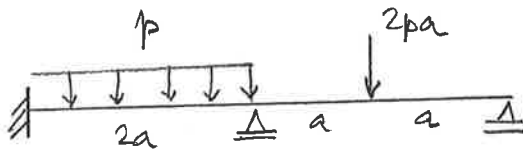


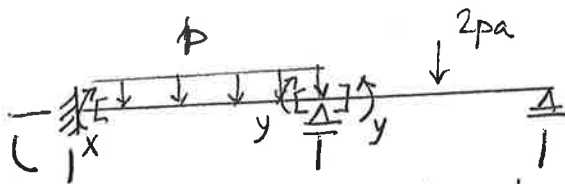
Ex. 1

Determine os diagramas de M e V . Considere $EI = \text{const.}$ e despreze as deformações longitudinais



Resolução

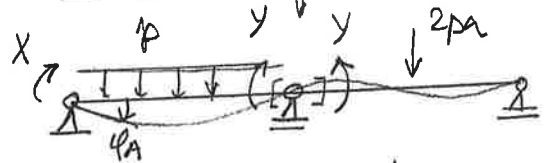
$GH = 5 - 3 = 2$



momentos nos ST^2 da hipercat.

eliminação de um vínculo interno

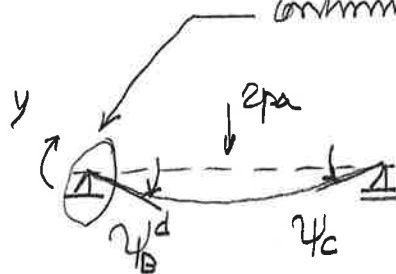
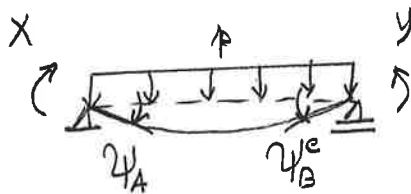
EIF



X e $Y \rightarrow$ momentos hiperestáticos (momentos associados aos vínculos eliminados).

compatível: $\varphi_A = 0$ $\varphi_B^{\text{esq}} = \varphi_B^{\text{dir}}$

A inestática fundamental comporta-se como uma sequência de apoios físicos em virtude da continuidade da viga!



compatível.

$$\psi_A = 0$$

$$-\psi_B^e = \psi_B^d$$

(φ_B^e) (φ_B^d)

$$\psi_A = \frac{X(2a)}{3EI} + \frac{p(2a)^3}{24EI} + \frac{Y(2a)}{6EI} = \frac{a}{EI} \left(\frac{2X}{3} + \frac{Y}{3} + \frac{pa^2}{3} \right)$$

$$\psi_B^e = \frac{X(2a)}{6EI} + \frac{p(2a)^3}{24EI} + \frac{Y(2a)}{3EI} = \frac{a}{EI} \left(\frac{X}{3} + \frac{2Y}{3} + \frac{pa^2}{3} \right)$$

$$\psi_B^d = \frac{Y(2a)}{3EI} + \frac{2pa(2a)^2}{16EI} = \frac{a}{EI} \left(\frac{2Y}{3} + \frac{pa^2}{2} \right)$$

$\psi > 0$ barriga p/barriga

Eq. de compatibilidade

$$\psi_A = 0 \Rightarrow 2X + Y = -pa^2$$

$$\psi_B^e = -\psi_B^d \Rightarrow X + 4Y = -\frac{5}{2}pa^2$$

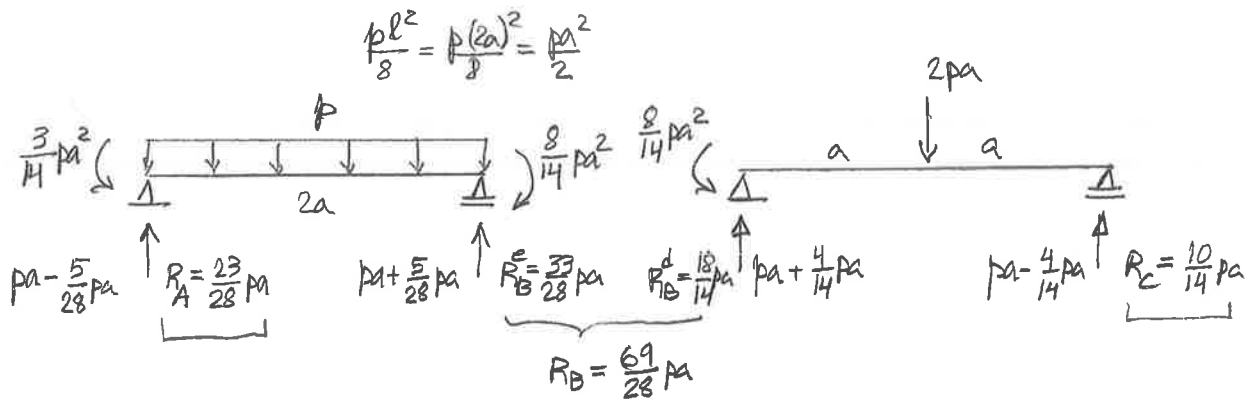
$$\Rightarrow \begin{aligned} -2X - Y &= pa^2 \\ 2X + 8Y &= -5pa^2 \\ \hline 7Y &= -4pa^2 \\ Y &= -\frac{4}{7}pa^2 \\ X &= \left(-\frac{5}{2} + \frac{16}{7}\right)pa^2 = -\frac{3}{14}pa^2 \end{aligned}$$

ou empregando determinantes,

$$\begin{cases} 2x + y = -pa^2 \\ x + 4y = -\frac{5}{2}pa^2 \end{cases}$$

$$x = \frac{\begin{vmatrix} -1 & 1 \\ -5/2 & 4 \end{vmatrix} pa^2}{\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}} = \frac{-4 + \frac{5}{2}pa^2}{8-1} = \frac{-\frac{3}{2}pa^2}{7} = -\frac{3}{14}pa^2 \quad (-0,2143pa^2)$$

$$y = \frac{\begin{vmatrix} 2 & -1 \\ 1 & -5/2 \end{vmatrix} pa^2}{7} = \frac{-5+1}{7}pa^2 = -\frac{4}{7}pa^2 \quad (-0,5714pa^2)$$



$$V(x) = \frac{23}{28}pa - px \Rightarrow V=0 \Rightarrow x = \frac{23}{28}a$$

$$M(x) = -\frac{3}{14}pa^2 + \frac{23}{28}pax - \frac{px^2}{2}$$

$$M_{\min} = M(x = \frac{23}{28}a) = -\frac{3}{14}pa^2 + \frac{23}{28}pa \times \frac{23}{28}a - \frac{p}{2}(\frac{23}{28}a)^2 = \frac{193}{1568}pa^2 = 0,123pa^2$$

Diagramas

