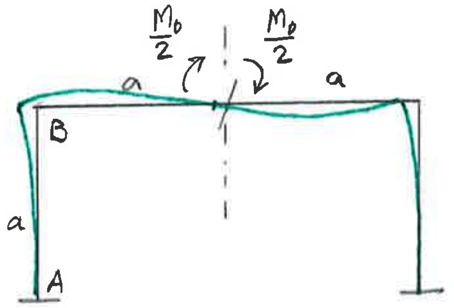
$$Y = \frac{\begin{vmatrix} 1/3 & 1/4 \\ 1/2 & 3/4 \end{vmatrix} Pa}{5/12} = \frac{\frac{1}{4} - \frac{1}{8}}{5/12} Pa = \frac{3}{10} Pa \Rightarrow Y = \frac{3}{10} Pa$$

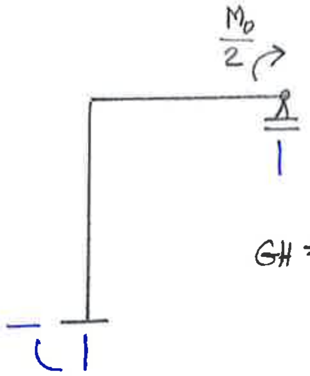


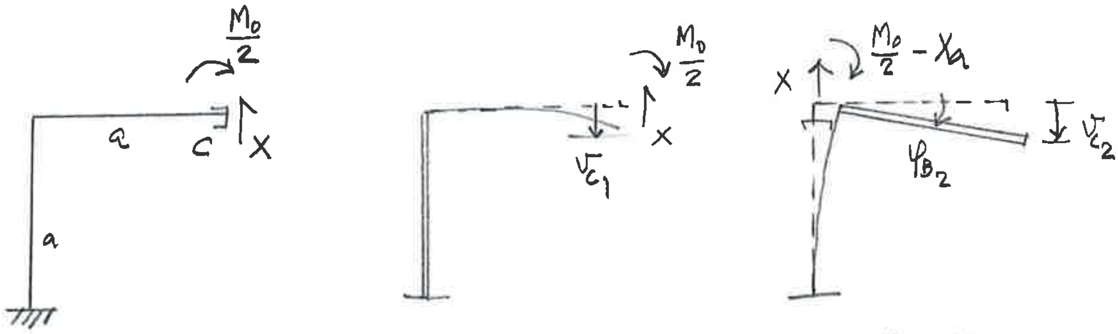



$GH = 4 - 3 = 1$

EIF

$v_C = 0$




$$v_{C_1} = \frac{\left(\frac{M_0}{2}\right) \cdot a^2}{2EI} - \frac{Xa^3}{3EI} = \frac{M_0 a^2}{4EI} - \frac{Xa^3}{3EI}$$

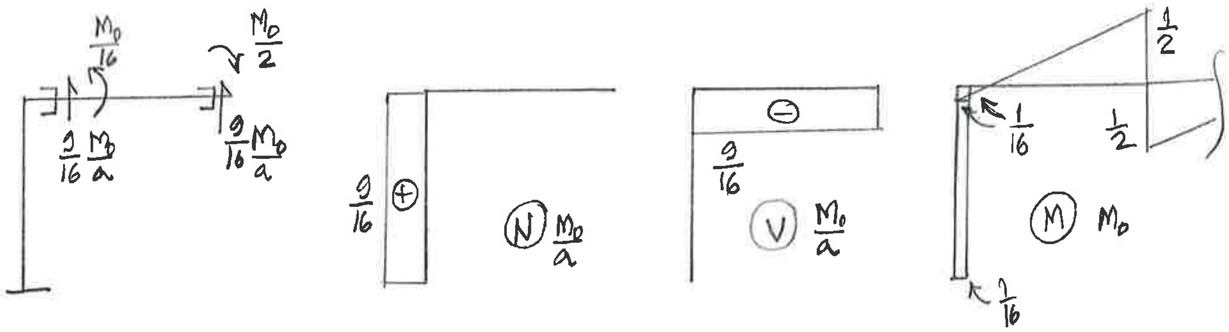
$$\varphi_{B_2} = \frac{\left(\frac{M_0}{2} - Xa\right) a}{EI} = \frac{M_0 a}{2EI} - \frac{Xa^2}{EI}$$

$$v_{C_2} = \varphi_{B_2} \cdot a = \frac{M_0 a^2}{2EI} - \frac{Xa^3}{EI}$$

Eq. de compatibilid.

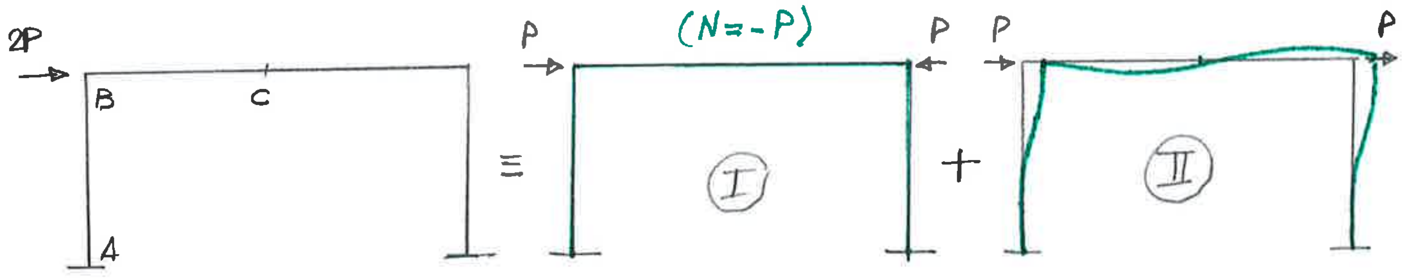
$$v_C = 0 \Rightarrow \left(\frac{1}{3} + 1\right) \frac{Xa^3}{EI} = \left(\frac{1}{4} + \frac{1}{2}\right) \frac{M_0 a^2}{EI} \Rightarrow \frac{4}{3} Xa = \frac{3}{4} M_0$$

$$X = \frac{9}{16} \frac{M_0}{a}$$



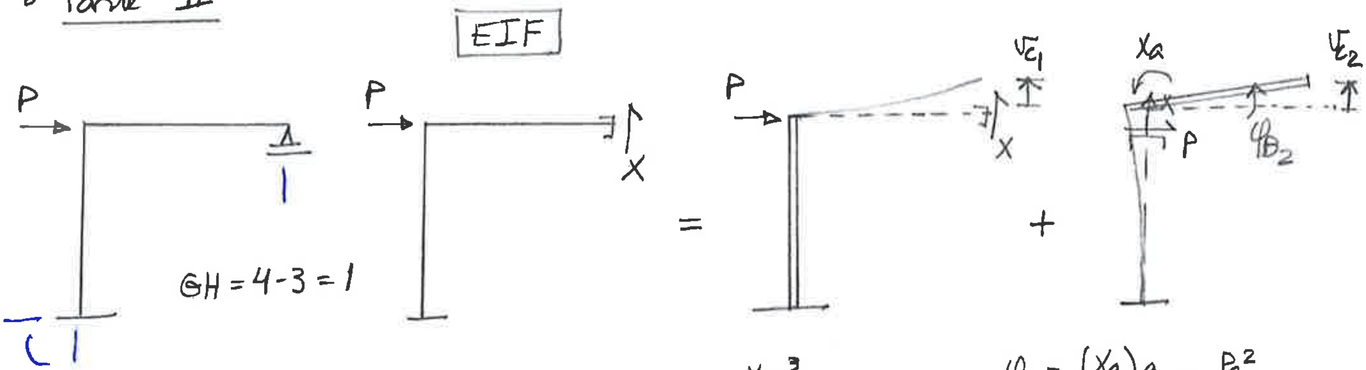

Deslocamento do ponto B

$$u_B = \frac{\left(\frac{M_0}{2} - Xa\right) a^2}{2EI} = \frac{-\frac{M_0}{16} a^2}{2EI} = -\frac{M_0 a^2}{32EI}$$




a resolução da parte simétrica é imediata e fornece $N^{I}_{BC} = -P$

• Parte II



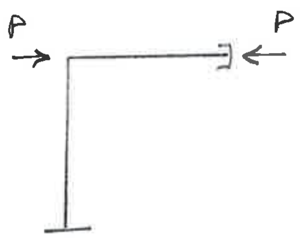

$$v_{C_1} = \frac{Xa^3}{3EI} \qquad \varphi_{B_2} = \frac{(Xa)a}{EI} - \frac{Pa^2}{2EI}$$

Equação de compatibili//  $\frac{4}{3}X = \frac{P}{2}$

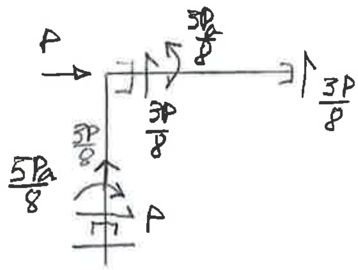
$$v_{C_2} = \varphi_{B_2} \cdot a = \frac{Xa^3}{EI} - \frac{Pa^3}{2EI}$$

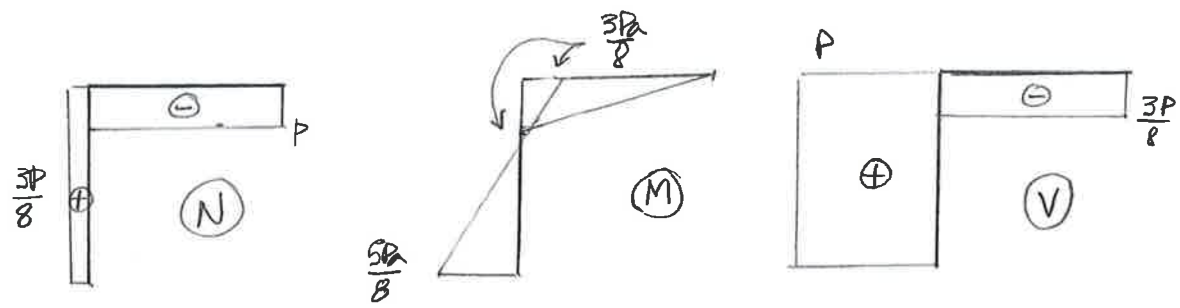
$$v_C = 0 \quad \Rightarrow \quad \left(\frac{1}{3} + 1\right)\frac{Xa^3}{EI} = \frac{Pa^3}{2EI} \quad \Rightarrow \quad X = \frac{3P}{8}$$

Parte I

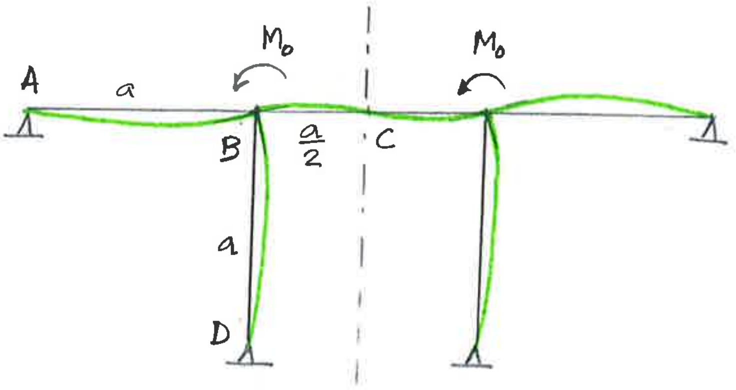


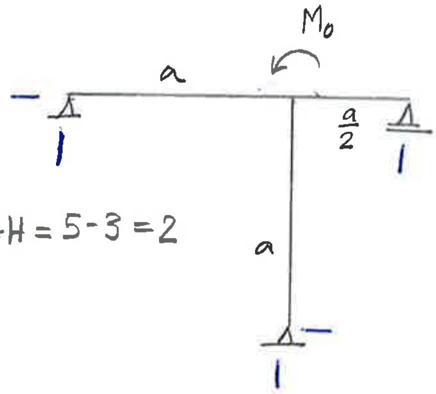

Parte II



[Diogo] 2015 mar 31

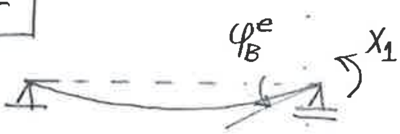


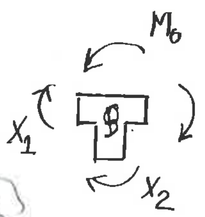



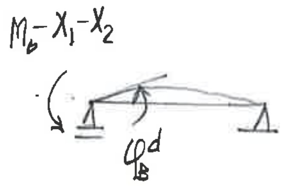

$GH = 5 - 3 = 2$

Como os nós B e C são indeslocáveis em decorrência dos efeitos das deformações longitudinais por M e N serem desprezíveis, podemos adotar uma EIF formada por vigas bi-apoiadas.

$\boxed{\text{EIF}}$



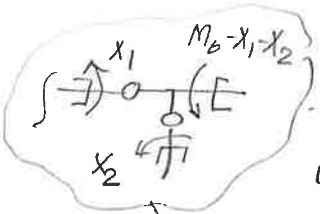



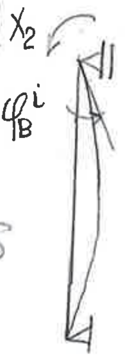

$$\varphi_B^e = \frac{X_1 a}{3EI}$$

$$\varphi_B^i = \frac{X_2 a}{3EI}$$

$$\varphi_B^d = \frac{(M_0 - X_1 - X_2)\frac{a}{2}}{3EI}$$



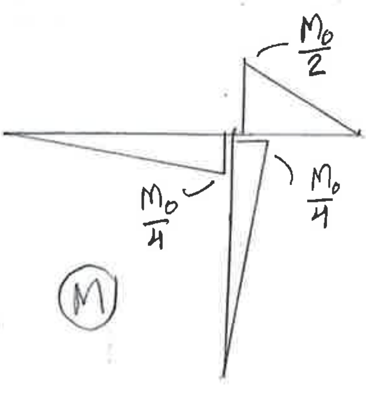

2 articulações

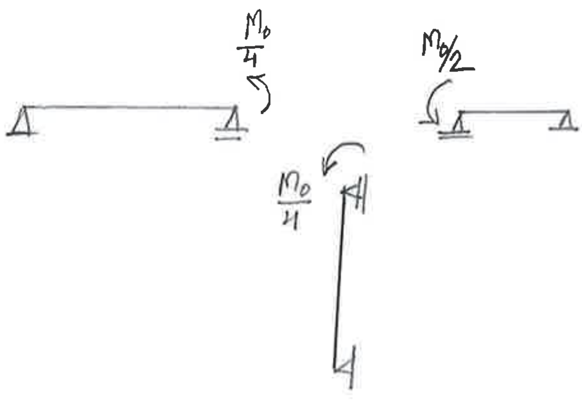


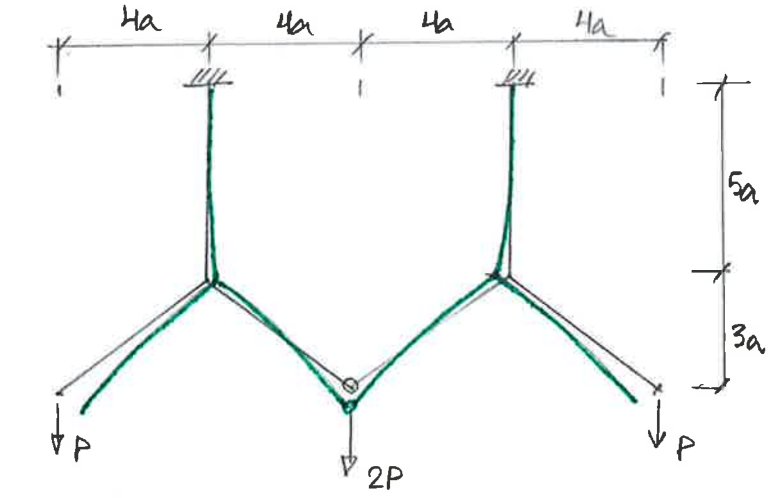

Eqs. de compatibilidade

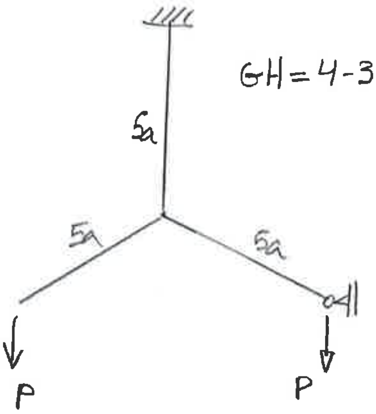
$$\underbrace{\varphi_B^e = \varphi_B^i = \varphi_B^d}_{\text{sentido anti-horário}}$$

$$\begin{cases} \varphi_B^e = \varphi_B^i \Rightarrow \dfrac{X_1 a}{3EI} = \dfrac{X_2 a}{3EI} \Rightarrow X_1 = X_2 \\ \varphi_B^e = \varphi_B^d \Rightarrow \dfrac{X_1 a}{\cancel{3EI}} = \dfrac{(M_0 - X_1 - X_2)\frac{a}{2}}{\cancel{3EI}} \Rightarrow 2X_1 = \dfrac{M_0}{2} \Rightarrow \underline{X_1 = \dfrac{M_0}{4} = X_2} \end{cases}$$



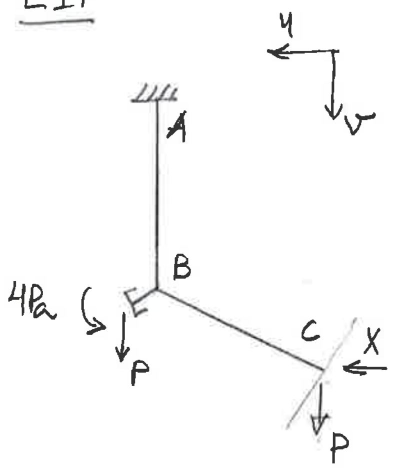



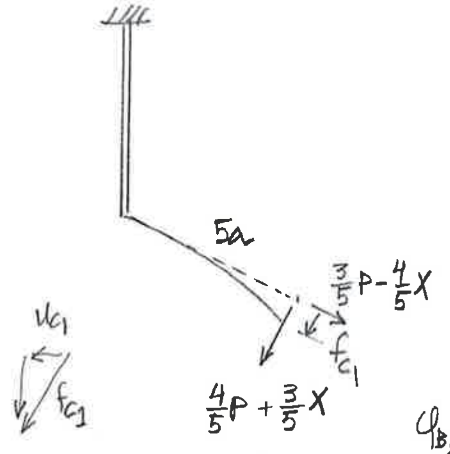


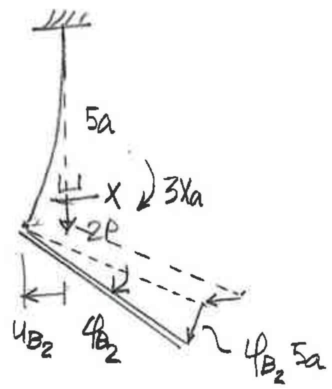




$GH = 4 - 3 = 1$

## EIF




$$\varphi_{B_2} = \frac{X(5a)^2}{2EI} + \frac{(3Xa)(5a)}{EI} = \frac{55}{2}\frac{Xa^2}{EI}$$

$$u_{B_2} = \frac{X(5a)^3}{3EI} + \frac{(3Xa)(5a)^2}{2EI} = \frac{475}{6}\frac{Xa^3}{EI}$$

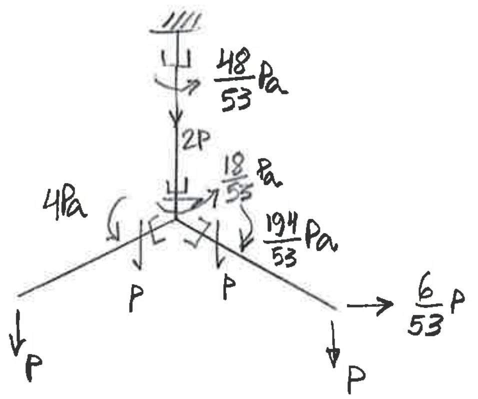
475
90
565

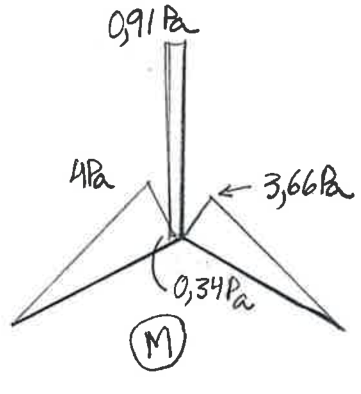
$$u_{c_1} = \frac{3}{5} f_{c_1} = \frac{\cancel{3}}{5}\frac{\left(\frac{4P}{5} + \frac{3X}{5}\right)(5a)^3}{\cancel{3}EI} = (20P + 15X)\frac{a^3}{EI}$$

$$u_{c_2} = u_{B_2} + \frac{3}{\cancel{5}}(\varphi_{B_2} \times \cancel{5}a) = \frac{475}{6}\frac{Xa^3}{EI} + 3 \times \frac{55}{2}\frac{Xa^3}{EI} = \frac{485}{3}\frac{Xa^3}{EI}$$

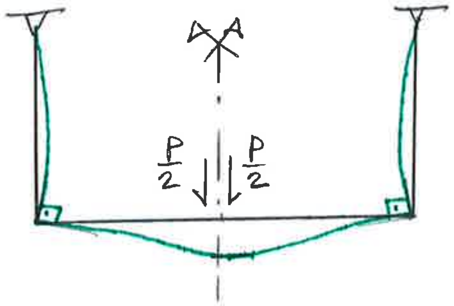
Equação de compatibili//

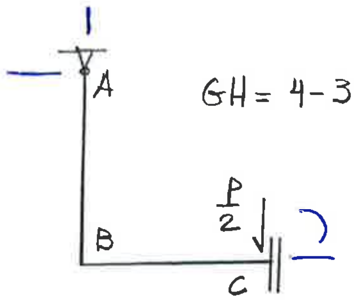
$$u_c = 0 \Rightarrow \left(15 + \frac{483}{3}\right)\frac{Xa^3}{EI} = -\frac{20Pa^3}{EI} \Rightarrow X = -\frac{6}{53}P$$



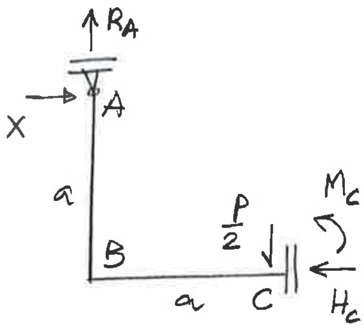

—//—




$GH = 4 - 3 = 1$

## EIF

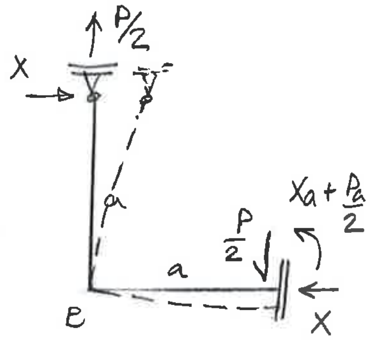



$\rightarrow \{ H_C = X$

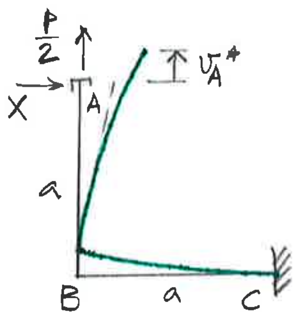
$\uparrow \{ R_A = \frac{P}{2}$

$\circlearrowleft A \{ -M_C + H_C \cdot a + \frac{Pa}{2} = 0$

$M_C = Xa + \frac{Pa}{2}$

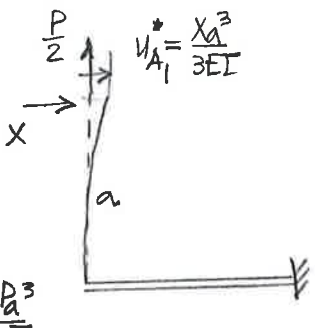



Estrutura auxiliar — Estrutura isostática igual à EIF exceto pela vinculação. Ela apresenta os mesmos esforços solicitantes mas os desloca/os diferem por um mov. de corpo rígido (translação e/ou rotação).

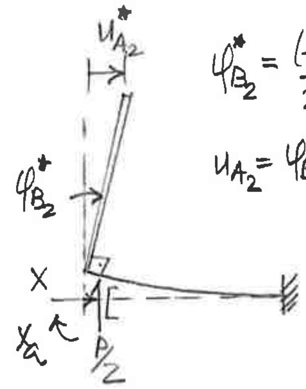



Os deslocamentos na estrutura ao lado diferem dos deslocamentos na EIF da transl. vertical $v(x) = v_A^*$

Portanto, temos $u_A = u_A^*$




$u_{A_1}^* = \frac{Xa^3}{3EI}$




$\varphi_{B_2}^* = \frac{(\frac{P}{2})a^2}{2EI} + \frac{(Xa)a}{EI}$

$u_{A_2} = \varphi_{B_2} \cdot a = \frac{Pa^3}{4EI} + \frac{Xa^3}{EI}$

Eq. de compatibili//

$u_A = 0 \Rightarrow \left(\frac{1}{3} + 1\right)\frac{Xa^3}{EI} = -\frac{Pa^3}{4EI}$

$X = -\frac{3}{16}P$

Voltando à EIF :