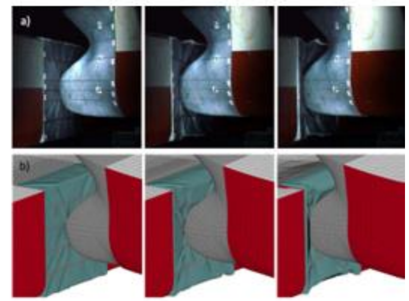
Axial impact on shells

MARCILIO NOV 2017



IMPACT ENGINEERING

Fundamentals

Experiments

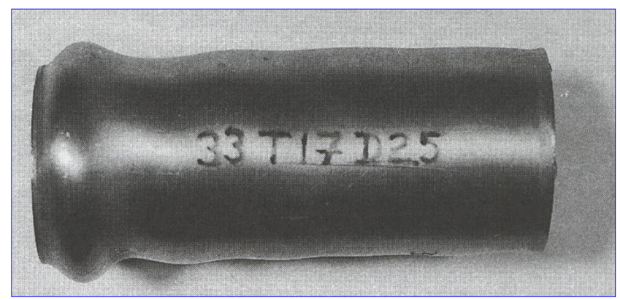
Nonlinear Finite Elements

Marcílio Alves

Axial impact on shells:
major application is absorption of kinetic energy

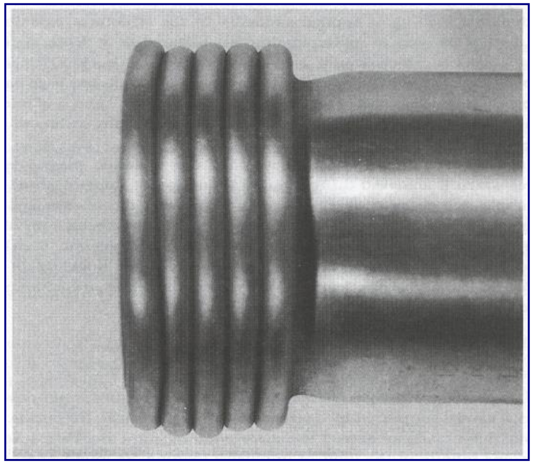
- Progressive buckling
  - Static (no strain rate effects)
  - Dynamic (material flow is affected by strain rate)

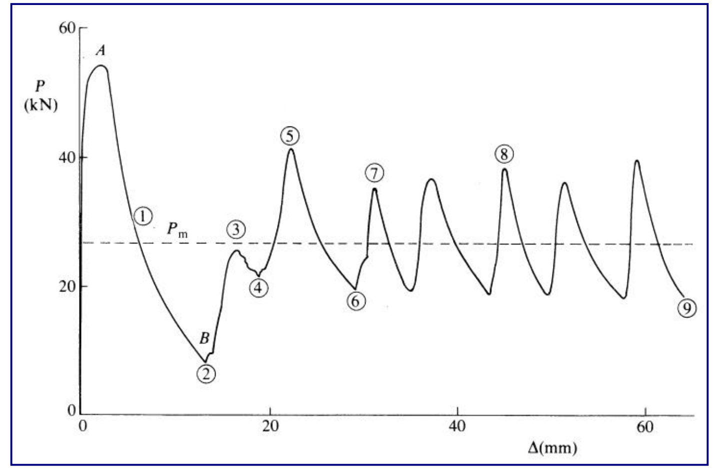
- Dynamic buckling
  - Inertia is very important



- Buckling of long tubes
  - Transition from global to progressive is very important

Progressive buckling of circular shells





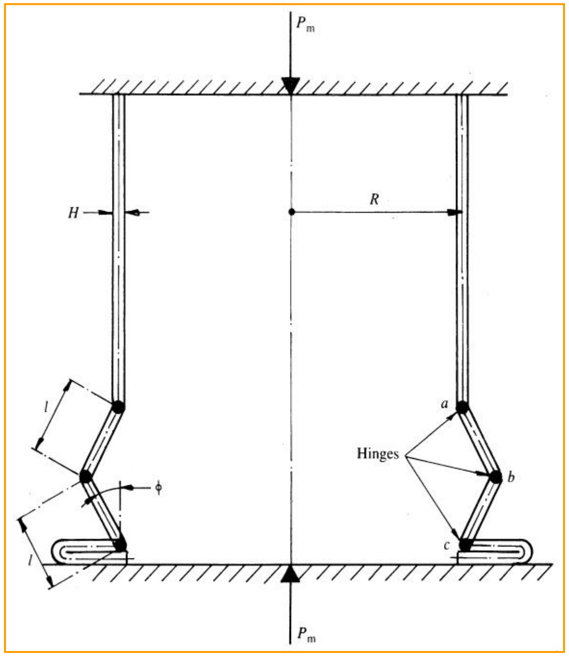
<u>Assumptions</u>:

- ✓ Rigid perfectly plastic material
- ✓ Assymmetric deformation pattern
- ✓ The external work is equal to the internal energy dissipated due to the plastic deformations

$$P_m \times 2l = D$$

N. Jones. Structural impact, Cambridge University press, 1989, Paperback edition, 1997.

Internal energy dissipation



$$D = D_1 + D_2 + D_3$$

$$D_1 = 2 \times 2\pi R M_0 \pi / 2, \quad M_0 = \left(2\sigma_0 / \sqrt{3}\right) H^2 / 4$$

$$dD_2 = 2\pi (R + l \sin\phi) M_0 (2 d\phi) \quad \rightarrow \quad D_2 = \int_0^{\pi/2} 4\pi (R + l \sin\phi) M_0 d\phi$$

$$D_2 = 4\pi M_0 (R\pi / 2 + l)$$

$$d\varepsilon_\theta = \frac{2\pi[(l/2)\sin(\phi + d\phi)] - 2\pi[(l/2)\sin(\phi)]}{2\pi R} \quad \rightarrow \quad d\varepsilon_\theta = l \cos\phi \, d\phi / (2R)$$

$$dD_3 = \sigma_0 d\varepsilon_\theta 2lH 2\pi R \quad \rightarrow \quad D_3 = \int_0^{\pi/2} \sigma_0 (l \cos\phi \, d\phi) 2lH\pi$$

$$D_3 = 2\sigma_0 l^2 H \pi$$

sen(a+b) = sen a · cos b + sen b · cos a

$$\boxed{D = 2\pi\sigma_0 H^2 (\pi R + l) / \sqrt{3} + 2\pi\sigma_0 l^2 H}$$

Axial crushing force

$$2lP_m = D$$

$$2lP_m = 2\pi\sigma_0 H^2(\pi R + l)/\sqrt{3} + 2\pi\sigma_0 l^2 H$$

$$P_m/\sigma_0 = \pi H\left\{H(\pi R/l+1)/\sqrt{3} + l\right\}$$

$$dP_m/dl = 0 \quad \rightarrow \quad H\left(-\pi R/l^2\right)/\sqrt{3} + 1 = 0$$

$$l = \left(\pi RH/\sqrt{3}\right)^{1/2}$$

outside wrinkles: $$P_m/M_0 = 4\sqrt[4]{3}\sqrt{\pi^3}\sqrt{R/H} + 2\pi$$

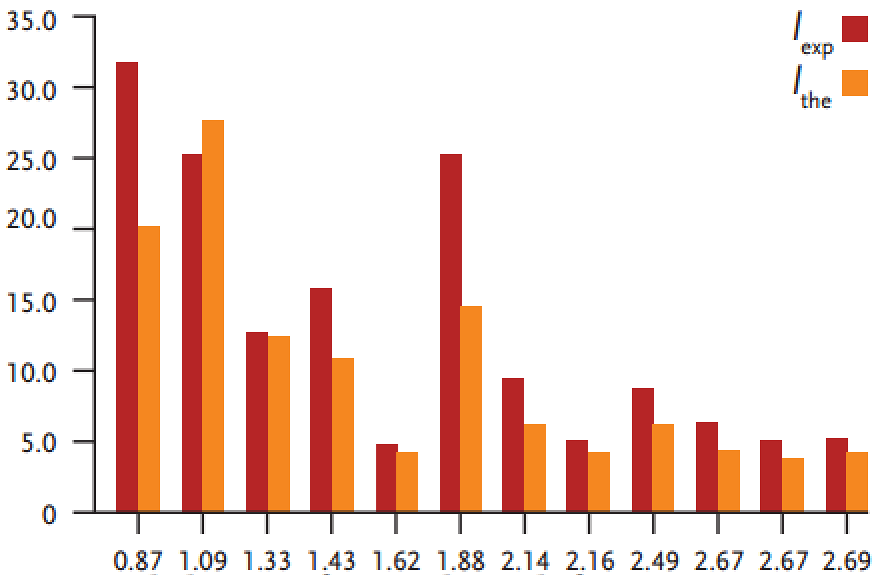
inside wrinkles: $$P_m/M_0 = 4\sqrt[4]{3}\sqrt{\pi^3}\sqrt{R/H} - 2\pi$$

$$P_m/M_0 = 4\sqrt[4]{3}\sqrt{\pi^3}\sqrt{R/H}$$

$$M_0 = \frac{2\sigma_0}{\sqrt{3}}\frac{H^2}{4}$$

$$\boxed{P_m = 2(\pi H)^{3/2} R^{1/2}\sigma_0/3^{1/4}}$$

$$\boxed{P_m = 6\sigma_0\sqrt{DH^3}}$$



Experimental and theoretical comparison for the fold length of various geometries of cylindrical shells.

Dynamic axial (progressive) crushing

$$\sigma_d = \sigma_0 \left\{ 1 + \left(\frac{\dot{\varepsilon}}{D} \right)^{1/q} \right\}$$

$$P_m = 2\left(\pi H^{3/2}\right) R^{1/2} \sigma_0 \left\{ 1 + \left(\dot{\varepsilon}/D\right)^{1/q} \right\} \Big/ 3^{1/4}$$

Strain rate (due to bending)

$$\varepsilon_\theta \cong l/(2R), \quad \dot{\varepsilon}_\theta = \varepsilon_\theta / T$$

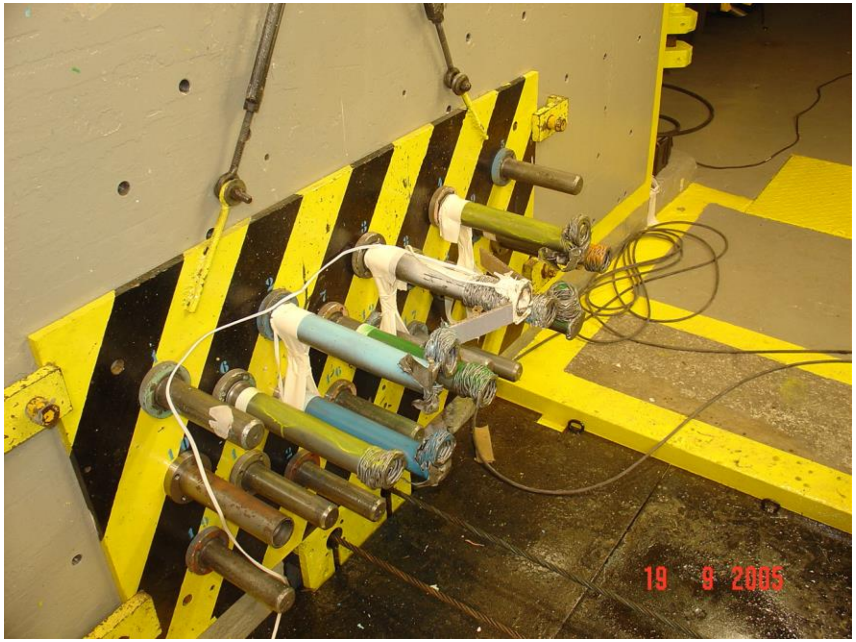
$$T = 2l/V_0, \qquad \dot{\varepsilon}_\theta \cong V_0/(4R)$$

$$\boxed{P_m = 2\left(\pi H^{3/2}\right) R^{1/2} \sigma_0 \left\{ 1 + \left(V_0/4RD\right)^{1/q} \right\} \Big/ 3^{1/4}}$$

Example

An energy-absorbing device, a nest of n circular tubes made of a rigid-plastic material, is required to arrest a mass M travelling with an impact velocity V_0.

Model this phenomenon and explore the results.



The mean load for one circular tube is

$$P_m = 2(\pi H)^{3/2} R^{1/2} \sigma_0 / 3^{1/4}$$

and the equation of motion

$$nP_m = -M\, dv/dt \qquad \text{leads to}$$

$$v = -nP_m t / M + V_0$$

Crushing of the tubes ends at

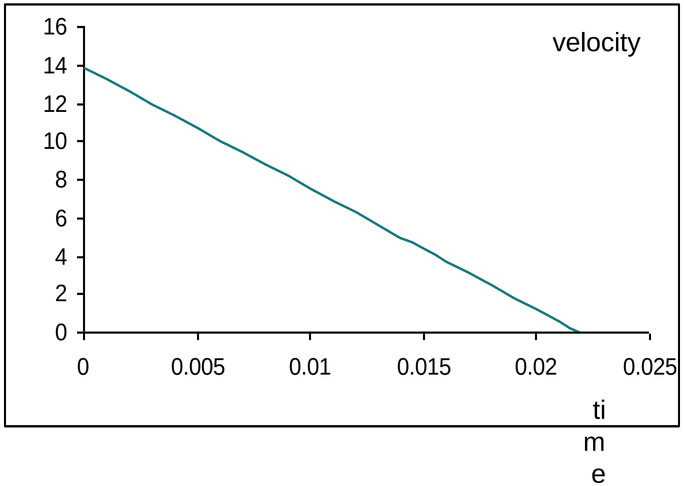
$$v = 0 \quad \rightarrow \quad t = \frac{MV_0}{nP_m}$$

Integration of the impact velocity gives the crushing distance

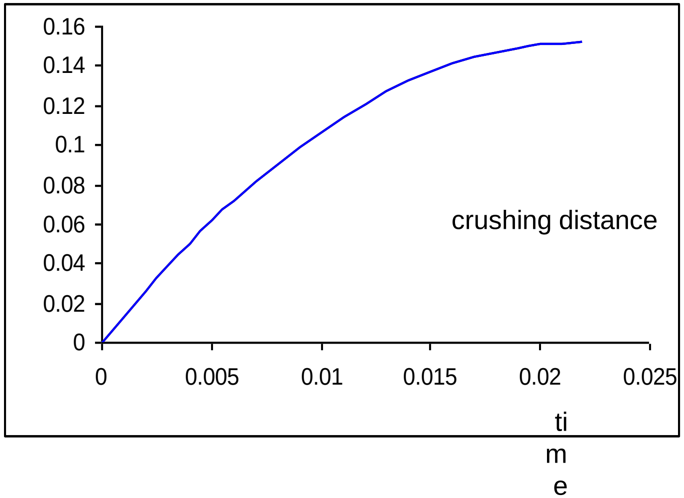
$$d = -\frac{nP_m t^2}{2M} + V_0 t$$

which, for this simple model, is the same as

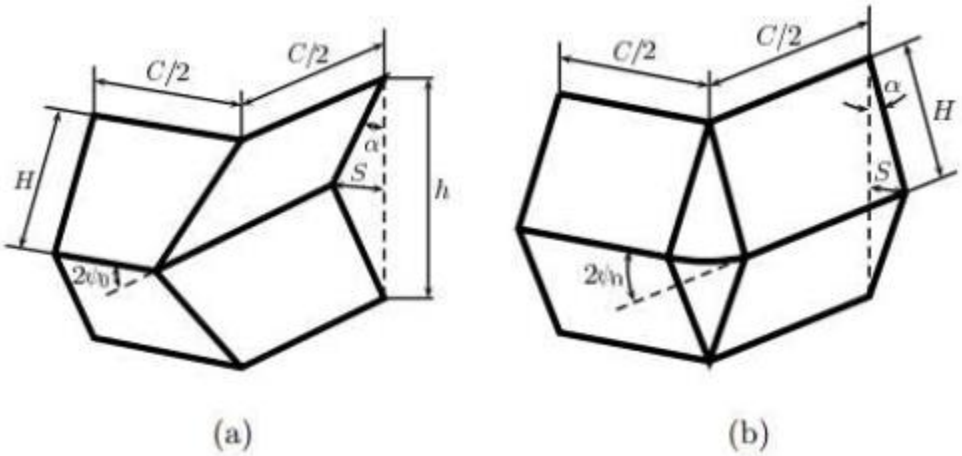
$$P_m d = \frac{1}{2} M V_0^{\,2}$$



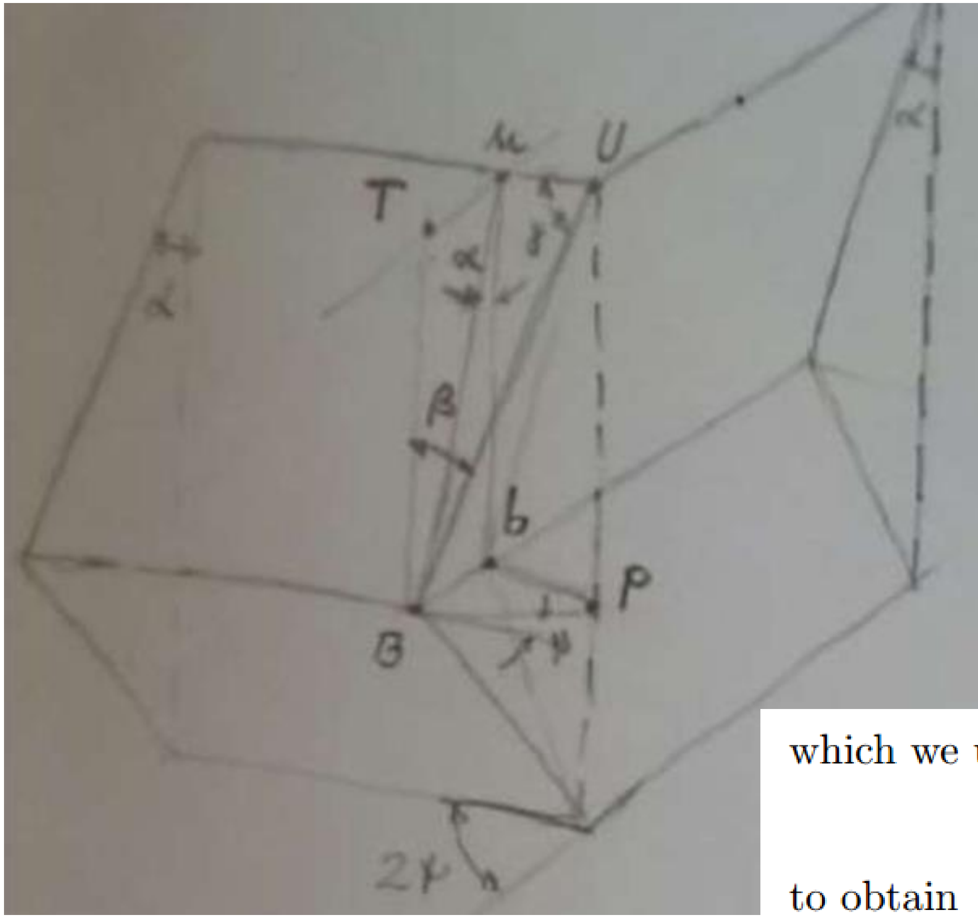
M=1000kg
V=50km/h
50 tubes of H=1mm and R=25mm
flow stress of 300MPa



Progressive buckling of square tubes



Corner of a square tube showing the (a) inextensional and (b) extensional buckling modes.



$$\sin\alpha = \frac{Bb}{Bu} = \frac{Uu}{Ub} = \frac{Pb}{Ub},$$

which we use in

$$\tan\gamma = \frac{Bu}{Uu}$$

to obtain

$$\tan\gamma = \frac{\tan\psi}{\sin\alpha}.$$

Also,

$$\tan\beta = \frac{BP}{PU} = \frac{TU}{BT} = \frac{\frac{Tu}{BU\sin\psi}}{\frac{Bu\sin\alpha}{BU}} = \frac{Tu}{Bu\sin\psi\sin\alpha}$$

or

$$\tan\beta = \frac{TU\sin\psi}{TU\cos\psi}\frac{1}{\sin\psi\sin\alpha} = \frac{\tan\psi}{\sin\psi\sin\alpha} = \frac{\tan\gamma}{\sin\psi}.$$

The crushing distance, δ, is related to α according to

$$\delta = 2H(1 - \cos\alpha).$$

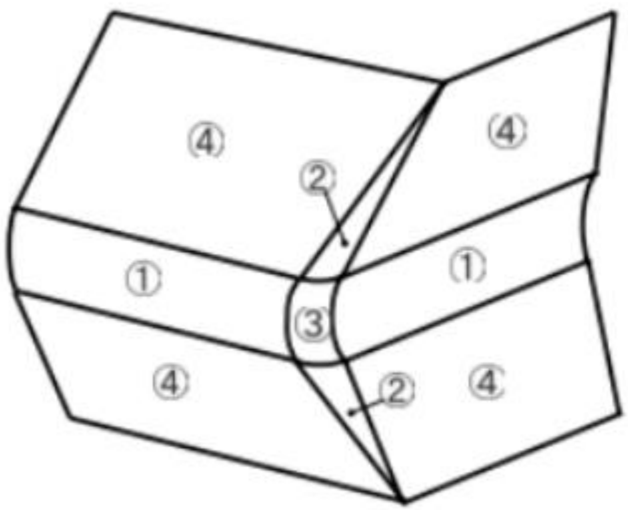
We also have that

$$S = H\sin\alpha,$$

so the time differentiation of these equations yield

$$\dot{\delta} = 2H\sin\alpha\dot{\alpha} \qquad \text{and} \qquad V = \dot{S} = H\cos\alpha\dot{\alpha}.$$

1. Energy dissipation in horizontal plastic hinge lines, W_1
2. Energy dissipation in the toroidal surface, W_2
3. Energy dissipation in inclined plastic hinge lines, W_3



Corner of a square tube and the associated main energy dissipation mechanisms.

This solution is due to T. Wierzbicki and W. Abramowicz, On the crushing mechanics of thin walled structures, Journal of Applied Mechanics, 50, p. 727–734, 1983. For square tubes with different wall thickness see X. Zhang and H. Zhang, Crush resistance of square tubes with various thickness configurations, International Journal of Mechanical Sciences, Volume 107, 2016, p 58-68. This section is based on G. Lu and T. Yu, Energy Absorption of Structures and Materials, CRC Press & Woodhead Publishing, 2003 and on Dai-heng Chen, Crush Mechanics of Thin-Walled Tubes, CRC Press, 2016.

In the dissipation mechanism 1, energy is consumed by the hinge lines AB and BC to an amount of

$$\dot{W}_1 \propto M_0\dot{\alpha} \to \dot{W}_2 = 2M_0 c\dot{\alpha},$$

or

$$W_1 = 2\int_0^{\pi/2} M_0 c \mathrm{d}\alpha = \pi M_0 c.$$

Mechanism 2 deals with the consumed energy in the toroidal surface. Consider the next figure which shows the toroidal surface. Let us assume a linear relation between coordinate ϕ and angle ψ,

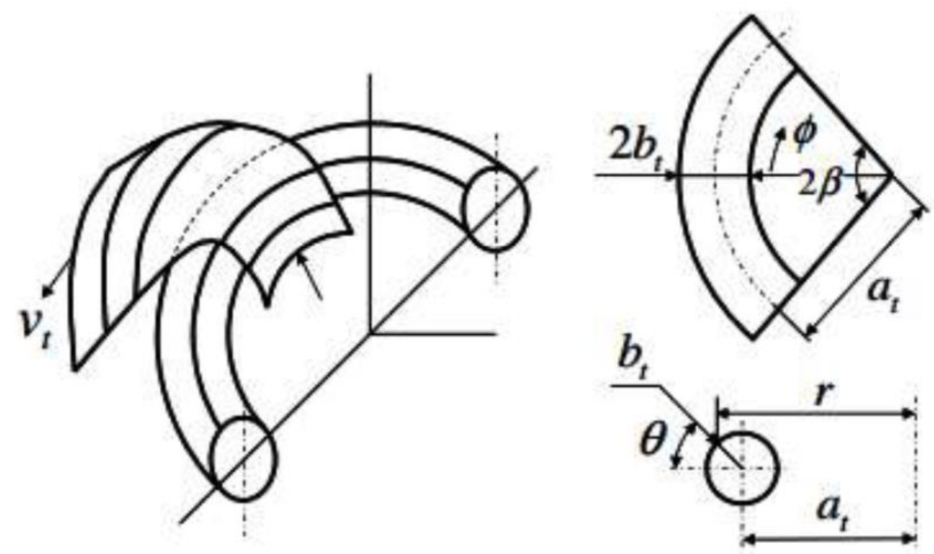
$$\psi = \psi_0 + \frac{\pi - 2\psi_0}{\pi}\phi.$$

Considering also the limits

$$\frac{\pi}{2} - \psi \leq \theta \leq \frac{\pi}{2} + \psi \qquad \text{and} \qquad -\beta \leq \phi \leq \beta$$

and the relation

$$r = b\cos\theta + a.$$



Now, the hinge BC moves inwardly with a velocity $\dot{S}$ so the points in the toroidal surface experiment a velocity of

$$v_t = \frac{\dot{s}}{\tan\phi_0},$$

with

$$\dot{s} = \frac{\mathrm{d}H\sin\alpha}{\mathrm{d}t} = H\cos\alpha\dot{\alpha},$$

such that the circumferential strain rate reads

$$\dot{\varepsilon}_\phi = \frac{V_t\sin\theta}{r} = \frac{H\cos\alpha}{\tan\psi_0}\frac{\sin\theta}{b\cos\theta + a}\dot{\alpha}.$$

As one moves along the ϕ coordinate, there will be a change in curvature but flow is here ruled by normal forces only, with the bending energy being zero.

The plastic dissipation rate in this toroidal surface reads then

$$\dot{W}_2 = \int_s N_0 \dot{\varepsilon}_\phi \mathrm{d}s = \int N_0 \dot{\varepsilon}_\phi r \mathrm{d}\phi \mathrm{d}b\theta,$$

which gives

$$\dot{W}_2 = 4N_0 bH \frac{\pi}{(\pi - 2\psi_0)\tan\psi_0} \cos\alpha\dot{\alpha}$$

$$\left[\cos\psi_0 - \cos\left(\psi_0 + \frac{\pi - 2\psi_0}{\pi}\beta\right)\right].$$

Integration of this equation leads to

$$W_2 = 4N_0 bHI_1(\phi_0) = \frac{16HbM_0}{h} I_1(\psi_0),$$

with

$$I_2 = \frac{\pi}{(\pi - 2\psi_0)\tan\psi_0} \int_0^{\pi/2} \beta\cos\alpha]\times$$

$$\left\{\sin\psi_0 \sin\left(\frac{\pi - 2\psi_0}{\pi}\right) + \cos\psi_0\left[1 - \cos\left(\frac{\pi - -2\psi_0}{\pi}\right)\right]\right\} \mathrm{d}\alpha,$$

which results in $I_2 = 0.58$ for a square tube, when noting that β and α are related as given before.

To obtain energy W_3, associated with the travelling inclined hinges, consider the next figure, where a strip of material is pushed through an anvil or radius ρ. The energy for unbending is proportional to the material strength, via M_0, to the unbent length and inverse to the bent radius so that

$$W = M_0(\pi - \beta) = AB\frac{M_0}{\rho},$$

so that, for the strip in the figure, energy is required for the segment AB to unbend, BC to bend and unbend it, for segments CD and DE for bending, giving

$$W = (AB + 2BC + CD + DE)\frac{M_0}{\rho} = \frac{2\Delta s}{\rho}M_0.$$

Take now Δs as the cone length, BB_1 so that the consumed power to bend and unbend the material is

$$\dot{W}_3 = 2\int_0^{BB_1} \frac{2V(s)\mathrm{d}s}{\rho}M_0,$$

with $V(s)$ being the velocity of a point s in the surface of radius ρ. Since

$$V(s) = \frac{s}{BB_1}V_t, \quad \rho = \frac{s}{BB_1}b \quad BB_1 = \frac{H}{\sin\gamma},$$

it follows that

$$\dot{W}_3 = \frac{4M_0H}{\sin\gamma}\frac{H\cos\alpha\dot{\alpha}}{b\tan\psi_0},$$

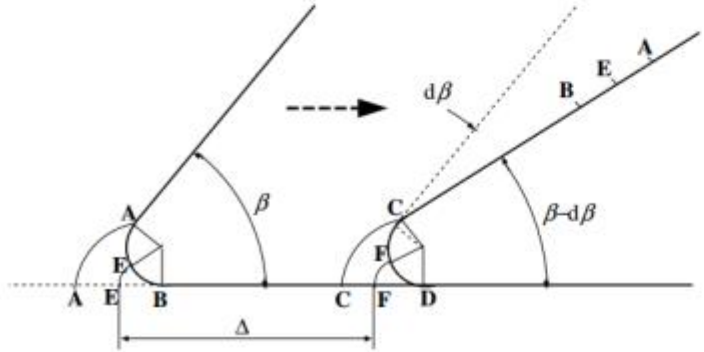
which gives

$$W_3 = \frac{4M_0H^2}{b}\frac{1}{\tan\psi_0}\int_0^{\pi/2}\frac{\cos\alpha}{\tan\psi_0}\frac{\sin\alpha}{\cos\alpha}\mathrm{d}\alpha = \frac{4M_0H^2}{b}I_3(\psi_0),$$

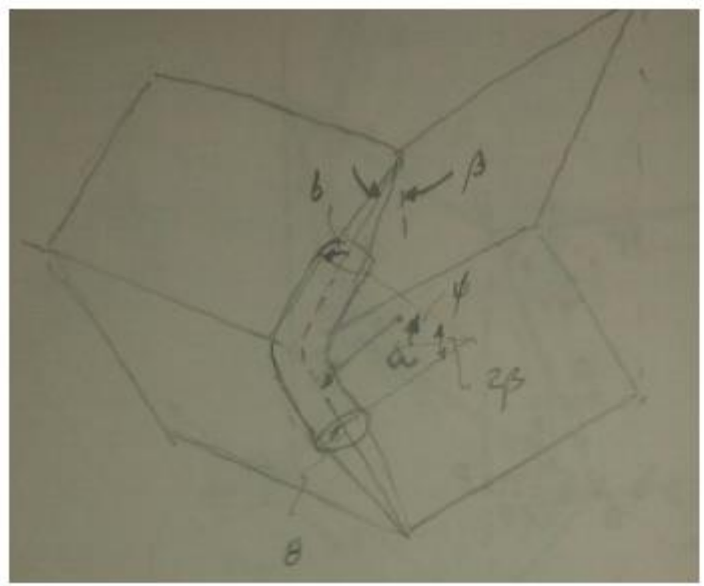
with

$$I_3(\psi_0) = \frac{1}{\tan\psi_0}\int_0^{\pi/2}\frac{\cos\alpha}{\sin\gamma}\mathrm{d}\gamma,$$

being $I_3 = 1,15$ for a square tube.



(a)



(b)

We now equate the external work, $2P_mH$, to the internal dissipation, $W_1+W_2+W_3$, to obtain

$$P_m = M_0\left(A_1\frac{b}{h}+A_2\frac{c}{H}+A_3\frac{H}{b}\right),$$

where $A_1 = 8I_1$, $A_2 = \pi/2$ and $A_3 = 2I_3$. The parameters b, the radius of the knee folding, and H, the height of the fold, are not known but they can be obtained from $\partial P_m/\partial H = 0$ and $\partial P_m/\partial b = 0$, such that

$$b = \sqrt[3]{\frac{A_2A_3}{A_1^2}ch^2} \qquad \text{and} \qquad H = \sqrt[3]{\frac{A_2^2}{A_1A_3}c^2h},$$

which are material independent for our model. We finally obtain,

$$P_m = 3M_0\sqrt[3]{A_1A_2A_3\frac{c}{h}},$$

with the three basic plastic dissipation mechanisms sharing equal contributions. The above equation becomes

$$P_m = 9.56\sigma_0\sqrt[3]{h^5c}$$

for a square tube of thickness h and side c.

There has been a large amount of studies in the so called kinetic energy absorbers, KEA. The next table presents some recent studies and ingenious ways of handling the kinetic energy.

These KEA are scrutinized in terms of efficiency and some parameters that are used are as follows:

- Structural effectiveness

$$\eta = \frac{P_m}{A\sigma_0}$$

- Solidity ratio

$$\phi = \frac{A}{A_c},$$

  A_c is the cross section area enclosed by the tube cross section. For a thin circular tube, $A_c = \pi R^2$ and $A = 2\pi RH$, so $\phi = 2H/R$.

- Specific energy

$$S_e = \frac{D_a}{m},$$

  where D_a is the total energy absorbed and m is the mass of the device.

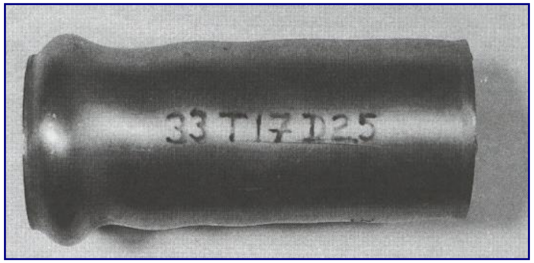
Dynamic plastic buckling of circular shells

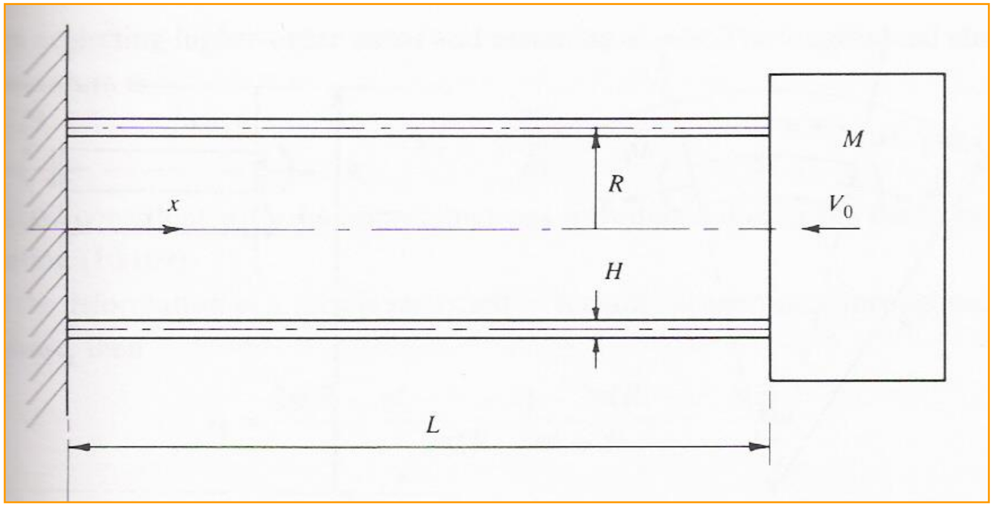
Assumptions:

✓Biaxial stress state is considered

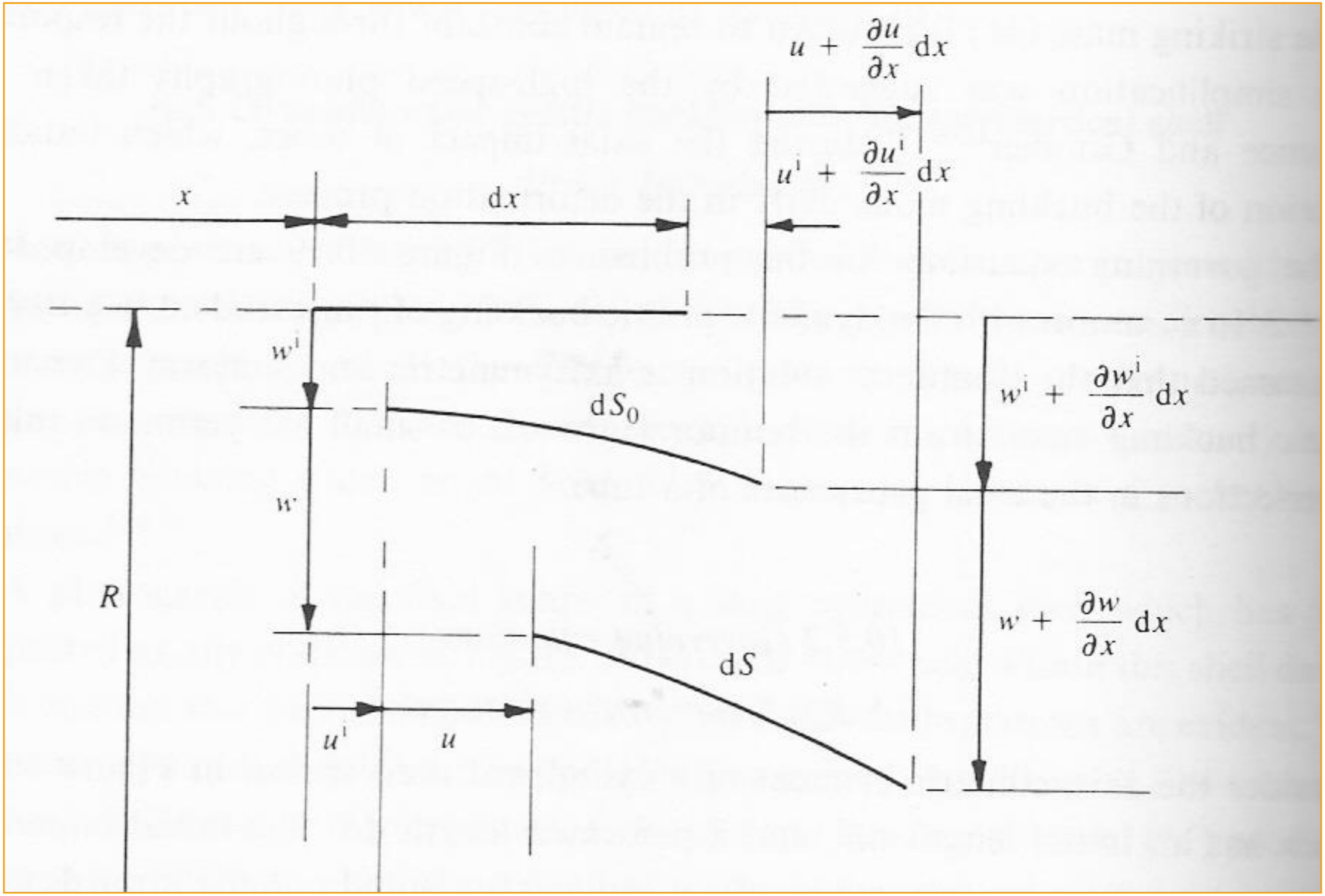
✓The axial velocity V_0 remains constant throughout the response

✓The shell deforms axisymetrically





Kinematics: strain



Kinematics: strains

$$dS^2 = (w' dx)^2 + (dx + u' dx)^2$$

Taylor series

$$\varepsilon_x = \frac{dS - dS_0}{dS_0}, \quad \varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w^i}{\partial x}\frac{\partial w}{\partial x}$$

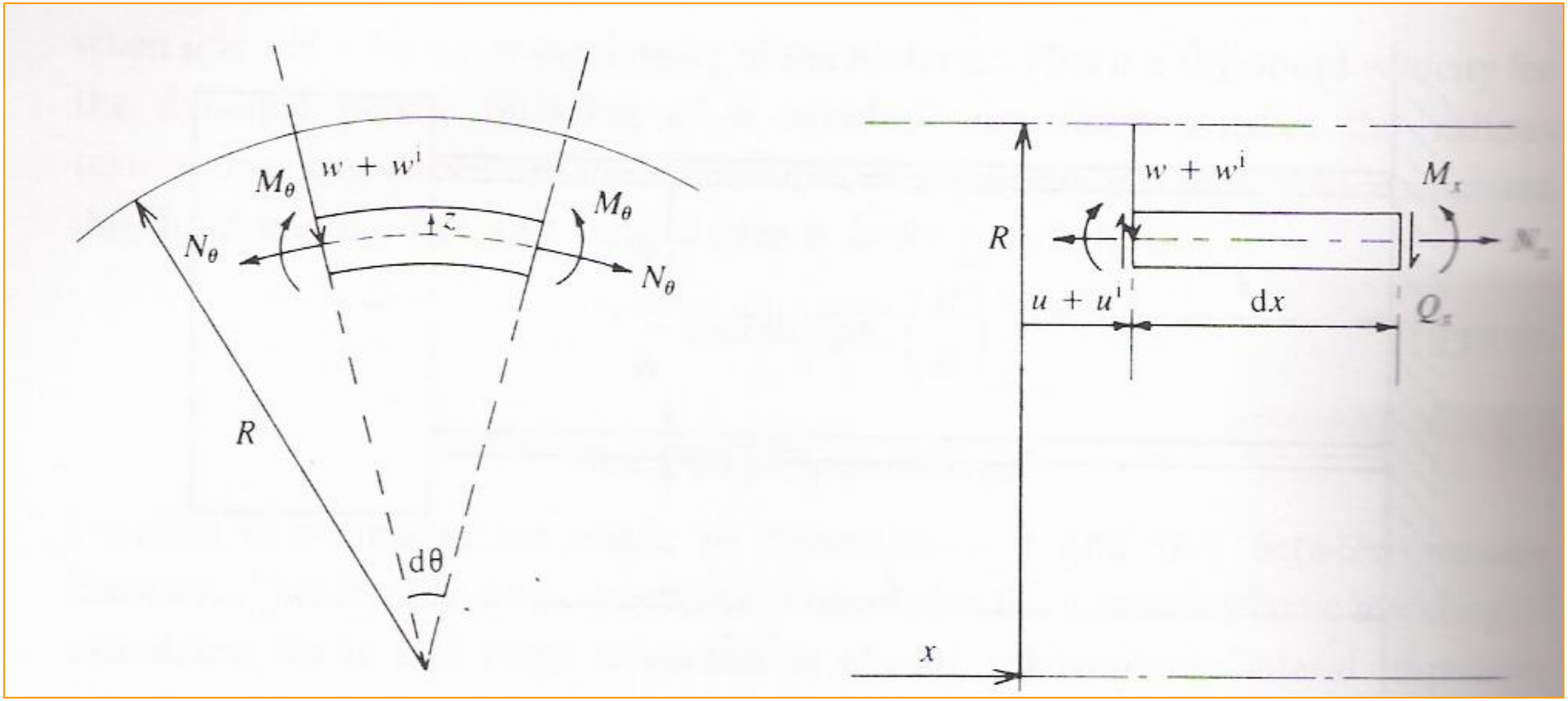
$$k_x = \frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_\theta = \frac{2\pi\left(R - w^i - w + z\right) - 2\pi\left(R - w^i + z\right)}{2\pi\left(R - w^i + z\right)}$$

$$\varepsilon_\theta \cong -\frac{w}{R}\left(1 + \frac{w^i}{R} - \frac{z}{R}\right)$$

$$k_\theta = 0$$

Equilibrium



Equilibrium

$$Q_x = \frac{\partial M_x}{\partial x}$$

$$\frac{\partial N_x}{\partial x} - \mu \frac{\partial^2 u}{\partial^2 t} = 0, \quad \mu = \rho H$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial}{\partial x}\left\{ N_x \left(\frac{\partial w}{\partial x} + \frac{\partial w^i}{\partial x} \right) \right\} + \frac{N_\theta}{R}\left(1 + \frac{w^i}{R}\right) - \mu \frac{\partial^2 u}{\partial^2 t} = 0$$

Constitutive equations:

Radial and longitudinal displacements are decomposed in dominant and perturbed components:

$$w(x,t) = \bar{w}(t) + w^*(x,t), \qquad u(x,t) = \bar{u}(t) + u^*(x,t)$$

Components of the dominant strain rates:

$$\frac{\partial \dot{\bar{u}}}{\partial x} = -\frac{V_0}{L} \quad \rightarrow \quad \dot{\bar{\varepsilon}}_x = -\frac{V_0}{L}$$

$$\dot{\bar{\varepsilon}}_\theta = \frac{V_0}{2L} \quad \text{(assuming } \sigma_\theta = 0\text{)}$$

$$\dot{\bar{\varepsilon}}_x + \dot{\bar{\varepsilon}}_\theta + \dot{\bar{\varepsilon}}_z = 0 \quad \rightarrow \quad \dot{\bar{\varepsilon}}_z = \frac{V_0}{2L}$$

Total strain rates $\dot{\varepsilon}_x = -\frac{V_0}{L} + z\frac{\partial^2 \dot{w}}{\partial x^2}, \quad \dot{\varepsilon}_\theta = \left(1 - \frac{z}{R}\right)\left(\frac{V_0}{2L} - \frac{\dot{w}}{R}\right), \quad \dot{\varepsilon}_z = -\dot{\varepsilon}_x - \dot{\varepsilon}_\theta$

Expressions for the equivalent strain rate, strain and stress as function of the impact velocity

$$\dot{\varepsilon}_e = \frac{2}{\sqrt{3}}\sqrt{\dot{\varepsilon}_x{}^2 + \dot{\varepsilon}_\theta{}^2 + \dot{\varepsilon}_x{}^2\dot{\varepsilon}_\theta{}^2}$$

$$\dot{\varepsilon}_e \cong \frac{V_0}{L}\left[1 + \frac{zL}{2V_0}\left(\frac{4\dot{w}^*}{3R^2} - 2\frac{\partial^2 \dot{w}^*}{\partial x^2}\right)\right], \qquad \varepsilon_e \cong \frac{V_0}{L}\left[t + \frac{zL}{2V_0}\left(\frac{4w^*}{3R^2} - 2\frac{\partial^2 w^*}{\partial x^2}\right)\right]$$

$$\sigma_x = -\sigma^0\left(1 + \frac{2L\dot{w}^*}{3V_0R}\right) + \frac{z\sigma^0 L}{3V_0}\left(\frac{\partial^2 \dot{w}^*}{\partial x^2} + \frac{4\dot{w}^*}{R^2} - \frac{V_0}{RL}\right) + zE_h\left(\frac{\partial^2 w^*}{\partial x^2} - \frac{2w^*}{3R^2}\right)$$

$$\sigma_\theta = -\sigma^0\frac{4L\sigma^0\dot{w}^*}{3V_0R} + \frac{z2L\sigma^0}{3V_0}\left(\frac{\partial^2 \dot{w}^*}{\partial x^2} + \frac{2\dot{w}^*}{R^2} - \frac{V_0}{RL}\right),$$

where $\sigma^0 = \sigma_0 + E_h\dfrac{V_0 t}{L}$

Definitions for the forces:

$$N_x = \int_{-H/2}^{H/2} \sigma_x (1 + z/R) dz \quad \rightarrow \quad N_x = \overline{N}_x + N_x^*$$

$$\overline{N}_x = -\sigma^0 H \quad \text{and} \quad N_x^* = -\frac{\sigma^0 2HL\dot{w}^*}{3V_0 R}$$

$$N_\theta = \int_{-H/2}^{h/2} \sigma_\theta dz, \qquad N_\theta = \overline{N}_\theta + N_\theta^*$$

$$\text{where} \quad \overline{N}_\theta = 0, \quad N_\theta^* = -\frac{4L\sigma^0 H}{3V_0 R} \dot{w}^*$$

$$M_\theta = -\int_{-H/2}^{H/2} \sigma_\theta z dz, \quad M_\theta = \overline{M}_\theta + M_\theta^*$$

$$\overline{M}_\theta = \frac{\sigma^0 H^3}{18R}, \quad M_\theta^* = -\frac{\sigma^0 H^3 L}{18V_0} \left(\frac{\partial^2 \dot{w}^*}{\partial x^2} + \frac{2\dot{w}^*}{R^2} \right)$$

$$M_x = \int_{-H/2}^{H/2} \sigma_x (1 + z/R) z dz$$

$$M_x = \overline{M}_x + M_x^*$$

$$\overline{M}_x = \frac{\sigma^0 H^3}{12R}$$

$$M_x^* = -\frac{\sigma_0 H^3 L}{36V_0} \left(\frac{\partial^2 \dot{w}^*}{\partial x^2} + \frac{2\dot{w}^*}{R^2} \right) - \frac{H^3 E_h}{36} \left(3\frac{\partial^2 w^*}{\partial x^2} - \frac{2w^*}{R^2} \right)$$

All forces are defined per unit length

Equations of motion

$$\frac{\partial}{\partial x}\left(\overline{N}_x + N_x^*\right) - \mu\left(\ddot{\overline{u}} + \ddot{u}^*\right) = 0$$

$$\frac{\partial}{\partial x}\left(\overline{M}_x + M_x^*\right) + \frac{\partial}{\partial x}\left\{\left(\overline{N}_x + N_x^*\right)\left(\frac{\partial w^*}{\partial x} + \frac{\partial w^i}{\partial x}\right)\right\} + \left(\frac{\left(\overline{N}_\theta + N_\theta^*\right)}{R}\right)\left(1 + \frac{w^i}{R}\right) - \mu\left(\ddot{\overline{w}} + \ddot{w}^*\right) = 0$$

$$\frac{\partial \overline{N}_x}{\partial x} - \mu\ddot{\overline{u}} = 0, \qquad \frac{\partial \overline{N}_x}{\partial x} = 0, \quad \frac{\partial N_x^*}{\partial x} - \mu\ddot{u}^* = 0$$

$$\frac{\overline{N}_\theta}{R}\left(1 + \frac{w^i}{R}\right) - \mu\ddot{\overline{w}} = 0$$

$$\frac{\partial^2 M_x^*}{\partial x^2} + \overline{N}_x\left(\frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w^i}{\partial x^2}\right) + \frac{N_\theta^*}{R}\left(1 + \frac{w^i}{R}\right) - \mu\ddot{w}^* = 0$$

$$\boxed{\begin{aligned} &\frac{\sigma^0 H^3 L}{36V_0}\frac{\partial^4 \dot{w}^*}{\partial x^4} + \frac{H^3 E_h}{12}\frac{\partial^4 w^*}{\partial x^4} + \frac{\sigma^0 H^3 L}{18V_0 R^2}\frac{\partial^2 \dot{w}^*}{\partial x^2} - \frac{H^3 E_h}{18R^2}\frac{\partial^2 w^*}{\partial x^2} + \\ &\sigma^0 H\left(\frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w^i}{\partial x^2}\right) + \frac{4\sigma^0 HL}{3V_0 R^2}\left(1 + \frac{w^i}{R}\right)\dot{w}^* + \mu\ddot{w}^* = 0 \end{aligned}}$$

Equations of motion

$$\ddot{v} + S_0\left\{\beta\dot{v}'''' + 2\alpha^2\beta^2\dot{v}'' + 48\alpha^2\left(1+v^i\right)\dot{v}\right\} + \gamma S_0\left(3v'''' - 2\alpha^2 v''\right) + 36S_0\left(v'' + (v^i)''\right) = 0$$

where

$$v = w^*/R, \quad v^i = w^i/R, \quad \alpha = L/R, \quad \beta = H/L, \quad \gamma = \beta^2 E_h/\sigma^0, \quad S_0 = \sigma^0/36\rho V_0^2,$$

$$\xi = x/L, \quad \tau = V_0 t/L, \quad \dot{(\,)} = \partial(\,)/\partial\tau, \quad (\,)' = \partial(\,)/\partial\xi.$$

Solution for the perturbed behaviour $v(\xi,\tau) = \sum_n^{\infty} v_n(\tau)\sin(n\pi\xi)$

Initial imperfections $v_i(\xi) = \sum_n^{\infty} a_n \sin(n\pi\xi)$

$$\sum_{n=1}^{\infty}\left\{\ddot{v}_n + \underbrace{S_0\left[48\alpha^2 + \beta^2(n\pi)^2\left((n\pi)^2 - 2\alpha^2\right)\right]}\dot{v}_n + \underbrace{S_0(n\pi)^2\left[2\alpha^2\gamma + 3\gamma(n\pi)^2 - 36\right]}v_n - \underbrace{36S_0(n\pi)^2}a_n\right\}\sin(n\pi\xi) = 0$$

$$Q_n = S_0\left[48\alpha^2 + \beta^2(n\pi)^2\left((n\pi)^2 - 2\alpha^2\right)\right]$$

$$\boxed{\ddot{v}_n + Q_n\dot{v}_n + R_n v_n = S_n a_n}$$

$$R_n = S_0(n\pi)^2\underbrace{\left[2\alpha^2\gamma + 3\gamma(n\pi)^2 - 36\right]}_{\text{CRITICAL NUMBER}}, \qquad S_n = 36S_0(n\pi)^2$$

Solution of the equation of motion for each n

$$v_n = E_n(\tau)a_n + F_n(\tau)b_n,$$

Initial conditions: $\dot{v}(0) = \dot{v}_i = \sum_{n=1}^{\infty} b_n \sin(n\pi\xi)$

where

$$E_n(\tau) = \frac{S_n}{R_n}\left(1 + \frac{\lambda n_2 e^{\lambda n_1 \tau} - \lambda n_1 e^{\lambda n_2 \tau}}{\lambda n_1 - \lambda n_2}\right), \qquad F_n(\tau) = \frac{e^{\lambda n_1 \tau} - e^{\lambda n_2 \tau}}{\lambda n_1 - \lambda n_2}$$

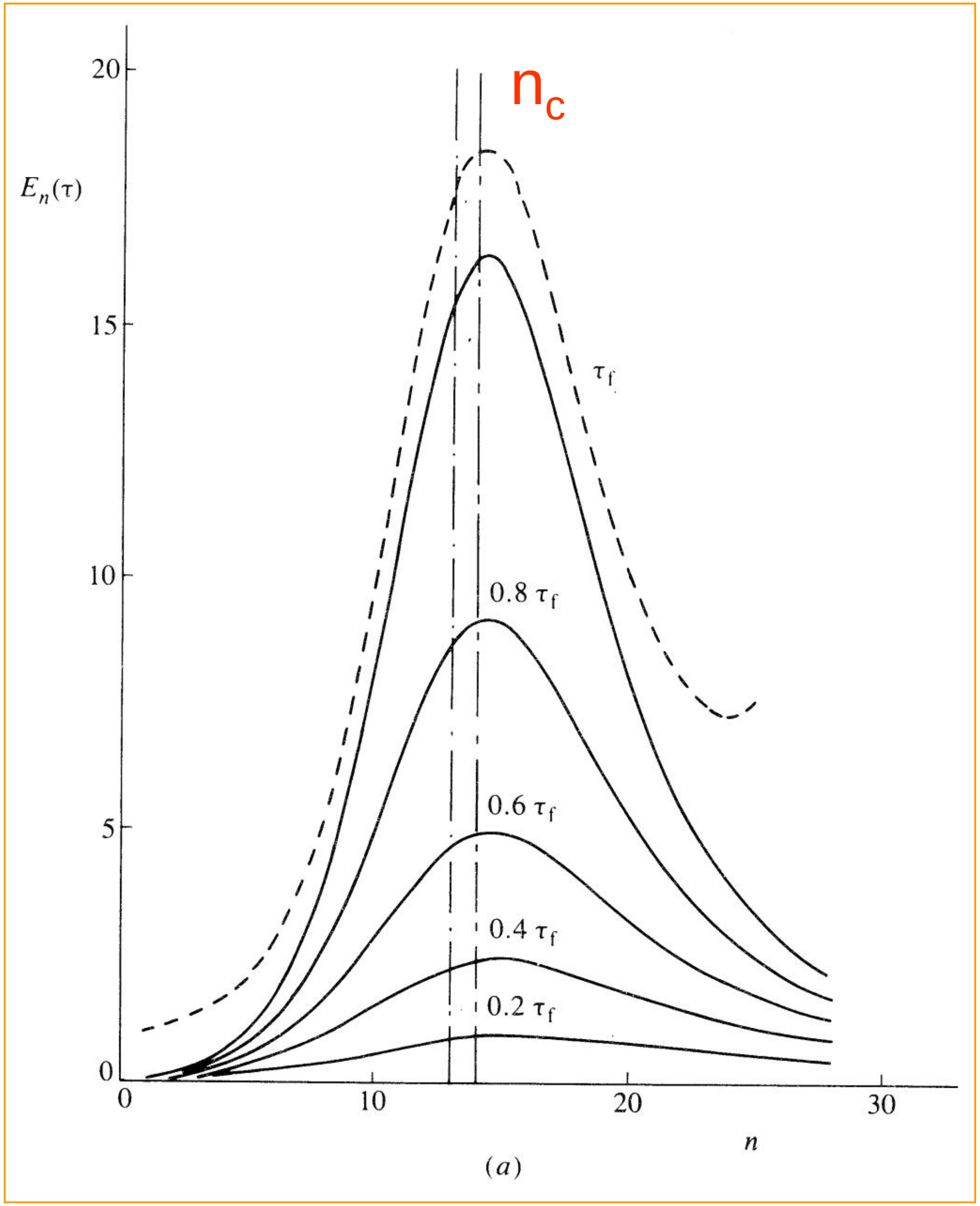
are the displacement and velocity amplification functions, respectively

$$\lambda n_1 = \left\{-Q_n + \left(Q_n^2 - 4R_n\right)^{1/2}\right\}/2, \qquad \lambda n_2 = -\left\{Q_n + \left(Q_n^2 - 4R_n\right)^{1/2}\right\}/2$$

$\lambda n_1 > 0$ for $R_n < 0$ when the amplification functions increase with time

Therefore, ***dynamic plastic buckling*** occurs for

$$n < \left(\frac{36 - 2\alpha^2\gamma}{3\gamma\pi^2}\right)^{1/2}$$



Critical mode number

E_h = 724 MPa, σ_0 = 307 MPa,
ρ = 2685 kg/m^3,
L = 101.6 mm, H = 2.54 mm,
R = 11.43 mm,
M = 120 g, V_0 = 170 m/s

Critical mode number n_c

Assumption : $4R_n / Q_n^2 << 1$

$$\lambda n_1 = Q_n \left\{-1 + 1 - 2R_n / Q_n^2 + \ldots\right\}/2 \cong -R_n / Q_n , \qquad \lambda n_2 \cong -Q_n$$

$$E_n(\tau) \cong -\frac{S_n}{R_n} e^{-(R_n/Q_n)\tau}, \qquad F_n(\tau) \cong \frac{e^{-(R_n/Q_n)\tau}}{Q_n}$$

For $R_n < 0$ the condition for buckling is satisfied and the largest value of R_n / Q_n is assumed to give the fastest growth of the amplification functions and therefore the critical value for n.

The condition $\partial(R_n / Q_n)/\partial n = 0$ leads to a quadratic equation for $(n_c\pi)^2$

$$\frac{\beta^2(9 - 2\alpha^2\gamma)}{24\alpha^2}(n_c\pi)^4 + 3\gamma(n_c\pi)^2 + \alpha^2\gamma - 18 = 0$$

if $\alpha^2\gamma << 4.5$ $\qquad n_c \cong \dfrac{2\alpha\sqrt{\gamma}}{\pi\beta}\left\{\left(1 + \dfrac{3\beta^2}{\alpha^2\gamma^2}\right)^{1/2} - 1\right\}^{1/2}$

$$\boxed{n_c \cong \frac{2\alpha}{\pi\beta}\left(\frac{\beta\sqrt{3}}{\alpha} - \gamma\right)^{1/2} \quad \text{when} \quad \frac{3\beta^2}{\alpha^2\gamma^2} >> 1}$$

Example

A thin-walled circular cylindrical shell with a mean radius R = 25 mm, H = 2.5 mm, L = 100 mm, σ_0 = 300 MPa, ρ = 2700 kg/m^3 and E_h/σ_0 = 2.5 is impacted at one end with a mass M = 100 g.

(a) What is the response duration?

(b) Determine the impact velocity, which produces dynamic plastic buckling (assuming that the critical mode amplifies the initial displacement imperfections lying in the critical mode).

(a) An estimate for the response duration , t_f, is made by equating the initial kinetic energy and the energy absorbed by plastic deformations in the tube

$$\int_0^{t_f} \sigma_e \dot{\varepsilon}_e (2\pi RHL) dt, \qquad \dot{\varepsilon}_e \cong V_0 / L$$

$$\frac{MV_0}{2} = 2\pi RHL\sigma_0 \int_0^{t_f} \frac{V_0}{L} dt \quad \rightarrow \quad t_f = \frac{M}{m}\frac{\rho V_0 L}{2\sigma_0}, \quad \tau_f = t_f \frac{V_0}{L} = \frac{M}{m}\frac{\rho V_0^2}{2\sigma_0}$$

(b) The displacement amplification function $E_n(\tau) \cong -\frac{S_n}{R_n} e^{-(R_n/Q_n)\tau}$ can be rearranged as

$$\log_e\left\{-\left(\frac{R_{nc}}{S_{nc}}\right)E_{nc}(\tau_f)\right\} = -\left(\frac{R_{nc}}{Q_{nc}}\right)\tau_f \quad \rightarrow \quad \boxed{V_0^2 = \frac{2\sigma_0}{\rho}\frac{m}{M}\left(-\frac{Q_{nc}}{R_{nc}}\right)\log_e\left\{-\left(\frac{R_{nc}}{S_{nc}}\right)E_{nc}(\tau_f)\right\}}$$

Example

$$\alpha^2\gamma = 0.0125 << 4.5 \quad \rightarrow \quad n_c \cong \frac{2\alpha\sqrt{\gamma}}{\pi\beta}\left\{\left(1+\frac{3\beta^2}{\alpha^2\gamma^2}\right)^{1/2} - 1\right\}^{1/2} = 9.8$$

$$V_0^2 = \frac{2\sigma_0}{\rho}\frac{m}{M}\left(-\frac{Q_{nc}(n_c)}{R_{nc}(n_c)}\right)\log_e\left\{-\left(\frac{R_{nc}(n_c)}{S_{nc}(n_c)}\right)E_{nc}\left(\tau_f\right)\right\}, \quad n_c = 10$$

$$V_0 = 149.8\, m/s, \quad t_f = 63.6\,\mu s$$

Exercise: How much is V using the Vaughan equation?

Exercise: Study the article IJIE, 24(2000) 1083-1115.

Stress waves and buckling

- Plastic stress waves in rods
- Plastic stress waves in circular shells
- Buckling of circular shells
- Comparison between the buckling modes in circular and square tubes

References:

W. Johnson. Impact strength of materials

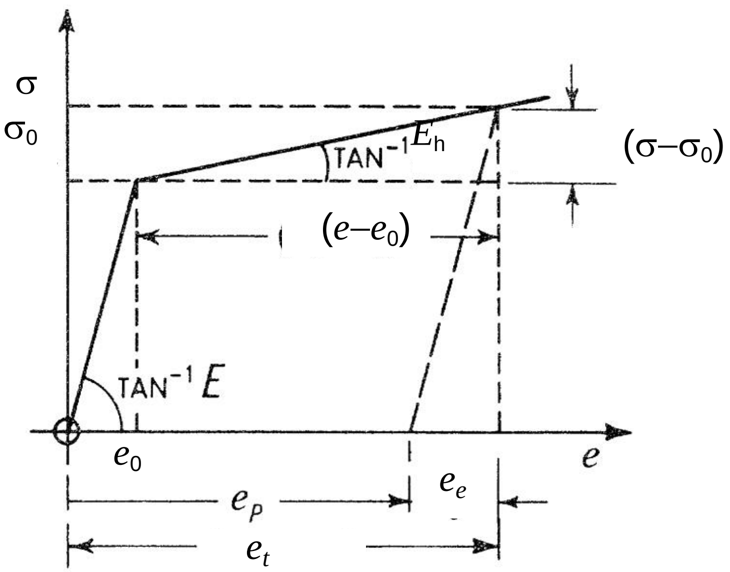
W. K. Nowacki. Stress waves in non-elastic solids, Pergamon Press, 1978

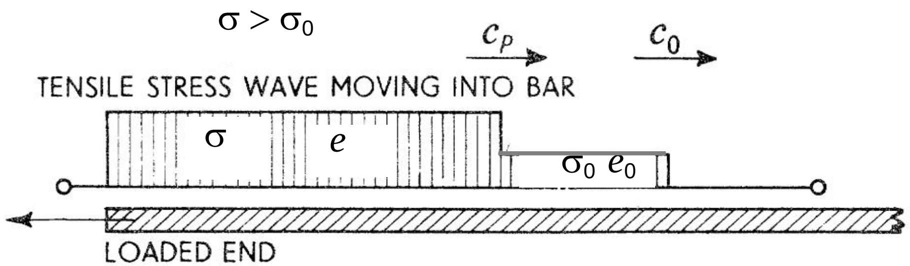
D. Karagiozova, M. Alves, N. Jones. Inertia effects in axisymmetrically deformed cylindrical shells under axial impact, *Int. J. Impact Eng*, 24 (2000) 1083)1115

D. Karagiozova, N. Jones. On dynamic buckling phenomena in axially loaded elastic–plastic cylindrical shells, *Int. J Non-linear Mech* 37 (2002) 1223 – 1238

D. Karagiozova, N. Jones. Dynamic buckling of elastic–plastic square tubes under axial impact-II: structural response, *Int. J. Impact Eng*, 30 (2004) 167–192

Plastic stress waves in rods





Consider a bar of idealised material possessing a nominal stress - engineering strain curve and subject to a tensile stress $\sigma > \sigma_0$. The stress will be transmitted by two waves, which start at the same instant from the loaded end of the bar but move at different speeds

$$c_0 = \sqrt{E/\rho} \quad \text{and} \quad c_p = \sqrt{E_h/\rho}$$

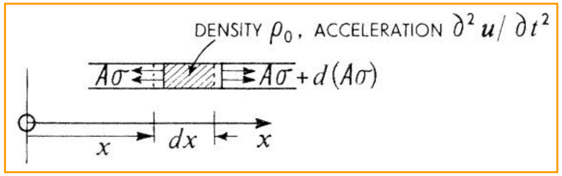
General stress-strain relationship

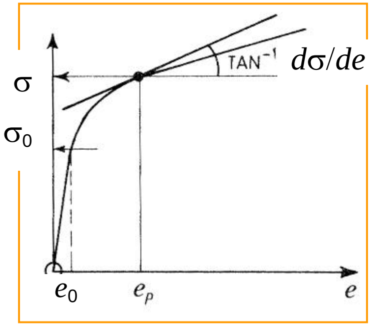
Equation of motion of an element with length dx (unstrained)

$$d(A\sigma) = \rho A dx . \partial^2 u / \partial t^2$$

$$\frac{d\sigma}{de} = \rho \frac{dx}{de}\frac{\partial^2 u}{\partial t^2}; \qquad e = \frac{\partial u}{\partial x}, \quad \frac{\partial e}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{d\sigma / de}{\rho}\frac{\partial^2 u}{\partial x^2} \quad \rightarrow \quad c_p = \sqrt{(d\sigma/de)/\rho}$$





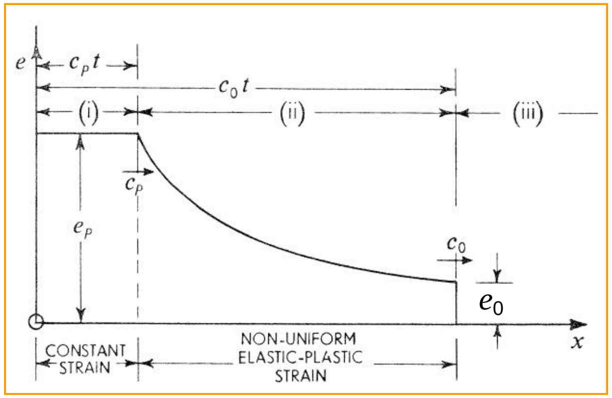
Strain distribution in a bar

(i) Between $x = 0$ and $x = c_p t$, the strain is constant at e_p;

$c_p = \sqrt{(d\sigma/de)/\rho}$ while σ is the largest stress imposed

(ii) Between $x = c_p t$ and $x = c_0 t$, there is a variable distribution of strains between e_p and e_0.

(iii) For $x > c_0 t$, i.e. ahead of the elastic wave the bar is not disturbed



Particle speed required to attain a given stress level

$dt = dx/c_p$ is the time to propagate force increment $d(A\sigma)$ at stress level σ.

$$dt = \frac{dx}{\sqrt{(d\sigma/de)/\rho}}$$

the increment in speed dv is

$$dv = \frac{d\sigma}{\rho\sqrt{\dfrac{d\sigma/de}{\rho}}}.$$

The total speed required by the element at $t = 0$ to attain stress level σ is

$$v = \int_0^{e_p} \sqrt{\frac{d\sigma/de}{\rho}}\, de = \int_0^{e_p} c_0 \sqrt{\frac{d\sigma/de}{E}}\, de$$

For a bilinear stress - strain curve the velocity is

$$v = e_0\sqrt{E/\rho} + (e_p - e_0)\sqrt{E_h/\rho}$$

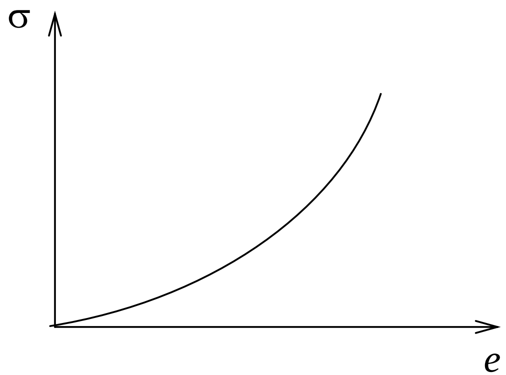
or

$$v = e_0 c_0 + (e_p - e_0) c_p$$

Shock waves

Shock waves can be observed in materials described by a convex curve towards the strain axis (nickel-chrome steel, polycrystalline magnesium).

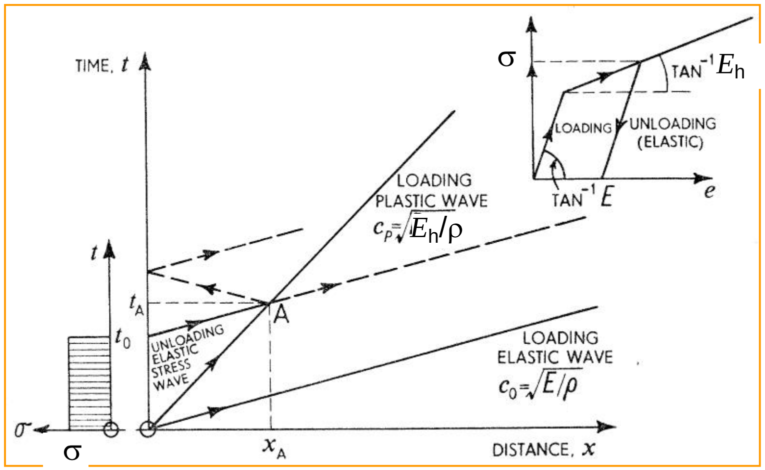
$$c_p = \sqrt{(d\sigma/de)/\rho}$$



When the end of a bar is loaded, the latest, and therefore largest strains is propagated at the faster rate than the early or lower strains, so that if a bar is sufficiently long, the following waves will overtake the early ones. Therefore, the distance between any last-to-be propagated (most intense stress wave) and the first (least intense wave) will be steadily reduced.

Unloading waves

The unloading wave for this particular loading case is always elastic



Impact of finite length uniform bar on a rigid flat anvil (1)

Stress wave speeds : $c_0 = \sqrt{E/\rho}$ and $c_p = \sqrt{E_h/\rho}$

Region I : Both elastic and plastic waves

Region II : Only elastic waves

Region III : Undesturbed

Minimum velocity to initiate plastic strains : $\sigma_0 / \rho c_0 = c_0 e_0$

The particle speed in region II is $v = V - c_0 e_0$

The compressive stress jump $(\sigma - \sigma_0)$ is

$\sigma - \sigma_0 = \rho c_1 (V - c_0 e_0)$

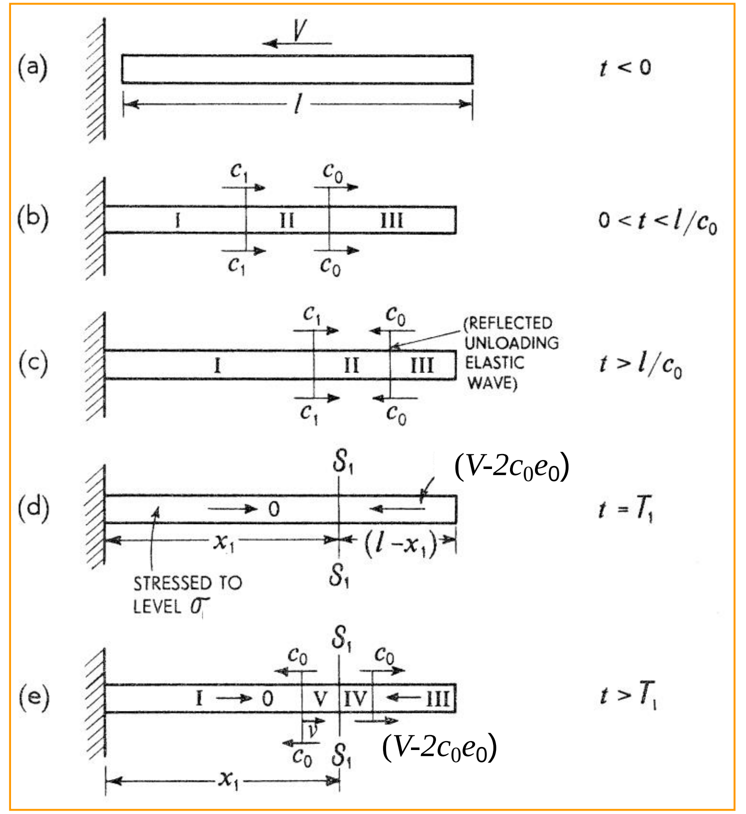
The compressive stress in region I (the plastic region) is

$\sigma = \sigma_0 + \rho c_1 (V - c_0 e_0)$

The total compressive strain is

$$e_t = e_0 + \frac{\sigma - \sigma_0}{E_h} = e_0 + \frac{V - c_0 e_0}{c_1}$$

The residual plstic strain is $e_P = e_t - e_e$



Impact of finite length uniform bar with a rigid flat anvil (2)

$$e_P = \left(e_0 + \frac{V - c_0 e_0}{c_1} \right) - \frac{\sigma}{E} =$$

$$= e_0 + \frac{V - c_0 e_0}{c_1} - \left(\frac{\sigma_0 + \rho c_1 (V - c_0 e_0)}{E} \right) = (V - c_0 e_0)\left(\frac{1}{c_1} - \frac{c_1}{c_0^2} \right)$$

$$= \frac{c_0^2 - c_1^2}{c_0^2 c_1}(V - c_0 e_0)$$

Unloading of region III: The velocity will decrease to

$(V - 2c_0 e_0)$

and the reflected elastic wave will approach

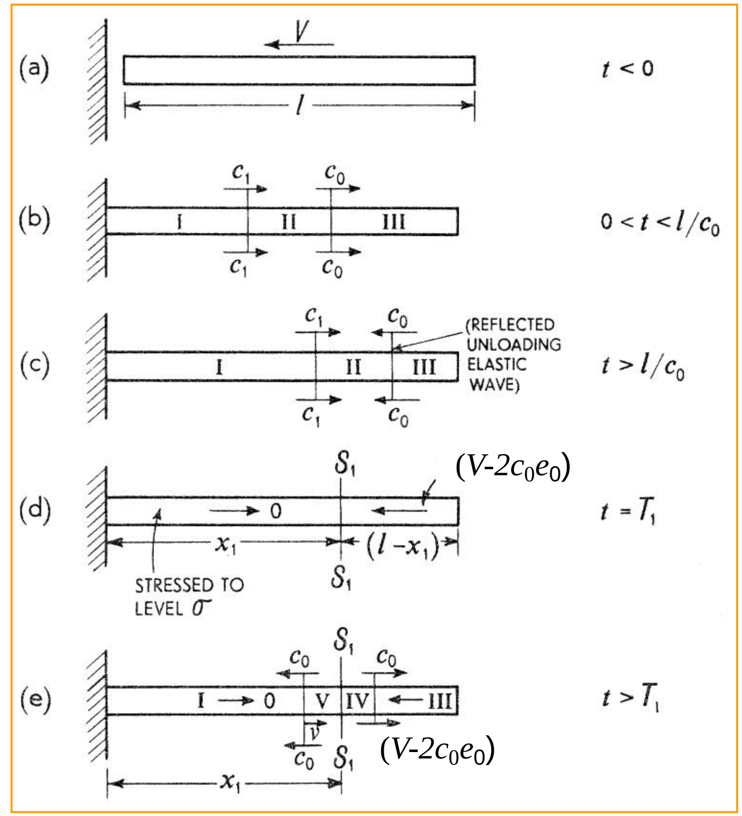
the advancing plastic wave at distance x_1 at time T_1

$$T_1 = \frac{x_1}{c_1} = \frac{2l - x_1}{c_0},$$

so that

$$\frac{x_1}{l} = \frac{2c_1 / c_0}{1 + c_1 / c_0} \quad \rightarrow \quad T = \frac{2l - x_1}{c_0 + c_1}$$

Length of plastically deformed region is : $x_1(1 - e_P)$



Impact of finite length uniform bar with a rigid flat anvil (3)

At time T_1 a bar of length $(l\text{-}x_1)$ and having a speed $(V\text{-}2c_0e_0)$ strucks a stationary bar of length x_1, which already has been subjected to a compressive stress ionis : σ_0

Region V :

The tensile wave elastically unloads the region by amount $\rho c_0 v$ to $\left[E_h(e_t - e_0) + Ec_0 - \rho c_0 v\right]$

Region IV :

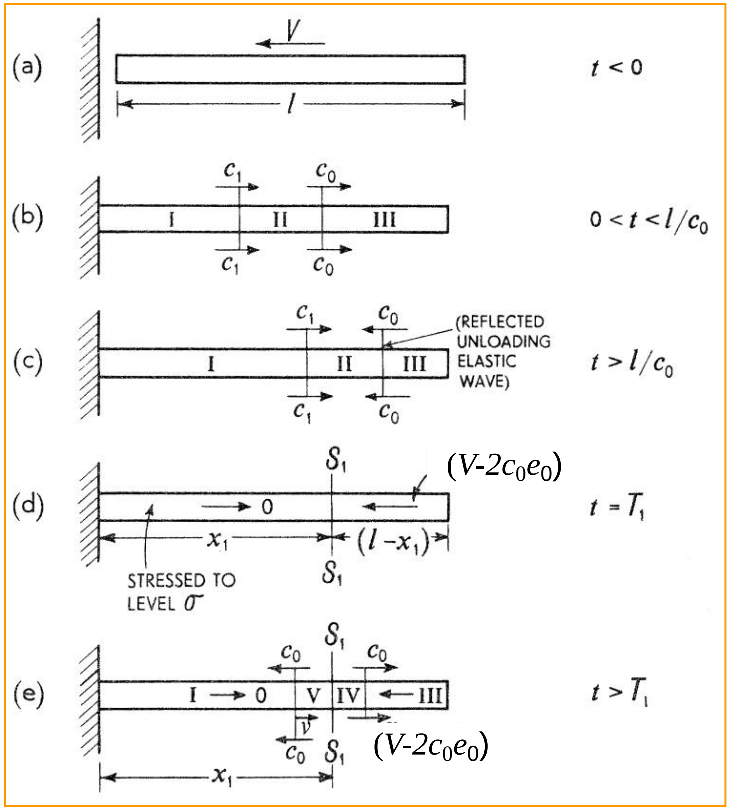
$\left[\rho c_0 (V - 2c_0 e_0 + v)\right]$

The equality between the forces at the interface S_1S_1 gives

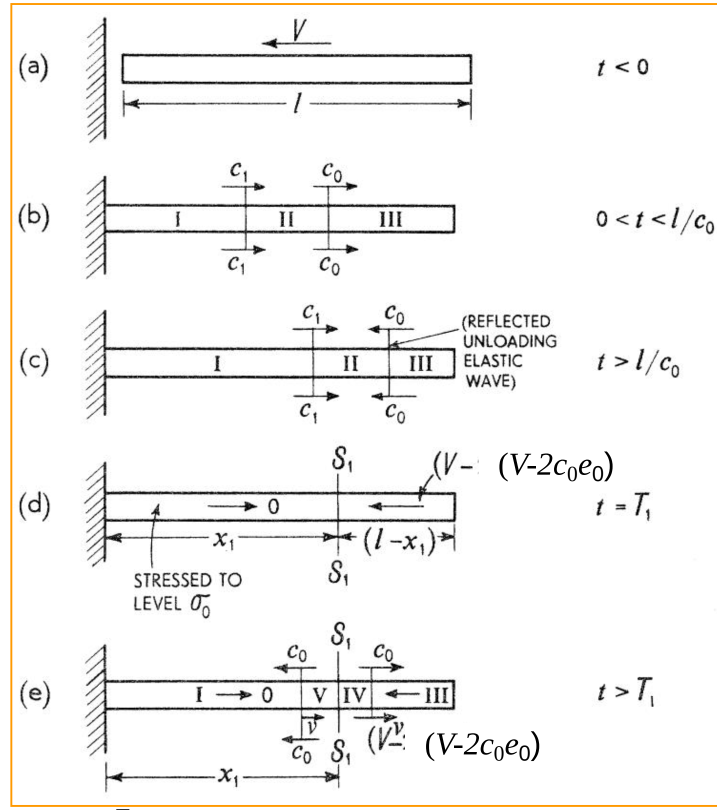
$$E_h(e_t - e_0) + Ee_0 - \rho c_0 v = \rho c_0 (V - 2c_0 e_0 + v)$$

The particles velocity is

$$v = \frac{(c_1 - 3c_0)(V - c_0 e_0)}{2c_0} + V$$



Impact of finite length uniform bar with a rigid flat anvil (4)



The resulting elastic strain is

$$e_1 = \frac{\rho c_0}{E}\left[\frac{(c_1 - 3c_0)(V - c_0 e_0)}{2c_0} + 2V - 2c_0 e_0\right] = \frac{(c_1 + c_0)(V - c_0 e_0)}{2c_0}$$

The greatest value of the strain in order Region IV to remain elastic is $e_1 = e_0$,

$$V = c_0 e_0\left(1 + \frac{2c_0}{c_1 + c_0}\right)$$

The compressive strain in region V results from the change of the compressive strain $e_0 + (V - c_0 e_0)/c_1$ by the amount of the tensile strain $\rho c_0 v / E$

$$e' = \left(e_0 + \frac{V - c_0 e_0}{c_1}\right) - \frac{\rho c_0 v}{E} = \left(e_0 + \frac{V - c_0 e_0}{c_1}\right) - \frac{\rho c_0}{E}\left[\frac{(c_1 - 3c_0)(V - c_0 e_0)}{2c_0} + V\right] =$$

$$= (V - c_0 e_0)\left[\frac{-c_1^2 + c_1 c_0 - 2c_0^2}{2c_0^2 c_1}\right]$$

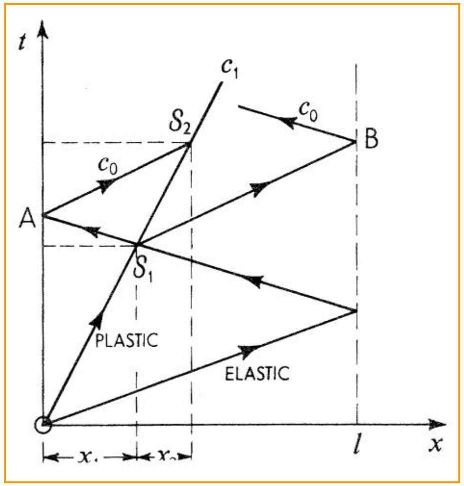
Impact of finite length uniform bar with a rigid flat anvil (5)

If only elastic waves leave the section S_1S_1, the plastic strains in region I will remain constant.

Thus, for impact velocities that satisfy

$$c_0 e_0 < V < c_0 e_0 \left(1 + \frac{2c_0}{c_0 c_1} \right)$$

the interface S_1S_1 is known as a ***stationary second order discontinuity in strain.***



Dynamic compression of a short cylinder between a constant speed rigid die and a stationary die

Velocity V remains constant for a period of time $nl_0/c_1,\ c_1 = \sqrt{E_h/\rho}$

The material engulfed by the plastic wave will move at speed V (the speed of the die) but the material in the elastic region will move at speed $u = c_0 e_0$.

The reflected wave must be a plastic wave and the material is under stress

$$\sigma - \sigma_0 = \rho c_1 . \Delta V$$

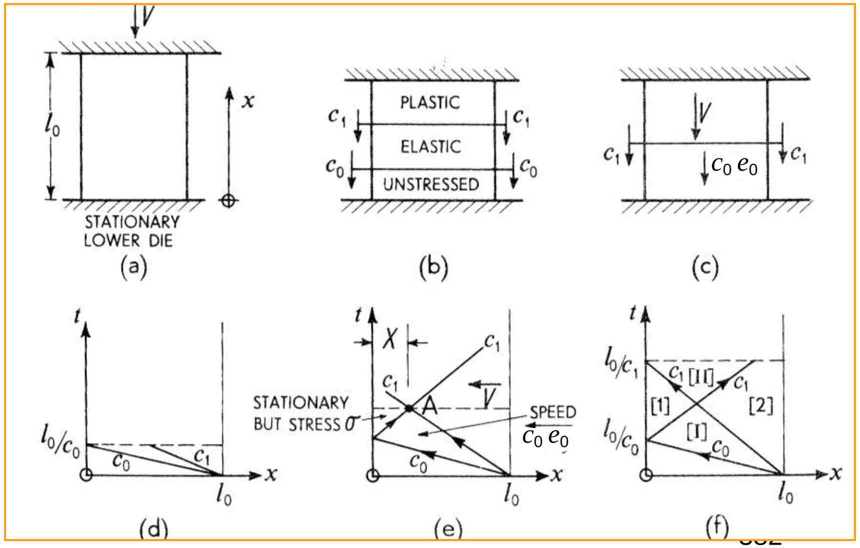
$$\sigma = \sigma_0 + \rho c_1 . c_0 e_0 = \sigma_0 \left(1 + c_1 c_0 \frac{\rho}{E} \right) = \sigma_0 \left(1 + \frac{c_1}{c_0} \right)$$

The incident and reflected plastic wave meet at distance X from the bottom die.

From the meeting, two incident plastic waves are produced.

At time $t = l_0/c_1$, the stress level in zone [2] is

$$\sigma_0 + (V - c_0 e_0)\rho c_1 = \sigma_0 \left(1 - \frac{c_1}{c_0} \right) + \rho c_1 V$$



The particle speed in zone [II] is w and considering zones [1] and [II], the stress in zone [II] is

$$\sigma_0\left(1-\frac{c_1}{c_0}\right)+\rho c_1 w$$

But, considering zones [2] and [II], the stress in zone [II] is

$$\sigma_0\left(1-\frac{c_1}{c_0}\right)+\rho c_1 V+\rho c_1(V-w).$$

However,

$$\sigma_0\left(1-\frac{c_1}{c_0}\right)+\rho c_1 w=\sigma_0\left(1-\frac{c_1}{c_0}\right)+\rho c_1 V+\rho c_1(V-w)$$

so that

$$w=V-c_0e_0.$$

Thus, the stress in zone [II] is

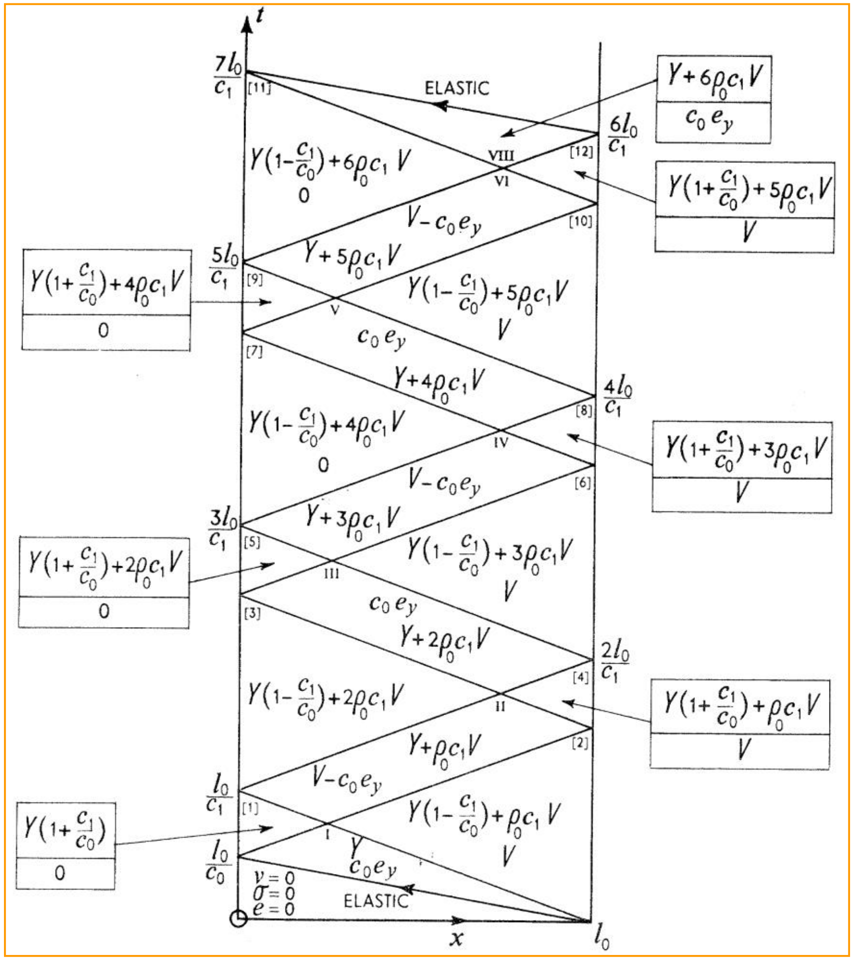
$$\sigma=\sigma_0\left(1+\frac{c_1}{c_0}\right)+\rho c_1(V-c_0e_0)=\sigma_0+\rho c_1 V$$

Assume $V=18.3$ m/s, $\sigma_0=172$MPa, $c_1/c_0=1/10$, and $c_0=5000$ m/s.

$\rho c_1 V=75$ MPa, $\sigma_0(1+c_1/c_0)=190$ MPa, $\sigma_0(1-c_1/c_0)=155$ MPa

Exercise

Calculate the stress levels in different zones if $V = 18.3$ m/s remains constant for a period of time $6l_0/c_1$, $c_1 = \sqrt{E_h/\rho}$, $\sigma_0 = 172\,MPa$, $c_1/c_0 = 1/10$, $c_0 = 5000\ m/s$. The location diagram is given below. (Note that in this diagram $Y = \sigma_0$, $e_y = e_0$ and $\rho_0 = \rho$)



Plastic stress waves in circular shells

Consider a thin-walled tube made of an elastic-plastic material with isotropic linear strain hardening subject to an axial loading. The biaxial stress state $\sigma_x \neq 0, \sigma_\theta \neq 0$ is assumed to obey the von Mises yield condition

Total strain rates: $\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{\mathrm{e}}_{ij} + \dot{\varepsilon}^{\mathrm{p}}_{ij}, \quad i, j = 1, 2$

Flow rules: $\dot{\varepsilon}^{\mathrm{p}}_x = \dfrac{\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{e}}}{2\sigma_{\mathrm{e}}}(2\sigma_x - \sigma_\theta), \quad \dot{\varepsilon}^{\mathrm{p}}_\theta = \dfrac{\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{e}}}{2\sigma_{\mathrm{e}}}(2\sigma_\theta - \sigma_x)$

$$\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{e}} = 2/\sqrt{3}((\dot{\varepsilon}^{\mathrm{p}}_x)^2 + (\dot{\varepsilon}^{\mathrm{p}}_\theta)^2 + \dot{\varepsilon}^{\mathrm{p}}_x \dot{\varepsilon}^{\mathrm{p}}_\theta)^{1/2}, \quad \sigma_{\mathrm{e}} = (\sigma_x^2 + \sigma_\theta^2 - \sigma_x \sigma_\theta)^{1/2}$$

$$\dot{\varepsilon}_e^p = \frac{(1-\lambda)}{\lambda E}\dot{\sigma}_e, \quad \lambda = E_h/E$$

Total strain rates: $\dot{\varepsilon}_x = \dfrac{1}{E}(\dot{\sigma}_x - \nu\dot{\sigma}_\theta) + \dfrac{\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{e}}}{2\sigma_{\mathrm{e}}}(2\sigma_x - \sigma_\theta),$

$$\dot{\varepsilon}_\theta = \frac{1}{E}(\dot{\sigma}_\theta - \nu\dot{\sigma}_x) + \frac{\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{e}}}{2\sigma_{\mathrm{e}}}(2\sigma_\theta - \sigma_x)$$

Equations of motion for a medium in biaxial stress state:

$$\sigma_{x,x} = \rho \dot{v}_x \qquad \frac{\sigma_\theta}{R} = \rho \dot{v}_r,$$

where $v_x = \partial u/\partial t$ and $v_r = \partial w/\partial t$,

Kinematic equations: $\dot{\varepsilon}_x = v_{x,x} \quad \dot{\varepsilon}_\theta = v_r/R.$

Constitutive equations:

The Prandtl-Reuss equations for an isotropic plastic material

$$\sigma_x = \frac{2\sigma_e}{3\dot{\varepsilon}_e}\left(2\dot{\varepsilon}_x + \dot{\varepsilon}_\theta\right), \quad \sigma_\theta = \frac{2\sigma_e}{3\dot{\varepsilon}_e}\left(2\dot{\varepsilon}_\theta + \dot{\varepsilon}_x\right) \quad \text{for a plane stress with} \quad \sigma_z = 0$$

Equivalent stress $\sigma_e = \sigma_0 + E_h \varepsilon_e$

Equivalent strain rate $\dot{\varepsilon}_e = \frac{2}{\sqrt{3}}\left(\dot{\varepsilon}_x^2 + \dot{\varepsilon}_\theta^2 + \dot{\varepsilon}_z^2\right)^{1/2}$

Governing equations for elastic-plastic stress waves

$$\boxed{A^t \boldsymbol{w}_{,t} + A^x \boldsymbol{w}_{,x} + \boldsymbol{b} = 0,}$$

where $\boldsymbol{w} = [\dot{u}, \sigma_x, \dot{w}, \sigma_\theta]^{\mathrm{T}}$

and

$$A^t = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & \alpha & 0 & \beta \\ 0 & 0 & \rho R & 0 \\ 0 & \beta & 0 & \gamma \end{bmatrix}, \quad A^x = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ -\sigma_\theta \\ -\frac{\dot{w}}{R} \end{bmatrix},$$

$$\alpha = \frac{1}{E}\left[1 + \frac{1-\lambda}{\lambda}\frac{(2\sigma_x - \sigma_\theta)^2}{(2\sigma_e)^2}\right],$$

$$\beta = \frac{1}{E}\left[-v + \frac{1-\lambda}{\lambda}\frac{(2\sigma_x - \sigma_\theta)(2\sigma_\theta - \sigma_x)}{(2\sigma_e)^2}\right],$$

$$\gamma = \frac{1}{E}\left[1 + \frac{1-\lambda}{\lambda}\frac{(2\sigma_\theta - \sigma_x)^2}{(2\sigma_e)^2}\right]$$

$$\lambda = E_h / E$$

Stress wave speeds

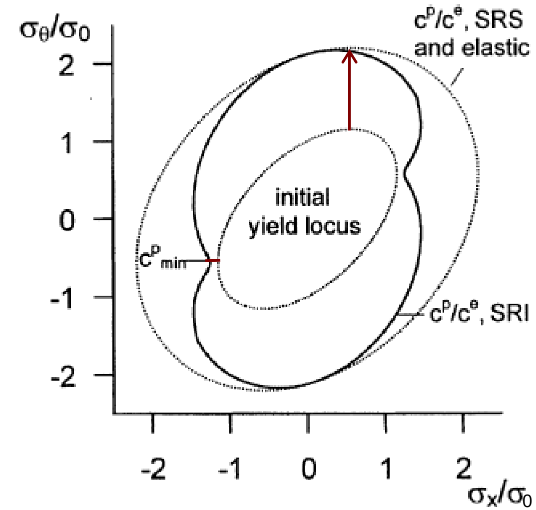
$$|-cA^t + A^x| = 0,$$

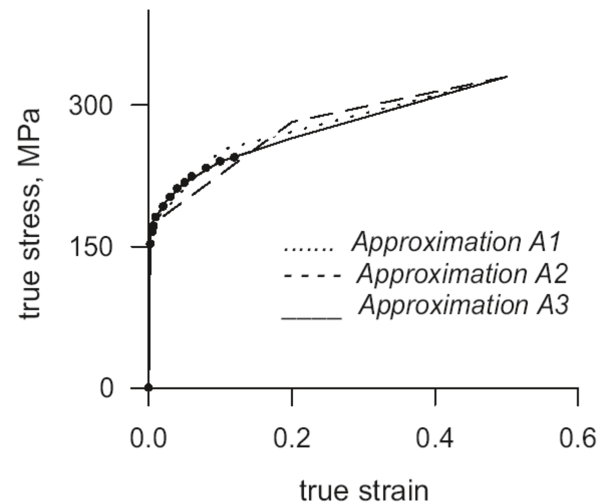
$$\begin{vmatrix} -\rho c & -1 & 0 & 0 \\ -1 & -\alpha c & 0 & -\beta c \\ 0 & 0 & -\rho R c & 0 \\ 0 & -\beta c & 0 & -\gamma c \end{vmatrix} = 0$$

$$\boxed{c = \pm\left(\frac{\gamma}{\rho(\alpha\gamma - \beta^2)}\right)^{1/2}}. \quad \boxed{c = \pm(E/\rho(1-\nu^2))^{1/2}}$$

$$c^{\mathrm{p}}_{\min} = \pm(E/\rho)^{1/2}\{4\lambda/[4\lambda(1-\nu^2)+3(1-\lambda)]\}^{1/2}$$
$$\left(\sigma_\theta = \frac{\sigma_x}{2}, \quad \dot\sigma_\theta = \nu\dot\sigma_x, \quad \sigma_x < 0, \quad \sigma_\theta < 0\right),$$

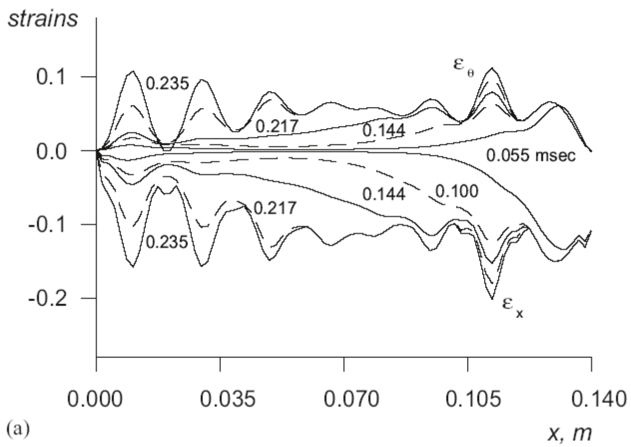
$$E = 70\,GPa, \quad E_h = 500MPa$$
$$c^p_{uniaxial} = \sqrt{E_h/\rho} = c_0\sqrt{\lambda} = 430\,m/s, \quad c^p_{\min} = 490\,m/s$$



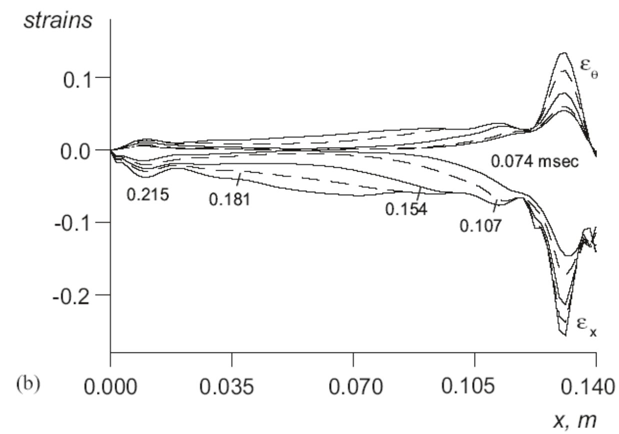


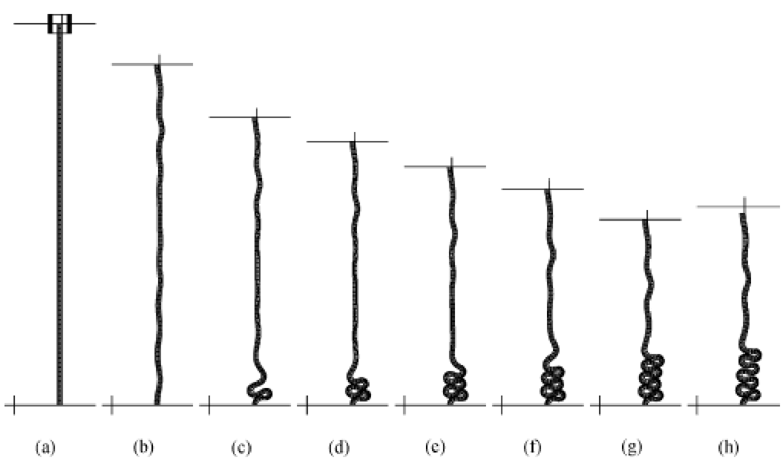
Stress wave propagation and buckling shapes

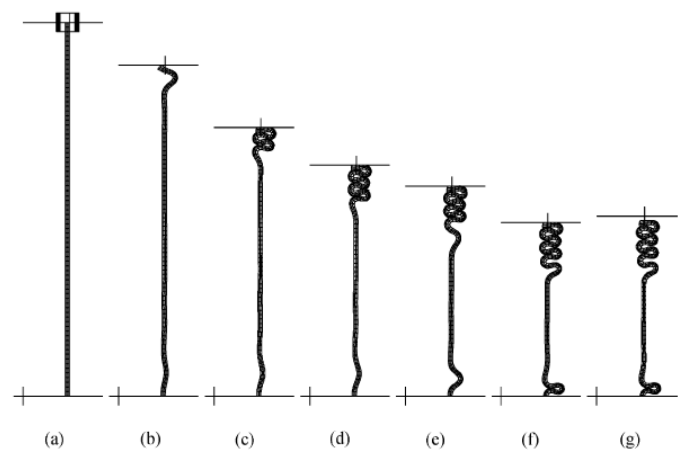
$\sigma_0 = 175$ MPa, $E_h = 600$ MPa, $c_p = 535$ m/s



$\sigma_0 = 175$ MPa, $E_h = 200$ MPa, $c_p = 312$ m/s



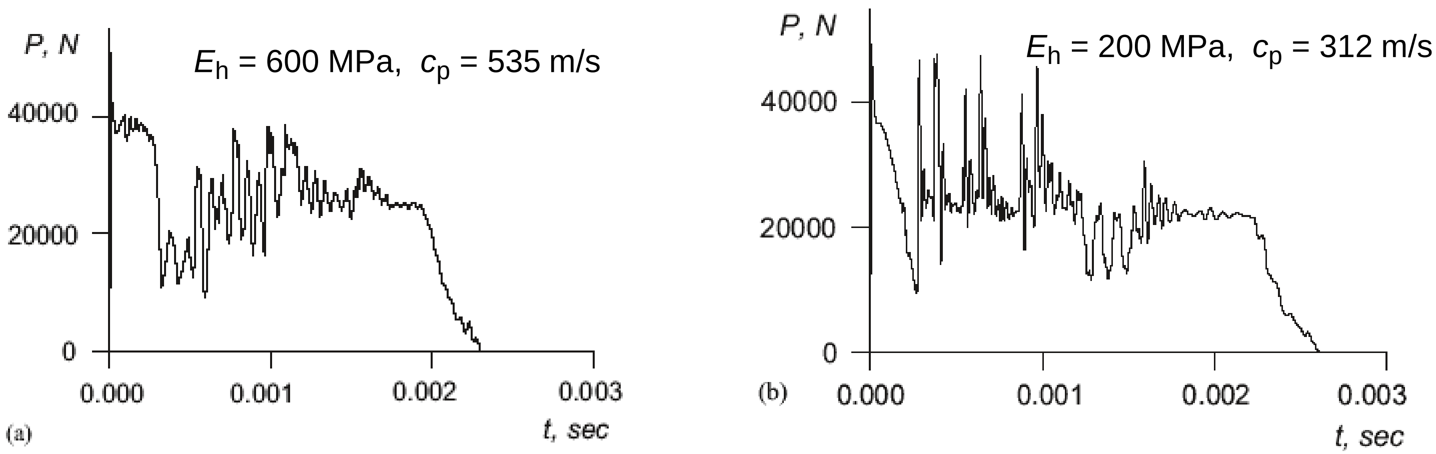




$D = 35$ mm, $h = 1.5$ mm, $L = 140$ mm; $G = 0.71$ kg, $V_0 = 75$ m/s

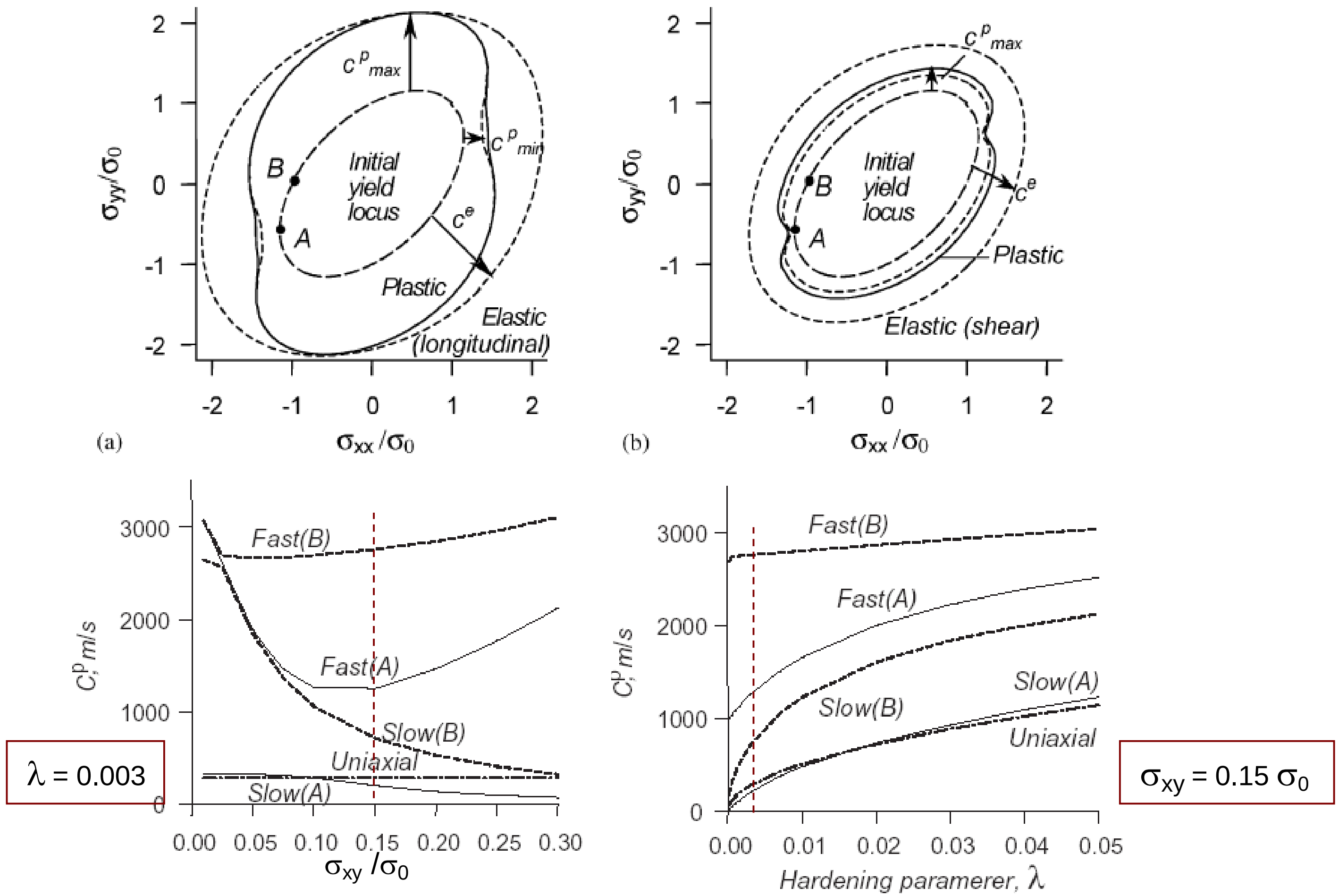
Discussion

D = 35 mm, h = 1.5 mm, L = 140 mm; G = 0.71 kg, V_0 = 75 m/s, σ_0 = 175 MPa



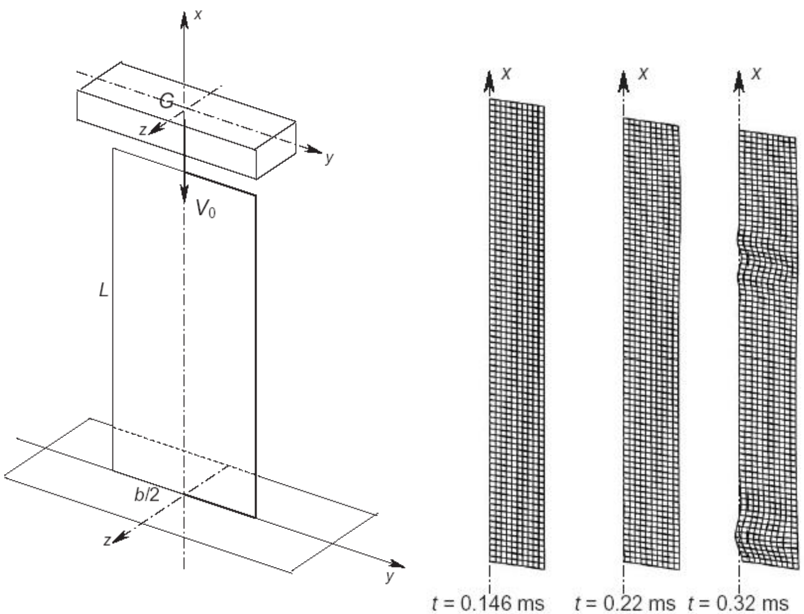
Dynamic plastic buckling can develop only within a sustained axial plastic flow when no unloading across the shell thickness occurs. Impact velocity causing stresses above the elastic limit of the material is the necessary condition for the development of small wrinkles along the entire shell. The sufficient condition can be formulated as a sufficient time for the axial plastic wave to traverse the entire shell length. This particular time depends on the wave speed (material strain hardening, respectively) and the inertia characteristics of the shell, including the shell geometry and the material density.

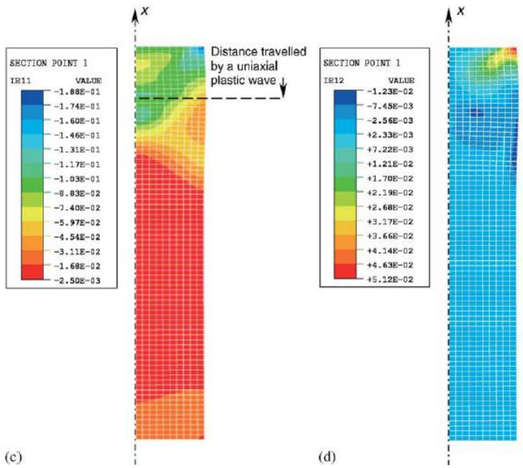
Comparison between the buckling modes in circular and square tubes



Stress waves in rectangular plates

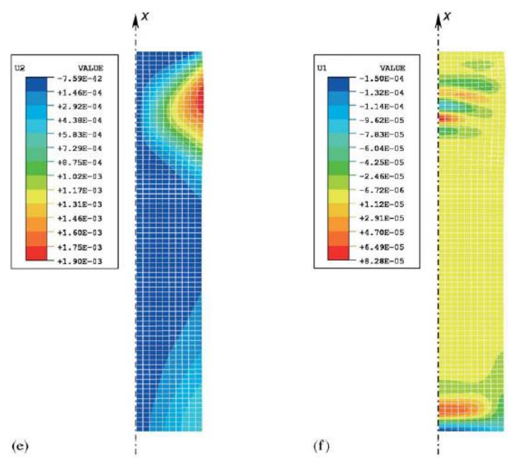
$\lambda = E_h/E = \ 0.003, V_0 = 60$ m/s





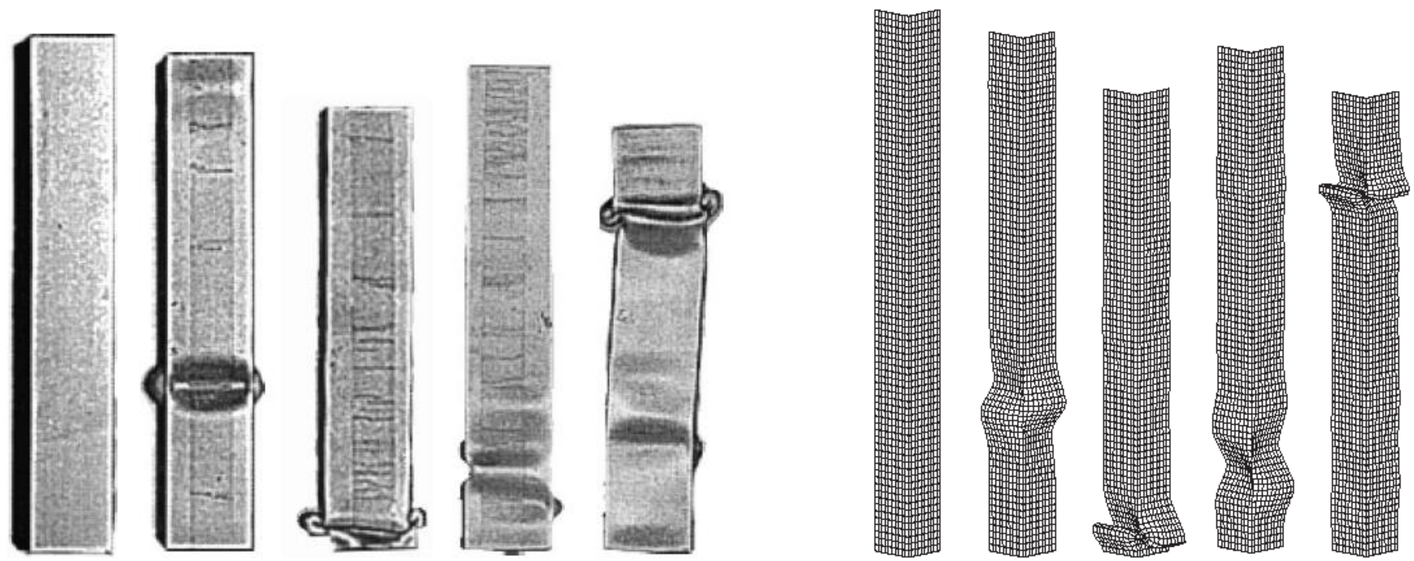
Axial and shear plastic strains

t = 0.146 ms



Lateral expansion and buckling

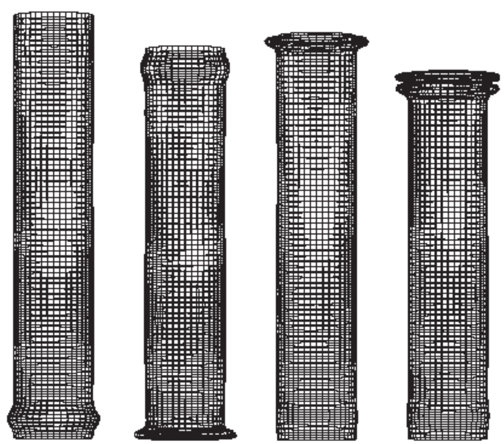
Buckling of square tubes

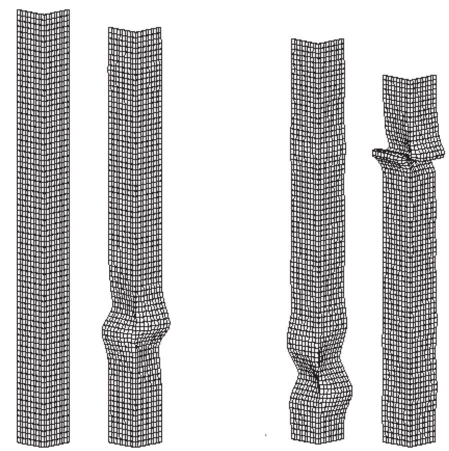


(a) (b) (c) (d) (e)   (a) (b) (c) (d) (e)

Infuence of the impact velocity - experimentally and numerically obtained buckling shapes. (a) Initial; (b) tube **N04**: V_0 = 15,91 m/s, G = 0.95 kg; (c) tube **N81**: V_0 = 35.35 m/s, G = 0.44 kg; (d) tube **N86**: V_0 = 64.62 m/s, G = 0,103 kg, and (e) tube **N82**: V_0 = 91.53 m/s, G = 0:103 kg.

Comparison between the buckling shapes of circular and square tubes





V_0 = 64.62 m/s, G = 0,103 kg