Guy Brousseau (English)

A prominent researcher in a central field for mathematics education
A life dedicated to the understanding and improvement of mathematical education and learning

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Guy Brousseau's career is inscribed in the history of the past forty years concerning the changes in mathematical education. Il is linked to the creation of the great paradigms that have structured fundamental research in this field. This becomes obvious when retracing the steps of his academic path, his contribution to research in mathematics education, his involvement in collective projects and international exchanges and finally the various dimensions of his influence.

An exceptional career

Guy Brousseau began his career as a student of a teachers training college in order to become a primary teacher. He remained a primary teacher for a couple of years before joining thanks to a secondment, all people that were involved, at the beginning of the sixties, in the launching of the general movement for change in mathematical education. With the support of the governing body, he completed his university career before being employed as an assistant at Bordeaux I University. It was at this same university, at the IREM[1], and with the continuing support of Professor Jean Colmez, that he carried out most of his research on mathematics education in compulsory education. He submitted his thesis in 1986. With the support of the academic authorities, he set up the COREM[2] that he was in charge from 1973 to 1998, before creating the LADIST[3] a laboratory linked to the COREM. In the meantime the creation of the IUFM enabled him to become a University Professor in 1992, until his retirement in 1998. He then became professor emeritus at the IUFM[4] which permitted him to continue his scientific work (supervising theses) in a new laboratory affiliated with Victor Segalen Bordeaux II University, the DAEST[5].
Guy Brousseau's first published work appeared in 1961 at the CIEAEM[6] conference in La Chataigneraie (Switzerland). This was followed by a text book aimed for first year elementary school (Grade1) (1965) and soon after came a succession of regular publications in the field of didactics from 1968 to the present day. The great intricacy of his personal work in teacher training at the IREM, also the specificity and originality of his research lead to the publication in university produced literature (Journal of the IREM from 1969 to 1978) of some essential articles to understand the fundamental theoretical instrument that is the theory of didactical situations. One can find these papers as well as others that were previously published in various journals such as RDM[7] in a volume published in English by Kluwer and entitled "*Theory of Didactical Situations in Mathematics*".

Profound and original scientific choices

Guy Brousseau's passion for mathematics education came from a double fascination: fascination for mathematics on the one hand, its explanatory ability and capacity to train the process of thought, and, on the other, fascination for the transfer and spread of knowledge, as well as the study of conditions that make it possible. Throughout his scientific career, in serving this double passion, he knew how to summon up constant and inexhaustible energy, faultless determination, limitless curiosity and extreme precision that led him to develop and propose the most thorough and coherent theory of the past thirsty years.

This approach and the way of thinking emerge, in their force and particularity, in the second half of the sixties. Brousseau at that time took an original and decisive theoretical decision that is presented in a fundamental paper: "*The process of mathematization*" given at a talk during the annual conference of the APMEP[8] in 1970. This paper was an important and major contribution. Its typicality and relevance cannot be denied.

Considering the student and the teacher as essential actors of teaching and learning, it would also be necessary to first of all focus one's attention to a third instance, "*the silent actor*": the situation in which they evolve and in which the student's and teacher's activity unfurls according to their respective goals: to learn and to teach. The situation is organized by one and experienced by the other, it evolves through the interplay of their interaction according to the rules, most often tacit, activated at the very heart of the didactical contract. The situation is designed as a model of the knowledge to be taught. It is at the same time, the condition for establishing a didactical relationship specific to the knowledge in question and the preferred instrument in the teaching-learning process. If one wants the situation to allow the learning of mathematics, it must not be arbitrary in the modalities of action it offers to the student.

One might characterize the irruption of the notion of situation as a central topic of research from two points of view:

- The first being that it consists in adopting, in a certain way, a dual position, compared to the experimenter who approaches and questions the students, with the help of specially designed tests concerning their conception of mathematical topics they have encountered, in education or in their diverse experiences in everyday life. The didactical project is something else entirely. Il consists in turning this perspective upside down and is concerned with the problems and situations themselves, for the way in which they inform us of the knowledge bring into play and that they activate. Thus, one no longer studies the subject "*in abstracto*" but instead the situation for the potential it must offer the student, whether this be in his mathematical activity or in the context of learning teaching process as a subject of a didactical institution.
- The second point of view takes as a starting point the consideration of the non didactical situation, in other words the context of employing mathematics whether that be in the work of mathematicians or a everyday user in an environment of specific practice. Indeed, the knowledge of mathematics could never be reduced to just the knowledge of theorems and algorithms, but requires the ability to recognize when necessary the conditions of their use. The meaning of a mathematical concept does not depend on an interplay of external obligations linked, for example, to the use of a piece of knowledge, a requirement that exists in all didactical commands. Based on this analysis, the main theoretical approach then consists in studying the conditions of setting up a didactical system of situations that involve the student as in non didactical ones. Guy Brousseau refers to these situations as "adidactical". For him, it is a matter of showing that it is possible to set up adidactical situations and to be aware of their function. Both on a theoretical level (the rule required relation to the knowledge in question) and in the contingency (by examining through observation the conditions of "didactical viability", in other words their creation within the constraints of the mathematical classroom.

Guy Brousseau puts in evidence that the success of establishing these conditions involves two aspects, which he decides to study more closely.

The first aspect concerns the setting up of the situation itself. This led him to propose a new concept, that of "*devolution*": if knowledge pre-exists to students, their understanding requires a common practice obviously expected by the teacher, but that cannot be imposed by him to the students,; that is

the paradox of devolution : "*If the teacher says what he wants from the student, he can no longer obtain it !*" (Brousseau, 1998). Brousseau initially endeavoured to study this paradox in the sixties by looking at the conditions in which this paradox can be overtaken by the devolution to the student of adidactical situations(Which basic strategies can students develop in this situation? Which retroactions will he get from it? What didactical variables are likely to keep the meaning of the target knowledge? The teacher attempts to ensure that the student's actions are carried out and justified purely by the demands of the *milieu* and not by the interpretation of the teacher's didactical behavior nor its expectation.
The second aspect is closely linked to the first since it concerns the conditions for maintaining the students commitment to the situation. Based on a clinical case today well known amongst the community of mathematical didacticians, "*The case of Gaël*", Brousseau studied the set of mutual obligations that each partner in the didactical situation imposes or believes to be imposed on the others, and those that are imposed on him or that he believes to be imposed on him, concerning the knowledge in question: this is the concept of "*didactical contract*". It corresponds to the outcome of an often implicit negociation of the setting up of the relationship between a student, a certain milieu, and an educational system. This is not a real contract : it is neither explicit nor consented to freely, since it relies upon knowledge necessarily unknown to the students. It positions the teacher and the student to face a truly paradoxical demand : if the teacher explains what he wants the student to do, he can only obtain it as the carrying out of an order and not through using knowledge and judgment. The reverse is also true; if the student accepts that the teacher shows him the solutions and the answers, he will not discover them by himself and therefore will not be able to appropriate it. Learning requires therefore the refusal of the contract in order to tackle the problem independently (devolution). Learning will therefore depend not only on the correct functioning of the contract, but also on the ruptures of it, hence the importance of studying the actual conditions of these ruptures more closely.
Moreover, as an actor in the situation the subject is aware of the knowledge, but this is not enough for it to be learned, because if the students experience is a necessary condition, the knowledge activated must also be recognized as such, then classified and incorporated into socially accepted knowledge. Guy Brousseau thus highlighted the need for "*institutionalization*" and paved the way for a new field of theorization of educational phenomena.

The theory put to the test by facts : the methods and the COREM[9]

A major concern of Guy Brousseau was to carry out experimental study of the phenomena of mathematical education, a scientific project that was born out of a general schema based on the interaction between the topic studied, understood within the frame work of an adapted theoretical paradigm. In this case, the theory cannot determine what it must be. He provides a model of the facts, summons and brings the phenomena to light in order for it to be analyzed and interpreted in a paper published in 1978, entitled "*The observation of didactical facts*", Guy Brousseau provides a solid foundation for the method that was at the heart of his work. Il is constructed around observation applied in the field of didactics: it is then a matter of putting together a collection of facts and constructing them as didactical phenomena, studying their reproducibility and their degree of generalization and consistency.
The COREM, the principle of which had been defined by Guy Brousseau at the end of the sixties and that he was able to be realized with the support of the authorities from 1972, enabled him to conduct this study. These research facilities, unfortunately unique in their kind, continued to function until the end of the nineties. The COREM was the product of joining together a primary school with facilities, welcoming the research and observation of classroom's situations proposed by the researcher. These situations were designed and constructed, using the theory of didactical situations, upon the questions and hypotheses particular to the research undertaken and on the expertise of the teachers that took the responsibility of the class. The theoretical notion and practice of "*didactical engineering*" takes into account the workings of a system that depends on a close collaboration between teachers and researchers.

Moreover, in order to back up this scientific research, Guy Brousseau contributed to the development of the use of statistics in research in mathematics education from a heuristics point of view (multidimensional analysis for example) and theoretical hypothesis testing (inferential statistics, descriptive statistics and the investigation of the facts). He contributed, in particular to the creation and use of implicative analysis in didactics. (Gras and Lerman)

The main notions developed in the field of didactics

- The fundamental notion is that of *situation*; it can be modeled as a formal game. The possibility of isolating in the specially constructed situations, like "*the race to twenty (20)*[10]" for example, moments of *action*, moments of *formulation*, moments oriented towards *validation* and the tools involved at each of these moments, and finally, moments for *institutionalization* constituted a major part of the work carried out for more than thirty years on various mathematical topics. This shows both the significance and heuristic value of this theorization and demonstrates the success of Guy Brousseau's scientific research project.
- The *didactical transposition* is a concept that was originally developed by Yves Chevallard to explain the transformations that mathematical subjects undergo when made to enter a didactical system. In the paradigm of the theory of situations, this concept is defined and activated by the notion of the *fundamental situation* for a knowledge, that constitutes a privileged study tool of phenomena involving transposition by defining the conditions for preserving the meaning of knowledge at the moment of transposition.
- The concept of *didactical contract*, central to the analysis of the workings of the didactical systems, was recently taken up again by Guy Brousseau himself, from the perspective of modeling different types of contracts. Other researchers have studied, from a different perspective, the didactical situations likely to explain why certain students prove to be more sensitive than others to the implicit factors raised by the contract, as well as the links this phenomenon of sensitivity to the contract has with the traditional question of educational differences. (B. Sarrazy)
- The concept of *obstacle*, taken from the work of the French epistemologist Gaston Bachelard, enabled original approaches to be developed concerning conceptual difficulties and analysis of students' errors. This concept has been particularly productive in the analysis of the difficulties experienced when moving from whole numbers to decimals.

The proposed distinction between the knowledge actuated in action (*C-knowledge*), the product of the subject's activity in his relationship with the milieu and the knowledge acquired in the institutions (*S-knowledge*) has opened up a new field of study related to the role of enumeration in the construction of numbers (J. Briand) and another concerning the treatment of relationships between spatial knowledge and Euclidean geometry (R. Berthelot, M.-H. Salin).
- The concept of *milieu* for action and its organization enables one to create a model of the necessary ruptures implemented in the subject's change of references in a didactical context (distinction between learning situation and didactical situation). This concept, introduced right at the beginning of the theorization of didactical facts, was taken up again and developed by C. Margolinas, in particular to analyze the teacher's action in ordinary classes.
- *Didactical memory* is a fundamental concept that enables one to explain phenomena linked to didactical time and it's progression: conversion of knowledge through the action of institutionalization (J. Centeno).
- The position and the role of *institutionalization* that consists in laying down components taken from knowledge developed in adidactical situations, contribute to the construction and explicit location of knowledge and thus ensures the establishment of consistency between learning and the teaching objectives set by the institution (A. Rouchier).
- The notion of *didactical assortment* is more recent. It enables one to study the structuring of the groups of activities and exercises brought together for teaching purposes (F. Genestoux).

The mathematical fields studied

Whether it be directly, through his own work or that of his students or even through work conducted in the paradigm that he set up, Guy Brousseau was interested in all areas of mathematics, notably those covering the curriculum of compulsory education.
The difficulties of learning some standard algorithms of multiplication and division, the aspects of other algorithms, from the point of view of both facilitating the learning process and their use, the early stages of teaching them : the meaning of the operation and construction of the algorithm (G. Brousseau).
The first lessons on numbers and numeration. The fundamental situation of numbers, averages in order to make a set “equipotent” to a given set combined with the use of didactic variables enables one to generate a large number of situations concentrating on action or communication, allowing one to successfully structure the early stages of learning.
The creation of a code of designation in a group context at kindergarten level.
Probably at the end of elementary school: meeting situations in which the early notions of probability are means of decision making (G. Brousseau).
Rational numbers and decimals: fundamental situations and complete early progression constructed following a program that lasts several years (G. Brousseau, N. Brousseau).
The required diversity of contexts and situations in which mathematical reasoning is specified: solving classroom arithmetic problems, situation of multiple choices, etc (P. Gibel, P. Orus, B. Mopondi).
Fixing the position of prior knowledge that has not been formalized and its effective treatment in education: the case of geometry (R. Berthelot, D. Fregona, M-H Salin) , enumeration (J. Briand) and that of reasoning (P. Orus).
The teaching of subtraction and the group of situations set out in the box game (G. Brousseau).
The study of the conditions of the transition from classroom arithmetic to algebra (D. Broin).
The notion of function and the role of graphical representation (P. Alson, I. Bloch, E. Lacasta)
The early stages of proportionality: a fundamental situation based on the notion of equitable share (E. Comin).

An active role in the commitments of a generation

Guy Brousseau’s commitment to mathematical education, and the study of the questions it raises wasn’t only noticeable in the realm of research.
On a national level, he played an extremely important role, notably as part of the Association of Mathematics Teachers, which enabled him to actively contribute to the creation and setting up of the IREM. These are institutions unique in the French institutional context, from which crossed collaboration was developed to serve mathematical education by supporting three areas : research , innovation and teachiers training. It was on his initiative that a national work group was created, which has united trainers of elementary school teachers for thirty years: the COPIRELEM (the Permanent Commission of the IREM for Elementary School).
He also took very active role in the creation of numerous instruments for collective scientific activity, dedicated to the training of young researchers; with the support of Professor Jean Colmez, Guy Brousseau created the first postgraduate course in Didactics in Mathematics in France, to the debate and circulation of ideas : amongst them, one must mention the scientific journal (RDM)[11], the association of scientists (ARDM)[12], the Summer School and the National Seminar on Didactics of Mathematics.
One can also notice his commitment on an international level, Guy Brousseau, developing on the work of Caleb Gattegno, Jean Piaget, Willy Servais, Zofia Krygowska, Lucienne Félix, Hans Freudenthal, Ephraïm Fishbein and many other major researchers, was the tireless driving force behind the CIEAEM which he was in charge of for several years and that he kept regular contact with during his summer trips ranging from Switzerland to Mexico, Hungary to Great Britain from 1960 to the beginning of the

nineties. Moreover the term of driving force doesn't fully express the diversity and depth of the work that had to be carried out in a structure that was subject as little as possible to institutional constraints, as was the CIEAEM during the sixties, seventies and eighties. Guy Brousseau equally played a central role in the initial launch of the international group "*Psychology of Mathematical Education*" at the International Conference of the ICME in 1976 in Karlsruhe. He has been and continues to be regularly invited to contribute to collective works and international scientific conferences concerning mathematical education. Guy Brousseau was awarded an Honorary Doctorate from the University of Montreal in June 1997.

Instruments for teacher activity and training and for the research

Guy Brousseau's influence goes beyond the realm of research. In the seventies for example in the INRP (National Institute of Educational Research) and in the IREM, numerous teams were formed to develop experimental products for teaching aiming to generalize through books for teachers and student text books.

These products focused mainly, on the one hand on the theoretical setting provided by the theory of didactical situations, and on the other, on numerous suggested situations and problems constructed and studied at the COREM. The recognition of the role and position of the mathematical exercise task as a likely driving force behind learning for the student, the consideration of epistemological and didactical obstacles, methodical focus, emphasis on fundamental situations, attention given to formulation are as much acquired as they are absorbed into the curriculum and practices of French teachers.

Teacher training has always been a concern for Guy Brousseau. The concepts he developed, proven in their ability to further the understanding of didactical actions, have strongly influenced the current syllabus for training elementary school teachers. One also finds this influence in the recruitment process. Indeed, students wishing to become teachers learn to analyze student output and educational documents through concentrating on categories of analysis taken from the theory of didactical situations. One also finds this influence in other moments of the course, moments when the young student teachers learn about other components of their profession: the construction of teaching and learning situations. Finally, through his contribution to the creation of COPIRELEM whose work he closely followed right from the start, he has enabled elementary school mathematics to have at its disposal a unique tool for national organization of teacher training, linked to the IREM and IUFM.

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- [3] Laboratoire Aquitain de Didactique des Sciences et Techniques.
- [4] Institut Universitaire de Formation des Maîtres
- [5] Didactique et Anthropologie des Enseignements Scientifiques et Techniques
- [6] Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques
- [7] Recherches en Didactique des Mathématiques
- [8] Association des Professeurs de Mathématiques de l'Enseignement Public
- [9] Centre pour l'Observation et la Recherche sur l'Enseignement des Mathématiques
- [10] One can find a presentation and an analysis of this situation in the introduction of the book, Theory of Didactics situations in Mathematics
- [11] Recherche en Didactique des Mathématiques
- [12] Association pour la Recherche en Didactique des Mathématiques