Modelling the applied vertical stress and settlement relationship of shallow foundations in saturated and unsaturated sands

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Abstract: The bearing capacity and settlement of foundations are determined experimentally or modelled numerically based on conventional soil mechanics for saturated soils. In both methods, bearing capacity and settlement are estimated based on the applied vertical stress versus surface settlement relationship. These methods are also conventionally used for soils that are in an unsaturated condition, ignoring the contribution of matric suction. In this study, a methodology is proposed to estimate the bearing capacity and settlement of shallow foundations in unsaturated sands by predicting the applied vertical stress versus surface settlement relationship. The proposed method requires soil parameters obtained under only saturated conditions (i.e., effective cohesion, effective internal friction angle, and modulus of subgrade reaction from model footing test) along with the soil-water characteristic curve (SWCC). In addition, finite element analyses are undertaken to simulate the applied vertical stress versus surface settlement relationship for unsaturated sands. The proposed method and finite element analyses are performed using an elastic – perfectly plastic model. The predicted bearing capacities and settlements from the proposed method and finite element analyses are compared with published model footing test results. There is good agreement between measured and predicted results.

Key words: unsaturated soil, model footing tests, settlement, bearing capacity, elastic – perfectly plastic model, finite element analysis.

Résumé : La capacité portante et le tassement de fondations sont déterminés expérimentalement ou numériquement d'après le modèle classique de la mécanique des sols pour les sols saturés. Pour les deux méthodes, la capacité portante et le tassement sont estimés d'après la relation entre la contrainte normale verticale appliquée en fonction du tassement de surface. Ces méthodes sont également utilisées conventionnellement pour les sols qui sont dans un état non saturé en ignorant la contribution de la succion matricielle. Dans la présente étude, une méthodologie est proposée afin d'estimer la capacité portante et le tassement des fondations peu profondes dans les sols sableux non saturés en prédisant la relation entre la contrainte verticale appliquée et le tassement de surface. La méthode proposée ne requiert que les paramètres du sol obtenus sous des conditions saturées (c.-à-d. cohésion effective, angle de friction interne effectif, et module de réaction du sol obtenu lors d'essais de semelle à échelle réduite) ainsi que sa courbe de rétention d'eau (« SWCC »). En outre, des analyses par éléments finis sont également entreprises afin de simuler la relation entre la contrainte verticale appliquée et le tassement de surface de sols sableux non saturés. La méthode proposée et les analyses par éléments finis sont effectuées en utilisant le modèle élastique – parfaitement plastique. La capacité portante et le tassement prédits par la méthode proposée et l'analyse par éléments finis sont comparés avec les résultats publiés des essais de semelles à échelle réduite. Il existe une bonne concordance entre les mesures et les résultats prédits.

Mots-clés : sols non saturés, essais sur semelle à échelle réduite, affaissement, capacité portante, modèle élastique – parfaitement plastique, analyse par éléments finis.

Introduction

Bearing capacity and settlement are two key parameters required in the design of foundations. There are several procedures or techniques available for the interpretation of bearing capacity and settlement behaviour of saturated soils (Poulos and Davids 2005). These procedures or techniques are also conventionally used by practicing engineers for estimating bearing capacity and settlement behaviour of soils that are in an unsaturated state. This approach is used due to the following two key reasons. First, the loss of suction due to precipitation can significantly reduce bearing capacity and may be a contributing factor to the instability of superstructures. Due to this reason, the design of foundations is usually undertaken using conventional soil mechanics assuming the ground below the foundation is in a saturated state. This assumption is believed to provide a conservative design approach in the assessment of bearing capacity of unsaturated soils. Second, there is a lack of a valid framework to interpret bearing capacity and settlement behaviour of unsaturated soils.

Steensen-Bach et al. (1987), Oloo et al. (1997), Costa et al. (2003), Mohamed and Vanapalli (2006), Rojas et al.

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(2007), and Vanapalli et al. (2007) demonstrated that the bearing capacity of unsaturated soils is significantly influenced by matric suction from their investigations on model footing or in situ plate load tests. Recently, Vanapalli and Mohamed (2007) provided a framework to both interpret and predict the variation of bearing capacity of unsaturated soils with respect to matric suction using saturated shear strength parameters (i.e., c', ϕ') and the soil-water characteristic curve (SWCC). There is a smooth transition between the proposed equation for unsaturated soils and the conventional Terzaghi's (1943) bearing capacity equation for saturated soils. In other words, the proposed equation will be the same as the conventional Terzaghi's (1943) bearing capacity equation when the matric suction value is set equal to zero.

In many cases, it is settlement behaviour — not bearing capacity — that governs the design of a foundation. This is particularly true in the case of coarse-grained soils such as sandy soils as settlement must be determined or estimated reliably due to two main reasons: (*i*) differential settlements in sandy soils are predominant in comparison with clayey soils because sand deposits are typically heterogeneous in nature and (*ii*) settlements in sandy soils are governed by elastic settlement that occurs quickly and can contribute to significant damage to superstructures (Maugeri et al. 1998).

The key parameter used in estimation of foundation settlements in coarse-grained soils is the modulus of elasticity, which is typically assumed to be constant both below and above the groundwater table in homogeneous deposits of soils. In other words, the influence of matric suction (i.e., unsaturated conditions) is not taken into account. A close examination of experimental results of the applied vertical stress versus surface settlement relationships for model footing tests conducted on unsaturated soils show that the modulus of elasticity is significantly influenced by matric suction (Steensen-Bach et al. 1987; Vanapalli and Mohamed 2007; Oh et al. 2009). Throughout this paper, the term "surface settlement" refers to the settlement below the center of a footing.

In this paper, two methods are presented for predicting the applied vertical stress versus surface settlement (hereafter referred to as stress versus settlement (SVS)) relationship of model footing tests in unsaturated sands. In the first method (proposed method), the SVS relationship was simplified using elastic – perfectly plastic model. In other words, the relationship was idealized using two straight lines, which represent elastic and perfectly plastic behaviour. These two straight lines were established extending the concepts proposed by Oh et al. (2009) and Vanapalli and Mohamed (2007) to predict the initial tangent modulus of subgrade reaction (i.e., k_{is}) and bearing capacity with respect to matric suction, respectively. In the second method, finite element analysis (FEA) was carried out also using the elastic – perfectly plastic model with the Mohr-Coulomb yield criterion (Chen and Zhang 1991). The procedures for modelling the model footing tests in unsaturated sands are presented in detail in a later section. The FEA was undertaken using SIGMA/W (Geostudio 2004), a software product of GEO-SLOPE (Krahn 2004). The predicted settlements and bearing capacities for unsaturated sand using the proposed method and the FEA were compared with those obtained from model footing tests performed in a specially designed apparatus at the University of Ottawa, Canada. The comparisons show that there is good agreement between measured and predicted elastic settlements and bearing capacities.

Background

Bearing capacity of unsaturated sandy soils

Fredlund et al. (1978) proposed the following shear strength equation for an unsaturated soil in terms of stress state variables:

$$[1] \qquad \tau_{\text{unsat}} = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$

where τ_{unsat} is shear strength of an unsaturated soil, c' is effective cohesion, $(\sigma_n - u_a)$ is net normal stress (σ_n is normal stress and u_a is pore-air pressure), ϕ' is effective internal friction angle, $(u_a - u_w)$ is matric suction (u_w is pore-water pressure), and ϕ^b is angle of shearing resistance relative to an increase in matric suction.

Several investigators have proposed empirical or semi-empirical procedures for predicting shear strength of unsaturated soils using the SWCC as a tool (for example, Fredlund et al. 1996; Vanapalli et al. 1996*a*; Öberg and Sällfours 1997; Bao et al. 1998; Khalili and Khabbaz 1998; Xu and Sun 2001; Tekinsoy et al. 2004; Xu 2004).

Vanapalli et al. (1996*a*) and Fredlund et al. (1996) proposed a semi-empirical procedure that is consistent with eq. [1] to predict the variation of shear strength with respect to matric suction using saturated shear strength parameters (i.e., c', ϕ') and the SWCC as given below.

$$[2] \qquad \tau_{\text{unsat}} = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)(S^{\kappa})\tan\phi'$$

where S is degree of saturation and κ is a fitting parameter.

The studies showed that the fitting parameter κ is a function of plasticity index, I_p, and $\kappa = 1$ is required to provide good comparison between the measured and predicted shear strength of unsaturated sandy soils (i.e., $I_p = 0$) (Vanapalli and Fredlund 2000; Garven and Vanapalli 2006).

Vanapalli and Mohamed (2007) suggested an equation to predict the variation of bearing capacity with respect to matric suction for surface footings on unsaturated sandy soils using saturated shear strength parameters (i.e., c', ϕ') and the SWCC (eq. [3]), extending the same approach for developing eq. [2]. A fitting parameter $\psi = 1$ is required for predicting bearing capacity of unsaturated sandy soils (i.e., $I_p = 0$), which is similar to the procedure of using $\kappa = 1$ as a fitting parameter for predicting the shear strength of unsaturated sandy soils.

$$[3] \qquad q_{\text{ult}} = [c' + (u_a - u_w)_b(1 - S^{\psi}\tan\phi') + (u_a - u_w)_{\text{AVR}}S^{\psi}\tan\phi'](N_c\xi_c) + 0.5B\gamma N_{\gamma}\xi_{\gamma}$$

where q_{ult} is ultimate bearing capacity, B is footing width, L is footing length, γ is soil unit weight, N_c is a bearing capacity factor from Terzaghi (1943), N_{γ} is a bearing capacity factor from Kumbhokjar (1993), $(u_a - u_w)_b$ is air-entry value, $(u_a - u_w)_{\text{AVR}}$ is average value of measured matric suction, and ξ_c and ξ_{γ} are two shape factors from Vesić (1973) defined as

$$\xi_c = 1.0 + \left(\frac{N_q}{N_c}\right)\left(\frac{B}{L}\right)$$
$$\xi_\gamma = 1.0 - 0.4\left(\frac{B}{L}\right)$$

Estimation of average matric suction value

The average matric suction value, $(u_a - u_w)_{AVR}$ in eq. [3], can be obtained by estimating the matric suction value corresponding to the centroid of the suction distribution diagram from 0 to 1.5B depth assuming a hydrostatic distribution profile (Fig. 1), which can be regarded as "representative suction value." The assumption of a hydrostatic suction profile above the groundwater table can be extended where suction measurement data are not available in practice. This is a safe assumption for sandy soils as results tend to be on the conservative side. This concept can be justified based on the following investigations.

Poulos and Davis (1974) suggested that when a load is applied to a shallow foundation, the stress transferred to the ground due to the load is predominant in the 0 to 1.5B depth region. The stress increment below a square footing at a depth deeper than 1.5B is less than 15% of the applied stress at the ground surface.

Agarwal and Rana (1987) performed model footing tests in sands to study the influence of the groundwater table on settlement. The results of the study show that the initially applied settlement starts increasing as the water table level below a footing increases (i.e., is deeper) and eventually reaches the depth of approximately 1.5B (Fig. 2). These results also indirectly support the assumption that the predominant zone of stress due to the load applied on the footing is limited from 0 to 1.5B depth below a footing.

Influence of air-entry value on bearing capacity

Unlike fine-grained soils, SWCCs of coarse-grained soils distribute in narrow range of matric suction values. Many sands rapidly desaturate from saturated conditions to close to dry conditions over a matric suction range of 0 to 10 kPa. This indicates that even low matric suction values of 1 kPa in coarse-grained soils can lead to significant differences in predicted bearing capacity values. For this reason, in the present study, three different methods were used to estimate the air-entry value in eq. [3] for the sand used. These details and sensitivity analyses are discussed in a later section.

Effective internal friction angle

The original bearing capacity equation proposed by Terzaghi (1943) was based on assuming a plane strain condition for continuous footings. Hence, the effective internal friction angle, ϕ', obtained from conventional laboratory tests (e.g., triaxial shear test) needs to be modified taking account the difference between plane strain (PS) and axisymmetric conditions in conventional triaxial compression (CTC) tests. In general, it is known that ϕ' from PS (ϕ'_{PS}) is typically higher than that of CTC (ϕ'_{CTC}) (Marachi et al. 1981; Alshibli et al. 2003; Wanatowski and Chu 2007).

Wanatowski and Chu (2007) showed that the difference in measured ϕ' values using PS and CTC increases with decreasing void ratio. The minimum ratio of ϕ'_{PS}/ϕ'_{CTC} was approximately 1.1 for the void ratio range they studied. The Danish code of practice DS 415 (DSCE 1984) suggests 1.1 as a ratio of ϕ'_{PS}/ϕ'_{CTC}. In addition, the following relationship is also suggested in this Danish code in terms of relative density, I_D:

[4] $$\phi'_{PS} = \phi'_{CTC}(1 + 0.163 I_D)$$

The ratio ϕ'_{PS}/ϕ'_{CTC} using eq. [4] for the average relative density used in the present study (i.e., 63.76%) is also estimated as 1.1.

Steensen-Bach et al. (1987) showed that there was a good comparison between measured and computed bearing capacities when the effective internal friction angle, ϕ', was increased by 10% to 15%.

Hence, in this study, the analyses were carried out using two effective internal friction angles, namely ϕ' (i.e., 35.3°) and 1.1ϕ' (i.e., 39°), for both computation of bearing capacity using eq. [3] and FEA for comparison purposes.

Bearing capacity factors

Bearing capacity factors proposed by Terzaghi (1943) for N_c and N_q are generally used in engineering practice. However, there is no consensus in the literature with respect to the use of appropriate values of N_γ. There are significant differences in N_γ values proposed by various investigators for effective internal friction angles, ϕ', greater than 30°; however, the differences are negligible in engineering practice for ϕ' less than 30° (Budhu 2006).

The bearing capacity equation for unsaturated soils proposed by Vanapalli and Mohamed (2007) (i.e., eq. [3]) adopts N_c and N_q from Terzaghi (1943), shape factors from Vesić (1973), and N_γ from Kumbhokjar (1993). The variation of bearing capacity factors with effective internal friction angle, ϕ', used in eq. [3] is shown in Fig. 3.

To investigate the reliability of the bearing capacity factors used in eq. [3], the bearing capacity values calculated using Terzaghi (1943), Meyerhof (1963), Vesić (1973), and Vanapalli and Mohamed (2007) equations were compared with measured bearing capacity values for the soil under saturated conditions. Table 1 provides a summary of the bearing capacity factors in eq. [3] and other bearing capacity factors used for comparison. Figures 4a and 4b show the comparison between the measured and calculated bearing capacities under saturated conditions for the model footing test results from Steensen-Bach et al. (1987) and Mohamed and Vanapalli (2006), respectively. The calculated bearing capacity values using eq. [3] are the closest to the measured model footing test results.

Modulus of elasticity of unsaturated soils

The modulus of elasticity, E, for plate load tests (or model footing tests) can be calculated using the equation given below (Timoshenko and Goodier 1951)

[5] $$E = \frac{(1-\nu^2)I_w}{(\Delta\delta/\Delta q_p)}B_p = k_s(1-\nu^2)I_w B_p$$

where ν is Poisson's ratio (a value of 0.3 was used for this study), I_w is influence factor (0.79 for circular plate and 0.88 for square plate), k_s is modulus of subgrade reaction (i.e.,

Fig. 1. Estimation of average matric suction value using the centroid of the suction distribution diagram. B, width of model footing; z, centroid of gravity of the suction distribution diagram; Ψ, matric suction.

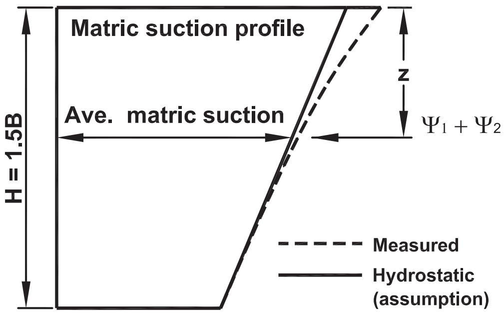


Fig. 2. Relationship between water table correction factor (C_w) and depth of water table below footing base (Agarwal and Rana 1987). δ, settlement applied initially; δ_1, increased settlement due to increased depth of water table.

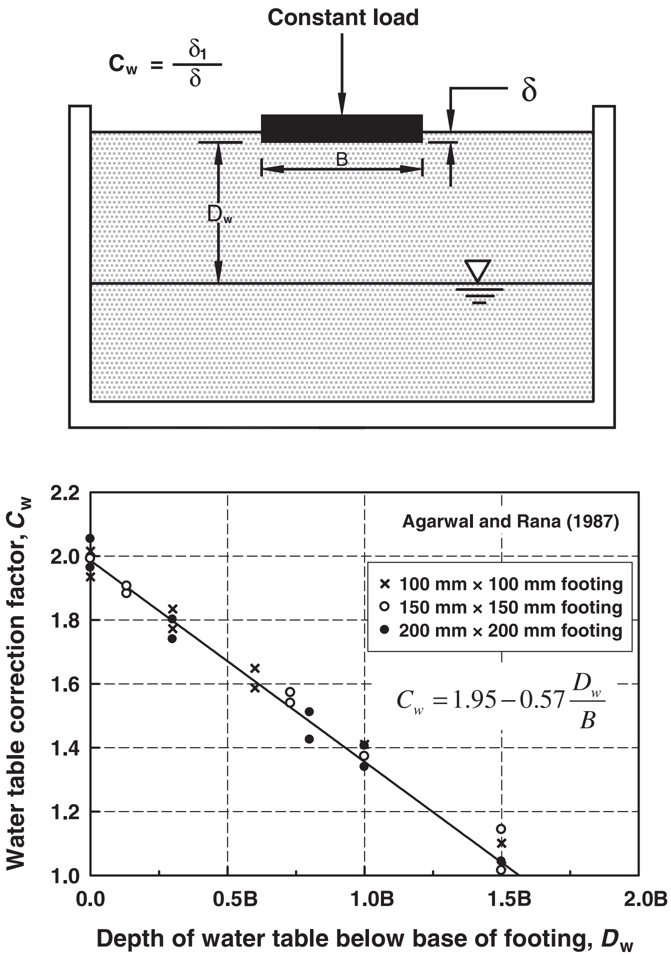
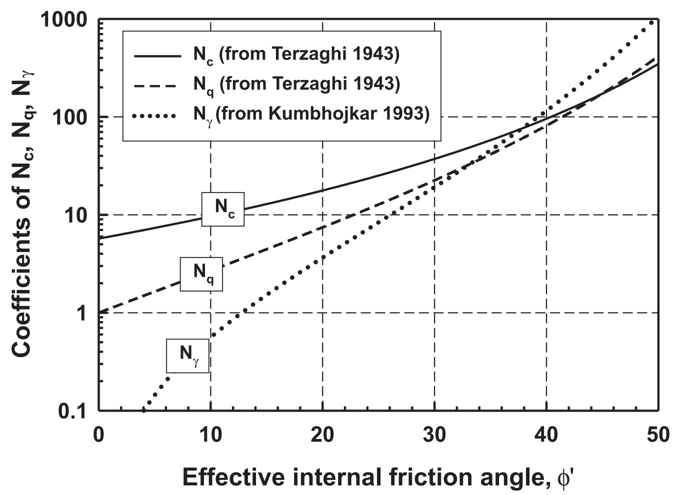


Fig. 3. Bearing capacity factors and effective internal friction angle relationship used for the equation proposed by Vanapalli and Mohamed (2007).



slope of applied vertical stress, Δq_p, versus surface settlement, $\Delta\delta$, relationship), and B_p is width or diameter of a plate.

Analyzing model footing test results on three different sands using different sizes of model footings (see Table 2), Oh et al. (2009) proposed a model for predicting the variation of initial tangent modulus of subgrade reaction, k_{is}, of unsaturated sands using the k_{is} in a saturated condition (i.e., $k_{is(sat)}$) and the SWCC as given below (eq. [6]). In this model, two fitting parameters, α and β were used

$$[6] \qquad k_{is(unsat)} = k_{is(sat)}\left[1 + \alpha\frac{(u_a - u_w)}{(P_a/101.3)}(S^\beta)\right]$$

where $k_{is(sat)}$ and $k_{is(unsat)}$ are initial tangent modulus of subgrade reaction under saturated and unsaturated conditions, respectively, and P_a is atmospheric pressure (i.e., 101.3 kPa).

In eq. [6], the terms S^β and α control the nonlinear variation of the modulus of elasticity. The term (P_a/101.3) is used for maintaining consistency with respect to the dimensions and units on both sides of the equation. A value of $\beta = 1$ (similar to using $\kappa = 1$ in eq. [2] and $\psi = 1$ in eq. [3] for predicting the shear strength and bearing capacity of unsaturated sandy soils (i.e., $I_p = 0$), respectively) is required for providing comparisons between measured and predicted moduli of elasticity of unsaturated sandy soils. It should be noted that the fitting parameter, α, is a function of width of the model footing.

Combining eqs. [5] and [6], the variation of the initial tangent modulus of elasticity of unsaturated soils can be estimated using eq. [7]

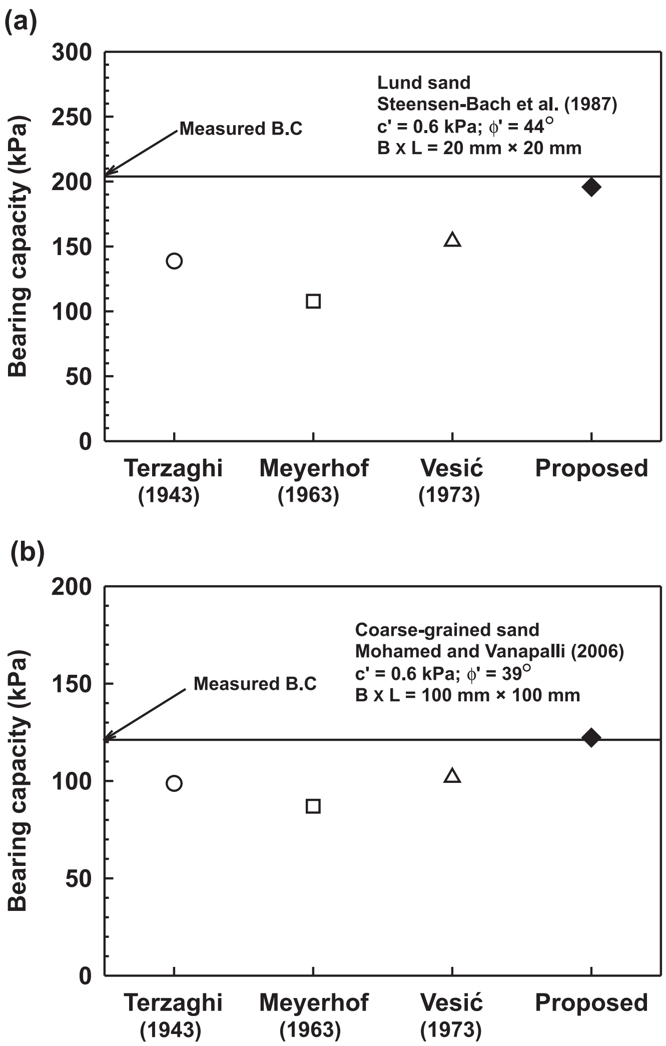
$$[7] \qquad E_{i(unsat)} = E_{i(sat)}\left[1 + \alpha\frac{(u_a - u_w)}{(P_a/100)}(S^\beta)\right]$$

Table 1. Summary of bearing capacity ($q_{ult} = cN_c\xi_c + 0.5B\gamma N_\gamma \xi_\gamma$ for surface footing in all cases) and shape factors used in the study for calculating bearing capacity.

Author	N_c	N_q	N_γ	ξ_c	ξ_γ
Terzaghi (1943)	$(N_q - 1)\cot\phi'$	$\dfrac{e^{2[(3\pi/4)-(\phi'/2)]\tan\phi'}}{2\cos^2[45+(\phi'/2)]}$	$\dfrac{\tan\phi'}{2}\left(\dfrac{K_p}{\cos^2\phi'} - 1\right)$	1.3 (square)	0.8 (square)
Meyerhof (1963)	Terzaghi (1943)	$e^{\pi\tan\phi'}\tan^2[45+(\phi'/2)]$	$(N_q - 1)\tan(1.4\phi')$	$1 + 0.2K_p\left(\dfrac{B}{L}\right)$	$1 + 0.1K_p\left(\dfrac{B}{L}\right)$
Vesić (1973)	Meyerhof (1963)	Meyerhof (1963)	$2(N_q + 1)\tan\phi'$	$1 + \left(\dfrac{N_q}{N_c}\right)\left(\dfrac{B}{L}\right)$	$1 - 0.4\left(\dfrac{B}{L}\right)$
Vanapalli and Mohamed (2007)	Terzaghi (1943)	Terzaghi (1943)	Kumbhojkar (1973)	Vesić (1973)	Vesić (1973)

Note: K_p, passive earth pressure coefficient.

Fig. 4. Comparison between measured and predicted bearing capacity under saturated conditions using different bearing capacity (B.C) equations on (*a*) Lund sand and (*b*) coarse-grained sand.



where $E_{i(sat)}$ and $E_{i(unsat)}$ are the initial tangent modulus of elasticity under saturated and unsaturated conditions, respectively.

More details on estimating initial tangent modulus of elasticity are illustrated graphically in Fig. 5. The terms "modulus of subgrade reaction" and "modulus of elasticity," hereafter, indicate initial tangent modulus of subgrade reaction (i.e., K_i) and modulus of elasticity (i.e., E_i).

Estimation of applied vertical stress versus surface settlement relationship of model footing test in unsaturated sands

In the present study, the applied vertical stress versus surface settlement (i.e., SVS) relationship of model footing tests was predicted using two methods.

First method (proposed method)

In this method, the SVS relationship was assumed to consist of two straight lines, L1 (elastic line) and L2 (plastic line) as shown in Fig. 5, assuming linear elastic – perfectly plastic behaviour. The line L1 in Fig. 5 represents linear elastic behaviour that has a slope equal to the modulus of subgrade reaction, k_{is}, that can be estimated using eq. [6]. The line L2 represents unrestricted perfectly plastic settlement behaviour at constant stress, which indicates the bearing capacity value estimated using eq. [3].

Second method (finite element analysis)

Finite element analysis (FEA) in unsaturated sands

The second method uses FEA to obtain the SVS relationships by simulating the model footing tests in unsaturated sands using the Mohr-Coulomb yield criterion. The yield function, F, of the elastic – perfectly plastic model (Chen and Zhang 1991) is given below

$$[8]\qquad F = \sqrt{J_2}\sin\left(\theta + \frac{\pi}{3}\right) - \sqrt{\frac{J_2}{3}}\cos\left(\theta + \frac{\pi}{3}\right)\sin\phi' - \frac{I_1}{3}\sin\phi' - c\cos\phi'$$

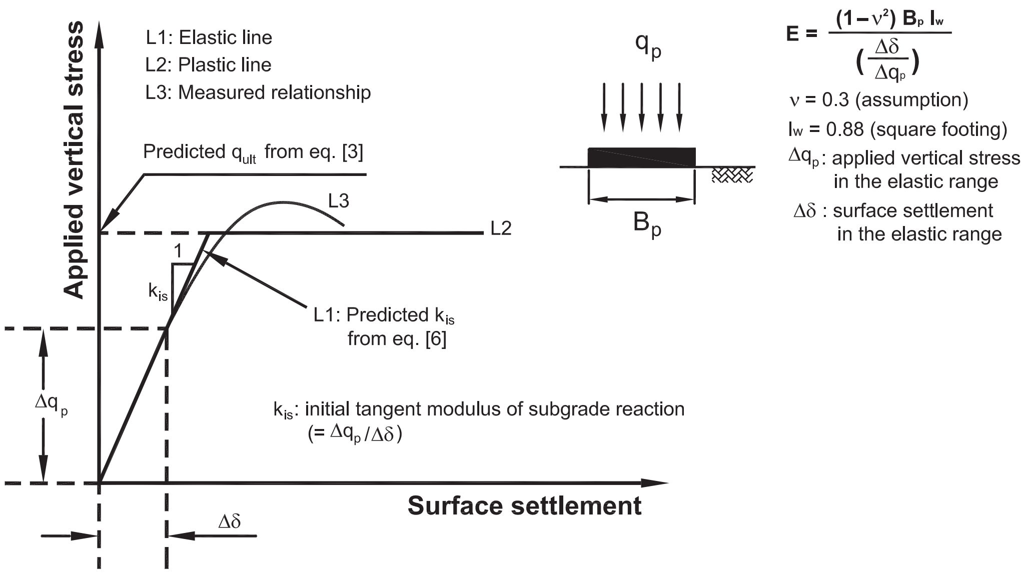
where J_2 is the second deviatoric stress invariant, θ is lode angle $\{=(1/3)\cos^{-1}[(3\sqrt{3}/2)(J_3/J_2^{3/2})]\}$, J_3 is the third deviatoric stress invariant, and I_1 is the first stress invariant.

It is well known that two independent stress state variables (i.e., suction and net normal stress) are necessary to reasonably interpret mechanical properties of unsaturated soils. For this reason, conventional elastoplastic models for unsaturated soils use suction as a primary stress variable along with the net normal stress (Alonso et al. 1990; Wheeler and Sivakumar 1995; Cui and Delage 1996). These constitutive

Table 2. Summary of data for the three different sands and fitting parameters.

	Vanapalli and Mohamed (2007)	Li (2008)	Steensen-Bach et al. (1987)	
Soil type	Unimin (7030) sand	Unimin (7030) sand	Sollerod sand	Lund sand
Mode of shear failure	General shear failure	General shear failure	General shear failure	General shear failure
B (mm) × L (mm)	100 × 100, 150 × 150	37.5 × 37.5	22 × 22	22 × 22
c' (kPa)	0.6	0.6	0.8	0.6
ϕ' (°)	35.3	35.3	35.8	44.0
Air-entry value, AEV (kPa)	4.0	4.0	5.7	1.1
α (in eq. [7])	1.5 (for 100 mm × 100 mm footing); 2.5 (for 150 mm × 150 mm footing)	0.5 (for 100 mm × 100 mm footing)	2.5	0.5
β (in eq. [7])	1 (i.e., I_p = 0)	1	1	1

Fig. 5. Schematic of measured and assumed applied vertical stresses versus surface settlement relationship in model footing tests.



models indicate that the variation of volumetric deformations due to swelling–shrinkage or net normal stress that results in change in degree of saturation should be taken into account when predicting the behaviour of unsaturated soils. Gallipoli et al. (2003) proposed a simple elastoplastic model using a single yield surface taking account of volumetric hardening based on the assumption that e/e_s at the same average skeleton stress is a unique function of the bonding variable, ξ (= $f(s)(1 - S_r)$, where e and e_s are void ratio in unsaturated and saturated conditions, respectively, at the same average net normal stress; $f(s)$ is a function of suction; and $(1 - S_r)$ is the degree of saturation of air). The development of elastoplastic models for unsaturated soils has reduced the number of tests required to determine parameters for the models; however, there are still difficulties in conducting tests for unsaturated soils as they require elaborate testing equipment.

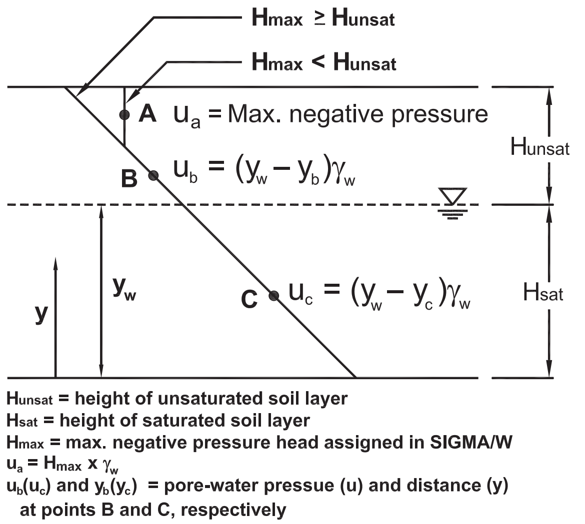
As such, a conventional elastoplastic model developed for saturated soils was used in the present study. The influence of matric suction on bearing capacity was incorporated as apparent cohesion in the model. The variation of suction value of the unsaturated soils below the model footing due to loading and specific volume change associated with desaturation was not taken into account because the model footing tests were conducted on unsaturated sand in a relatively short period of time.

The FEA was performed as an axisymetric problem with equivalent area although the model footing test results used in the present study were obtained for a square footing. This can be justified based on the following experimental and numerical studies published in the literature. For example, Cerato and Lutenegger (2006) conducted model footing tests on a sand with both square and circular footings (width and diameter = 102 mm). The bearing capacity values using a square footing were higher than those using a circular footing by a mean value of 1.25 times. This is attributed to the use of a square footing whose width was equal to the diameter of the circular footing (i.e., not an equivalent area). Gourvenec et al. (2006) showed that the bearing capacity of a square footing is less than that of an equivalent circular footing by 3% based on FEA results.

Negative pore-water pressures in unsaturated soils

In SIGMA/W (Geostudio 2004), the variation of pore-water pressure with depth can be simulated by defining an initial water table and maximum negative pressure head on

Fig. 6. Calculation of pore-water pressure using SIGMA/W. y_w, height of water table; γ_w, unit weight of water.



the assumption that it varies hydrostatically with distance above and below the initial water table as shown in Fig. 6 (Krahn 2004). For instance, if maximum negative pressure head, H_{max}, is lower than the height of the unsaturated soil layer, H_{unsat} (i.e., $H_{max} < H_{unsat}$), the negative pore-water pressure is constant up to ground surface beyond maximum negative pressure head. On the other hand, if maximum negative pressure head is greater than the height of unsaturated soil layer (i.e., $H_{max} > H_{unsat}$) the negative pore-water pressure increases hydrostatically up to the ground surface.

Cohesion and internal friction angle

Based on the hydrostatic suction distribution diagram, the contribution of matric suction, ϕ^b, towards total cohesion, c, is calculated using eq. [9] as given below

$$[9] \qquad c = c' + (u_a - u_w)\tan\phi^b$$

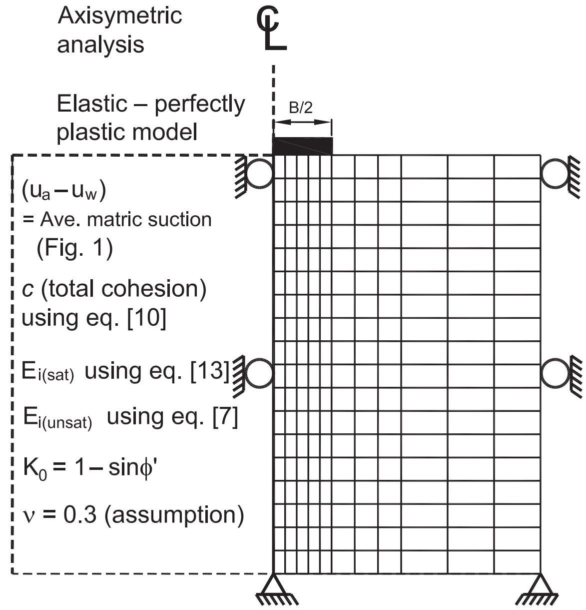
The contribution of matric suction, ϕ^b can be specified as an input parameter in SIGMA/W, which is the FEA program of Geostudio 2004 (Krahn 2004). However, the determination of ϕ^b from laboratory tests (i.e., modified triaxial compression or modified direct shear test) for unsaturated soils is time consuming and needs expensive equipment.

To overcome this limitation, the contribution of matric suction, ϕ^b, is included in the total cohesion, c (i.e., the parameter, ϕ^b is set as zero for the analysis in SIGMA/W). The total cohesion, c, can be calculated using eq. [10], which is derived from eq. [2] with $\kappa = 1$ (for sandy soils with $I_p = 0$) and setting the value of $(\sigma_n - u_a)$ as zero (i.e., surface footing). In this case, the matric suction value, $(u_a - u_w)$, and degree of saturation, S, in eq. [10] corresponds to an average suction value.

$$[10] \qquad c = c' + (u_a - u_w)(S^{\kappa})\tan\phi'$$

The advantage of the above methodology is that it can be extended even in commercial finite element software in which the provision of including the shear strength contribution due to matric suction, ϕ^b, is not available.

Fig. 7. Soil properties and boundary conditions for FEA. K_0, coefficient of earth pressure at rest.



Estimation of modulus of elasticity for FEA

The modulus of elasticity for FEA is mostly estimated using triaxial test results (except unconsolidated undrained test for saturated soils). Janbu (1963) showed that the initial tangent modulus of elasticity, E_i, increases with the confining pressure (eq. [11])

$$[11] \qquad E_i = KP_a\left(\frac{\sigma_3}{P_a}\right)^n$$

where K is modulus number, σ_3 is confining pressure, and n is an exponent determining the rate of variation of E_i with σ_3.

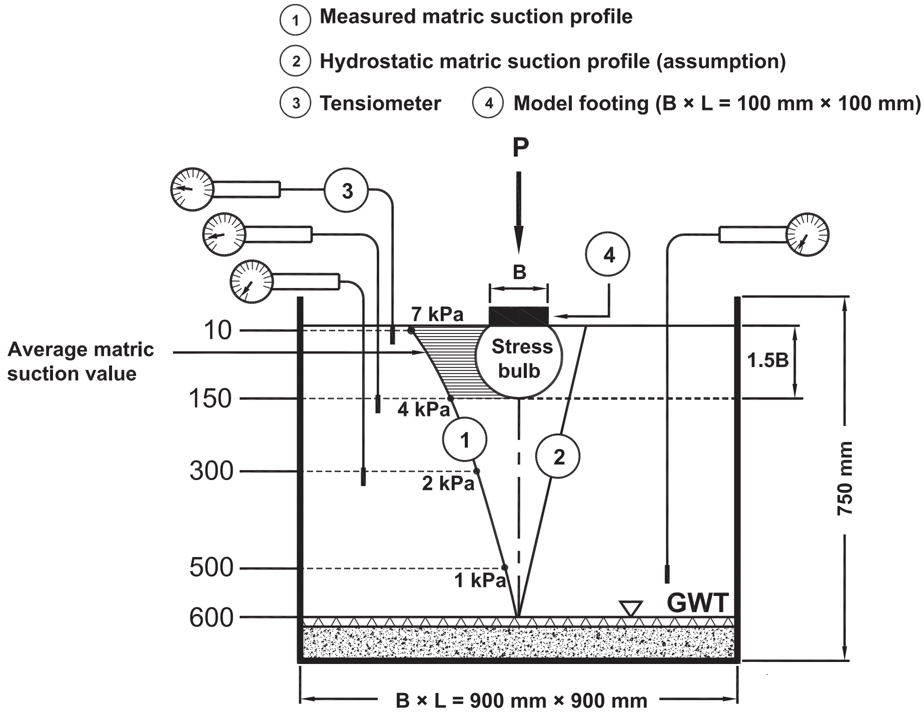
The concept in eq. [11] indicates that different values of E_i should be assigned to elements of the FEA meshes to obtain a reasonable SVS relationship. However, conducting triaxial tests with different confining pressures to estimate K and n in eq. [11] are time consuming. Hence, in the present study, a different approach was used to estimate E_i for the FEA.

Preliminary studies were undertaken to estimate reasonable E_i values for the FEA. The E_i values estimated using eq. [5] were found to be significantly underestimated for the FEA. This can be attributed to the fact that the influence of Poisson's ratio, v, and the size of footing (i.e., I_w) are included as input parameters in the FEA. Hence, the E_i values were estimated using eq. [12] without considering v and I_w.

$$[12] \qquad E_i = \frac{B_p}{(\Delta\delta/\Delta q_p)} = \frac{\Delta q_p}{(\Delta\delta/B_p)}$$

The E_i values calculated using eq. [12], however, also did not provide a good comparison between the measured and predicted SVS relationship in the elastic range. However, when $1.5B_p$ was used instead of B_p in eq. [12] (i.e., eq. [13]), good agreement was observed between the measured and predicted SVS relationship in the elastic range.

Fig. 8. Variation of matric suction values with depth in the UOBCE along with assumed hydrostatic matric suction distribution. Data in the figure corresponds to experimental results with average matric suction of 6 kPa in the stress bulb zone (modified from Mohamed and Vanapalli 2006). Values shown at left side of figure are in millimetres. GWT, groundwater table.



$$[13] \qquad E_i = \frac{1.5B_p}{(\Delta\delta/\Delta q_p)} = \frac{\Delta q_p}{(\Delta\delta/1.5B_p)}$$

It is of interest to note that $1.5B_p$ in eq. [13] represents the depth at which the stress below a foundation is predominant as discussed in the section titled “Bearing capacity of unsaturated sandy soils”. These observations are consistent with the findings of Poulos and Davis (1974) and Agarwal and Rana (1987). Therefore, in the present study, E_i values estimated using eq. [13] were used for the FEA.

Modelling model footing tests on unsaturated sandy soil

Procedures used in the present study for modelling the model footing tests on unsaturated sands can be summarized as follows:

(1) The tested soil is modelled as a single soil layer as described in “Negative pore-water pressure in unsaturated soils” (see section titled “Second method (finite element analysis)”).
(2) The average matric suction value is determined from the suction distribution diagram using procedures described in “Estimation of average matric suction value” (see the section titled “Bearing capacity of unsaturated sandy soils” and Fig. 1).
(3) Total cohesion, c, is calculated using eq. [10] (the value of ϕ^b is set as zero in SIGMA/W).
(4) Moduli of elasticity, $E_{i(sat)}$ and $E_{i(unsat)}$, are determined using eqs. [13] and [7], respectively.
(5) Appropriate boundary conditions are applied.

The procedure of modelling is schematically illustrated in Fig. 7.

Testing program

Model footing tests in unsaturated soils

Mohamed and Vanapalli (2006) carried out model footing tests in a specially designed bearing capacity tank (University of Ottawa Bearing Capacity Equipment (UOBCE) 900 mm × 900 mm × 750 mm), which has provisions to simulate both saturated and unsaturated conditions in the tank. A V-shaped hopper was used to place sand in the tank by spreading the sand from a free-fall height of 1 m to achieve a maximum density index. The placed sand was first saturated by increasing the water table above the soil surface and then lowering it to obtain the targeted matric suction values. Levels of the water and matric suction values were monitored using a piezometer and conventional tensiometers. The model footing was placed on the soil surface and loaded vertically with a constant rate when the matric suction values in the soil reached equilibrium conditions.

Figure 8 shows a schematic of the tank along with four conventional tensiometers. Lines (1) and (2) show the measured suction distribution profile using conventional tensiometers and the assumed hydrostatic suction distribution profile for the average matric suction value of 6 kPa, respectively. The average matric suction value was estimated using the procedures described in the section titled “Bearing capacity of unsaturated sandy soils” along with Fig. 1. Experimental data from the UOBCE is summarized in Table 3.

Soil properties and shear strength parameters

The soil description and properties of the tested sand in the UOBCE are shown in Table 4. The effective cohesion, c', and the effective internal friction angle, ϕ', from direct

Table 3. Summary of experimental data from the UOBCE tank for different average suction values (Mohamed 2006).

D (mm)	γ_t (kN/m^3)	γ_d (kN/m^3)	e	S (%)	$(u_a - u_w)$ (%)	$(u_a - u_w)_{AVR}$ (kPa)
10	18.17	15.94	0.63	14.0	58	6
150	18.75	15.85	0.64	18.3	76	4
300	19.27	16.07	0.62	20.0	86	2
500	19.40	15.77	0.64	23.0	94	1
600	19.74	15.95	0.63	23.8	100	0

Note: Specific gravity, G_s = 2.65; average relative density index, I_D = 63.75%. D, depth from the surface of compacted sand; γ_t, total unit weight; γ_d, dry unit weight.

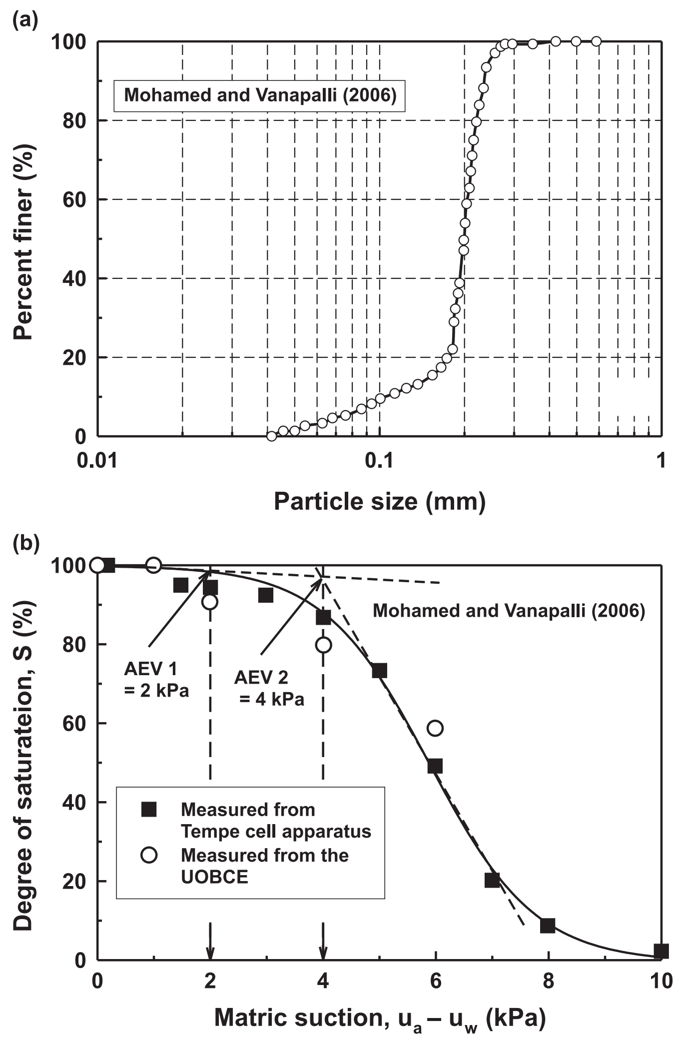
Table 4. Soil description and properties.

Property	Value
Specific gravity, G_s	2.65
D_{60} (mm)	0.22
D_{30} (mm)	0.18
D_{10} (mm)	0.12
Coefficient of uniformity, C_u	1.83
Coefficient of curvature, C_z	1.23
Average dry density of the compacted soil in the UOBCE (kN/m^3)	16.05
USCS*	SP

Note: D_m, grain size corresponding to m% finer; SP, poorly graded sand.

*Unified Soil Classification System (ASTM 2006).

Fig. 9. (*a*) Grain-size distribution curve and (*b*) SWCC for the tested sand with air-entry values measured using two different procedures.



shear tests under saturated conditions were 0.6 kPa and 35.3°, respectively (see Table 2). The total cohesion, c, for unsaturated conditions were estimated using eq. [10], as detailed in the section titled "Second method (finite element analysis)". A constant value of effective internal friction angle, ϕ', was used regardless of the matric suction value as ϕ' is not influenced by matric suction (Vanapalli et al. 1996*b*; Wang et al. 2002; Nishimura et al. 2007).

Matric suction measurement and air-entry value (AEV) of the soil

The SWCC for the tested soil is shown in Fig. 9 along with its grain-size distribution curve. Data points used for establishing the SWCC in the matric suction range of 0 to 10 kPa were obtained both from a Tempe cell apparatus and by measuring matric suction and degree of saturation at several levels of depth in the UOBCE (Mohamed and Vanapalli 2006). The air-entry value (AEV) of the tested sand was estimated from the SWCC using two methods: (*i*) matric suction value corresponding to the point at which the initial constant slope portion terminates on the SWCC (AEV 1 in Fig. 9) and (*ii*) matric suction value corresponding to the intersection of the two linear slope segments of the SWCC (AEV 2 in Fig. 9) using the procedure detailed in Vanapalli et al. (1999).

Analysis

Parameters used (i.e., effective and total cohesion, effective internal friction angle, initial tangent modulus of subgrade reaction, and initial tangent modulus of elasticity) for predicting SVS relationships using the proposed approach and FEA are summarized in Table 5.

Comparison between measured and predicted SVS relationship

There is good agreement between the measured and predicted $K_{is(unsat)}$ values (Table 5). This indicates that the reliability of the proposed method for predicting the SVS relationship is dependent on predicted bearing capacity val-

Table 5. Effective and total cohesion, effective internal friction angle, and modulus of elasticity for each average suction value used in the FEA.

Matric suction (kPa)	c' or c (kPa)	ϕ' (°)	$1.1\phi'$ (°)	k_{is}_1 (kN/m^3)	k_{is}_2 (kN/m^3)	E_i (kPa)
0	0.60	35.3	39.0	17 726	17 726	2659
2	2.56	35.3	39.0	75 007	69 840	11 250
4	4.08	35.3	39.0	112 500	110 255	16 875
6	4.20	35.3	39.0	91 410	91 111	13 712

Note: c from eq. [10]. k_{is}_1, from model footing test results; k_{is}_2, $k_{i(sat)}$ from model footing test result and $k_{i(unsat)}$ using eq. [6]; E_i, $E_{i(sat)}$ using eq. [13] and $E_{i(unsat)}$ using eq. [7].

Table 6. Comparison between measured and predicted bearing capacity values for various average suctions using different air-entry values (AEV) and effective internal friction angles.

			Predicted bearing capacity					
			AEV 1 (2 kPa)		AEV 2 (4 kPa)		AEV 3 = (AEV 1 + AEV 2)/2 (3 kPa)	
Matric suction (kPa)	Measured bearing capacity (kPa)	Maximum stress* (kPa)	ϕ' (case 1)	$1.1\phi'$ (case 2)	ϕ' (case 1)	$1.1\phi'$ (case 4)	ϕ' (case 1)	$1.1\phi'$ (case 3)
0	121	79	86	122	86	122	86	122
2	570	370	241	403	265	439	253	421
4	715	378	370	595	411	656	391	625
6	840	431	438	694	509	800	474	747

*Maximum stress in elastic range in Fig. 5.

Fig. 10. Variation of bearing capacity with respect to matric suction using eq. [3] for four different cases with different air-entry values and effective friction angles.

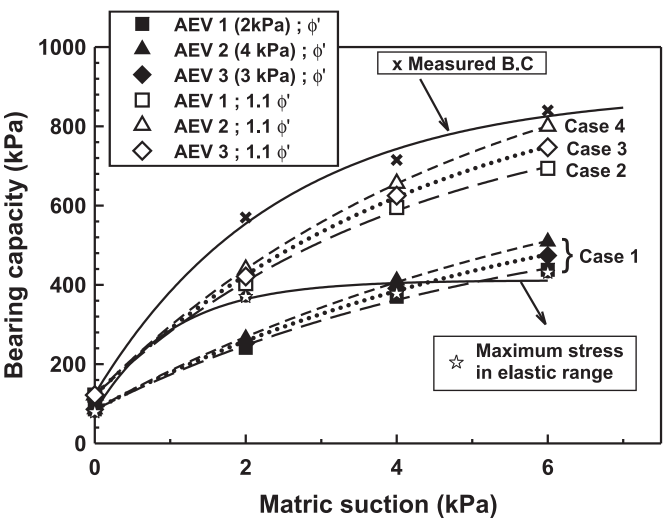
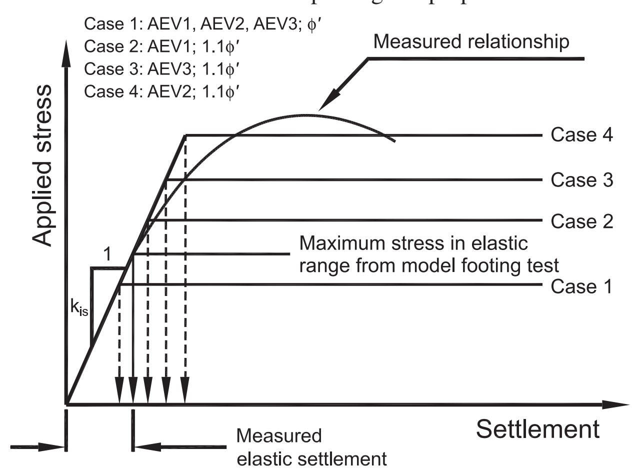


Fig. 11. Different scenarios of predicted applied vertical stress versus surface settlement relationship using the proposed method.

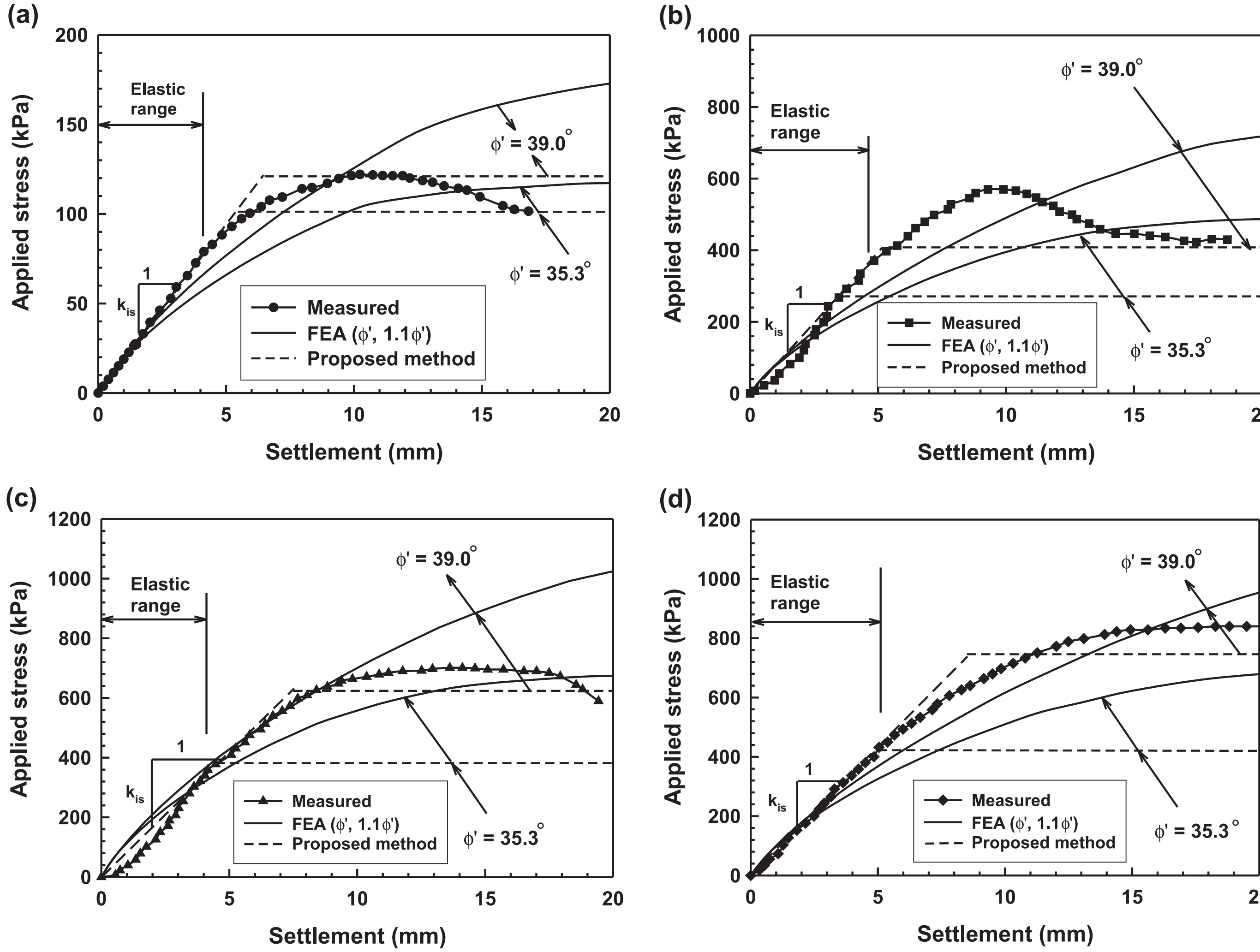


ues using eq. [3]. For this reason, in the present analysis, three different air-entry values — namely, AEV 1 (2 kPa), AEV 2 (4 kPa) (see Fig. 9), and average air-entry value, AEV 3 (3 kPa) [= (AEV 1 + AEV 2)/2] — were considered for studying the sensitivity of the air-entry value with respect to the predicted SVS relationships. In addition, the influence of effective internal friction angle, ϕ', on the SVS relationship was also studied using two different effective internal friction angles (i.e., ϕ' and $1.1\phi'$) as discussed earlier. The focus of this analysis is to provide practical guidelines that can be useful in the reliable prediction of the SVS relationship for both saturated and unsaturated sand.

Table 6 and Fig. 10 show the measured and predicted bearing capacities obtained using eq. [3] for different matric suction values with three different air-entry values (i.e., AEV 1, AEV 2, and AEV 3) and two effective internal friction angles (i.e., ϕ' and $1.1\phi'$). Predicted bearing capacity values obtained using AEV 2 and $1.1\phi'$ are closest to the measured values. However, bearing capacities obtained using ϕ' are underestimated regardless of the AEV. From these results, two important observations can be made as follows:

(1) The methodology of estimating air-entry value (i.e., AEV 1, AEV 2, and AEV 3) can lead to recognizable differences in predicted bearing capacity values (maximum 150 kPa) using eq. [3] for the sand used in the present study. For instance, 2 kPa of difference in air-entry value (i.e., AEV 1 and AEV 2) increased predicted

Fig. 12. Comparison of applied vertical stress versus surface settlement relationship obtained from model footing tests, FEA, and idealized behaviour: $(u_a - u_w)$ = (*a*) 0 kPa (saturated condition); (*b*) 2 kPa; (*c*) 4 kPa; (*d*) 6 kPa.



bearing capacity values by 15% at the highest matric suction value used in this study (i.e., 6 kPa).

(2) Predicted bearing capacities are more reasonable when $1.1\phi'$ is used.

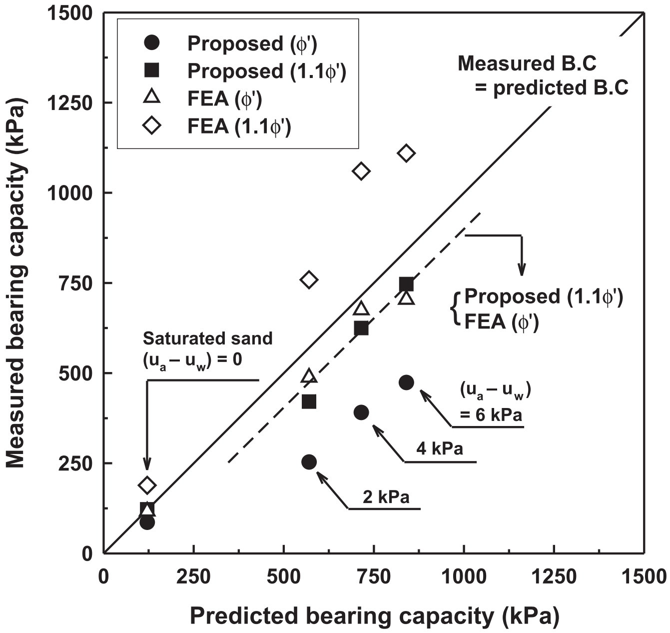
Results in Fig. 10 were grouped into four categories (case 1 to case 4) based on comparisons between the measured and predicted bearing capacity values. Each case can be interpreted with the aid of Fig. 11 as follows:

- ***Case 1*** (using ϕ' and three different AEV values (AEV1, AEV2, AEV3); see Table 6) — Bearing capacity values are significantly underestimated compared with measured values, which can lead to uneconomical design of foundations.
- ***Case 2*** (AEV = AEV 1 and effective internal friction angle = $1.1\phi'$) — Estimated settlement is close to the measured value, but bearing capacity is underestimated. This case is also not reasonable from an engineering practice point of view as it results in uneconomical design of foundations.
- ***Case 3*** (AEV = AEV 3 and effective internal friction angle = $1.1\phi'$) — This case can be considered to be more reasonable as both the estimated bearing capacity and settlement are close to measured values, but are on the conservative side.
- ***Case 4*** (AEV = AEV 2 and effective internal friction angle = $1.1\phi'$) — Estimated bearing capacity is close to the measured value, but settlement is overestimated.

In summary, using AEV 3 (= (AEV 1 + AEV 2)/2) and $1.1\phi'$ provides the most reasonable SVS relationship (i.e., Case 3). Hence, in the present study, average AEV (i.e., AEV 3 that is equal to 3 kPa) and two effective internal friction angle values (i.e., ϕ' and $1.1\phi'$) were used for the comparison between the measured and FEA results.

Figure 12 shows the comparison between the measured and predicted SVS relationships. Bearing capacity values using the FEA with ϕ' shows good agreement compared with measured values, while those obtained with $1.1\phi'$ are significantly overestimated. These results imply that it is not necessary to consider the effect of stress–strain condition (PS and CTC) on ϕ' in the FEA. This behaviour can be attributed to the difference in failure mechanism between eq. [3] (limit equilibrium approach) and the FEA (Mohr–Coulomb criterion).

Fig. 13. Comparison between measured and predicted bearing capacity values.



Comparison between measured and predicted bearing capacity and settlement

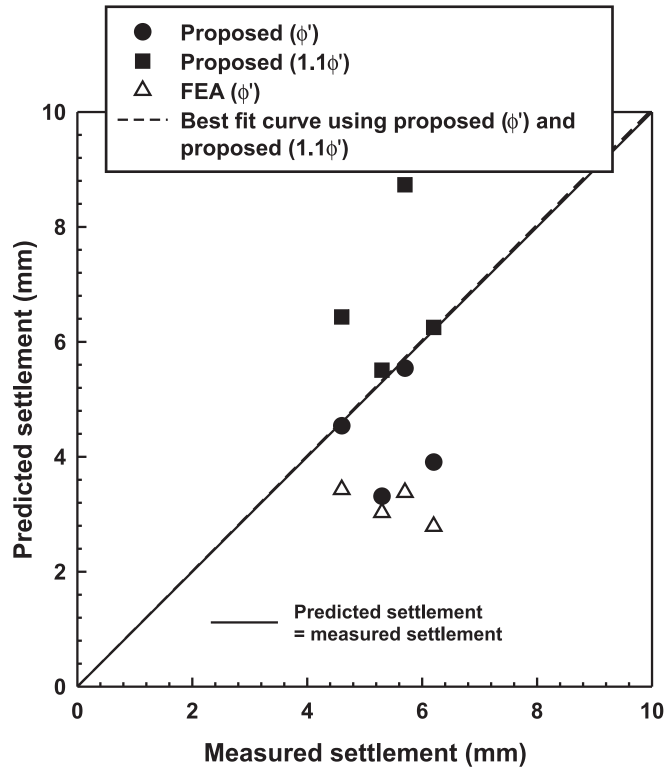
Figure 13 shows the comparison between measured bearing capacity values and those predicted using the proposed method and FEA. As explained in the section titled "Comparison between the measured and predicted SVS relationship", bearing capacity values from the proposed method using an internal friction angle of $1.1\phi'$ and the FEA with ϕ' shows good agreement in comparison with measured values.

Settlements using the proposed method with $1.1\phi'$ were overestimated, while those using both the proposed method and FEA with ϕ' were underestimated as shown in Fig. 14. This phenomenon can be explained using Fig. 11, which shows that the settlement for Case 3 (proposed ($1.1\phi'$) in Fig. 11) is overestimated, whereas for Case 1 (proposed (ϕ') in Fig. 11) it is underestimated. Hence, it can be expected that the best-fit curve using predicted settlements from both "proposed (ϕ')" and "proposed ($1.1\phi'$)" (i.e., short-dash line in Fig. 14) can provide reasonably good settlement estimates.

Summary and conclusions

In this study, a method was proposed to predict the applied vertical stress versus surface settlement (i.e., SVS) relationship of both saturated and unsaturated sands based on the assumption that their behaviour is elastic – perfectly plastic. The SVS relationship is established as segments of two straight lines (Fig. 5) in this method: (*i*) the first segment is the linear elastic line that has a slope (i.e., initial tangent modulus of subgrade reaction) obtained using eq. [6] and (*ii*) the second segment is the plastic line that is parallel to the settlement axis and starts on the elastic line with a value equal to the bearing capacity obtained using eq. [3]. Both lines can be estimated using the parameters under saturated conditions (c', ϕ', and $k_{i(sat)}$) and the SWCC.

Fig. 14. Comparison between measured and predicted settlements.



In addition to the proposed method, finite element analysis (i.e., FEA) was also carried out to show how the SVS relationship in unsaturated sands can be simulated using the elastic – perfectly plastic model.

Results show that the bearing capacity and settlement behaviour of sands under both saturated and unsaturated conditions can be reasonably estimated using the proposed method. Predicted bearing capacity values are conservative in comparison with measured values when average air-entry value AEV 3 (= (AEV 1 + AEV 2)/2) is used. The best-fit curve for settlements obtained using the proposed method with both ϕ' and $1.1\phi'$ can be used as a tool for predicting settlement reliably. Bearing capacities obtained from the FEA show good agreement with measured values, while settlements were underestimated.

Three important observations from this study are summarized below:

(1) The methodology of estimating air-entry value (i.e., AEV 1, AEV 2, and AEV 3) can lead to recognizable differences in predicted bearing capacity values for coarse-grained soils.

(2) Bearing capacities can be predicted with greater reliability when effective internal friction angle, ϕ', is increased by 10%.

(3) A different form of equation (eq. [13]) should be used for estimating modulus of elasticity from model footing tests for the FEA instead of the conventional equation proposed by Timoshenko and Goodier (1951) (eq. [5]).

The proposed method in the present study is based on model footing test results using a footing size of 100 m on sand. More studies using different sizes of footings and soil types would be useful to understand the SVS relationships in field conditions, such that the proposed method can be used in geotechnical engineering practice.

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