

Yves Chevallard (English)

THE ANTHROPOLOGICAL THEORY OF THE DIDACTIC

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Yves Chevallard's importance in the field of the didactics of mathematics comes from the singular way he considers teaching and learning of mathematics as much as from the types of empirical data he offers to study. By enlarging the analysis' scope of didactical phenomena he managed to point out the main constraints that bear on the educational system and built fruitful theoretical and methodological tools. Furthermore, one of his great achievements is to have shown the necessity to associate the analysis of mathematical knowledge with the study of institutional practices, in which these elements of knowledge are created, developed, used, spread, taught and learned.

Trained as a logician, Yves Chevallard started his career as a mathematics researcher in this field in the beginning of the 70s. However, he rapidly focused his interest on questions about the teaching of mathematics, a field of investigation that he discovered while attending a conference by Guy Brousseau in 1976. Inspired by his reading of Michel Foucault, Pierre Bourdieu and Louis Althusser – whom he discovered while attending his lecture at the École Normale Supérieure in Paris – Yves Chevallard chose right from the beginning of his work, to built a didactic theory clearly in the line of the Theory of Didactic Situations, that Guy Brousseau was developing at that time. His originality is to try to take into account the institutional relativity of knowledge, on which he bases his analysis of didactical phenomena. His work in the 80s bears on phenomena that he interprets in the light of the didactic transposition, that will be enlarged from the 90s into the Anthropological Theory of the Didactic (ATD).

At the origins: an emancipative theory

The creative power of Yves Chevallard's research work lies at first in his epistemological and institutional emancipative positioning with regard to the institutions, in which the elements of knowledge studied by the didactics of mathematics "live". For him, indeed, things cannot be considered as "here since or for ever", elements of knowledge are products of human constructions, their place and function vary according to places, societies, and periods of time. An engineer modelling activity on a production chain, a journalist commenting on recent pools, an architect calculating the resistance of some hardware, a teacher teaching addition... all participate socially in the diffusion of mathematical knowledge or know-how, among different groups. In this context, mathematics is made of human activities, produced, spread, managed, taught, among a large variety of social institutions.

Objects studied by researchers in the didactics of mathematics live within institutions, of which they are themselves subjects. This situation necessitates a critical position in order not to assume as given what indeed needs to be questioned. In this sense, the emancipation offered by the ATD lies in its rejection to validate intellectual products naturalised in common culture, in its attention to the relativity of contents and forms of knowledge and its claim that researcher in the didactics of mathematics necessarily need to make a step aside, in order to analyse institutions, of which they are themselves subjects. The ATD offers modelling and analysis tools for these human activities, which allow a control of the implicit constraints that any institution imposes on any practice that it shelters. This explicit

search for an epistemological break is what allowed to point out phenomena that could be interpreted in terms of didactic transposition. “Where does knowledge present in different didactical systems originate?” was the first question, whose study gave birth in the 80s to the theory of didactical transposition, for which Yves Chevallard’s name is now famous all around the world (Chevallard, 1985a).

At the origins: the theory of didactic transposition

The theory of didactic transposition questions what seems obvious, about knowledge present in didactical systems (and therefore breaks a certain illusion of transparency), about the fact that identical objects could live under different names, or more generally about the inclination to see only what institutions point out as worth of interest. Looking from a certain distance is the only way to see the effects of the institutions accurately. Mathematical knowledge is most often produced outside school and is subject to a series of adaptations before being accepted for teaching: mathematical objects created by mathematicians are not the ones taught in school. The object of the theory of didactic transposition is precisely to describe and explain the phenomena of transformation of knowledge from its production up to its teaching (Bosch & Gascón, 2005).

This is how the theory of didactic transposition allows the distinction between academic knowledge produced, for instance, by mathematicians, knowledge to be taught defined by the educational system, knowledge taught by the professor and finally knowledge learnt by students. This work of transposition is a social construction made by lots of different persons within various institutions: political authorities, mathematicians, teachers and their associations define the issues of teaching and choose what should be taught, as well as under which form. This level of institutional organisation is what Chevallard calls the “noosphere”, it sets up the limits, redefines and reorganises the knowledge in socially, historically or culturally determined contexts, which make possible or not certain choices. Beside the reference book that Chevallard published for the first time in 1985, *La transposition didactique – Du savoir savant au savoir enseigné*, which has been reedited and translated in Spanish, several works have studied phenomena of didactic transposition about various mathematical domains: elementary algebra (Chevallard 1985b, Kang 1990, Coulange 2001), proportionality (Bolea et al. 2001, Comin 2002, Hersant 2005), volume (Menotti 2001), geometry (Tavignot 1991, Chevallard & Jullien 1991, Matheron 1993, Bolea 1995), irrational numbers (Assude 1992, Bronner 1997), functions and calculus (Artigue 1993, 1998 ; Ruiz Higuera 1994, 1998 ; Chauvat 1999 ; Amra 2004 ; Barbé et al. 2005), linear algebra (Ahmed and Arsac 1998, Dorier 2000, Gueudet 2000), arithmetic (Ravel 2002), proof (Arsac 1989, Cabassut 2005), modelling (García 2005), statistics (Wozniak 2005), mathematics in economy (Artaud 1993, 1995) ; but also in other disciplines as different as physics (e.g. Johsua 1994), music (e.g. Beaugé 2004), or sport (e.g. Barbot 1998).

Furthermore, the wish to fight against the illusion of transparency has motivated, from the mid 80s, the introduction of the “ecology approach” in didactic of mathematics (Rajoson, 1988). This approach is based upon a set of persistent questions: What does or does not exist? What should exist? What could exist? What are the conditions, which favour, allow or on the contrary make difficult or even prevent the existence of such object? (Artaud, 1997). The answers given to such questions bring to light conditions of existence of mathematics in the educational system, which bear on mathematics itself, as well as on the systems in which they live. Bringing in the notion of ecosystem makes it possible for the researcher in didactic of mathematics to consider in relation with mathematics several new objects outside mathematics. The ecology viewpoint is today an essential positioning in the use of techniques of analysis with tools from the ATD. Its field of intervention has been enlarged and enriched. The various works consisting in determining ecological conditions of existence of mathematical objects have finally led to a structuring schema in nine levels, called levels of didactical codetermination going from the most specific (subject, theme, sector, domain, discipline) to the most generic (pedagogy, school, society, civilisation). This structuring schema has proven to be most productive recently, while bringing into light the most determining elements constraining the didactical systems (Wozniak 2007).

A didactic anthropological theory

The type of questioning generating the theory of didactic transposition calls for a more accurate distinction between objects which seem to be the same, but do not live in the same manner from one institution to another, since they are not used to do the same thing. Moreover, to describe and analyse the genesis and evolution of elements of knowledge in a given institution, as well as personal and institutional relations to these elements, it is necessary to design a model of these elements of knowledge or know-how. The difficulty is that no elements of knowledge can be totally isolated, but is rather always part of an aggregate. Within the ATD, a significant breakthrough came with the modelling of such aggregates in terms of praxeologies made of the two components: praxis and logos. This model came initially from an attempt to describe the mathematical activity in relation with the concept of institutional relations and with use of the notion of ostensif (Chevallard 1994, Bosch & Chevallard 1999).

The notion of praxeology insists on the techniques, which allows to accomplish certain types of tasks, bringing to light the plurality of techniques for one type of task, hidden within the subjection to a didactical system. On the other hand, it insists on the technological function of knowledge (for producing, justifying and making techniques comprehensible). This points out a system of conditions and constraints bearing on the existence or absence of such technique, in such institution. An element of knowledge is before all a discourse making possible to justify, produce, make comprehensible techniques and not only what the culture designates as obvious under the label “knowledge”. In this sense, the praxis refers to the practice, the know-how in some ways, while the logos refers to the theory, the discourse describing, legitimising, explaining, the praxis. Therefore, a praxeology does not encompass the study of human practice, but the “science”, personal or institutional, of a certain practice. It is thus relative to the person using it or to the institution in which it can live. The use of the notion of praxeology gives a fundamental model in order to apprehend the elements of knowledge, to study their transformations, and to give account of what is done with them in any particular institution. It makes explicit the epistemological model of reference, which nourishes the analyses of transposition phenomena.

From the profession of teacher to the epistemological refoundation

Chevallard’s first works, centred on the study of didactical transposition phenomena and the use of the ecology viewpoint, immediately produced elements of knowledge on didactical systems and contents for mathematics teachers’ training. Yves Chevallard developed these contents during in service training sessions, in the context of the IREM^[4] of Aix-Marseille, with a constant care for answering the needs of the profession of mathematics teacher. This attention to what is now called the problems of the profession (Cirade, 2006) leads, along with the constitution of a clinic of didactical phenomena, to a development of the theory as well as its practical realization.

As soon as he got a position as professor in the IUFM^[5] of Aix-Marseille – into the creation and development of which he has been strongly involved – he set up most of his work in the context of mathematics teachers’ pre-service training, that he has been supervising for the last 15 years. The various didactical designs that he has set up have allowed, through the years, the constitution of a text of professionally orientated knowledge, in terms of “archives for training”.

The research design that he conjointly sets up is one of the main originalities of Yves Chevallard’s activity in research. It is common for a researcher in the didactics of mathematics to use the classroom as a “laboratory” for the study of didactic engineering, by experimenting the didactic situations that he elaborates. Based on his experience as teachers’ trainer in a IUFM, Yves Chevallard sets up, more than a laboratory, a clinic for mathematics classes, their teachers and their students. Innovative training designs are created (Chevallard, 2006), such as one called “the questions of the week”: each trainee-teacher is invited to raise an issue in relation with his own teaching practice. Some of these issues are then debated and studied within the whole group of trainees. These questions of the week, adding up to

around a thousand every year, reveal the major problems of a profession in mutation, especially for these repeated year after year.

The whole set of data produced by teachers-trainees and Yves Chevallard's seminar, – about 450-500 pages every year – constitute the “archives for training” and give to researchers clinical data, which have allowed recently the development of what has become the “clinic for training” (Chevallard 2007, Cirade 2007) in relation with a new approach known as the dialectic of medias and milieux (Chevallard, 2006). One plays against a system, which is not free of didactical intention. The goal is to point out among the “responses” of the system, the elements, which have some chance of not sustaining any intentional strategy, but are only here, like any symptom which is not commanded.

This position of trainer, open to problems of the profession, led Yves Chevallard in the second half of the 90s, to introducing the model of the didactic moments, as a means of analysis of the didactic praxeologies. This means studying and analysing the difficulties of teachers while implementing a new teaching design (called modules) imposed by the French institution (Ministry of Education). Indeed, how can one describe the diffusion and in particular the difficulties of diffusion of didactic praxeologies in an institution, especially school? How can one explain that a didactic situation cannot ‘live’ in school, that the conditions and constraints on the teacher or school prevent that such didactic situation can ‘live’ in the class? An essential condition is that the elements of knowledge be apprehended from the viewpoint of the *raison d'être*. Why, for instance, should one teach the properties of triangles? What are the questions that this subject allows to study? In order for school to be able to let these questions live as generative of knowledge, one must act in two directions: epistemological and didactical. Yves Chevallard's indefectible care for answering the needs of the profession of teacher and of the society led him to the exploration of these two ways (Chevallard, 2002a, 2002b). The first way consists in developing a functional way to approach an element of knowledge, that Yves Chevallard structures in terms of Study and Research Activities (SRA) and more recently in terms of Study and Research Path (SRP). In doing so, he meets one of the central aspect of the Theory of Didactic Situations developed by Guy Brousseau, precisely the conception of fundamental situations. Moreover, the study of didactical systems, leads to the emergence of the notion of moments of the study, each of them corresponding to one specific didactical function in the process of the study. The didactical moments then appear as some types of task for the study. The model of the mathematical organisations in terms of praxeologies and of didactical organisations in terms of moments of the study allows the study of the didactical systems, from the viewpoint of knowledge as well as its activation. Today, the study of the didactical praxeologies constitutes one of the most promising vehicles of development for the ADT, especially in the specific context of the use of new technologies.

In conclusion, one can say that three different ingredients are therefore essential in the theorisation that Yves Chevallard has been conducting in the last thirty years:

- a deep anchorage within mathematics,
- a willingness for breaking the illusion of transparency (not trusting what the institution put into light and pointing out the conditions explaining what exists or not),
- a clinical approach to didactical phenomena, in articulation with their theorisation which complete the experimental approach like most research programs in mathematics education.

Devotion to the community of research in didactic of mathematics

Yves Chevallard has always cared to create the conditions for production and diffusion of research in the didactics of mathematics to the widest audience. In this sense, he has been director of the IREM of Aix-Marseilles between 1984 and 1991. He also took a great part in the creation of the IUFM of Aix-Marseilles, in 1991, being a member of the administrative board from the beginning, as well as director of the scientific and pedagogic council from 1991 until 1999 and director of research and development from 1991 until 1997. He also created and directed the scientific journal *Skholé*, and has been head of the mathematics department since 1991. Recently he enlarged his audience in the context

of the department of education science in the university, in order to claim that “the didactic care is an eminent social duty”.

Yves Chevallard has also been chief editor of the international journal *Recherches en didactique des mathématiques* from 2000 until 2002, member of the scientific board of the collection *Raisons éducatives* published by the Faculty of Psychology and Education Science of Geneva University, member of the editorial board of the journal *Éducation et didactique* recently created. His care for the diffusion of the theoretical framework he has created takes shape in his important participation to juries for doctorates and his electronic publications through Internet (<http://yves.chevallard.free.fr/>). Yves Chevallard is indeed a prolific researcher, whose list of publication covers over 13 pages: 3 books in French, one translated in Spanish, 1 book in Spanish, also translated in Portuguese, 15 participations to collective books, 36 articles in international journals more than 60 communications in international congress, and many in various seminars.

Outside French speaking countries, Yves Chevallard has close cooperation with Spanish and latino-american countries. The Spanish translation of his book about didactic transposition in Argentina in 1997 has widely contributed to the diffusion of this approach in all parts of education. His book in Spanish (Chevallard, Bosch et Gascón 1997) is about to be diffused by the Ministry of Education in all Mexican schools in pocket edition. The ADT today represents a spreading field of research regrouping about 200 French or Spanish speaking researchers over four continents, Europe, America, Asia and Africa. The two international congresses on the ADT (Baeza, Spain, 2005 and Uzès, France, 2007) are proofs of the dynamics and importance of the projects around which a community of research of ADT is being built. A teachers' training program set up in Marseilles (since 1990); a project of curricular development encouraged by the Ministry of Education in Chile that mobilises since 2002 a whole team of researchers working with teachers and students of 300 primary schools; a research team about renovation of secondary and tertiary education using mathematical modelling in Spain; and research teams working on different subjects in Latin America, Canada, Vietnam, North Africa, South Africa, and Europe (Belgium, Denmark, France, Italy, Sweden, Switzerland).

Je ne saurais pas dire tout ce que la collaboration avec Yves m'a apporté comme idées et comme plaisir. Sa culture, la précision de sa pensée, son écoute aussi m'ont vraiment « éduqué » sans jamais infléchir mes propres démarches.

These words of recognition addressed by Guy Brousseau to Yves Chevallard during the first international congress on the ADT in Baeza reveal, beyond the friendship of these two exceptional didacticians, the close and original relation that bounds their two theories and therefore the essential place of each of them in French Didactic of mathematics but also in the world of research in Mathematics education.

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