

Cabos e Risers

Modos Naturais de Vibrar

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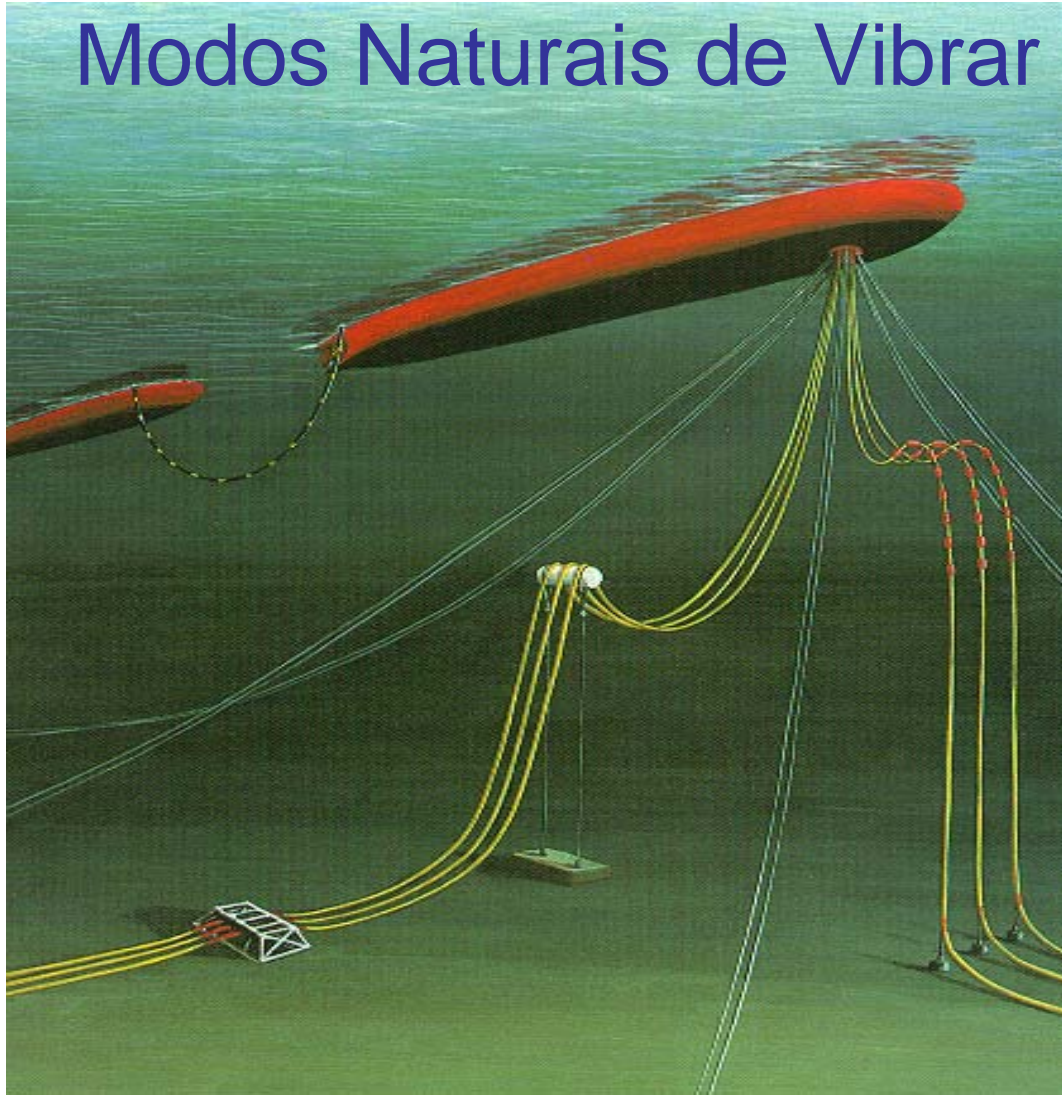
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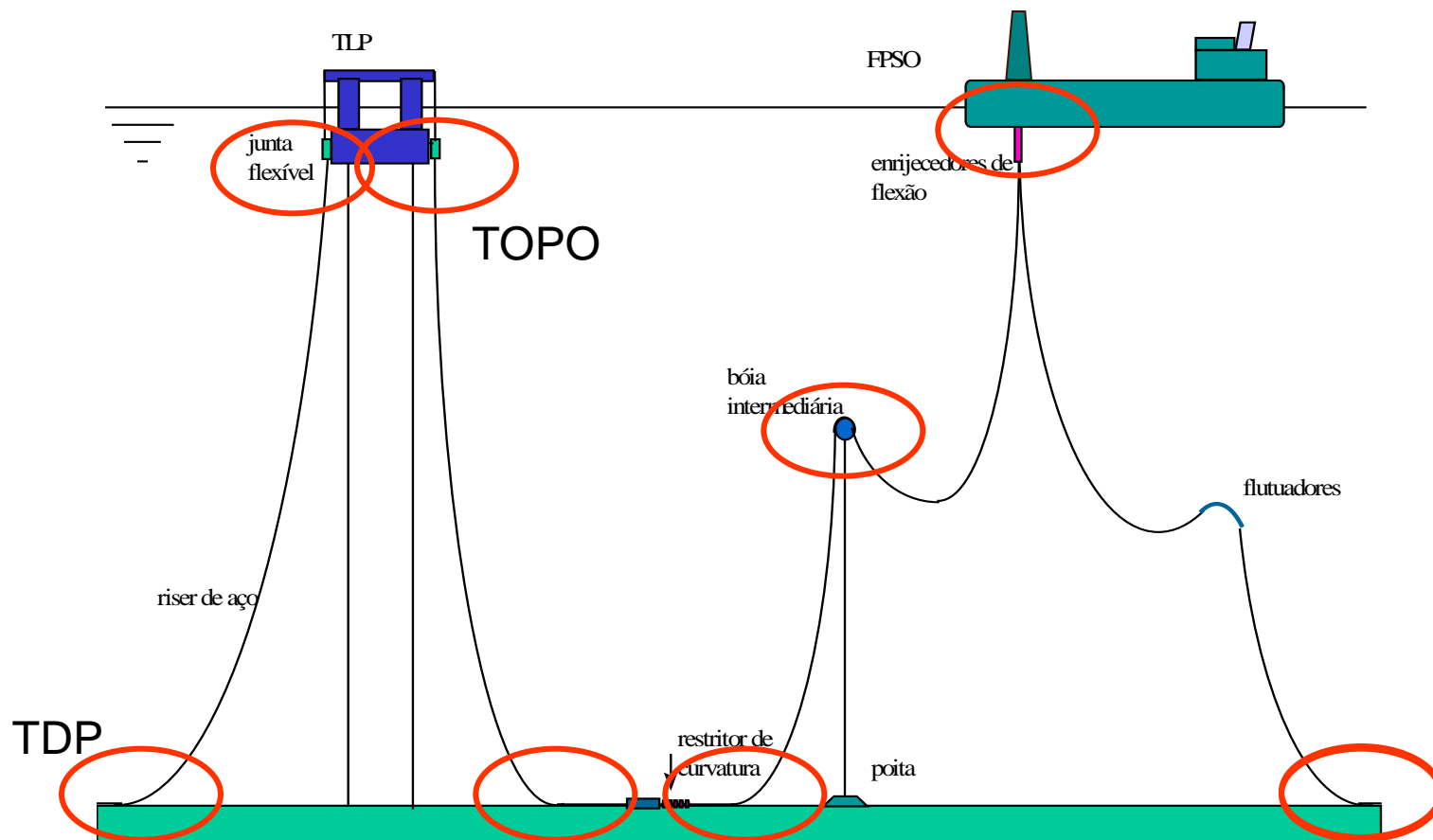
Prof. Associado em Mecânica

Problema Dinâmico

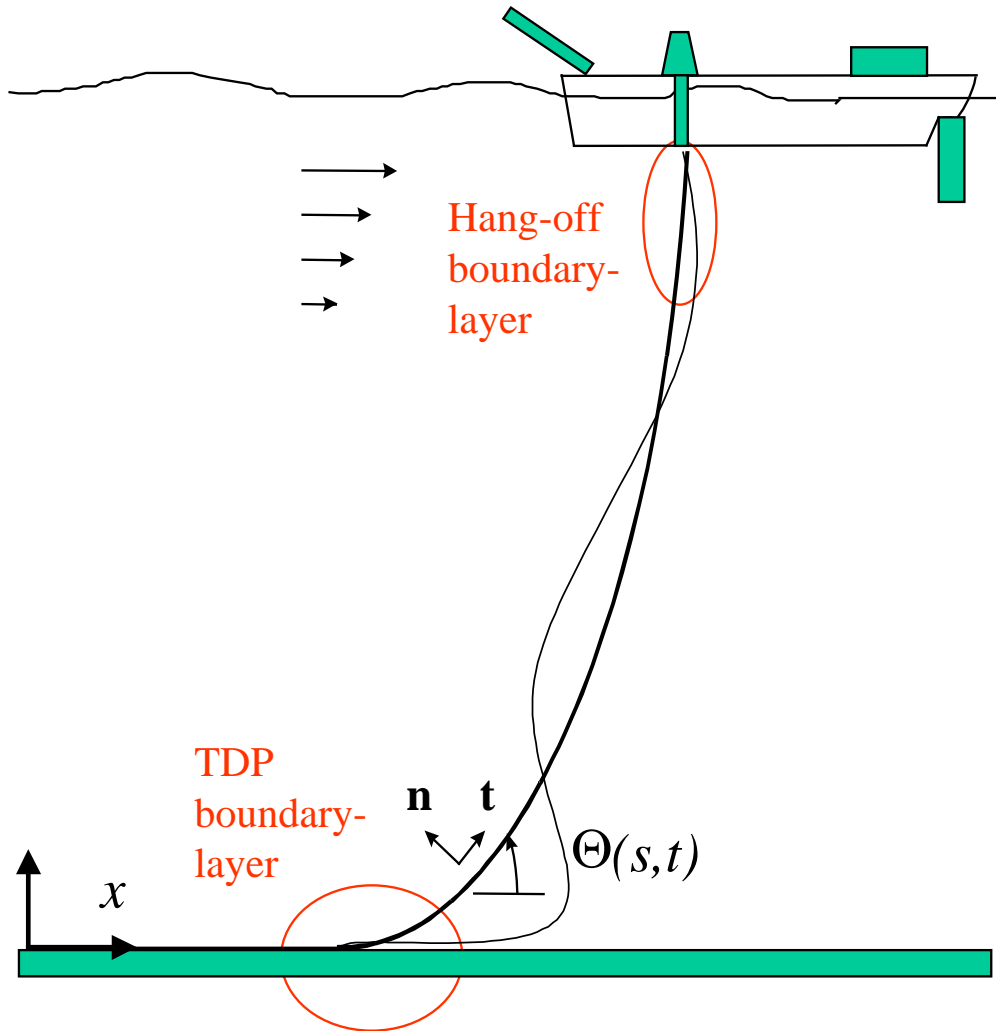
Modos Naturais de Vibrar



Cabos Umbilicais e Tubos Submersos



O Problema Dinâmico no Plano



- A dinâmica global é regida pela rigidez de catenária.
- **O efeito da rigidez flexional é importante junto às extremidades** ou quando os modos de vibrar tem comprimento comparável ao comprimento de flexão local.
- Existem diversas escalas de tempo que regem a dinâmica da linha.

Procedimento de Análise Dinâmica no Plano

- Solução estática de cabo-extensível, sob correnteza no plano.
 - Solução dinâmica de cabo-extensível, *no domínio da frequência*, sob correnteza, ondas e movimento imposto ao topo, considerando:
 - extremidades articuladas (TDP e TOPO);
 - articulação no TDP ligada a mola linear que simula extensão de trecho apoiado no fundo.
 - Correção da curvatura, ângulo e curva elástica junto às extremidades, incorporando efeito de rigidez flexional - **a posteriori** - via soluções assintóticas, conseguidas através da aplicação da técnica da camada-limite:
 - articulação no TOPO ligada a mola torsional que simula o *bending-stiffner*.
 - **Modos naturais de vibrar podem ser avaliados analiticamente através da equação dinâmica de cabo inextensível.**
-

O Efeito da Rigidez Flexional

Definindo:

$$\lambda = \sqrt{\frac{EI}{T_0}}$$

$$\varepsilon = \frac{\lambda}{L}$$

Comprimento de flexão
no TDP

$$\chi_0 = \frac{q}{T_0}$$

Curvatura estática
no TDP

$$\hat{t} = \frac{c_0}{L} t$$

Celeridade de onda
associada à rigidez
geométrica, no TDP

$$c_0 = \sqrt{\frac{T_0}{m + m_a}}$$

A equação na direção normal fica então:

$$-\varepsilon^2 \frac{\partial^2 \hat{\chi}}{\partial \hat{s}^2} + \hat{T} \hat{\chi} + \left(\frac{\partial \hat{T}}{\partial \hat{s}} - \varepsilon^2 \frac{d\theta}{d\hat{s}} \frac{\partial \hat{\chi}}{\partial \hat{s}} \right) \gamma + \hat{c}_n + \hat{h}_n - \hat{\chi}_0 \cos \theta = \frac{\partial^2 \hat{u}_n}{\partial \hat{t}^2}$$

OBS: Escalas de Tempo

$$t_1 = \frac{L}{c_g}$$

$$\bar{c}_g = \sqrt{\frac{\bar{T}}{(m + m_a)}}$$

Celeridade de onda associada à rigidez geométrica

$$t_2 = \frac{\lambda}{c_g}$$

$$\bar{c}_f^{(i)} = \frac{2\pi}{\lambda_f^{(i)}} \sqrt{\frac{EI}{(m + m_a)}}$$

Celeridades de ondas associada à rigidez flexional

$$t_3^{(i)} = \frac{L}{c_f^{(i)}}$$

$$t_4^{(i)} = \frac{\lambda}{c_f^{(i)}}$$

$$c_a = \frac{EA}{m}$$

Celeridade de onda associada à rigidez axial

$$t_5 = \frac{L}{c_a}$$

$$\lambda(s) = \sqrt{\frac{EI}{T(s)}}$$

Comprimento de flexão local

O Efeito da Rigidez Flexional

Risers rígidos: $\varepsilon \approx O(10^{-2})$

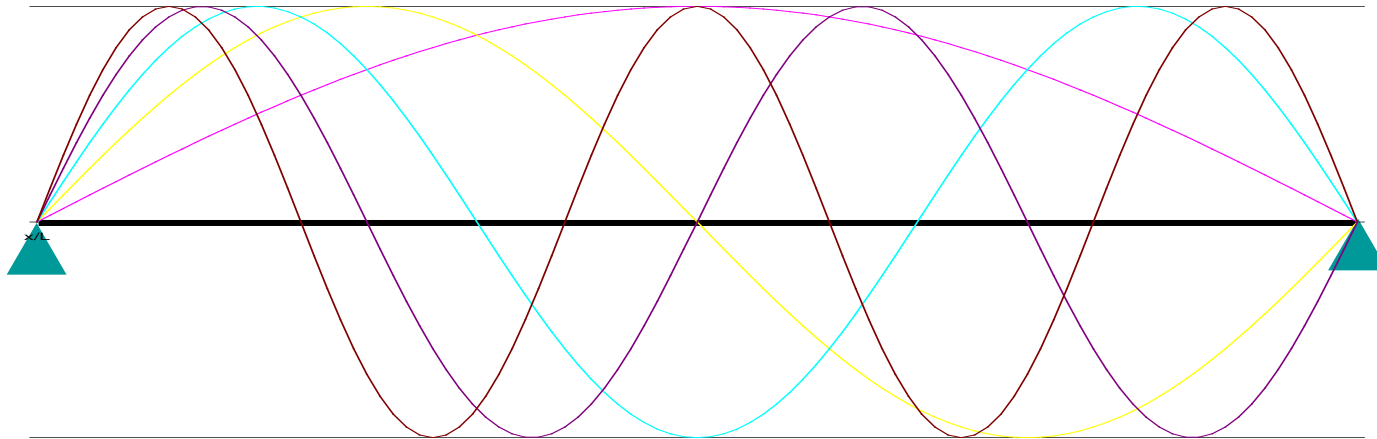
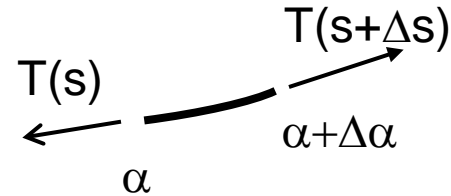
Risers flexíveis: $\varepsilon \approx O(10^{-3})$

Desprezando a rigidez flexional, a equação na direção normal fica então regida pela tração:

$$\hat{T}\hat{\chi} + \frac{\partial \hat{T}}{\partial \hat{s}} \gamma + \hat{c}_n + \hat{h}_n - \hat{\chi}_0 \cos \theta = \frac{\partial^2 \hat{u}_n}{\partial \hat{t}^2} (1 + O(\varepsilon^2))$$

O problema da corda vibrante

Fio inextensível



$$\alpha(x) \ll 1 \Rightarrow s \approx x \text{ e } \tan \alpha \cong \sin \alpha \cong \frac{dy}{dx}$$

$$\mu(x)\Delta x \frac{\partial^2 y}{\partial t^2} = T(x + \Delta x) \sin(\alpha(x + \Delta x)) - T(x) \sin(\alpha(x))$$

$$\Delta x \rightarrow 0 \quad \longrightarrow$$

$$\boxed{\mu(x) \frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial x} \left(T(x) \frac{\partial y}{\partial x} \right)}$$

O problema da corda vibrante

$$T(x) = T_0 \text{ e } \mu(x) = \mu = \text{cte} \Rightarrow$$

$$\boxed{\mu \frac{\partial^2 y}{\partial t^2} = T_0 \frac{\partial^2 y}{\partial x^2}} \quad \text{ou} \quad \boxed{\frac{\partial^2 y}{\partial t^2} = c_0^2 \frac{\partial^2 y}{\partial x^2}} \quad \text{com} \quad \boxed{c_0 = \sqrt{\frac{T_0}{\mu}}}$$

Solução do tipo $y_n(x, t) = \varphi_n(x) e^{i\Omega_n t}$

$$\frac{\partial^2 y_n}{\partial t^2} = -\Omega_n^2 \varphi_n(x) e^{i\Omega_n t} \text{ e } \frac{\partial^2 y_n}{\partial x^2} = \varphi_n'' e^{i\omega_n t} \Rightarrow \left(c_0^2 \varphi_n'' + \Omega_n^2 \varphi_n(x) \right) e^{i\omega_n t} = 0$$

Levando a $\boxed{\left(c_0^2 \varphi_n'' + \Omega_n^2 \varphi_n(x) \right) = 0}$

Soluções do tipo: $\varphi_n(x) = A_n e^{ik_n x} \Rightarrow \left(-k_n^2 + \frac{\Omega_n^2}{c_0^2} \right) \varphi_n(x) = 0$

O problema da corda vibrante

Equação característica:
$$\left(-k_n^2 + \frac{\Omega_n^2}{c_0^2} \right) = 0$$

Condições de contorno:
$$[\varphi_n]_{x=0} = [\varphi_n]_{x=L} \equiv 0 \Rightarrow k_n L = n\pi$$

$$\therefore k_n = \frac{n\pi}{L}$$

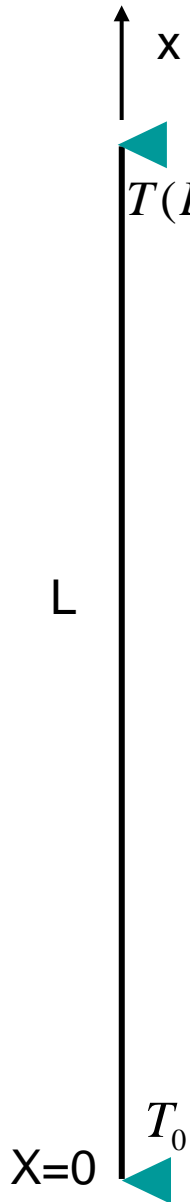
E portanto:

$$\Omega_n = c_0 k_n = n c_0 \frac{\pi}{L} = n \frac{\pi}{L} \sqrt{\frac{T_0}{\mu}}$$

$$\varphi_n(x) = A_n \sin(k_n x) = A_n \sin\left(n\pi \frac{x}{L}\right)$$

$$\rightarrow y(x, t) = \sum_{n=1}^{\infty} \left[C_n \varphi_n(x) e^{i(\Omega_n t + \theta_n)} \right]$$

O problema da corda vertical



$$T(L) = T_0 + \gamma L$$

$$\mu(x) \frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial x} \left(T(x) \frac{\partial y}{\partial x} \right)$$

com,

$$\mu(x) = m(x) + m_a(x)$$

$$T(x) = T_0 + \int_0^x \gamma(x) dx$$

Solução do tipo: $y(x, t) = \varphi(x)\eta(t)$

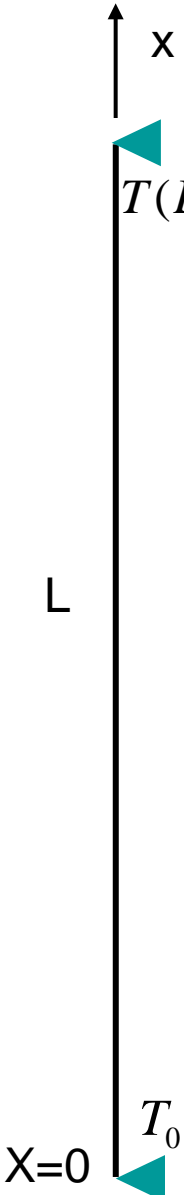
Como: $\frac{\partial^2 y}{\partial t^2} = \varphi \ddot{\eta}$ e $\frac{\partial y}{\partial x} = \varphi' \eta \rightarrow \frac{\ddot{\eta}}{\eta} = \frac{1}{\mu \varphi} \frac{d}{dx} (T(x) \varphi') = -\Omega^2$

Assim: $\ddot{\eta} + \Omega^2 \eta = 0 \rightarrow \eta_n(t) = e^{i(\Omega_n t + \theta_n)}$ E:

$$\frac{d}{dx} (T(x) \varphi') + \mu(x) \Omega^2 \varphi = 0$$

$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda vertical


$$T(L) = T_0 + \gamma L \quad \gamma(x) = \gamma = \text{cte} \Rightarrow T(x) = T_0 + \gamma x$$

$$\mu(x) = \mu = \text{cte} \Rightarrow (T_0 + \gamma x)\varphi'' + \gamma\varphi' + \mu\Omega^2\varphi = 0$$

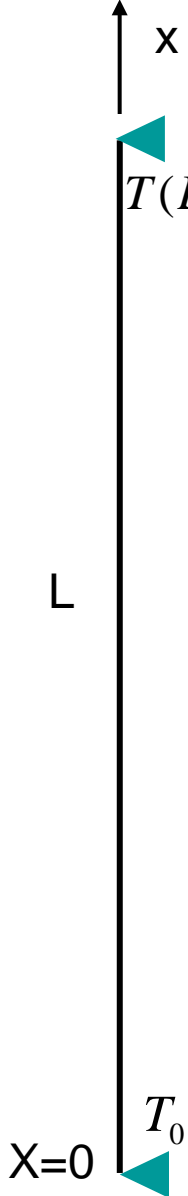
Definindo: $\xi = \frac{T_0}{\gamma} + x$ e $\beta^2 = \frac{\mu}{\gamma}\Omega^2$

Vem: $\xi\varphi'' + \varphi' + \beta^2\varphi = 0$

Esta equação pode ser transformada em uma Equação de Bessel modificada, cuja solução é conhecida.

$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda vertical


$$T(L) = T_0 + \gamma L$$

De fato, definindo uma nova variável:

$$\xi = \zeta^2$$

Tal que:

$$d\zeta = \frac{1}{2} \xi^{-1/2} d\xi = \frac{1}{2} \frac{1}{\zeta} d\xi$$

Vem que:

$$\frac{d\varphi}{d\xi} = \frac{d}{d\zeta} [\varphi(\xi(\zeta))] \frac{d\zeta}{d\xi} = \frac{1}{2\zeta} \frac{d\varphi}{d\zeta}$$

e:

$$\begin{aligned} \frac{d^2\varphi}{d\xi^2} &= \frac{d}{d\xi} \left[\frac{d\varphi}{d\xi} \right] = \frac{d}{d\zeta} \left[\frac{1}{2\zeta} \frac{d\varphi}{d\zeta} \right] \frac{d\zeta}{d\xi} = \\ &= \left[\frac{1}{2\zeta} \frac{d^2\varphi}{d\zeta^2} - \frac{1}{2\zeta^2} \frac{d\varphi}{d\zeta} \right] \frac{1}{2\zeta} = \frac{1}{4\zeta^2} \left[\frac{d^2\varphi}{d\zeta^2} - \frac{1}{\zeta} \frac{d\varphi}{d\zeta} \right] \end{aligned}$$

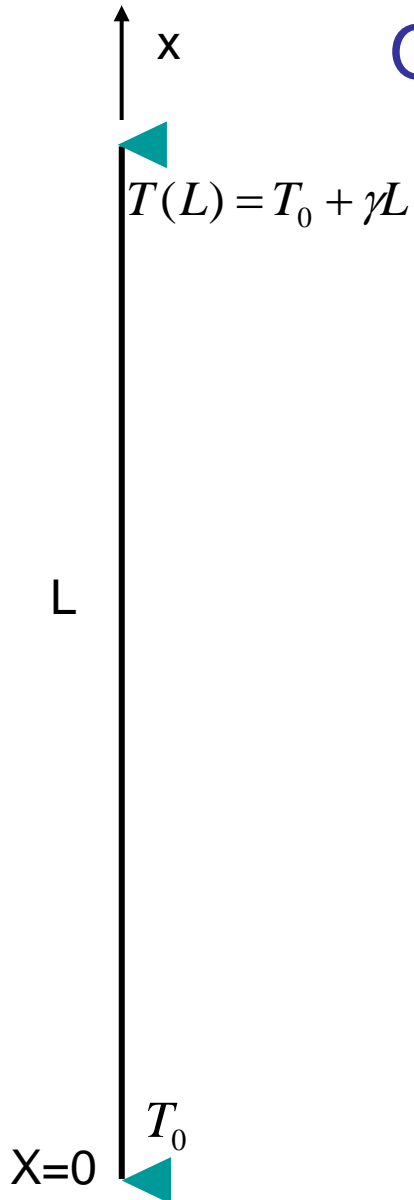
$$x=0 \quad T_0$$

resultando:

$$\xi \varphi'' = \frac{1}{4} \left[\frac{d^2\varphi}{d\zeta^2} - \frac{1}{\zeta} \frac{d\varphi}{d\zeta} \right]$$

$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda vertical



E, de $\xi \varphi'' + \varphi' + \beta^2 \varphi = 0$



$$\frac{1}{4} \left[\frac{d^2 \varphi}{d\zeta^2} - \frac{1}{\zeta} \frac{d\varphi}{d\zeta} \right] + \frac{1}{2\zeta} \frac{d\varphi}{d\zeta} + \beta^2 \varphi = 0$$



$$\frac{d^2 \varphi}{d\zeta^2} + \frac{1}{\zeta} \frac{d\varphi}{d\zeta} + 4\beta^2 \varphi = 0$$



$$\zeta^2 \frac{d^2 \varphi}{d\zeta^2} + \frac{d\varphi}{d\zeta} + 4\beta^2 \zeta^2 \varphi = 0$$

$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda vertical

Equação indicial

$$T(L) = T_0 + \gamma L$$

$$\zeta^2 \frac{d^2 \varphi}{d\zeta^2} + \frac{d\varphi}{d\zeta} + 4\beta^2 \zeta^2 \varphi = 0$$

Que tem solução dada na forma de Funções de Bessel de ordem zero:

$$\varphi(\zeta) = C_1 J_0(2\beta\zeta) + C_2 Y_0(2\beta\zeta)$$

ou:

$$\varphi(\xi) = C_1 J_0(2\beta\xi^{1/2}) + C_2 Y_0(2\beta\xi^{1/2})$$

$$X=0 \quad T_0$$

L

x

O problema da corda vertical

$$T(L) = T_0 + \gamma L$$

Lembrando que: $\xi = \frac{T_0}{\gamma} + x$ e $\beta^2 = \frac{\mu}{\gamma} \Omega^2$

$$\varphi(x) = C_1 J_0 \left(2\Omega \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + x \right)^{1/2} \right) + C_2 Y_0 \left(2\Omega \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + x \right)^{1/2} \right)$$

Condições de contorno nas extremidades: $\varphi_n(0) = \varphi_n(L) = 0$



$$\det \begin{bmatrix} J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} \right)^{1/2} \right) & Y_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} \right)^{1/2} \right) \\ J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + L \right)^{1/2} \right) & Y_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + L \right)^{1/2} \right) \end{bmatrix} \equiv 0$$

Equação característica

X=0

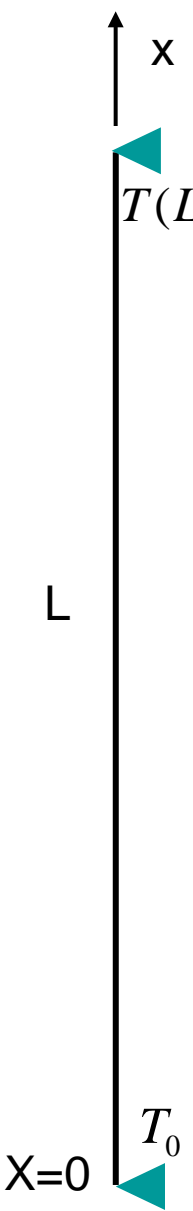
T₀

L

x

$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda vertical



$T(L) = T_0 + \gamma L$

Determinados os $\Omega_n = \Omega_n(T_0, \mu, \gamma, L)$

os modos naturais de vibrar são dados por:

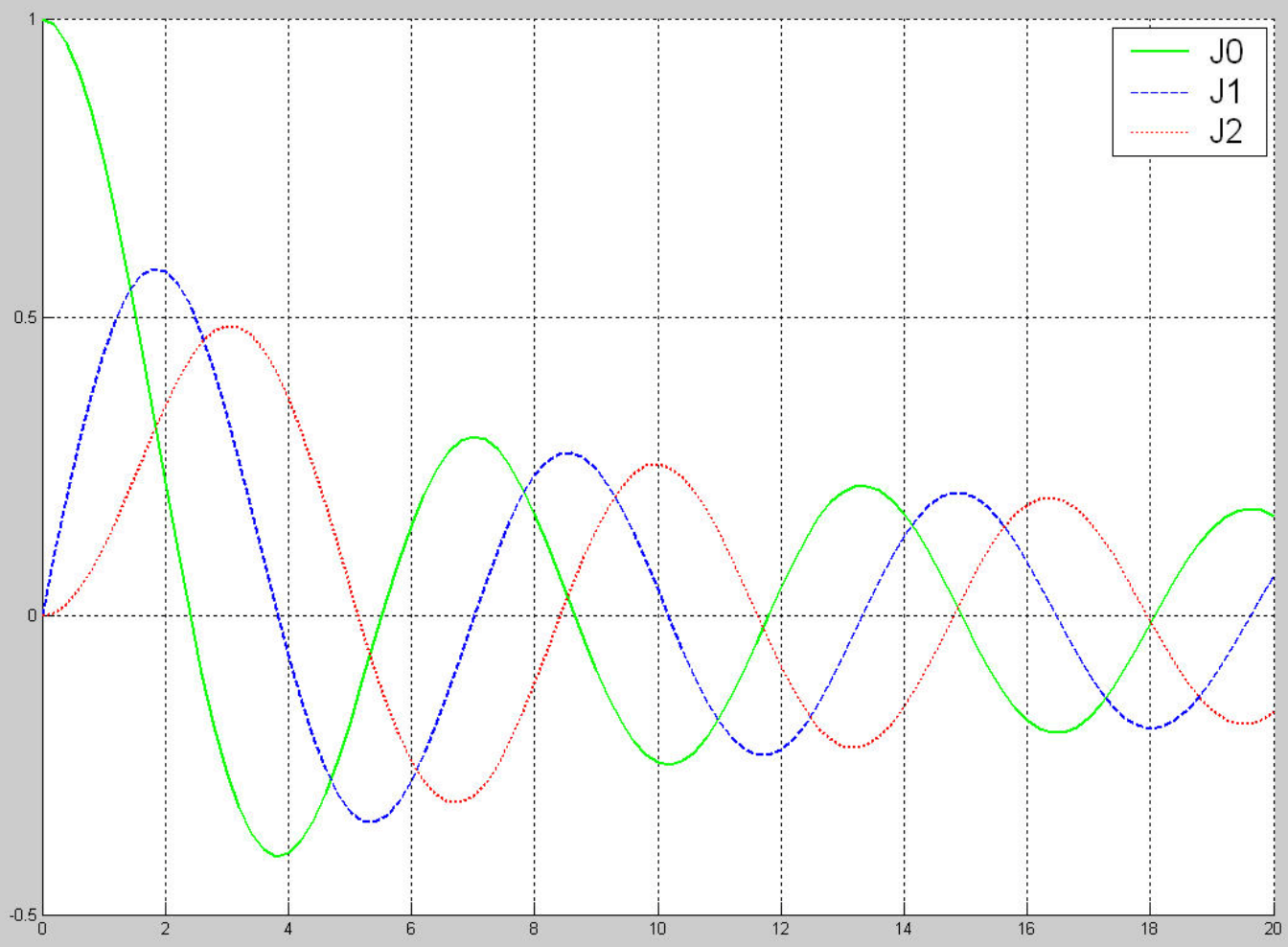
$$\varphi_n(x) = C_{1n} J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + x \right)^{1/2} \right) + C_{2n} Y_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + x \right)^{1/2} \right)$$

com:

$$C_{2n} = -C_{1n} \frac{J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} \right)^{1/2} \right)}{Y_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} \right)^{1/2} \right)} = -C_{1n} \frac{J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + L \right)^{1/2} \right)}{Y_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + L \right)^{1/2} \right)}$$

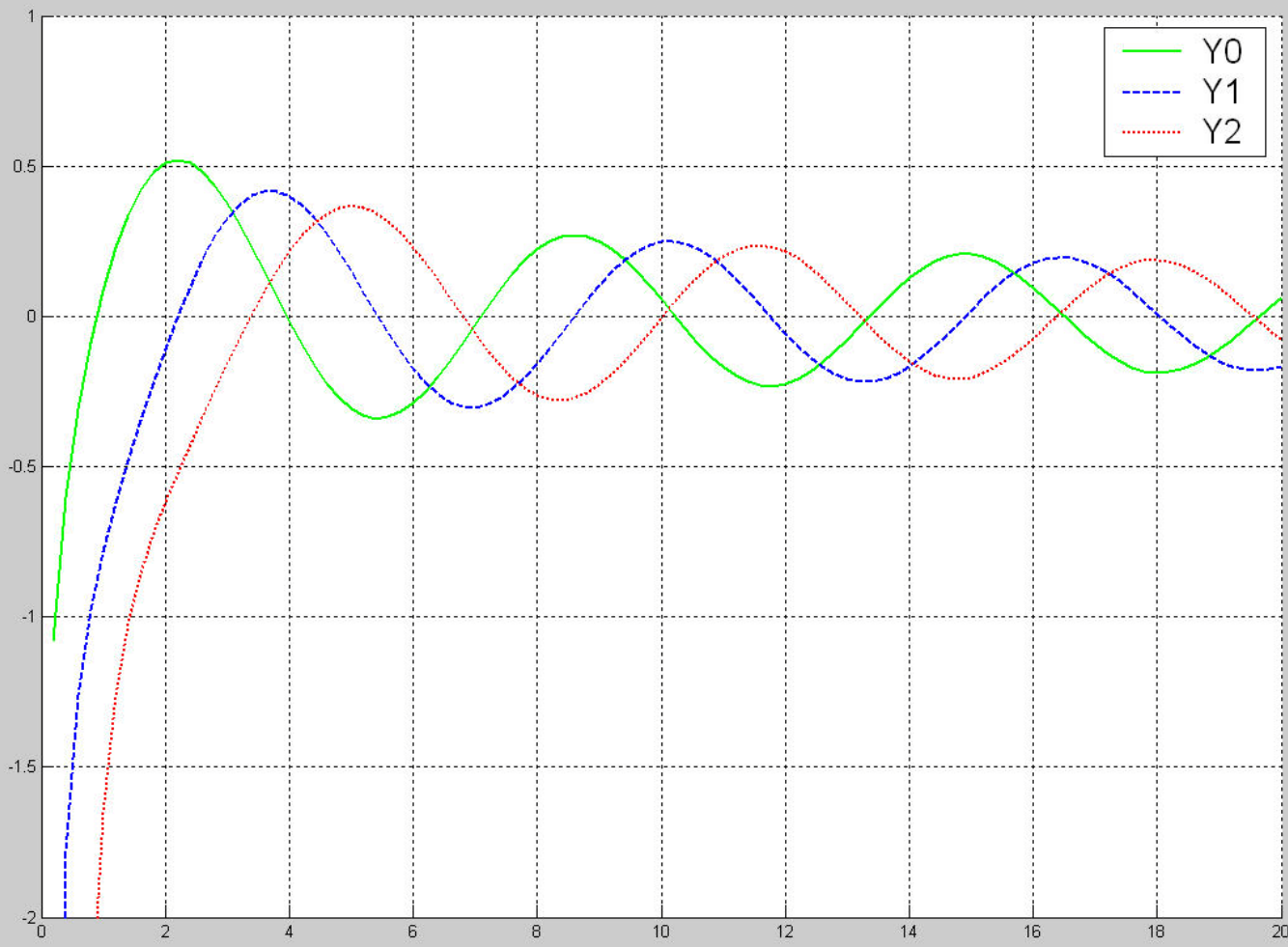
$x=0$

T_0



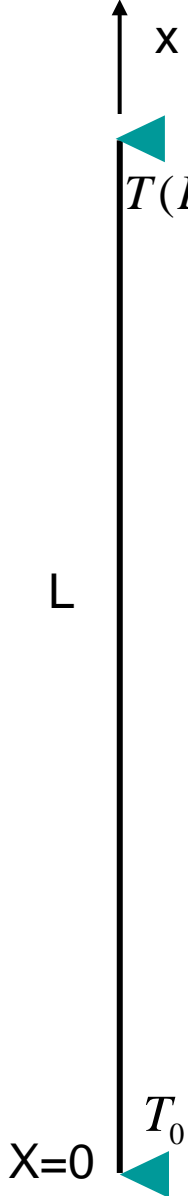


LIFE&MO



$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda vertical


$$T(L) = T_0 + \gamma L$$

Caso Singular: $T_0 = 0$ porque $[Y_0(0)] \rightarrow \infty$

Sim, pois, para que a singularidade fosse removida seria necessário que:

$$C_{2n} \equiv 0$$

mas como:

$$C_{2n} = -C_{1n} \frac{J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + L \right)^{1/2} \right)}{Y_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} \left(\frac{T_0}{\gamma} + L \right)^{1/2} \right)}$$




$$C_{1n} = C_{2n} \equiv 0$$

Solução trivial

$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda vertical


$$T(L) = T_0 + \gamma L$$

Caso Singular: $T_0 = 0$ porque $[Y_0(0)] \rightarrow \infty$

Interpretação física: quando a tração se anula a celeridade da onda transversal se anula:

$$c(x) = \sqrt{\frac{T(x)}{\mu}}$$

Deve-se incluir, *localmente*, o efeito da rigidez flexional, e compatibilizar as soluções nos domínios interior e exterior (técnica da camada-limite).

$X=0$ T_0

$$\xi = \frac{T_0}{\gamma} + x$$

O problema da corda suspensa

x

$$T(L) = T_0 + \gamma L$$

Caso Particular: $T_0 = 0$

Como $[Y_0(0)] \rightarrow \infty$ deve-se ter $C_{2n} \equiv 0$



$$\varphi_n(x) = C_{1n} J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} (x)^{1/2} \right)$$

Da condição de contorno: $\varphi_n(L) \equiv 0$

$$T_0 = 0$$

$$J_0 \left(2\Omega_n \left(\frac{\mu}{\gamma} \right)^{1/2} (L)^{1/2} \right) = 0$$

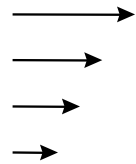
Equação característica

X=0

L

O problema do cabo sob ação de correnteza

Equações dinâmicas:



$$-T \frac{d\theta}{ds} \left(\frac{\partial v}{\partial s} + u \frac{d\theta}{ds} \right) + \varpi_u = m \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial}{\partial s} \left(T \left(\frac{\partial v}{\partial s} + u \frac{d\theta}{ds} \right) \right) + \varpi_v = m \frac{\partial^2 v}{\partial t^2}$$

Hipóteses:

- problema no plano;

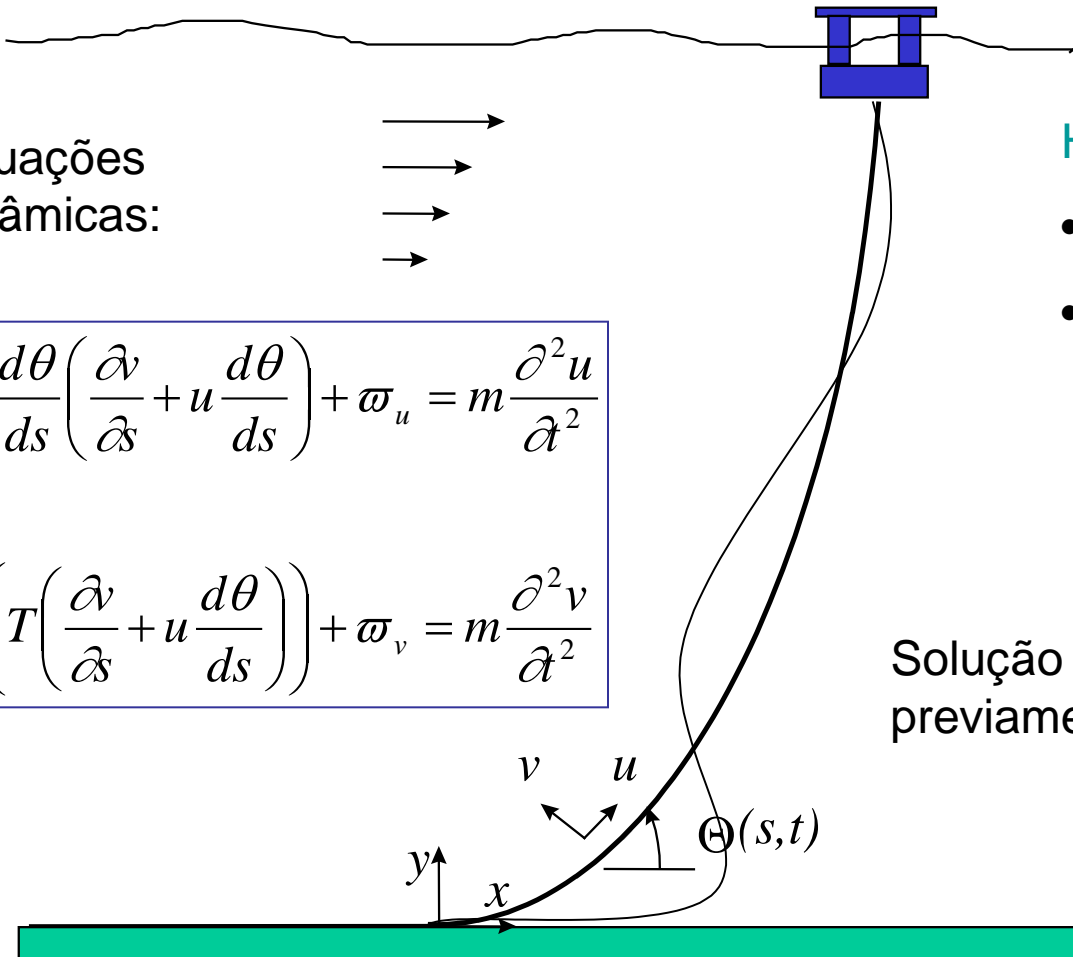
- cabo:

- ***inextensível;***

- ***infinitamente flexível.***

Solução estática, previamente determinada:

$$T(s), \theta(s)$$



O problema do cabo sob ação de correnteza

Na forma adimensional e na ausência de forças dinâmicas externas:

$$F(\xi) \left(\frac{\partial \eta}{\partial \xi} + \nu \frac{d\theta}{d\xi} \right) \frac{d\theta}{d\xi} + \frac{1}{1+a} \frac{\partial^2 \nu}{\partial t^2} = 0$$

$$-\frac{\partial}{\partial \xi} \left(F(\xi) \left(\frac{\partial \eta}{\partial \xi} + \nu \frac{d\theta}{d\xi} \right) \right) + \frac{\partial^2 \eta}{\partial t^2} = 0$$

onde:

$$\xi = s/L; \quad \nu = u/L; \quad \eta = v/L$$

$$t = c_0 t/L; \quad c_0 = \sqrt{T_0/(m + m_a)} \quad \text{e} \quad a = \frac{m_a}{m}$$

com,

$$F(\xi) = \frac{T(\xi)}{T_0} = \left(\frac{c(\xi)}{c_0} \right)^2$$

Tração adimensional

O problema do cabo sob ação de correnteza

Desprezando termos de segunda ordem na curvatura estática:

Linearmente
proporcional a φ'

$$\psi(\xi) = \frac{1}{\omega^2} [(1+a)F(\xi)\chi(\xi)]\varphi'(\xi)$$

$$(F(\xi)\varphi')' + \omega^2\varphi = 0$$

Problema de Sturm-Liouville
(de auto valor clássico)

$$\omega = \Omega \frac{L}{c_0}$$

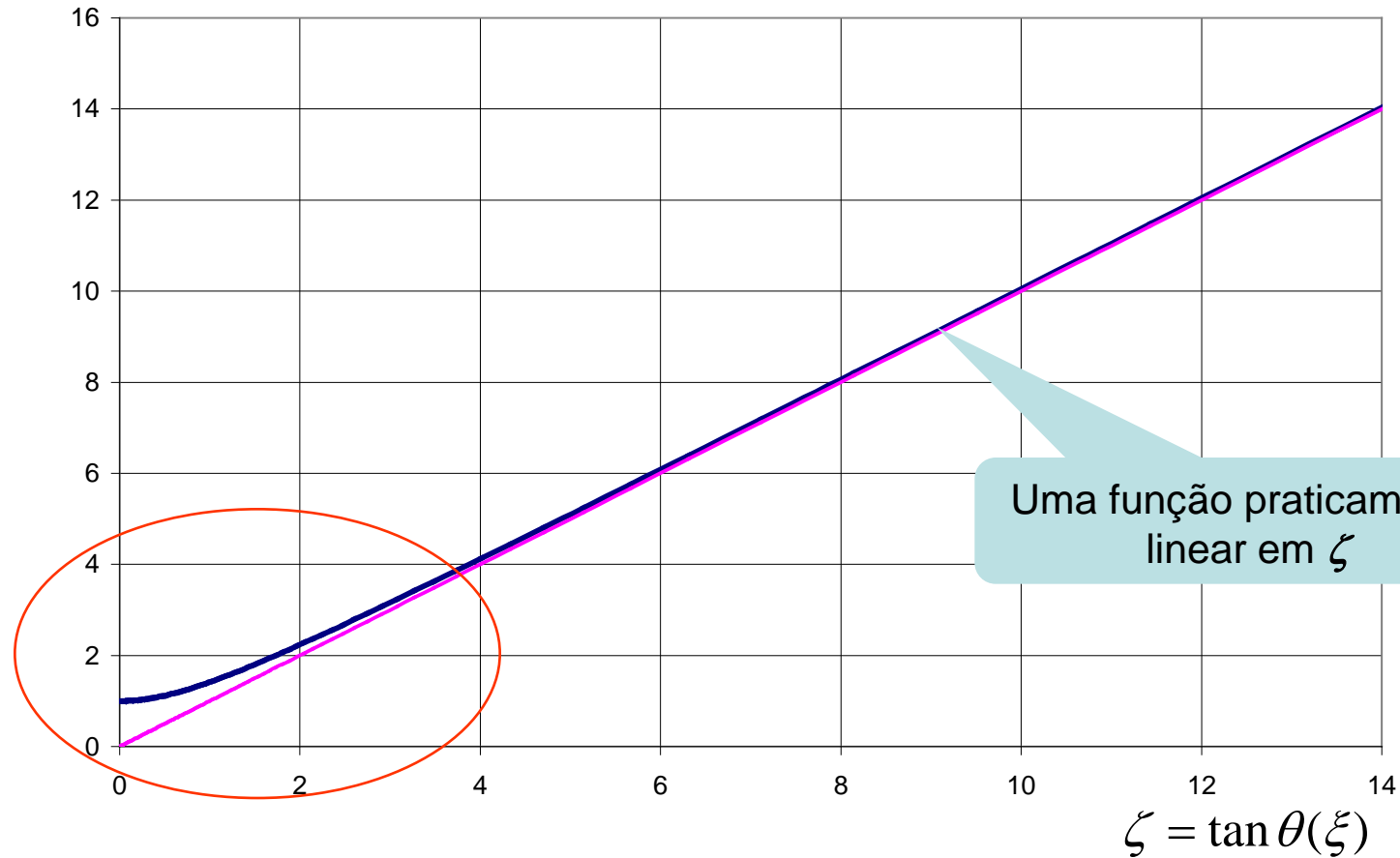
Se a tração
fosse linear, a
solução seria
dada por
funções de
Bessel

$$v(\xi, t) = \psi(\xi)e^{i\omega t}$$
$$\eta(\xi, t) = \varphi(\xi)e^{i\omega t}$$

Por separação de variáveis

O problema do cabo sem ação de correnteza

$$F_c(\zeta) = \sqrt{1 + \zeta^2}$$



$$F_c(\xi) = \sec \theta(\xi) = \sqrt{1 + \tan^2 \theta(\xi)}$$



$$F_c(\zeta) = \sqrt{1 + \zeta^2}$$

com $\zeta = \tan \theta(\xi)$

O problema do cabo sem ação de correnteza

Aproximando por mínimos quadrados:

$$F(\zeta) \approx \alpha^2 + b\zeta \quad \text{com} \quad \beta = b/\alpha^2$$

$$\text{E definindo: } z^2 = 1 + \beta\zeta = 1 + (b/\alpha^2)\zeta$$

Obtem-se uma Equação de Bessel modificada na forma:

$$z^2 \varphi'' + z\varphi' + 4K^2 z^2 \varphi = 0$$

$$K = \frac{\omega}{\alpha\beta \tan \theta_L}$$

Com solução do tipo:

$$\varphi(z) = C_1 J_0(2Kz) + C_2 Y_0(2Kz)$$

O problema do cabo sem ação de correnteza

Para um riser em catenária direta:

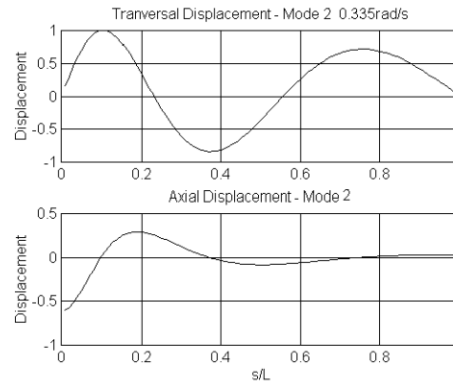
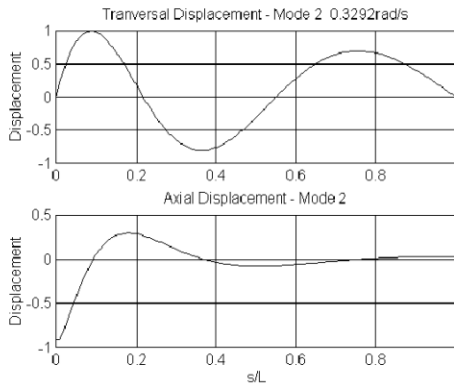
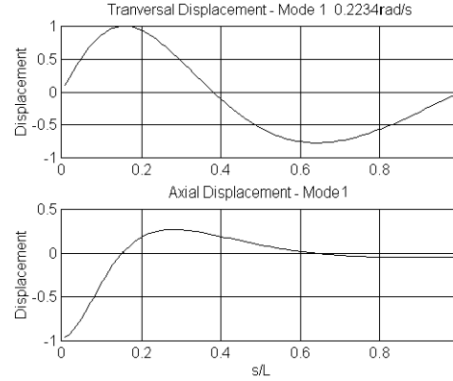
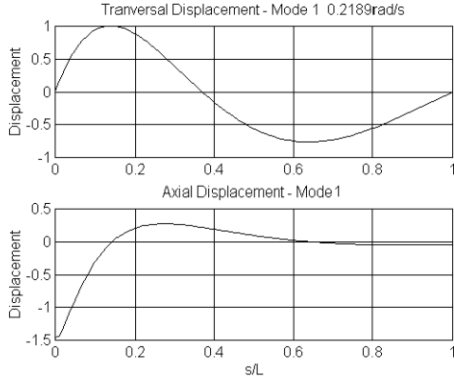
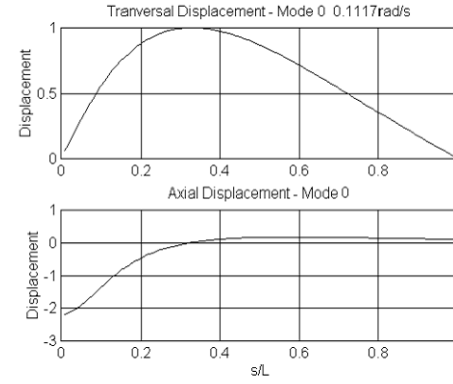
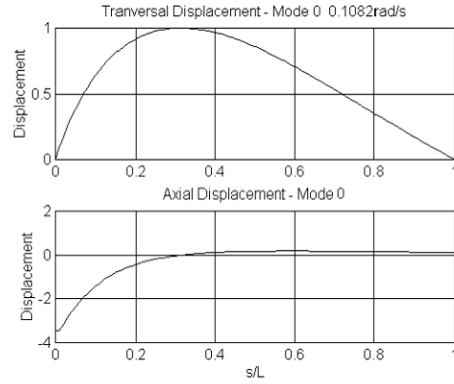
$$\begin{aligned} \varphi_n(\xi) = & \\ = J_0\left(2\frac{\omega_n}{\alpha\beta \tan \theta_L}(1 + \beta \tan \theta_L \xi)^{1/2}\right) & + \frac{J_0\left(2\frac{\omega_n}{\alpha\beta \tan \theta_L}\right)}{Y_0\left(2\frac{\omega_n}{\alpha\beta \tan \theta_L}\right)} Y_0\left(2\frac{\omega_n}{\alpha\beta \tan \theta_L}(1 + \beta \tan \theta_L \xi)^{1/2}\right) \end{aligned}$$

Satisfazendo a equação característica:

$$\begin{aligned} J_0\left(2\frac{\omega}{\alpha\beta \tan \theta_L}(1 + \beta \tan \theta_L)^{1/2}\right) Y_0\left(2\frac{\omega}{\alpha\beta \tan \theta_L}\right) = & \\ = -J_0\left(2\frac{\omega}{\alpha\beta \tan \theta_L}\right) Y_0\left(2\frac{\omega}{\alpha\beta \tan \theta_L}(1 + \beta \tan \theta_L)^{1/2}\right) \end{aligned}$$



Bessel's $\mu=5.7$ WKB



Para um riser em catenária direta:

$$\tan \theta_L = 5.7$$
$$\theta_L = 80^\circ$$

O problema do cabo sob ação de correnteza

$$\psi(\zeta) = \frac{(1+a)}{\Lambda^2} F(\zeta) \chi(\zeta) \varphi'(\zeta)$$

$$\varphi'' + \frac{F'}{F} \varphi' + \frac{\Lambda^2}{F} \varphi = 0$$

Forma propícia para aplicação do método assintótico

WKB, se:

$$\Lambda^2 \gg 1$$

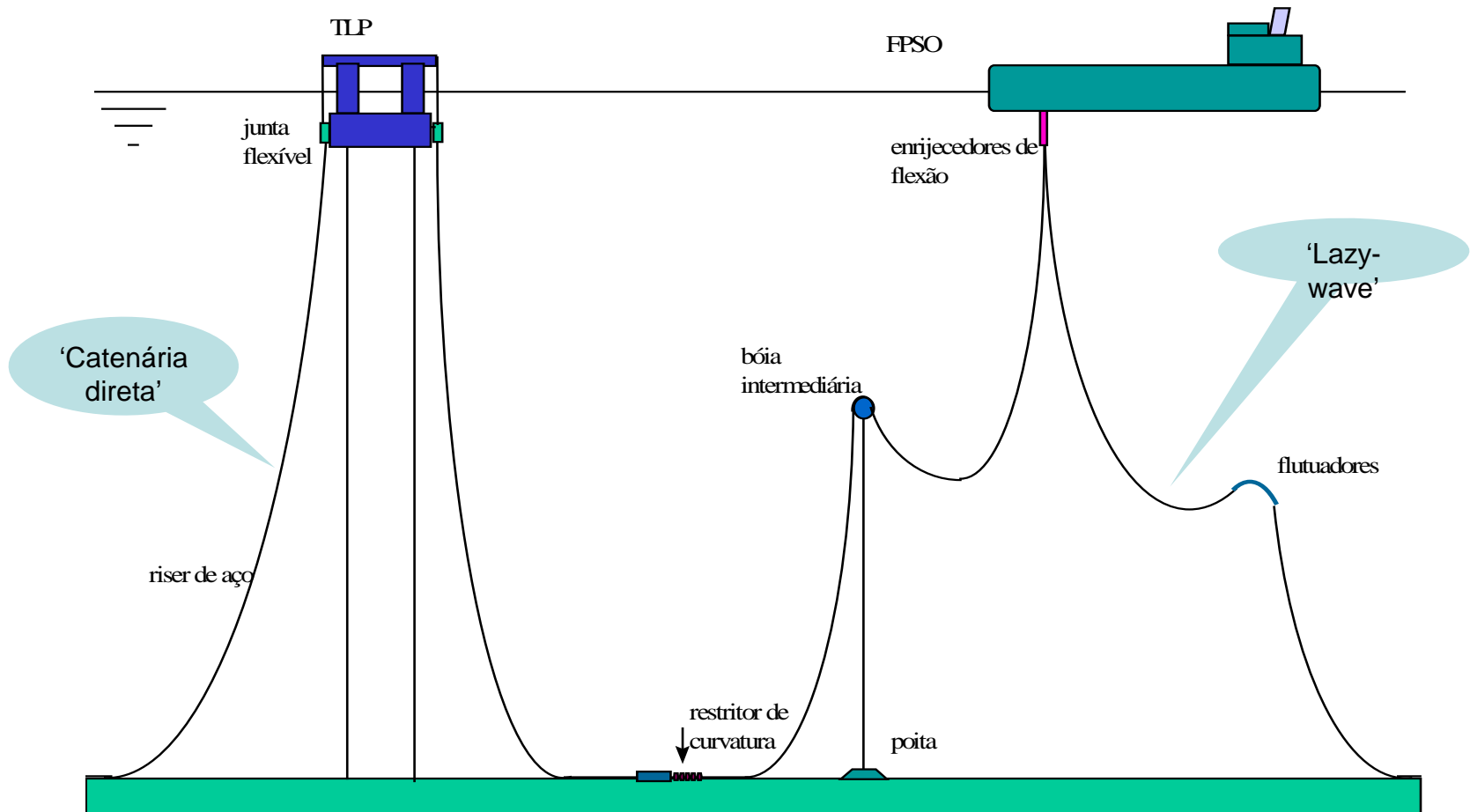
Problema singular se $F(\zeta)=0$

$$\Lambda = \frac{\omega}{\tan \theta_L}; \quad \zeta = \tan \theta(\xi)$$

$$F(\zeta) = \left(\frac{c(\zeta)}{c_0} \right)^2 = \frac{T\zeta}{T_0}$$

WKB: *Wentzel, Kramers, Brillouin* (bem como *Rayleigh* e *Jeffreys*)

O problema do cabo sob ação de correnteza



O problema do cabo sob ação de correnteza

Aproximação WKB clássica
(ver Bender & Orszag, pg. 490)

$$\varphi(\zeta) \cong F^{1/4}(\zeta) \left[C_1 \sin\left(\Lambda \int^\zeta F^{-1/2}(u) du\right) + C_2 \cos\left(\Lambda \int^\zeta F^{-1/2}(u) du\right) \right]$$

$$\Lambda^2 \gg 1$$

$$\Lambda = \frac{\omega}{\tan \theta_L}$$

Modos de vibrar são funções trigonométricas moduladas em fase e amplitude e 'lembram' funções de Bessel

'Turning Point' se $\Lambda=0$

$$\phi(\zeta) = \Lambda \int^\zeta F^{-1/2} d\zeta$$

Fase local

$$\kappa = \frac{d\phi}{d\zeta} = \frac{\Lambda}{\sqrt{F(\zeta)}}$$

Número de onda local

$$\frac{c(\zeta)}{c_0} = \frac{\Lambda}{\kappa(\zeta)}$$

Celeridade local

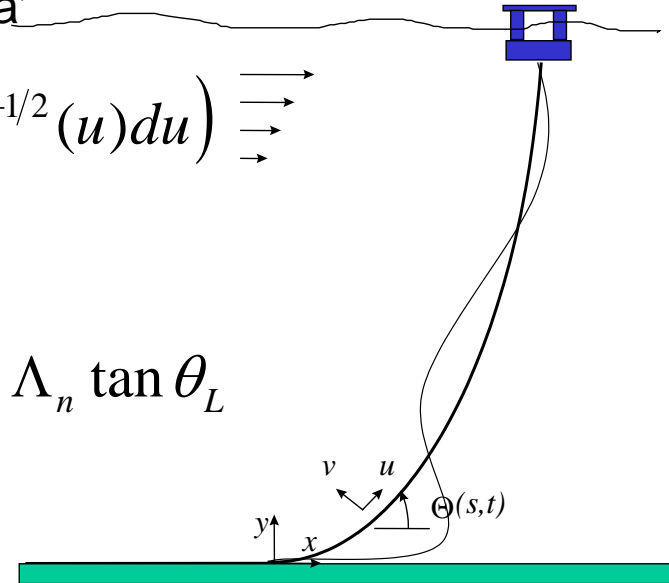
O problema do cabo sob ação de correnteza

Auto-valores são simples quadratura da solução estática!!!!

Riser em 'Catenária Direta'

$$\varphi_n(\zeta) \cong A_n F^{1/4}(\zeta) \sin\left(\Lambda_n \int_0^\zeta F^{-1/2}(u) du\right)$$

$$\Lambda_n \cong n\pi \left(\int_0^\mu \frac{d\zeta}{\sqrt{F(\zeta)}} \right)^{-1}; \quad \omega_n = \Lambda_n \tan \theta_L$$



Lineares em n

Modos de vibrar são funções senoidais moduladas em fase e amplitude e 'lembra' funções de Bessel

$$\phi_n(\zeta) = \Lambda_n \int^\zeta F^{-1/2} d\zeta$$

Fase local

$$\kappa_n = \frac{d\phi}{d\zeta} = \frac{\Lambda_n}{\sqrt{F(\zeta)}}$$

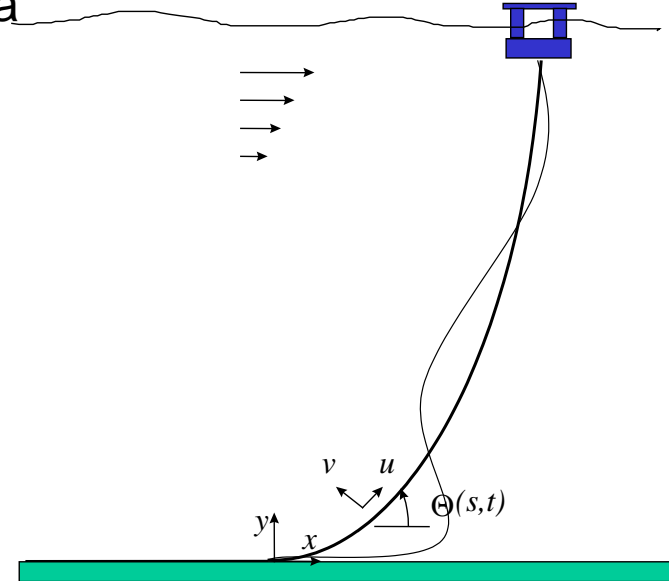
Número de onda local

$$\frac{c(\zeta)}{c_0} = \frac{\Lambda_n}{\kappa_n(\zeta)}$$

Celeridade local

O problema do cabo sob ação de correnteza

Riser em 'Catenária Direta'



Freqüências Naturais Dimensionais

$$\Omega_n = \omega_n \frac{c_0}{L} =$$

$$= \Lambda_n \tan \theta_L \frac{1}{L} \sqrt{\frac{T_0}{(m + m_a)}}$$

Solução Analítica, em forma fechada

O problema do cabo sem ação de correnteza

Riser em 'Catenária Direta'

$$\varphi_n(\theta; \theta_L) \cong A_n (\cos \theta)^{-1/4} \sin \left\{ \Lambda_n \int_0^{\theta_L} \frac{d\theta}{(\cos \theta)^{3/2}} \right\}$$

$$\Lambda_n = \Lambda_n(\theta_L) \cong \frac{n\pi}{\int_0^{\theta_L} \frac{d\theta}{(\cos \theta)^{3/2}}}$$

como $T_0 = \frac{qL}{\tan \theta_L} \rightarrow \Omega_n \cong \Lambda_n \sqrt{\frac{q \tan \theta_L}{(m + m_a)L}}$

O problema do cabo sem ação de correnteza

Riser em 'Catenária Direta'

Usando $m_a \cong \rho \pi D^2 / 4 \longrightarrow q \cong (m - m_a)g$

e, com $a = m_a / m \longrightarrow \Omega_n \cong \Lambda_n \sqrt{\tan \theta_L \frac{(1-a)}{(1+a)} \sqrt{\frac{g}{L}}}$

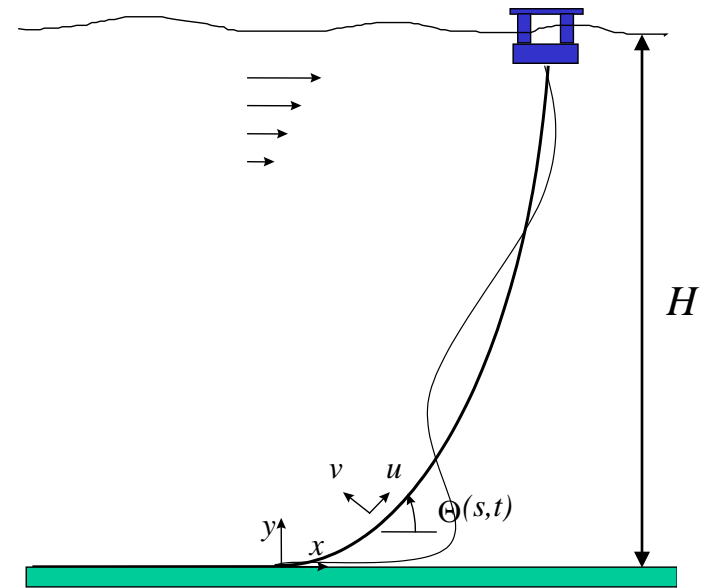
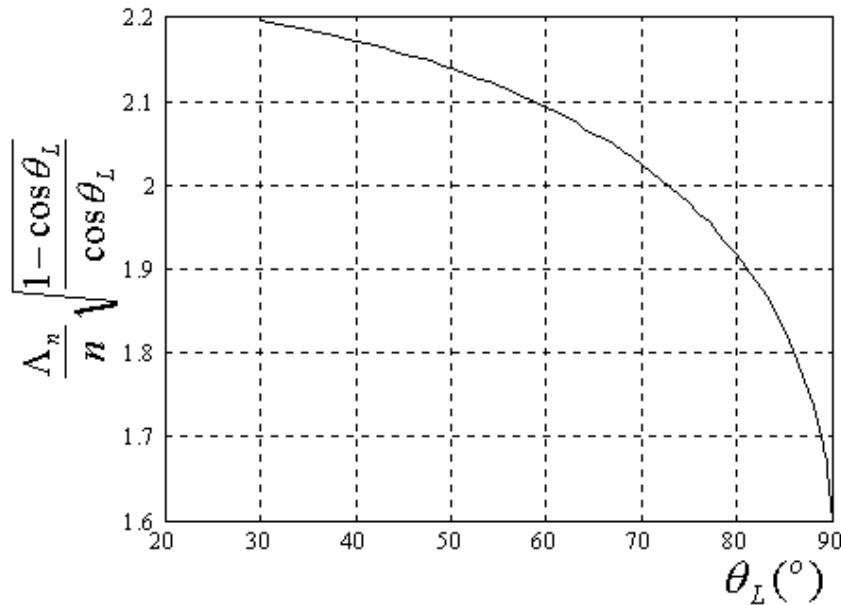
mas $L = H \sin \theta_L / (1 - \cos \theta_L)$



$$\Omega_n \cong \Lambda_n \sqrt{\frac{(1 - \cos \theta_L)}{\cos \theta_L} \sqrt{\frac{(1-a)}{(1+a)} \sqrt{\frac{g}{H}}}}$$

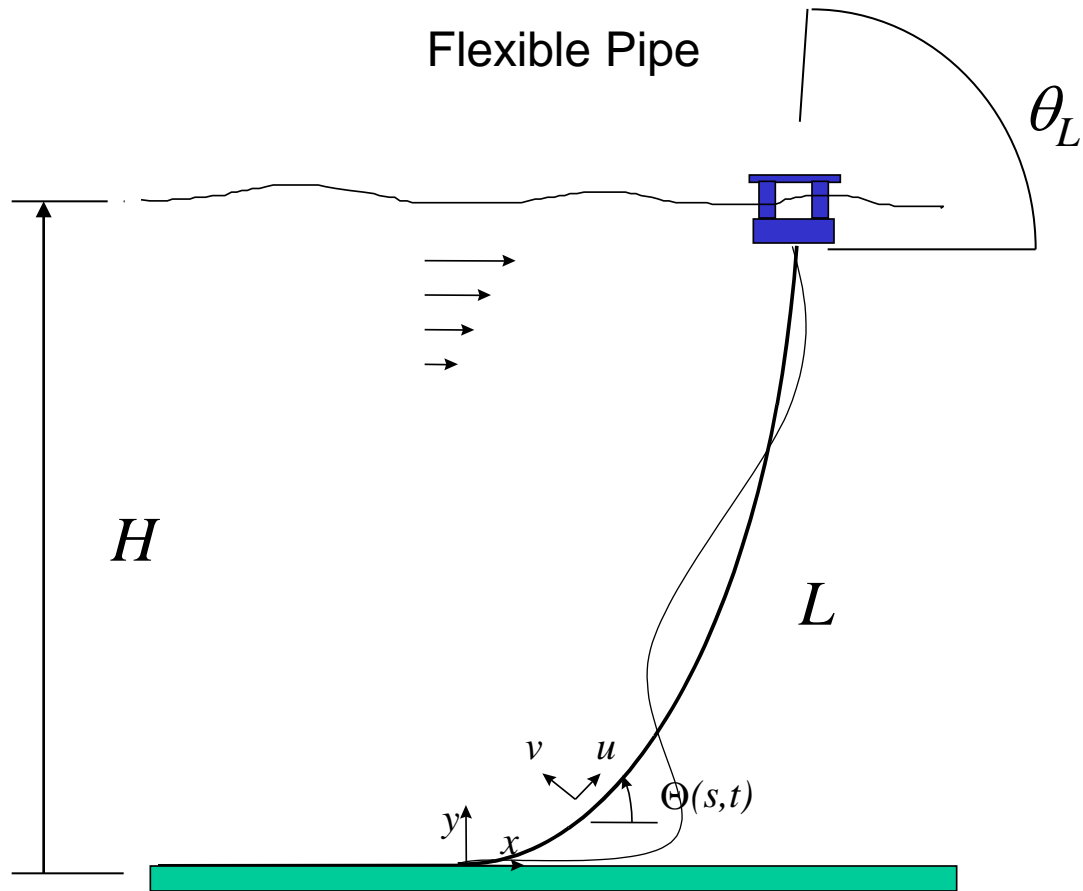
O problema do cabo sem ação de correnteza

Riser em 'Catenária Direta'



$$\Omega_n \cong \Lambda_n \sqrt{\frac{(1 - \cos \theta_L)}{\cos \theta_L}} \sqrt{\frac{(1 - a)}{(1 + a)}} \sqrt{\frac{g}{H}}$$

WKB vs. POLIFLEX



WKB vs. POLIFLEX

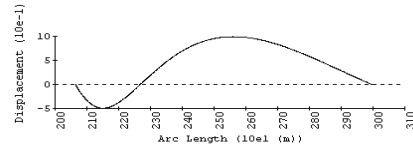
Flexible Pipe

EA (kN)	312500
EJ (kNm ²)	49.61
q (kN/m)	0.914
m (t/m)	0.218
D (m)	0.3934
H (m)	785
Total length (m)	3000
L (m) for $\theta_L = 80^\circ$	935.5
L (m) for $\theta_L = 60^\circ$	1359.6

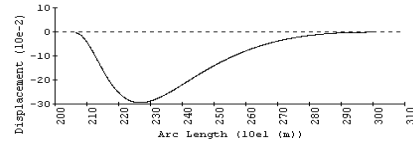
POLIFLEX

WKB

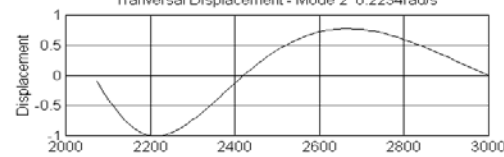
Transversal Displacement - Mode 2 $\omega=0.17\text{rad/s}$



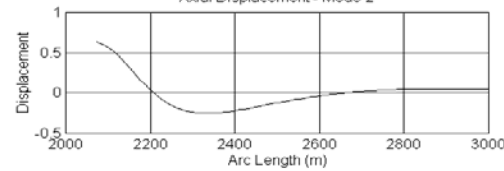
Axial Displacement - Mode 2



Transversal Displacement - Mode 2 0.2234rad/s



Axial Displacement - Mode 2

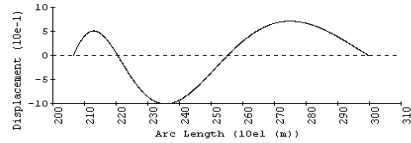


Flexible Pipe

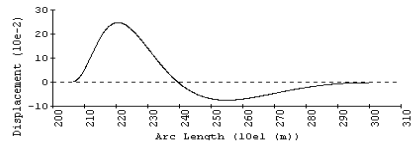
$$\tan \theta_L = 5.7$$

$$\theta_L = 80^\circ$$

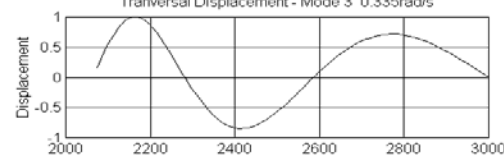
Transversal Displacement - Mode 3 $\omega=0.30\text{rad/s}$



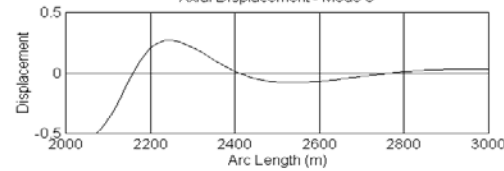
Axial Displacement - Mode 3



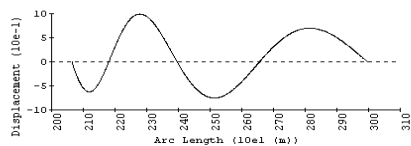
Transversal Displacement - Mode 3 0.335rad/s



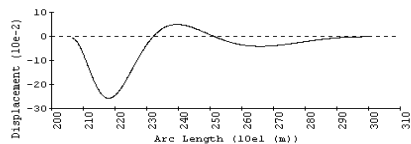
Axial Displacement - Mode 3



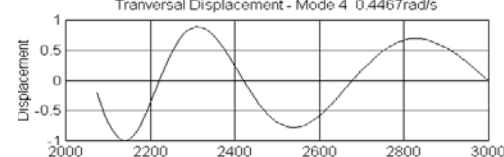
Transversal Displacement - Mode 4 $\omega=0.42\text{rad/s}$



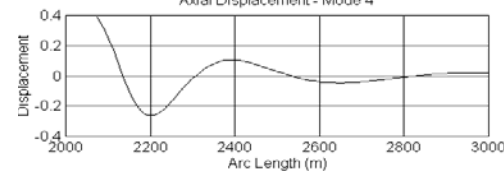
Axial Displacement - Mode 4



Transversal Displacement - Mode 4 0.4467rad/s



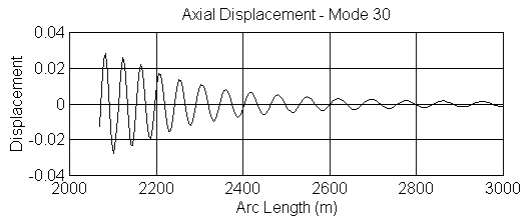
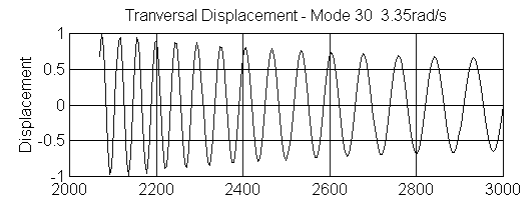
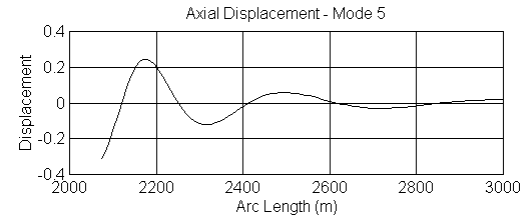
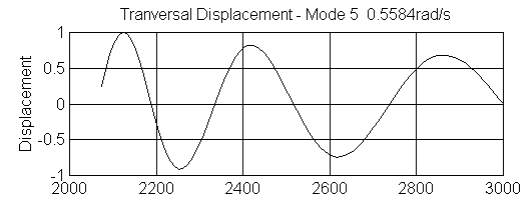
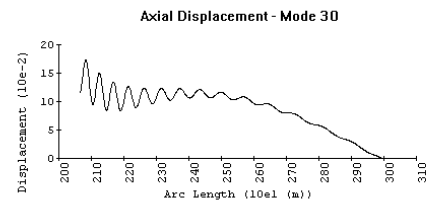
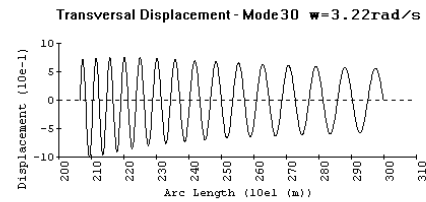
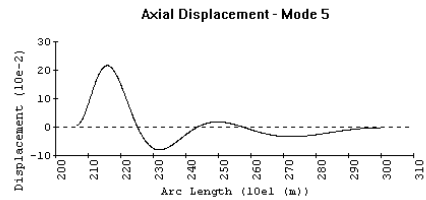
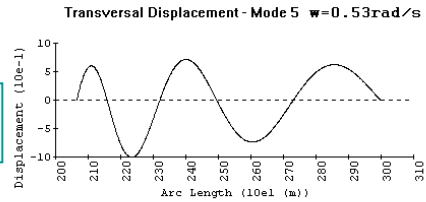
Axial Displacement - Mode 4



POLIFLEX

WKB

Flexible Pipe



$$\tan \theta_L = 5.7$$

$$\theta_L = 80^\circ$$

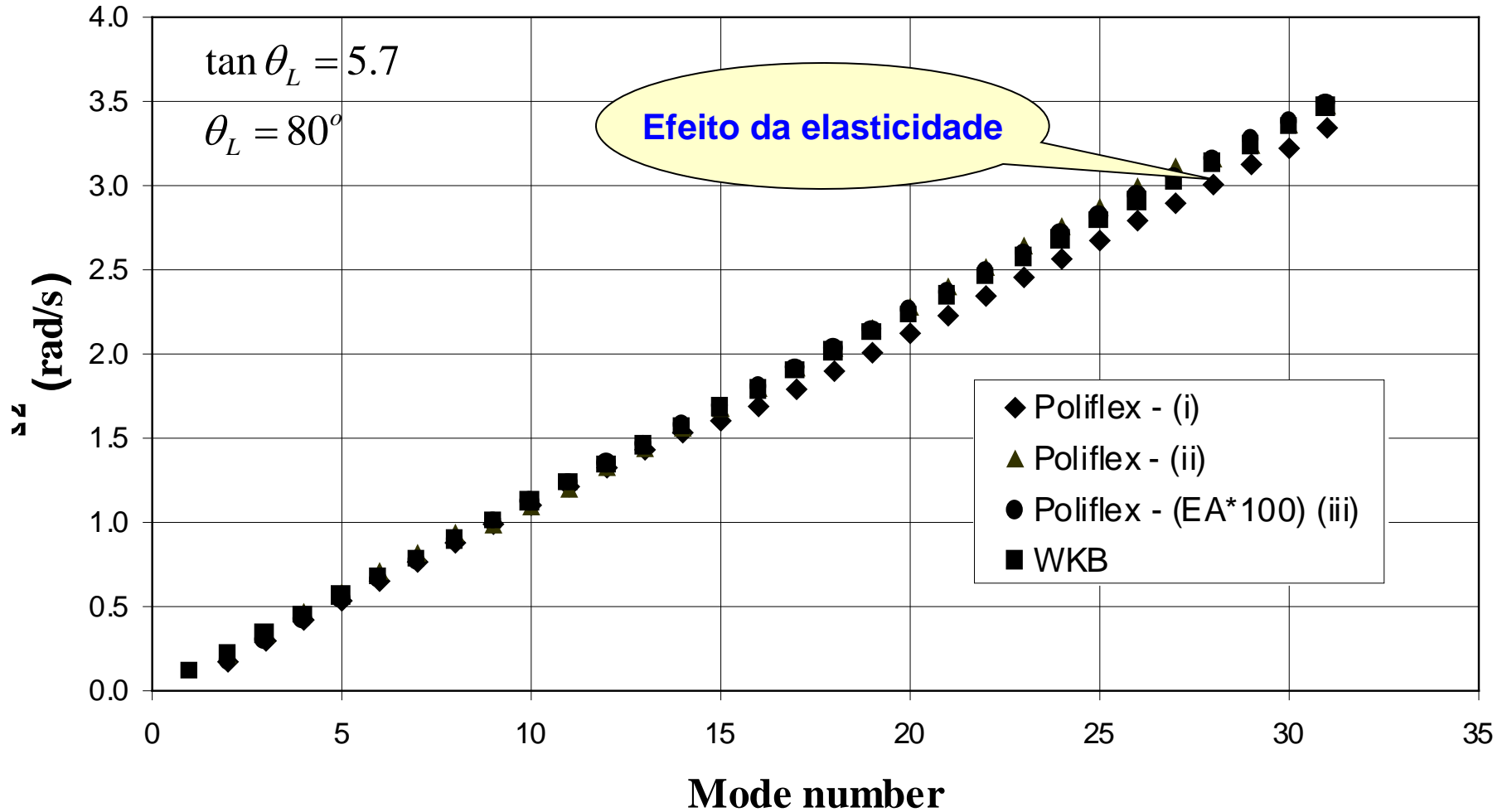
WKB vs. POLIFLEX

Flexible Pipe

$$\mu = \frac{\sigma}{\tan L} = 5.7$$

$$\tan \theta_L = 5.7$$

$$\theta_L = 80^\circ$$



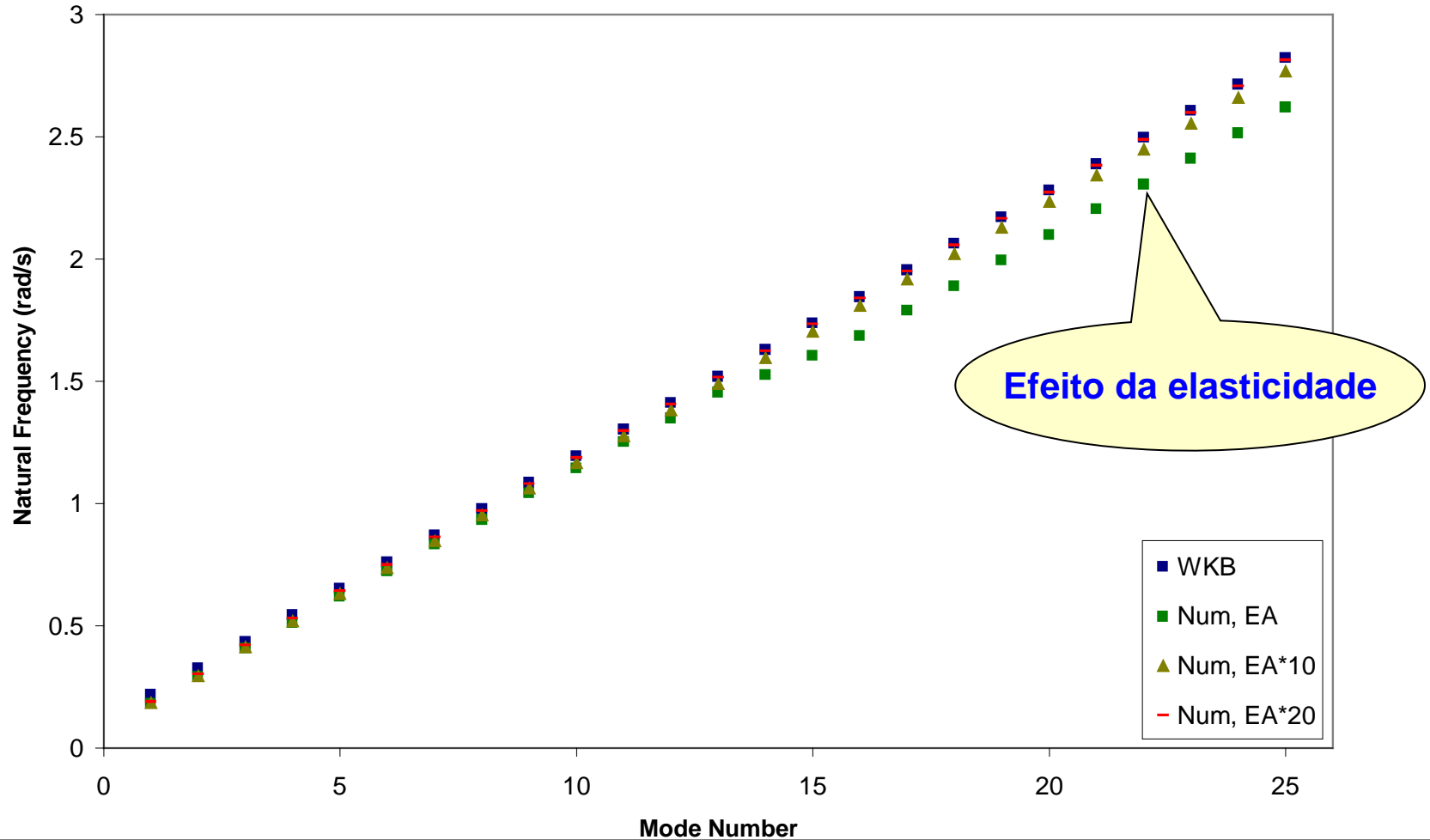
SCR Típico

Axial Rigidity, EA (kN)	2.314×10^6
Bending Stiffness, EI (kNm ²)	9915
Immersed weight, q (kN/m)	0.727
m (kg/m) (filled with water)	108.0
External diameter, D (m)	0.2032
Thickness (mm)	19.05
Depth H (m)	1800
Total length (m)	5047
Angle at top, θ_L (°) (no current)	70 (w.r.t. horizontal)
Soil Rigidity, k (kN/m/m)	466.37
Suspended length, L (m)	2571
Static tension at TDP, T_0 (kN)	680.55
Flexural length, λ (m)	3.82
Curvature at TDP, χ_0 (m ⁻¹)	1.077E-03
Nondimensional curvature at TDP, $X_0 = \chi_0 \lambda$	4.114E-03
Local scale, $\varepsilon = \lambda/L$	1.486E-03
Nondimensional soil rigidity parameter, $K = kEI/T_0^2$	10

WKB vs. POLIFLEX

SCR

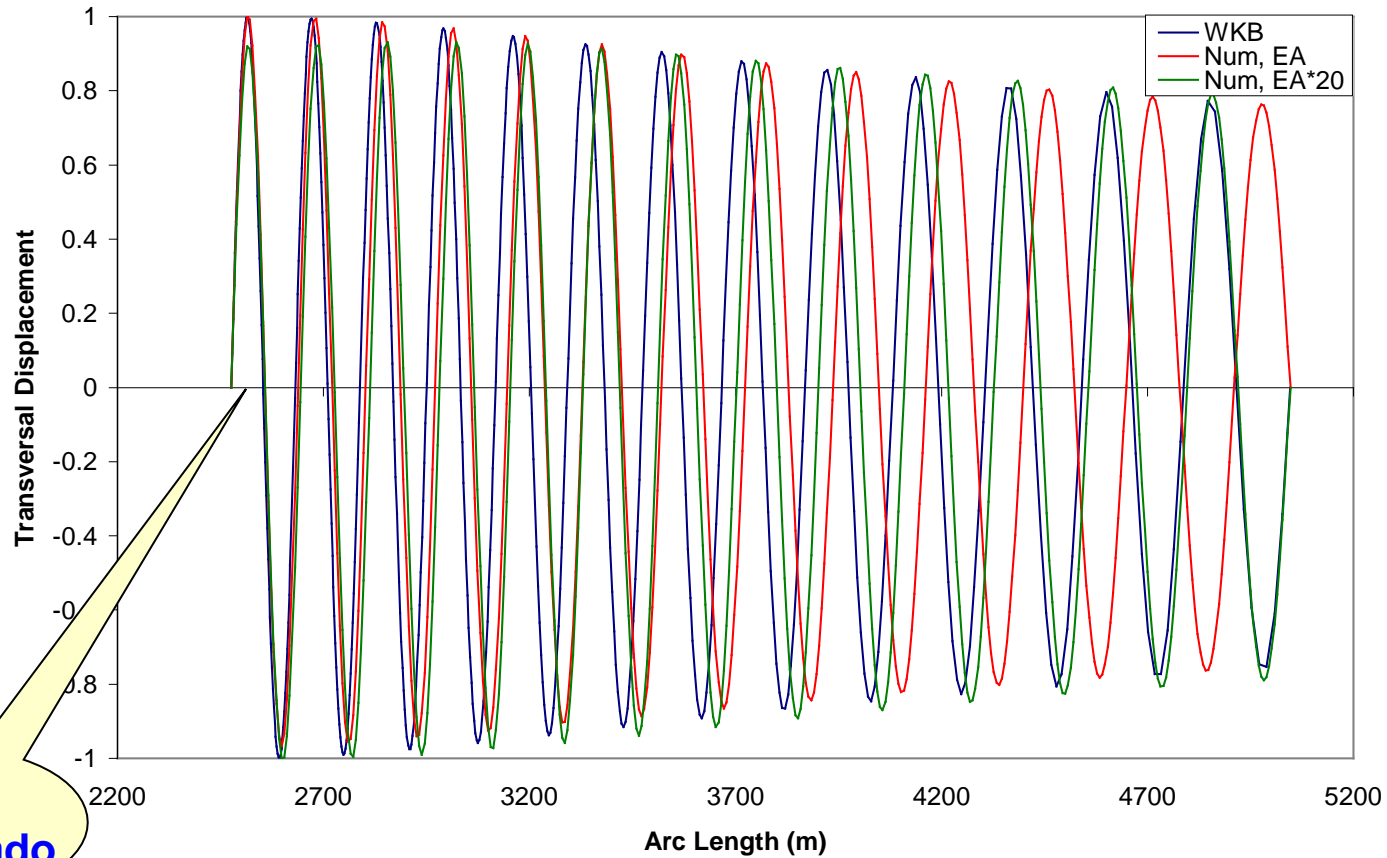
Natural Frequencies for a Catenary Riser; WKB compared to numerical approach



WKB vs. POLIFLEX

SCR: Modo 25

Transversal Displacement - Mode 25



TDP articulado

Sumário

- A dinâmica global é regida pela rigidez de catenária.
 - O efeito da rigidez flexional é importante junto às extremidades ou quando os modos de vibrar tem comprimento comparável ao comprimento de flexão local.
 - Existem diversas escalas de tempo que regem a dinâmica da linha.
 - É possível construir uma solução em forma fechada, via técnica WKB, para o problema de uma linha no plano.
 - Para o caso de uma catenária pura, na ausência de correnteza, a solução é analítica.
 - A solução numérica via POLIFLEX, que incorpora o efeito da elasticidade, recai assintoticamente na solução analítica quando a rigidez axial é aumentada.
-