2014 mar 13

05PRQ3 Ex. 1

$\circlearrowright \square \quad (M_B^{dir}) = 0$

$\{R_D \times 3a - Pa = 0 \Rightarrow R_D = \frac{P}{3}$

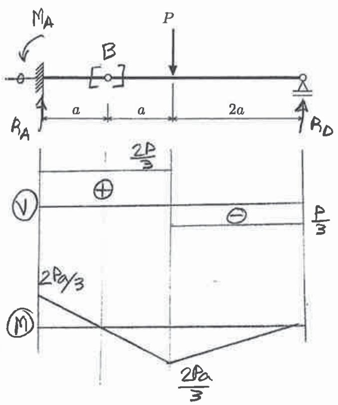
$\uparrow \{ R_A = \frac{2P}{3}$

$\circlearrowright \square \quad (M_B^{esq}) = 0$

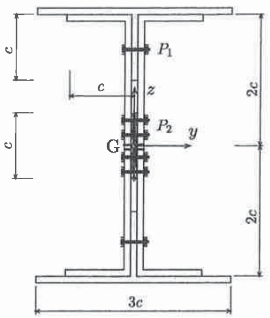
$\{-M_A + \frac{2P}{3}a = 0 \Rightarrow M_A = \frac{2Pa}{3}$

verif.

$\circlearrowright D \{ -\frac{2Pa}{3} + \frac{2P}{3} \times 4a - P \times 2a = 0$



Seção Transversal:



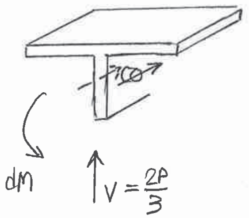
$P = 150\ kN$ $\quad \delta = 0{,}5\ cm$

$a = 200\ cm$ $\quad I_y = 56\delta c^3$

$c = 10\ cm$ $\quad \bar{\tau} = 7{,}5\ \frac{kN}{cm^2}$

$s_1 = s_2 = 10\ cm$

Parafuso P_1



$\bar{S}_1 = (3c\delta) \times 2c + c\delta \times \frac{3c}{2} = \frac{15}{2}\delta c^2$

$q_1 = \frac{V\bar{S}_1}{I_y} = \frac{\frac{2}{3}P \times \frac{15}{2}\delta c^2}{56\delta c^3} = \frac{5}{56}\frac{P}{c}$

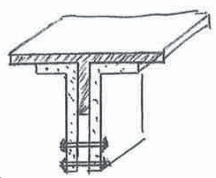
força longitudinal por unidade de comprimento que deve ser resistida por P1

resultante em s_1 $\quad$ área efetiva

$q_1 \times s_1 = \bar{\tau}_{par} \times A_{ef} = \bar{\tau}_{par} \times (2A_1)$

$A_1 = \frac{q_1 s_1}{2\bar{\tau}} = \frac{\left(\frac{5}{56} \times \frac{150}{10}\right) \times 10}{2 \times 7{,}5} = \frac{50}{56} = 0{,}89\ cm^2 \quad (R = 0{,}53\ cm)$

Parafuso P_2

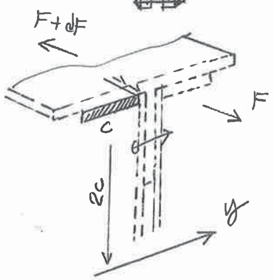


$\bar{S}_2 = \bar{S}_1 + 2(c\delta \times 2c + 2c\delta \cdot c) = \left(\frac{15}{2} + \frac{16}{2}\right)\delta c^2 = \frac{31}{2}\delta c^2$

$q_2 = \frac{V\bar{S}_2}{I_y} = \frac{\frac{2P}{3} \times \frac{31}{2}\delta c^2}{56\delta c^3} = \frac{31}{168}\frac{P}{c}$

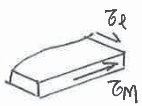
$q_2 s_2 = \bar{\tau}_{par}(4A_2) \Rightarrow A_2 = 0{,}92\ cm^2$

$(R = 0{,}54\ cm)$



Ponto M

$\bar{S}_y^M = (c\delta)\,2c = 2\delta c^2$



$\tau^M = \frac{V\bar{S}_y}{b\,I_y} = \frac{\frac{2P}{3} \times 2\delta c^2}{\delta \times 56\delta c^3} = \frac{P}{42\delta c} = 0{,}71\ \frac{kN}{cm^2}$

2013 mar 13

96P3Q2 (Ex. 2)

Seção mais solicitada:

$\uparrow \{ R_B + R_D = 300$

$\circlearrowright B \quad \{ -100 \times 2 + 500 + 200 \times 3 - R_D \times 6 = 0$

$6R_D = 900 \Rightarrow R_D = 150$

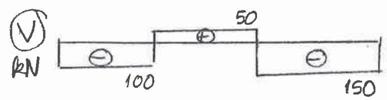
$R_B = 150$

verif

$\circlearrowright D \quad \{ \underbrace{-100 \times 8}_{-800} + \underbrace{150 \times 6}_{900} - \underbrace{200 \times 3}_{-600} + 500 = 0$ ✓

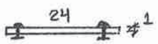
Força cortante admissível no parafuso.

$\bar{X} = \bar{\tau}_{par} \times A_{par} = 12 \times \pi \times 0{,}5^2 = 9{,}425\ kN$



Parafusos (trecho CD)

P_1)



$\bar{S}_1 = (24 \times 1) \times 19{,}5 = 468\ m^3$

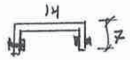
$q_1 = \frac{V\bar{S}_1}{I_y} = \frac{150 \times 468}{47936} = 1{,}464\ \frac{kN}{cm}$

$q_1 s_1 = 2\bar{X} \Rightarrow s_1 = \frac{2 \times 9{,}425}{1{,}464} = 12{,}9\ cm$

$nf = \frac{300}{12{,}9} = 23{,}3$ $\quad$ Adotando 2x24 paraf.

$s_1^* = 12{,}5\ cm$

P_2)



$\bar{S}_2 = (14 \times 1) \times 18{,}5 + 2[(1 \times 6) \times 15]$
$= 439\ cm^3$

$q_2 = \frac{V\bar{S}_2}{I_y} = \frac{150 \times 439}{47936} = 1{,}374\ \frac{kN}{m}$

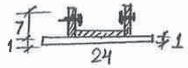
$q_2 s_2 = 2\bar{S} \Rightarrow s_2 = 13{,}7\ cm$

$nf = \frac{300}{13{,}7} = 21{,}9$ $\quad$ Adotando 2x22 paraf.

$s_2^* = 13{,}6\ cm$

P_4) $\quad \bar{S}_4 = \bar{S}_1$ $\quad$ $s_4^* = s_1^* = 12{,}5\ cm$

P_3)



$\bar{S}_3 = \bar{S}_1 + \bar{S}_2 = 468 + 439 = 907\ cm^3$

$q_3 = \frac{150 \times 907}{47936} = 2{,}838\ \frac{kN}{m}$

$q_3 s_3 = 2\bar{S} \Rightarrow s_3 = 6{,}64\ cm$

$nf = \frac{300}{6{,}64} = 45{,}2$ $\quad$ Adotando 2x46 paraf.

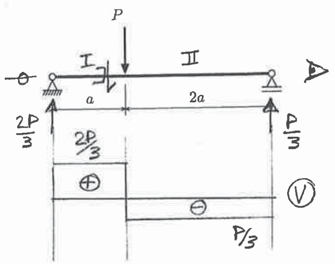
$s_3^* = 6{,}5\ cm$

Solução prática considerando a fabricação do perfil.

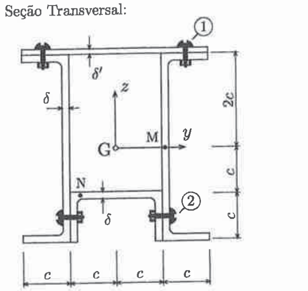
P1 :	48	parafusos	$s = 12{,}5\ cm$
P2 :	48	"	$s = 12{,}5\ cm$
P3 :	96	"	$s = 6{,}25\ cm$
P4 :	48	"	$s = 12{,}5\ cm$

Ex. 3 2014 rot 13

98PSQ2



Seção Transversal:



$\delta' = \frac{5}{8}\delta$ (chapa sup.)

a) Espaçamentos longitudinais

P_1: $\bar{S}_1 = 4c\left(\frac{5}{8}\delta\right)2c = 5\delta c^2 \Rightarrow q_1 = \frac{\frac{2P}{3}\,5\delta c^2}{\frac{130}{3}\delta c^3} = \frac{P}{13c}$

$\boxed{q_1 s_1 = \bar{\tau}_p (2A_p)} \Rightarrow s_1 = \frac{2\bar{\tau}_p A_p}{P}\times 13c = \frac{2\times 750\times 1{,}15}{45000}\times 13\times 15 = 7{,}475\,cm$ $(s_1 = 7{,}5\,cm)$

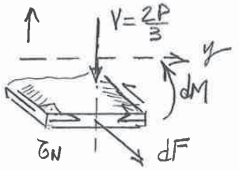
$s_1^* = 7{,}5\,cm$ { 80 parafusos em I
80 parafusos em II $(s_1 = 15\,cm)$

P_2: $\bar{S}_2 = (2c\delta)c + 2(c\delta\times 1{,}5c) = 5\delta c^2$

como $\bar{S}_2 = \bar{S}_1$, $s_2^* = 7{,}5\,cm$ { 80 parafusos em I $(s_2 = 7{,}5\,cm)$
80 parafusos em II $(s_2 = 15\,cm)$

b) Tensões tangenciais nos pontos M e N

Ponto N (fazendo 2 cortes longitudinais simétricos)



$\bar{S}_N = (2c\delta)c = 2\delta c^2$

$\tau_N = \frac{V\bar{S}_N}{b I_y} = \frac{\frac{2P}{3}\times 2\delta c^2}{(2\delta)\times\frac{130}{3}\delta c^3} = \frac{P}{65\delta c} = \frac{45000}{65\times 1{,}2\times 15}$

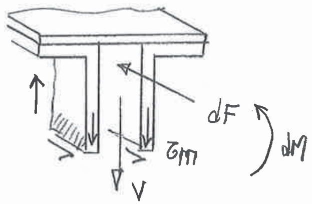
2 seções com tensão $\tau_m = \tau$

$\tau_N = 38{,}5\ \frac{kgf}{cm^2}$ (→)

Ponto M (fazendo 2 cortes longitudinais simétricos)

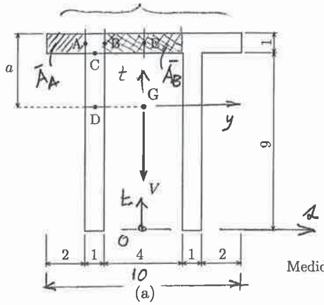
$\bar{S}_M = \bar{S}_1 + 2(c\delta\times 2c + 2c\delta c) = 13\delta c^2$

$\tau_M = \frac{\frac{2P}{3}\times 13\delta c^2}{(2\delta)\times\frac{130}{3}\delta c^3} = \frac{P}{10\delta c} = \frac{45000}{10\times 1{,}2\times 15} = 250{,}0\ \frac{kgf}{cm^2}$ (↓)

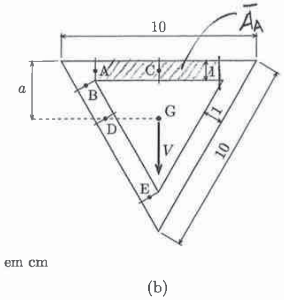


Ex. 4

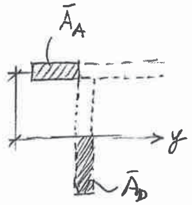
PEF2201 L7-1



Medidas em cm



Seção (a)



$t_G = \frac{10\times 10\times 5 - 8\times 9\times 4{,}5}{100 - 72} = 6{,}2857\,cm \Rightarrow a = 3{,}7143\,cm$

$I_z = \frac{10\times 10^3 - 8\times 9^3}{3} = 1389{,}33\,cm^4 \Rightarrow I_y = I_z - 28\times t_G^2 = 283{,}05\,cm^4$

$\tau = \frac{V\bar{S}_y}{b I_y} = 3{,}533\times 10^{-3}\,\frac{V\bar{S}_y}{b}$

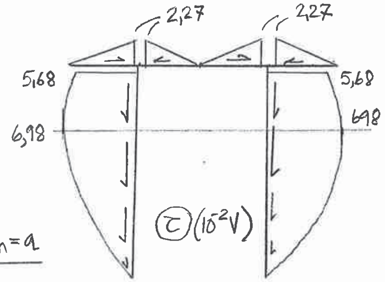
$\bar{S}_A = (2\times 1)\times 3{,}214 = 6{,}428\,cm^3 \Rightarrow \tau_A = V\bar{S}_A/(1\times I_y) = 2{,}27\times 10^{-2}V$

$\bar{S}_B = (4\times 1)\times 3{,}214 = 12{,}856$ " $\quad \tau_B = V\bar{S}_B/(2\times I_y) = 2{,}27\times 10^{-2}V$

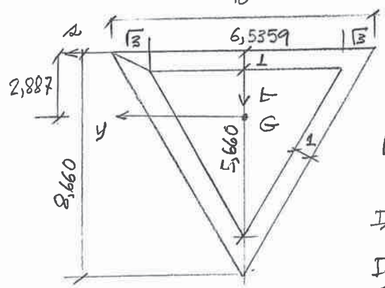
toda a mesa → $\bar{S}_C = (10\times 1)\times 3{,}214 = 32{,}14$ " $\quad \tau_C = V\bar{S}_B/(2\times I_y) = 5{,}68\times 10^{-2}V$

$\bar{S}_D = -\frac{1\times 6{,}2857^2}{2} = -19{,}75$ " $\quad \tau_D = V\bar{S}_D/(1\times I_y) = 6{,}98\times 10^{-2}V$

$\bar{S}_E = 0 \Rightarrow \tau_E = 0$



Seção (b)



Como os triângulos são concêntricos*:

$t_G = \frac{8{,}660}{3} = 2{,}887\,cm = a$

$I_z = \frac{10\times 8{,}66^3}{12} - \left(\frac{6{,}5359\times 5{,}660^3}{36} + 18{,}50\times 2{,}887^2\right) = 354{,}1\,cm^4$

$I_y = I_z - 24{,}80\times(2{,}887)^2 = 147{,}4\,cm^4$

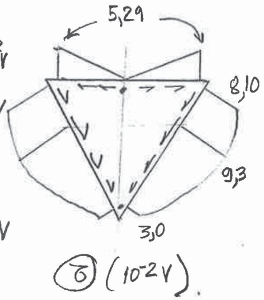
$\bar{S}_A = 6{,}5359\times 2{,}387 = 15{,}60\,cm^3 \Rightarrow \tau_A = V\bar{S}_A/(2I_y) = 5{,}29\times 10^{-2}V$

$\bar{S}_B \cong (10\times 1)\times 2{,}387 = 23{,}87\,cm^3 \quad \tau_B = V\bar{S}_B/(2I_y) = 8{,}10\times 10^{-2}V$

$\bar{S}_C = 0 \quad \tau_C = 0$

$\bar{S}_D \cong \bar{S}_B + 2(1{,}887\times 1)\times\left(\frac{1{,}887}{2}\right) = 27{,}43\,cm^3 \quad \tau_D = V\bar{S}_D/(2I_y) = 9{,}3\times 10^{-2}V$

$\bar{S}_E \approx 2\left(\frac{\sqrt{3}\times 1}{2}\right)\left(\frac{2}{3}8{,}66 - \frac{2}{3}\right) = 8{,}84\,cm^3 \quad \tau_E = V\bar{S}_E/(2I_y) = 3{,}0\times 10^{-2}V$



* compensa calcular o mom. de inércia em relação a y'

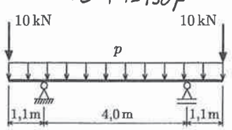
$I_{y'} = \frac{10\times 8{,}66^3}{36} - \frac{6{,}5359\times 5{,}66^3}{36} = 147{,}5\,cm^4$ ✓

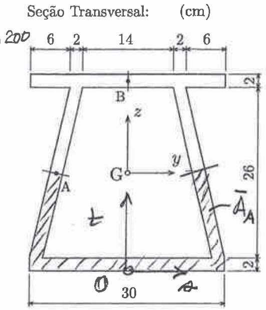
Ex. 5

2014 nov 13

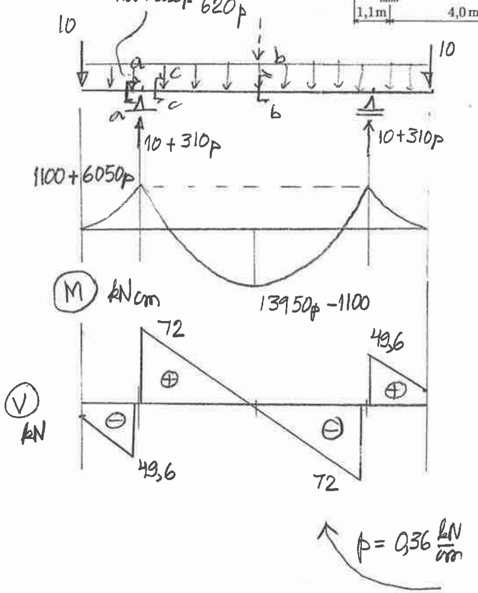
$\boxed{\text{PEF 2201 \quad L7-2}}$

$M = -10\times310 - p\frac{310^2}{2} + (10+310p)\,200 = -1100 + 13950p$





$10\times110 + \frac{p\,110^2}{2} = 1100 + 6050p$



$p\ \left[\frac{kN}{cm}\right]$

$t_G = 15\,cm$ (almas são paralelogramos)

$I_y = \frac{30\times30^3}{12} - \frac{26\times26^3}{12} = 29418{,}7\,cm^4$

$W' = W'' = \frac{I_y}{15} = 1961\,cm^3$

Como $\bar{\sigma}_c = 5\,\frac{kN}{cm^2}$ e $\bar{\sigma}_t = 2\,\frac{kN}{cm^2}$

as fibras mais tracionadas nas seções a-a e b-b são críticas.

$\sigma'_{at} = \frac{1100+6050p}{1961} \le 2 \Rightarrow p \le 0{,}466\,\frac{kN}{cm}$

$\sigma''_{bt} = \frac{13950-1100}{1961} \le 2 \Rightarrow p \le 0{,}360\,\frac{kN}{cm}$

$p = 0{,}36\,\frac{kN}{cm}$

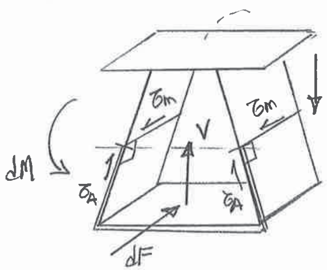
Tensões tangenciais:

Seção c-c: ($V = 72\,kN$)

$\bar{S}_A = (30\times15)\frac{15}{2} - (26\times13)\frac{13}{2} = 1178\,cm^3$

$\tau_A = \frac{72\times1178}{(2\times1{,}95)\,29418{,}7} = 0{,}739\,\frac{kN}{cm^2}$ (↑)

$\tau_B = 0$ (simetria)



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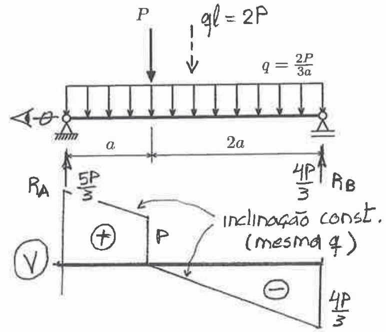
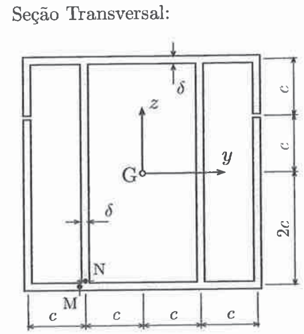
2015 out 22

Para a seção mais solicitada à força cortante da viga da figura, determine as tensões tangenciais nos pontos M e N segundo o ponto de vista indicado na figura.





a) Reações

$\uparrow\ \{\ R_A + R_B = 3P$

$\circlearrowright B\ \{\ R_A\cdot3a - P\cdot2a - 2P\cdot\frac{3a}{2} = 0$

$R_A = \frac{5P}{3} \qquad R_B = \frac{4P}{3}$

verif $\circlearrowright A\ \{\ P\cdot a + 2P\cdot\frac{3a}{2} - \frac{4P}{3}\cdot3a = 0$ ✓

b) Momento de inércia ($\delta \ll c$)

$I_y = 2(\ 4c\ \delta\ ) + 4(\ \delta\ 4c\ )$

$= 2\left(4\delta c\,(2c)^2\right) + 4\left(\frac{\delta(4c)^3}{12}\right)$

$= \left(32 + \frac{64}{3}\right)\delta c^3 = \frac{160}{3}\delta c^3$

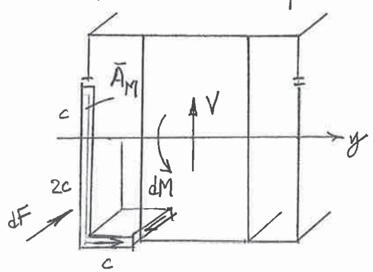
c) Tensão tangencial em M

seção vista da esquerda

$\bar{S}_M = (3c\delta)\left(-\frac{c}{2}\right) + c\delta(-2c) = -\frac{7}{2}\delta c^2$

$\tau = \frac{V\bar{S}}{b\,I_y} = \frac{\frac{5P}{3}\times\bar{S}}{b\,\frac{160}{3}\delta c^3} = \frac{P}{32\delta c^3}\,\frac{\bar{S}}{b}$

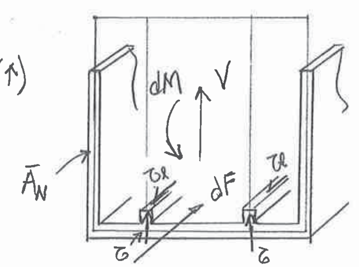
$\tau^M = \frac{P}{32\delta c^3}\,\frac{\frac{7}{2}\delta c^2}{\delta} = \frac{7}{64}\,\frac{P}{\delta c}$ (→)



d) Tensão tangencial em N

$\bar{S} = \cancel{2}(3c\delta)\left(-\frac{c}{\cancel{2}}\right) + (4c\delta)(-2c) = -11\,\delta c^2$

$\tau^N = \frac{P}{32\delta c^3}\,\frac{\bar{S}}{b} = \frac{P}{32\delta c^3}\,\frac{11\delta c^2}{(2\delta)} = \frac{11}{64}\,\frac{P}{\delta c}$ (↑)



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2014 nov 15 15/10/22

YY - Diogo

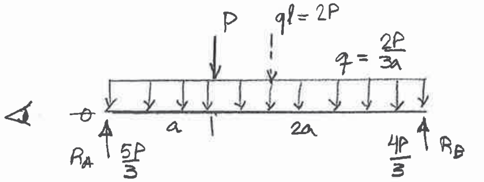
Ex. 7

a) Reações

$\uparrow \; \{ \; R_A + R_B = 3P$

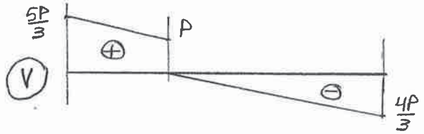
$\circlearrowleft B \; \{ \; R_A \cdot 3a - P \cdot 2a - \cancel{2}P \cdot \frac{3}{\cancel{2}}a = 0$

$R_A = \frac{5P}{3} \qquad R_B = \frac{4P}{3}$



verif.

$\circlearrowleft A \; \{ \; P \cdot a + \cancel{2}P \cdot \frac{3}{\cancel{2}}a - \frac{4P}{3} \cdot 3a = 0$ ✓



b) Momento de inércia (seção delgada)

$$I_y = 2\left[4bc\,(2c)^2\right] + 4\left[\frac{b(4c)^3}{12}\right] = \left(32 + \frac{64}{3}\right) bc^3 = \frac{160}{3} bc^3$$

Diagrama de τ considerando a simetria:

Cortes longitudinais:

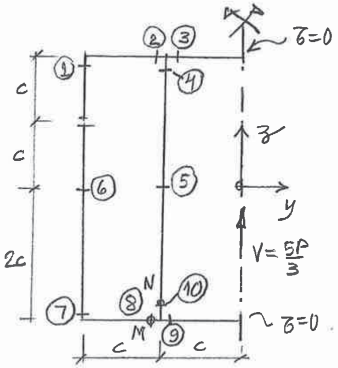
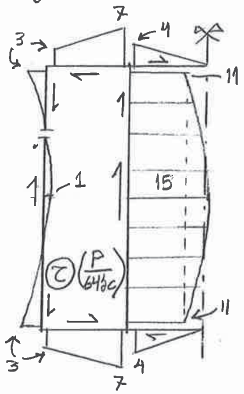
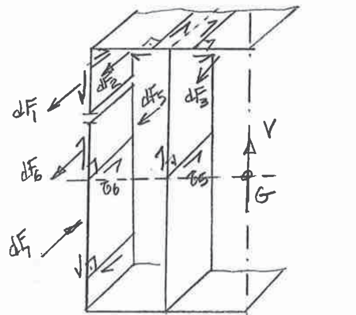


Diagrama de τ



Determinação dos sentidos



$\bar{S}_1 = bc \cdot \frac{3}{2}c = \frac{3}{2}bc^2 \longrightarrow \tau_1 = \frac{V\bar{S}}{b I_y} = \frac{\frac{5P}{3} \times \bar{S}}{b \times \frac{160}{3} bc^3} = \frac{P\bar{S}}{32bc^3} = \frac{3}{64}\frac{P}{bc}$

$\bar{S}_2 = \bar{S}_1 + bc \cdot 2c = \frac{7}{2}$ " $\qquad \tau_2 = \frac{7}{64}\frac{P}{bc}$

$\bar{S}_3 = (bc) \cdot 2c = 2$ " $\qquad \tau_3 = \frac{4}{64}$ "

$\bar{S}_4 = \bar{S}_2 + \bar{S}_3 = \frac{11}{2}$ " $\qquad \tau_4 = \frac{11}{64}$ "

$\bar{S}_5 = \bar{S}_4 + 2bc \cdot c = \frac{15}{2}$ " $\qquad \tau_5 = \frac{15}{64}$ "

$\bar{S}_6 = bc \cdot \frac{c}{2} = \frac{1}{2}$ " $\qquad \tau_6 = \frac{1}{64}$ "

$\bar{S}_7 = (3bc) \cdot \left(-\frac{c}{2}\right) = -\frac{3}{2}$ " $\qquad \tau_7 = \frac{3}{64}$ "

$\bar{S}_M \left[\bar{S}_8 = \bar{S}_7 + bc \cdot (-2c) = -\frac{7}{2} \right.$ " $\qquad \tau_8 = \frac{7}{64}$ "

$\bar{S}_9 = -\bar{S}_3 = -2$ " $\qquad \tau_9 = \frac{4}{64}$ "

$\bar{S}_N \left[\bar{S}_{10} = -\bar{S}_4 = -\frac{11}{2} \right.$ " $\longrightarrow \tau_{10} = \frac{11}{64}$ "

simetria

$\tau_3 = \frac{\frac{5P}{3}\bar{S}_3}{(2b) \frac{160}{3} bc^3} = \frac{1}{32}\frac{\bar{S}P}{b\,bc^3}$

no plano de simetria $\tau = 0$

$A = (2 \times 4 + 4 \times 4)\,bc = 24bc$

$\tau_5 = \frac{45}{8}\frac{P}{A}$

Tensões bem mais elevadas que $\tau_m = P/A$.

2014 nov 12

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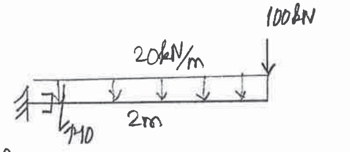
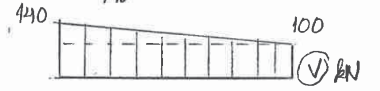
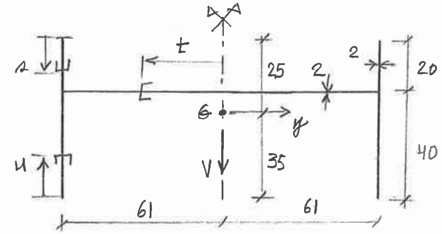
Ex. 8

a) Distribuição das tensões na seção de engaste





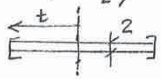
Considerando a ST delgada



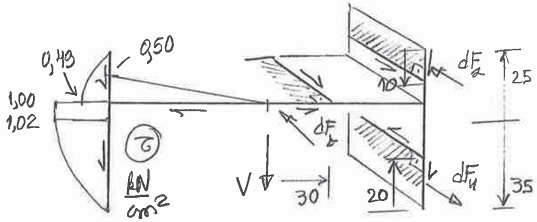
$q = \frac{V\bar{S}_y}{I_y} = \beta \bar{S}_y \qquad \tau = \frac{q}{b}$

$\bar{S}(s) = 2s \times \left(25 - \frac{s}{2}\right) \Rightarrow q(s)$: $s = 0$: $q = 0$, $\tau = 0$; $s = 19$: $q = 589\beta$, $\tau = \frac{589\beta}{2} = 294{,}5\beta$

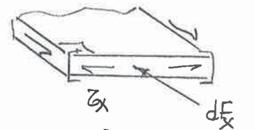
$\bar{S}(u) = 2u \times \left(35 - \frac{u}{2}\right) \Rightarrow q(u)$: $u = 0$: $q = 0$, $\tau = 0$; $u = 35$ (G): $q = 1225\beta$, $\tau = 612{,}5\beta$; $u = 39$: $q = 1209\beta$, $\tau = 604{,}5\beta$



$\bar{S}(t) = 2(2t) \times 5 = 20t \Rightarrow q(t)$: $t = 0$: $q = 0$, $\tau = 0$; $t = 60$: $q = 1200\beta$, $\tau = \frac{1200\beta}{4} = 300\beta$



$\beta = \frac{V}{I_y} = \frac{140}{84080} = 1{,}665 \times 10^{-3}$



b) Cálculo das tensões nos pontos X, Y e Z $\qquad \beta = 1{,}665 \times 10^{-3}$

$\bar{S}_x = 2(30 \times 2) \times 5 = 600\ cm^3 \Rightarrow \tau_x = \frac{140 \times 600}{(2 \times 2)\,84080} = 0{,}25\ \frac{kN}{cm^2}$ (←)

$\bar{S}_y = (2 \times 10) \times 20 = 400\ cm^3 \Rightarrow \tau_y = \frac{\beta \times 400}{2} = 0{,}33\ \frac{kN}{cm^2}$ (↓)

$\bar{S}_z = (2 \times 20) \times 25 = 1000\ cm^3 \Rightarrow \tau_y = \frac{\beta \times 1000}{2} = 0{,}83\ \frac{kN}{cm^2}$ (↓)