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ARTICLE

Resilient modulus of unsaturated subgrade soil: experimental and theoretical investigations

C.W.W. Ng, C. Zhou, Q. Yuan, and J. Xu

Abstract: The resilient modulus, M_R, of subgrade soil is an important stiffness parameter for analysing fatigue cracking in either the asphalt or concrete layer of a pavement. Although subgrade soil is often unsaturated and subject to seasonal variations of moisture content and hence suction in the field, effects of soil suction on the resilient modulus are generally not accounted for in existing testing methods. In this study, M_R values of a subgrade soil under various stress and suction conditions were investigated using a suction-controlled cyclic triaxial apparatus. To enhance the accuracy of measurements, Hall-effect transducers were employed to monitor the local axial and radial deformation of each specimen. It was found that M_R increases with number of load applications when a soil contracts, but decreases slightly when a soil dilates. When suction increases, the soil response tends to change from contractive to dilative due to suction-induced dilatancy. Moreover, the measured M_R is highly dependent on the stress state. It decreases with cyclic stress due to the nonlinearity of the soil stress–strain behaviour, but increases significantly with suction due to the presence of water tension. At the same stress and suction conditions, M_R measured along the wetting path is generally larger than that measured along the drying path. A new semi-empirical equation representing the stress-dependency of M_R is proposed and was verified using experimental results of four different soils.

Key words: unsaturated subgrade soil, cyclic loading–unloading, resilient modulus, net stress, matric suction, wetting and drying.

Résumé : Le module de résilience, M_R, du sol de fondation est un paramètre important de la rigidité pour l'analyse des fissures de fatigue dans l'asphalte ou la couche de béton d'une chaussée. Même si le sol de fondation sur le terrain est souvent non saturé et soumis aux variations saisonnières de teneur en eau, et conséquemment de succion, les effets de la succion du sol sur le module de résilience ne sont généralement pas considérés dans les méthodes d'essais existantes. Dans cette étude, les M_R d'un sol de fondation soumis à différentes conditions de contraintes et succion ont été étudiés à l'aide d'un appareil triaxial cyclique contrôlé par la succion. Pour améliorer la précision des mesures, des capteurs à effet de Hall ont été utilisés pour suivre les déformations axiales et radiales de chaque échantillon. Il a été observé que M_R augmente avec le nombre d'applications de charges lorsque le sol se contracte, mais diminue légèrement lorsque le sol se dilate. Quand la succion augmente, le comportement du sol tend à changer de contractif à dilatant en raison de la dilatation induite par la succion. De plus, le M_R mesuré est fortement dépendant de l'état des contraintes. Il diminue avec les contraintes cycliques en raison de la non-linéarité du comportement en contrainte-déformation du sol, mais augmente significativement avec la succion à cause de la présence de la tension de l'eau. Pour les mêmes conditions de contrainte et succion, le M_R mesuré en mouillage est généralement plus grand que celui mesuré en séchage. Une nouvelle équation semi-empirique représentant la dépendance de M_R sur les contraintes a été proposée et vérifiée à l'aide de résultats expérimentaux obtenus sur quatre sols différents. [Traduit par la Rédaction]

Mots-clés : sol de fondation non saturé, chargement-déchargement cyclique, module de résilience, contrainte nette, succion matricielle, mouillage et séchage.

Introduction

Fatigue cracking in either the asphalt or concrete layer of a pavement is of great concern to pavement designers and users. Its incidence may be caused by a number of factors, such as increase in traffic volume, deterioration of asphalt and concrete, rutting of unbound granular materials, and differential settlement of subgrade soils (Brown 1997). Previous researchers have found that any nonuniform deformation of subgrade soils under cyclic traffic loads plays an important role in crack generation and thus the performance of a pavement (Seed et al. 1962; Brown 1996). The resilient modulus (M_R), defined by Seed at al. (1962) as the ratio of repeated deviator stress to axial recoverable strain in a cyclic triaxial test, is widely used as a stiffness parameter in pavement engineering to determine soil deformation under cyclic traffic loads (Li and Selig 1994; Brown 1996; Kim and Kim 2007).

Subgrade soil is often unsaturated and subject to seasonal variations of moisture content and hence suction in the field (Jin et al. 1994; Brown 1996; Khoury and Zaman 2004; Yang et al. 2008). It is generally recognized that the behaviour of unsaturated soils is governed by two stress state variables, namely net normal stress ($\sigma - u_a$) and matric suction ($u_a - u_w$), where σ, u_a, and u_w are total normal stress, pore-air pressure, and pore-water pressure, respectively (Coleman 1962; Fredlund and Morgenstern 1977). By controlling these two stress state variables in the cyclic triaxial test, Yang et al. (2008) observed that a change of suction from 50 to 450 kPa at a cyclic stress of 103 kPa results in an increase of 200% in measured M_R. Although matric suction is very important for understanding resilient modulus of subgrade soil, it is rarely controlled or measured in current resilient modulus tests. This is possibly due to difficulties in suction control and measurement.

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In addition, suction-controlled tests on unsaturated soil are generally time-consuming.

Laboratory tests previously carried out by other researchers have shown that M_R decreases when soil water content increases (Seed et al. 1967; Jin et al. 1994; Lekarp et al. 2000). It is difficult to quantify the relationship between water content and M_R because the relationship is highly soil type–dependent. Therefore, some researchers have interpreted measured data in terms of matric suction rather than water content. They found that M_R increases with increasing matric suction (Fredlund et al. 1977; Brown et al. 1987; Khoury and Zaman 2004; Yang et al. 2005). In their studies, soil specimens were compacted at different water contents and sample suctions were measured using the filter paper technique or deduced from a soil-water characteristic curve (SWCC) under no confinement. This approach has certain limitations. First, specimens re-compacted to the same dry density but at different initial water contents may not be considered as "identical." Some researchers reported that different compaction water contents could induce different inherent soil structures (Lambe 1958; Ng and Pang 2000; Mancuso et al. 2002). On the other hand, Li and Selig (1994) assumed that soil structure is independent of compaction water content with the same dry density when developing semi-empirical equations for M_R. So far, the influence of compaction water content on soil structure is still not fully understood. Second, suction determination in these studies may not be accurate, as the filter paper technique is very user-dependent (Ng and Menzies 2007). Moreover, Ng and Pang (2000) proposed and verified that SWCC is stress-dependent. Soil suction determined under no confinement may not represent the one during the resilient modulus test. More recently, Yang et al. (2008) developed a suction-controlled triaxial apparatus to investigate suction effects on M_R. They observed that when cyclic stress increases, M_R decreases at low matric suction but increases at high matric suction. More suction-controlled tests and theoretical considerations are needed to verify and explain resilient modulus characteristics of unsaturated soil.

On the other hand, many experimental results, such as those reported by Mancuso et al. (2002) and Ng and Yung (2008), have demonstrated that matric suction significantly affects the very small–strain shear modulus (G_0) of an unsaturated soil. Ng et al. (2009) demonstrated that G_0 is also affected by wetting and drying history. As far as the authors are aware, both laboratory and theoretical studies on unsaturated soil stiffness have focused on G_0. No systematic investigation of the influence of matric suction on M_R has been reported that is similar to the one presented in this paper.

In this study, a suction-controlled triaxial apparatus adopting the axis-translation technique (Hilf 1956) was used to determine M_R of a subgrade soil. The influence of (*i*) two stress-state variables (matric suction and net stress) and (*ii*) wetting and drying history on M_R were investigated. Effects of the number of load applications were also studied. The measured results were interpreted using a newly proposed semi-empirical equation for M_R of saturated and unsaturated soils.

Theoretical considerations

Resilient modulus (M_R) is equivalent to secant Young's modulus following its definition and it is sometimes called resilient Young's modulus (Brown et al. 1987; Li and Selig 1994). For an isotropic elastic material, Young's modulus is linked to shear modulus G by $E = 2G(1 + \nu)$, where ν is Poisson's ratio. Thus, M_R can be determined from G and ν. Many researchers (Mancuso et al. 2002; Ng and Yung 2008; Ng et al. 2009) measured G of unsaturated soils using bender elements or a resonant column at very small strains (or at elastic state). Ng and Yung (2008) proposed a semi-empirical equation for G_0 of unsaturated soil

$$[1] \qquad G_0 = C_{ij}^2 F(e)\left(\frac{p}{p_r}\right)^{2n}\left(1+\frac{s}{p_r}\right)^{2k}$$

where C_{ij} is a constant reflecting the inherent soil structure in the ij plane (i.e., plane of shear), with units of m/s; $F(e)$ is a void ratio function relating shear modulus to void ratio; p and s are net mean stress and matric suction ($u_a - u_w$), respectively; p_r is reference pressure for normalizing p, assumed as 1 kPa for simplicity; n and k are regression parameters. The validity of this equation for representing the variation of G_0 with p and s was verified using experimental data (Ng and Yung 2008). For cyclic tests, this equation may be modified to predict M_R.

The resilient modulus, M_R, of a subgrade soil is dependent on various factors, including grain-size distribution, density, stress level, stress history, loading frequency, and cyclic number. Experimental results obtained by other researchers have demonstrated that stress level is the most important factor (see Lekarp et al. (2000) for a summary). Numerous efforts have been devoted to establish the relationship between M_R and stress level. One of the most widely used formulations was proposed by Uzan (1985) as follows:

$$[2] \qquad M_R = a\left(\frac{\sigma_1+\sigma_2+\sigma_3}{p_{atm}}\right)^b\left(\frac{q_{cyc}}{p_{atm}}\right)^c$$

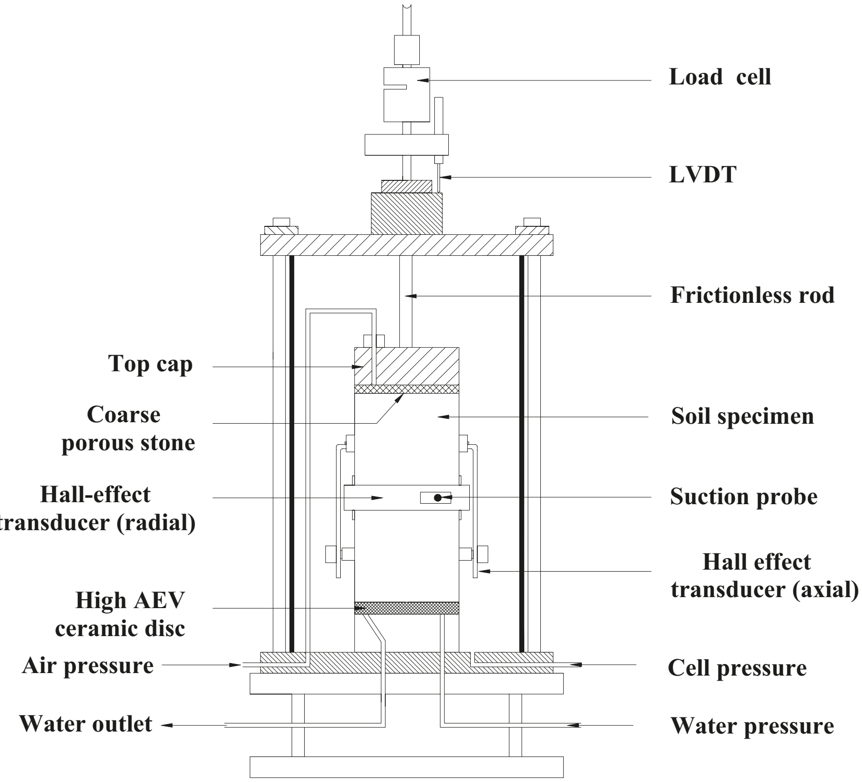
where σ_1, σ_2, and σ_3 are principal stresses; q_{cyc} and p_{atm} are cyclic stress (i.e., the amplitude of change in deviator stress during cyclic loading–unloading) and atmospheric pressure, respectively; a, b, and c are regression coefficients. Clearly, this equation does not consider seasonal variations of M_R, as it ignores soil suction. It can be modified by adopting two stress state variables.

Equation [1] incorporates net confining pressure and matric suction, whereas eq. [2] considers net confining pressure and cyclic deviator stress. Both equations can be modified to represent M_R of unsaturated soil. To completely describe M_R of unsaturated soil, a new equation is proposed by employing the advantages of each equation

$$[3] \qquad M_R = M_0\left(\frac{p}{p_r}\right)^{k_1}\left(\frac{q_{cyc}}{p_r}\right)^{k_2}\left(1+\frac{s}{p}\right)^{k_3}$$

where net mean stress, p, is defined as $[(\sigma_1 + \sigma_2 + \sigma_3)/3 - u_a]$; k_1, k_2, and k_3 are regression exponents. The first term on the right-hand side denotes the resilient modulus at the reference stress state where $p = p_r$, $q_{cyc} = p_r$, and $s = 0$. The second term quantifies the influence of net mean stress on M_R. Numerous experimental studies have demonstrated that soil stiffness including M_R increases with confinement (Houlsby and Wroth 1991; Viggiani and Atkinson 1995; Lekarp et al. 2000). Thus, the empirical exponent k_1 should be positive. The third term reflects variation of M_R with cyclic stress. For a linearly elastic material, the exponent k_2 should be equal to 0. For a soil specimen characterized by the nonlinearity of stress–strain behaviour, the exponent k_2 is negative because soil stiffness decreases with increasing strain (Viggiani and Atkinson 1995). The fourth term accounts for the influence of suction on M_R. Experimental results such as those reported by Ng and Yung (2008) have shown that G_0 of an unsaturated soil increases significantly with matric suction. Similarly, M_R of an unsaturated soil is expected to increase with matric suction. Therefore, parameter k_3 should be positive. Equation [3] allows for a smooth transition between an unsaturated soil and a saturated soil. When matric suction is zero, the fourth term reduces to 1.0. Then this equation can be applied to determine M_R of a saturated soil from effective confining pressure and cyclic stress.

Fig. 1. Suction-controlled cyclic triaxial apparatus for testing unsaturated soils (modified after Ng and Yung 2008).



When q_{cyc} approaches 0 (i.e., at very small strains), $q_{cyc}^{k_2}$ becomes a very large value because the exponent k_2 is negative. Based on eq. [3], a very large M_R can be predicted. The limitation of this equation can be simply overcome by replacing the term (q_{cyc}/p_r) by ($1 + q_{cyc}/p_r$). Then, this equation can be rewritten as

$$[4] \qquad M_R = M_0\left(\frac{p}{p_r}\right)^{k_1}\left(1 + \frac{q_{cyc}}{p_r}\right)^{k_2}\left(1 + \frac{s}{p}\right)^{k_3}$$

This new equation is proposed to represent the influence of net stress and suction on M_R of unsaturated soil. Its validity is verified experimentally in the next section.

Experimental study

Testing apparatus and measuring device

In this study, a suction-controlled cyclic triaxial system was adopted to investigate M_R of subgrade soils. Figure 1 shows a schematic diagram of this cyclic triaxial system. To test unsaturated soils and prevent potential cavitation in the water drainage system, the axis-translation technique (Hilf 1956) was employed to control the matric suction of the soil specimen. This technique imposes ($u_a - u_w$) on a soil specimen by controlling u_a and u_w independently. u_a is controlled through a coarse, low air-entry value (AEV) porous stone on top of each soil specimen. u_w is controlled through a saturated high AEV (i.e., 3 bar (1 bar = 100 kPa)) ceramic disc sealed to the pedestal of the triaxial cell. The saturated high AEV ceramic disc allows the exchange of water across it, but prevents the passage of free air as long as ($u_a - u_w$) is lower than its AEV. However, dissolved air in water may pass through the ceramic disc and accumulate either underneath the ceramic disc or in the water drainage system. In this study, any accumulated air bubble was flushed and collected once every 24 h, using a diffused air volume indicator proposed by Fredlund (1975). The volume of collected air was used to correct measured soil water content change. Readers may refer to Ng and Yung (2008) for details of this cyclic triaxial system.

In addition to the conventional external measurement of axial strain using a linear variable differential transformer (LVDT), the cyclic triaxial system is equipped with three Hall-effect transducers (Clayton et al. 1989) for measuring local soil deformation at the mid-height of each specimen, as shown in Fig. 1. One of these Hall-effect transducers is used to measure radial deformation, while the other two are used to measure axial deformation independently. The resolution and accuracy of each Hall-effect transducer are about 1 and 3 μm, respectively, corresponding to a strain of about 0.001% and 0.003% (Ng and Xu 2012). The percent difference (Wilson and Hernandez-Hall 2004) between measured axial strains using the two Hall-effect transducers is less than 10% in this study, depending on various factors such as soil stiffness and background noise. As expected, the axial strain obtained using an LVDT (external device) is generally larger than that obtained using a Hall-effect transducer (local device), as the former measures the overall deformation of a soil specimen together with bedding errors and compliance of the system (Jardine et al. 1984), whereas the latter records the actual displacement of the soil specimen. To obtain reliable data, axial strain measured using a Hall-effect transducer is used to calculate M_R.

As the Hall-effect transducers measure the local strain at the middle half of each specimen, it is sensible to also measure pore-water pressure within this region. Apart from measuring pore-water pressure at the bottom of each soil specimen using a conventional pore-water pressure transducer; it was also measured at the mid-height of each specimen using a suction probe as illustrated in Fig. 1. The suction probe was a modified Druck PDCR 81 miniature pore-water pressure transducer, which mainly consists of a ceramic tip, small water reservoir, and a diaphragm connected to a transducer. The original low AEV ceramic tip was replaced by a ceramic tip with a higher AEV (500 kPa). After this modification, the transducer is able to measure a negative pore-water pressure of up to 480 kPa, close to the AEV of the ceramic tip used in the probe. More details of modification, saturation procedures, and performance of the suction probe are given by Xu (2011).

Soil type, specimen preparation, and SWCC

The material tested is a completely decomposed tuff (CDT) sampled from Hong Kong. It is described as silt (ML) according to the Unified Soil Classification System of the American Society of Testing and Materials (ASTM 2006), and as A-7-6 following the American Association of State Highway and State Highway Officials (AASHTO 2000) classification. Figure 2 shows the particle-size distribution of the CDT as determined by sieve and hydrometer analyses (British Standards Institution 1990). The material is yellowish-brown, slightly plastic, and has very small percentages of fine sand and coarse sand. The physical properties of the CDT are summarized in Table 1.

To obtain soil specimens with identical fabric, all specimens were prepared following the same method. De-aired water was added to oven-dried soil to obtain the desired water content of 16.3%, corresponding to the optimum water content as determined in the Standard Proctor compaction test (British Standards Institution 1990). The soil and water were mixed thoroughly and large aggregates of soil formed during mixing were crushed using a pestle. Thereafter, the mixed soil was sieved through a 2 mm sieve again and any remaining lumps of soil were again crushed with the pestle. To minimize the loss of moisture, the process of sieving and grinding was done as quickly as possible. The soil that passed through the sieve was transferred to a plastic bag, which was then sealed and kept in a temperature- and moisture-controlled room for 48 h for moisture equalization. Then each soil specimen, 76 mm in diameter and 152 mm in height, was statically compacted in 10 layers in a mould at a loading rate of 1.5 mm/min. To prevent excessive densification of the lower layers during compaction of the upper layers, the process of compaction was stress-controlled with a maximum compaction pressure of about 1162 kPa. After sample compaction, the initial dry density of each soil specimen was found to be about 1.76 g/cm^3, which corresponds to a dry density ratio of 100%. The initial matric suction, which was measured using the suction probe, was about 95 kPa.

Figure 3 shows the SWCC of the compacted CDT specimen. The SWCC was measured using a triaxial pressure plate system (Ng et al. 2011). It can be seen that volumetric water content decreases as suction increases. The AEV is estimated to be 60 kPa along the drying path. At a given suction, the equilibrium water content along the drying path is higher than that along the wetting path. The observed hydraulic hysteresis is mainly caused by an "ink-bottle" effect and the difference in the receding and advancing contact angles (Hillel 1982; Ng and Pang 2000; Ng et al. 2009). Details of testing procedures and more test results are given by Ng and Xu (2012).

Test program and procedures

Three series of cyclic triaxial tests were conducted to investigate the effects of (*i*) two stress state variables (i.e., net stress and matric suction) and (*ii*) wetting and drying history on M_R of a subgrade soil. Figure 4 shows the stress paths of the three series of tests. After compaction, each specimen was set up in the triaxial system. The initial stress state of each specimen is indicated by point A in the figure. First, each specimen was compressed isotropically to a net confining stress of 30 kPa at constant water content (A→B). Then, specimens were subjected to different suction paths at the same net confining pressure of 30 kPa. Finally, cyclic loading–unloading was carried out on each specimen to determine M_R. More details of testing conditions are summarized in Table 2.

In series 1 tests, three specimens — W60, W30, and W0 — were wetted by decreasing soil suction from 95 kPa to 60, 30, and 0 kPa (B→C, B→D, and B→E), respectively. Measured results were compared to investigate effects of suction magnitude on M_R along a wetting path. In series 2 tests, three specimens D100, D150, and D250 were dried to suctions of 100, 150, and 250 kPa (B→F, B→G, and B→H), respectively. Measured M_R was compared to study suction effects on M_R along a drying path. In series 3 tests, suction of specimen WD60 was decreased from 95 to 0 kPa and then increased to 60 kPa (B→E→C). To investigate the effect of wetting and drying history, M_R of this specimen was compared with that of specimen W60 from series 1 (B→C). Similarly, the suction of

Fig. 2. Particle-size distribution of CDT (data from Ng and Yung 2008).

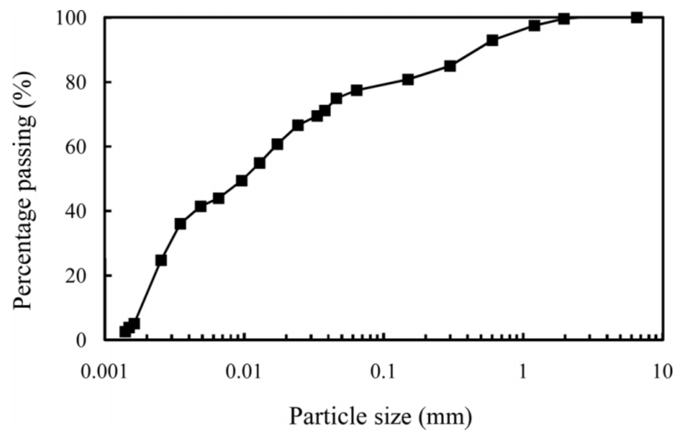


Table 1. Index properties of CDT (data from Ng and Yung 2008).

Index test	Measured value
Standard compaction tests	
Maximum dry density (kg/m^3)	1760
Optimum water content (%)	16.3
Grain-size distribution	
Percentage of sand (%)	24
Percentage of silt (%)	72
Percentage of clay (%)	4
D_{10} (mm)	0.003
D_{30} (mm)	0.006
D_{60} (mm)	0.015
Coefficient of uniformity, D_{60}/D_{10}	4.55
Coefficient of curvature, $(D_{30})^2/(D_{10}D_{60})$	0.61
Specific gravity	2.73
Atterberg limits (grain size <425 μm)	
Liquid limit (%)	43
Plastic limit (%)	29
Plasticity index (%)	14

Fig. 3. A soil-water characteristic curve (SWCC) of recompacted CDT.

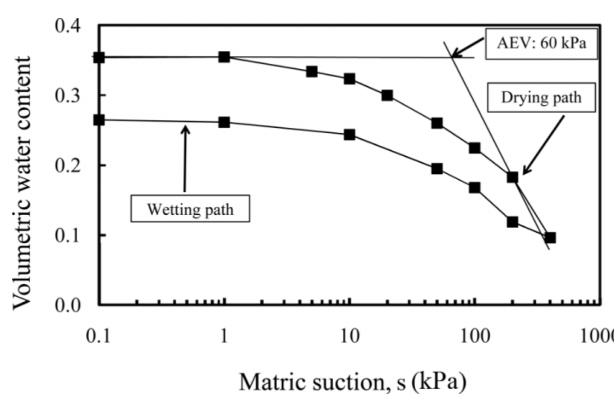


Fig. 4. Stress path of each soil specimen during suction equalization stage and variations of axial stress during cyclic loading–unloading.

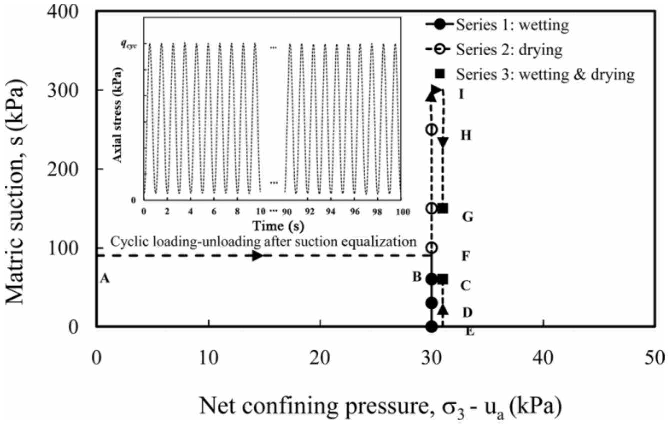


Table 2. Details of experimental program.

Specimen identity	Matric suction (kPa)	Monotonic shear strength (kPa)	Equalization time (days)
Series 1: wetting path			
W0	95→0	81	12
W30	95→30	113	7
W60	95→60	146	4
Series 2: drying path			
D100	95→100	189	4
D150	95→150	243	7
D250	95→250	351	13
Series 3: wetting and drying			
WD60	95→0→60	146	14
DW150	95→300→150	243	17

specimen DW150 in series 3 was increased from 95 to 300 kPa and then decreased to 150 kPa (B→I→G). The measured M_R of this specimen was compared with that of specimen D150 from series 2 (B→G).

Soil suction is imposed on the soil specimen by controlling a pore-water pressure of 50 kPa at the bottom and applying a predefined u_a from the top. After applying predefined u_a and u_w at the top and bottom, respectively, an equalization stage is needed to ensure that the whole specimen reaches the desired suction (u_a – u_w). The equalization stage is considered to be completed when water flow of the specimen is less than 0.5 cm^3 within 24 h, corresponding to a rate of water content change of less than 0.04% per day (Sivakumar 1993). The equalization time of each specimen, which falls in the range of 4 to 13 days, is given in Table 2. In this study, the volumetric strain measured during the suction equalization stage is smaller than 0.05% in each test. This implies that the difference between the dry densities of specimens is very small, after the stage of suction equalization.

Once a specimen had equalized at a given net stress and matric suction, it was subjected to cyclic loading–unloading to determine its M_R. In each cyclic test, net confining pressure was maintained constant at 30 kPa while applied axial stress was varied with time following a haversine form. For clarity, variations of axial stress during the first and last 10 cycles are shown in an insert in Fig. 4. The difference between the maximum and minimum axial stresses is defined as cyclic stress, q_{cyc}. According to AASHTO's (2003) standard for the resilient modulus test, four levels of cyclic stress (i.e., 30, 40, 55, and 70 kPa) were considered and applied to each specimen in succession. At each level of q_{cyc}, 100 cycles of loading–unloading at 1 Hz were applied. The conditioning procedure that is included in the AASHTO standard is not applied in this study to eliminate any influence of overconsolidation ratio (OCR) on M_R.

Monotonic shear strength of unsaturated soil can be described by the extended Mohr–Coulomb shear strength formulation (Fredlund et al. 1978). The shear strength parameters for CDT (c' = 0 kPa, ϕ' = 35°, and ϕ^b = 16°) were obtained from suction-controlled direct shear tests (Tse 2008). Using these parameters, the monotonic shear strength of each specimen was deduced and is provided in Table 2. It can be seen that the cyclic stresses applied are all lower than monotonic shear strength, which increases with matric suction.

During cyclic loading–unloading, a constant water content condition is simulated for the subgrade soil because the dissipation rate of excess pore-water pressure is low compared to the rate of repeated traffic loads in the field. The drainage valve for water was closed and pore-water pressure was measured at the base and mid-height of each specimen. Figure 5 shows variations of pore-water pressure measured at the base and mid-height during a typical cyclic loading–unloading test. For clarity, only the first and last 10 cycles are shown in the figure. It should be noted that the variations of pore-water pressures during the remaining 80 cycles are similar to those in these 20 cycles. It can be seen that pore-water pressures measured at the base and mid-height vary with applied deviator stress in a similar manner. The magnitude of variation of measured pore-water pressure is about 10 kPa at the base and 5 kPa at mid-height. Previous researchers found that pore-water pressure measurement at mid-height is more representative, as it is not affected by end restraint (Hight 1982). On the other hand, pore-air pressure applied from the top of each specimen is maintained constant during cyclic loading–unloading. Considering that the coefficient of air permeability of unsaturated soil is high when the air phase is continuous, it may be reasonable to assume that excess pore-air pressure is negligible throughout the soil specimen.

It should be noted that M_R measured in this study is based on a constant water content condition. This approach is different from those suggested by the AASHTO (2003) standard, in which water drainage is open to the atmosphere. Given the low permeability of unsaturated fine-grained soil and high rate of cyclic loading–unloading, the measured M_R should be quite similar for these two drainage conditions.

Experimental results

Influence of number of load applications on resilient modulus

At each cyclic stress, there are 100 cycles of loading–unloading. The resilient modulus, which is defined as the ratio of repeated deviator stress to axial recoverable strain, is determined from each cycle. To investigate the influence of number of load applications on resilient modulus, the resilient modulus from the Nth cycle (M_r^N) is normalized by the resilient modulus from the first cycle (M_r^1).

Figure 6 shows the relationship between normalized resilient modulus (M_r^N/M_r^1) and number of load applications (N) after wetting to zero suction (A–B–E in Fig. 4). It can be seen that M_r^N/M_r^1 increases with N at each cyclic stress (30, 40, 55, and 70 kPa). This is a consequence of progressive densification resulting from the application of repeated cyclic stress (Dehlen 1969). In this study, volume change was determined from axial and radial strain measured using Hall-effect transducers. At zero suction, contractive volumetric strains measured after 100 cycles of loading–unloading are 0.03%, 0.04%, 0.09%, and 0.25%, corresponding to cyclic stresses of 30, 40, 55, and 70 kPa, respectively. The decreasing volume and hence increasing dry density of each specimen results in an increase in M_r^N/M_r^1 with N.

Figure 6 also reveals that the rate of increase in M_r^N/M_r^1 with N is dependent on cyclic stress. The influence of N on M_r^N/M_r^1 is more

Fig. 5. Pore-water pressures measured at the base and mid-height of sample during a cyclic loading–unloading test.

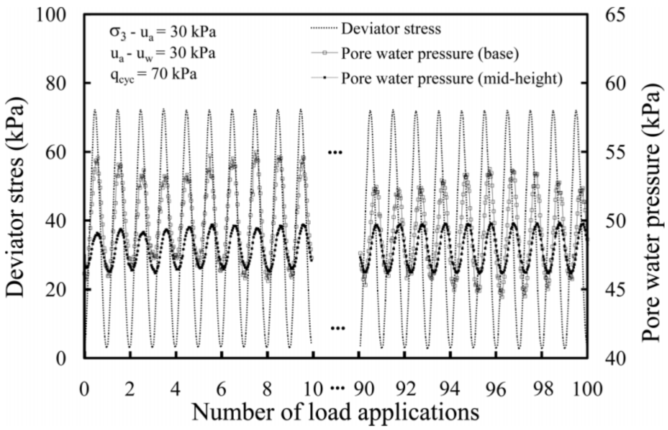
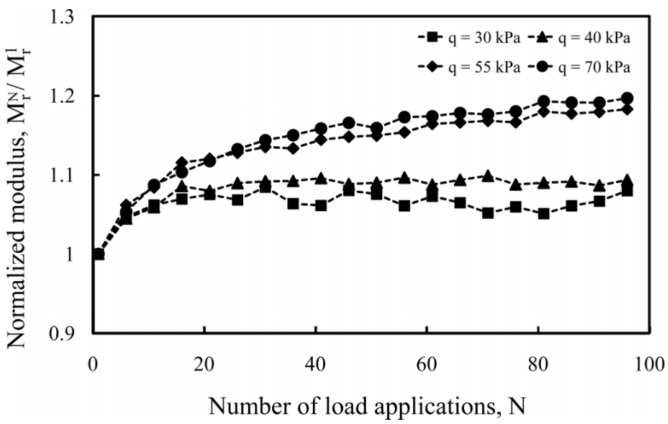


Fig. 6. Relationship between normalized resilient modulus and number of load applications at zero suction in series 1 tests.

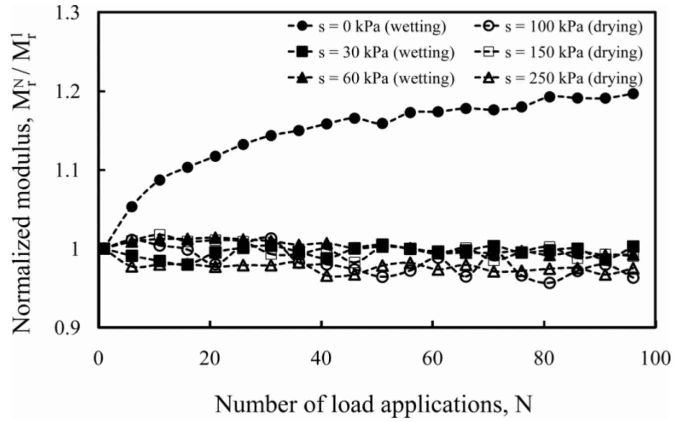


significant at higher cyclic stress. When cyclic stress is 30 and 40 kPa, M_r^N/M_r^1 increases by about 10% during the first 20 cycles of loading–unloading. After the first 20 cycles, M_r^N/M_r^1 almost keeps constant. For the cases with cyclic stress of 55 and 70 kPa, M_r^N/M_r^1 increases continuously during the 100 cycles of loading–unloading, but at a decreasing rate after the first 20 cycles. This is due to the fact that accumulated contractive volumetric strain during cyclic loading–unloading increases with cyclic stress at an increasing rate. Due to larger contractive volumetric strain, the densification effect on M_R is more significant at higher cyclic stresses.

Coupled effects of number of load applications and suction on resilient modulus

Figure 7 shows the relationship between M_r^N/M_r^1 and N at the same cyclic stress (i.e., 70 kPa), but different suctions (0, 30, 60, 100, 150, and 250 kPa). This figure clearly reveals two types of soil response. At zero suction, M_r^N/M_r^1 increases continuously with N. The total increase during the 100 cycles of loading–unloading is up to 20%. On the other hand, when suction is equal to or larger than 30 kPa (s = 30, 60, 100, 150 or 250 kPa), M_r^N/M_r^1 varies only slightly with N. One reason is that contractive volumetric strain during cyclic loading–unloading is much smaller when suction is equal to or larger than 30 kPa. For example, contractive volumetric strain under cyclic loading–unloading measured at a cyclic stress of 70 kPa decreases from 0.25% to 0.03% when matric suction increases from 0 to 30 kPa. Given such a small volumetric strain, the variation of M_r^N/M_r^1 with N becomes insignificant. At a high suction such as 100 kPa, there is even a slight reduction in

Fig. 7. Relationship between normalized resilient modulus and number of load applications at a cyclic stress of 70 kPa (s = 0, 30, and 60 in series 1 tests; s = 100, 150, and 250 in series 2 tests).



M_r^N/M_r^1 with N. This is because soil dilation rather than contraction occurs during cyclic loading–unloading. Dilative volumetric strain of –0.03% is measured during cyclic loading–unloading at a cyclic stress of 70 kPa and matric suction of 100 kPa. Soil density and hence resilient modulus decreases with an increase in number of load applications. Suction effects on soil behaviour under cyclic loads are further discussed in the subsection "Influence of suction on resilient modulus".

It is revealed in Figs. 6 and 7 that the variation of M_r^N/M_r^1 is negligible when the number of load applications is larger than 20, except when cyclic stress is larger than 55 kPa and the soil is at zero matric suction. An unsaturated CDT specimen generally achieves a stable resilient modulus within 100 loading–unloading cycles.

Influence of cyclic stress on resilient modulus

Figure 8 shows the relationship between M_R and q_{cyc} at different suctions (0, 30, 60, 100, 150, and 250 kPa). M_R is the average resilient modulus of the last five cycles at each stress state (i.e., N = 96–100). It can be seen from this figure that M_R decreases significantly with increasing q_{cyc} at all suctions except s = 0. For instance, M_R decreases by about 40% when q_{cyc} increases from 30 to 70 kPa at a suction of 30 kPa. The observed decrease of M_R with q_{cyc} is due to the nonlinearity of the soil stress–strain relationship. Previous studies have demonstrated that soil stiffness is high at small strains, but it decays with an increase in strain level (Atkinson 2000). In resilient modulus tests, strain level increases with q_{cyc}, hence measured M_R decreases with an increase in q_{cyc}. The nonlinearity of soil stress–strain behaviour is captured by the term $(1 + q_{cyc}/p_r)^{k_2}$ in eq. [4]. As M_R decreases with q_{cyc}, the parameter k_2 should be negative. For a soil specimen with a larger degree of nonlinearity, the reduction of M_R with q_{cyc} should be more significant and parameter k_2 should be smaller.

Most experimental data obtained by other researchers have shown that M_R decreases with q_{cyc} (Fredlund et al. 1977; Loach 1987; Khoury and Zaman 2004). On the contrary, some experimental tests have revealed the possibility that M_R increases with increasing q_{cyc} (Seed et al. 1962; Kim and Kim 2007; Yang et al. 2008). These controversial observations may be explained consistently using eq. [4]. Based on this equation, M_R increases with net mean stress. P, but decreases with q_{cyc}. In these resilient modulus tests, cyclic loading is applied under constant confinement. When axial stress increases, both p and q_{cyc} increase. Thus the measured M_R may either increase or decrease with q_{cyc}, depending on the stress level and parameters k_1, k_2, and k_3.

Fig. 8 also reveals that the slope of each curve is generally larger at higher suction. The influence of q_{cyc} on M_R is more significant at

Fig. 8. Influence of cyclic stress on resilient modulus (s = 0, 30, and 60 in series 1 tests; s = 100, 150, and 250 in series 2 tests).

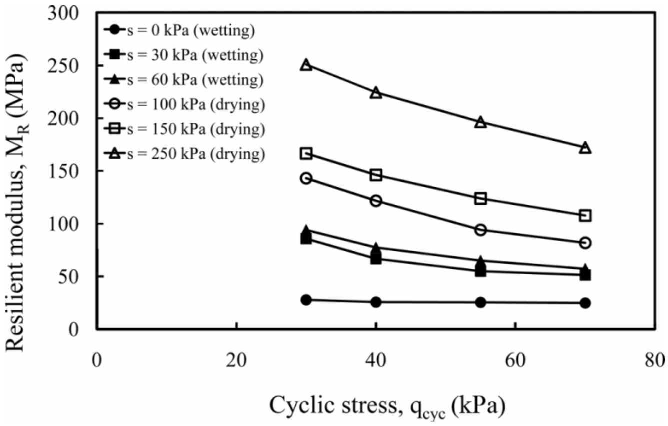
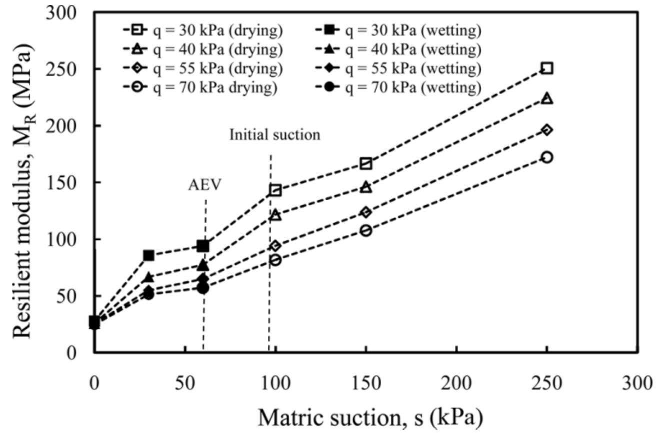


Fig. 9. Influence of suction on resilient modulus (s = 0, 30, and 60 in series 1 tests; s = 100, 150, and 250 in series 2 tests).



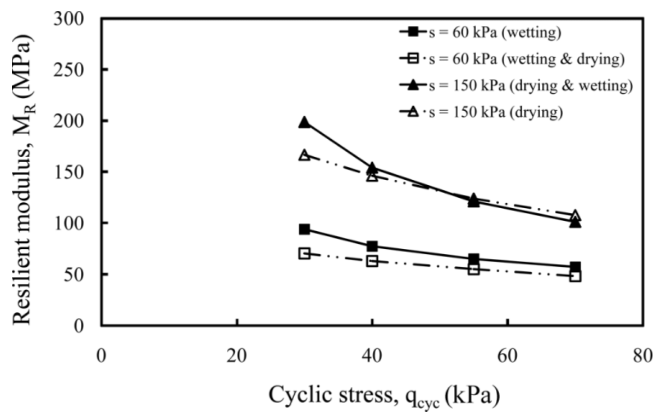
higher suction. This is because the degree of nonlinearity of the soil stress–strain relationship is more significant at higher suctions (Xu 2011). Suction effects on the resilient modulus are further discussed in the next subsection.

Influence of suction on resilient modulus

Figure 9 shows the relationship between M_R and s at different cyclic stresses (30, 40, 55, and 70 kPa). Irrespective of whether it is along a drying or wetting path, M_R increases significantly with increasing s. At a cyclic stress of 30 kPa, M_R increases by about 10 times when s increases from 0 to 250 kPa. The beneficial effects of s on M_R arise due to at least two possible reasons. First, when a soil specimen becomes unsaturated, voids are partly filled with water and partly occupied by air, resulting in an air–water interface in each void. When there is an increase in matric suction, the radius of an air–water interface decreases and hence induces a larger normal interparticle contact force (Fisher 1926; Mancuso et al. 2002; Wheeler et al. 2003; Ng and Yung 2008). This normal interparticle contact force provides a stabilizing effect on an unsaturated soil by inhibiting slippage at particle contacts and enhancing the shear resistance of the unsaturated soil (Wheeler et al. 2003). Second, an increase in s induces shrinkage of the soil specimen (Ng and Pang 2000). Due to the stronger internormal force between particles and higher density, M_R measured during cyclic loading–unloading is larger at higher suctions. The influence of s on M_R is captured by the term $(1 + s/p)^{k_3}$ in eq. [4]. As M_R increases with s, the parameter k_3 should be positive. In most state-of-the-art standards for testing resilient modulus, seasonal variation of soil moisture is not taken into account. In AASHTO (2003), each soil specimen is prepared and tested at the in situ soil moisture level. The resilient characteristic of an unsaturated subgrade soil is always represented by a single M_R. The usage of a single M_R cannot reflect its seasonal variation. Observed significant increases in M_R with s from this figure demonstrate that M_R of a subgrade soil is very likely to be underestimated in a dry season and overestimated in a wet season. To appropriately describe resilient characteristics of an unsaturated subgrade soil at a given stress state, a suction equalization stage is necessary prior to cyclic loading–unloading.

Further inspection of this figure reveals that the relationship between M_R and s is highly nonlinear along a wetting path (in series 1 tests). Given the same increase in s, the percentage of increase in M_R is much larger in the lower suction range. At a cyclic stress of 30 kPa, M_R doubles when s increases from 0 to 30 kPa. On the other hand, when s increases from 30 to 60 kPa, the percentage increase is only 10%. In series 2 tests, it can be seen that the increase rate of M_R with s is almost constant along the drying path. The different results observed in different suction ranges are probably because the bulk water effects dominate soil behaviour when suction is lower than the AEV of the soil (here about 60 kPa, see Fig. 3) in series 1 tests and meniscus water effects dominate soil behaviour when suction exceeds the AEV in series 2 tests (Ng and Yung 2008).

Fig. 10. Influence of wetting and drying history on resilient modulus (in series 3 tests).



Comparisons of the slope of each curve in Fig. 9 reveal that suction effects on M_R are generally more obvious at low cyclic stress levels (i.e., low strain levels). This observation is also illustrated in Fig. 8 and can be explained by experimental results from conventional triaxial compression tests. Nyunt et al. (2009) conducted constant water content triaxial compression tests to investigate the influence of s on the stiffness–strain relationship of an unsaturated soil. They found that suction effects on the secant axial Young's modulus decrease with axial strain. It is therefore evident that suction effects on M_R decrease slightly with increasing strain level and cyclic stress.

Influence of wetting and drying history on resilient modulus

Figure 10 illustrates the influence of wetting and drying history on measured M_R at two suctions (i.e., 60 and 150 kPa) in series 3 tests. At a suction of 60 kPa, M_R measured along a wetting path is larger than that measured along a drying path. Observed differences between these two paths decrease slightly with an increase in cyclic stress. Similarly, Ng et al. (2009) found that G_0 measured along a wetting path is obviously larger than that measured along a drying path.

At a suction of 150 kPa, M_R measured along a wetting path is also larger than that measured along a drying path when cyclic stress

Table 3. Summary of soil properties and regression coefficients of proposed semi-empirical equation.

Material	AASHTO (2000) classification	Specific gravity	Plastic limit	Liquid limit	Plasticity index	M_0	k_1	k_2	k_3	R^2	Se/Sy
CDT	A-7-6	2.73	29	43	14	8.32	1.00	−0.65	1.01	0.98	0.14
Keuper Marl	A-7-6	2.69	18	37	19	6.32	1.00	−0.65	1.01	0.66	0.60
Gault clay	A-7-5	2.69	25	61	36	0.61	1.00	−0.36	1.31	0.98	0.14
London clay	A-7-5	2.73	23	71	48	0.53	1.00	−0.36	1.31	0.96	0.21

is low (30 and 40 kPa). The influence of wetting and drying history on soil stiffness is due to the coupling effect between mechanical and hydraulic behaviour. Experimental studies have concluded that an unsaturated soil deforms upon suction change and irreversible volume change may occur during cyclic wetting and drying (Ng and Pang 2000; Wheeler et al. 2003). In this study, when suction increases from 150 to 300 kPa and then decreases to 150 kPa, plastic shrinkage occurs and soil density increases. The soil specimen along a wetting path may behave like an overconsolidated soil and result in a larger stiffness, at least in the low cyclic stress range. When cyclic stress increases to 50 kPa at a suction of 150 kPa, M_R measured along a drying path becomes even larger than that measured along a wetting path. One possible reason is that the wetting and drying history not only induces effects of overconsolidation, but also affects the equilibrium soil water content. Under the same stress and suction condition, the soil specimen along the drying path has a larger water content as shown in Fig. 3. At higher water content, the number of air–water interfaces is larger and hence the average skeleton force is higher. Therefore, M_R measured along the drying path could be even larger, when the effects of overconsolidation become relatively less important at high cyclic stress levels.

A comparison of Figs. 6 to 10 demonstrates that the stress and suction conditions impose a much more pronounced influence on the M_R than number of load applications and wetting and drying history. This observation is taken into account in the new semi-empirical eq. [4]. Although this equation does not consider the effects of number of load applications and the wetting and drying history, it can still represent the resilient modulus of an unsaturated subgrade soil with good accuracy.

Verification of the proposed equation

To verify the validity of the newly proposed eq. [4] for resilient modulus at a quantitative level, the measured and calculated resilient modulus of four different soils, i.e., CDT, Keuper Marl, Gault clay, and London clay (Brown et al. 1987), are compared. According to the AASHTO (2000) soil classification, CDT and Keuper Marl are classified as A-7-6 soils, while Gault clay and London clay are classified as A-7-5 soils. Resilient moduli measured from CDT tests are first used to fit eq. [4] to derive parameters M_0, k_1, k_2, and k_3. The derived parameters k_1, k_2, and k_3 are then adopted to predict M_R of Keuper Marl, which is in the same category of A-7-6 soils, while M_0 is fitted. This is because M_0 is affected by various factors such as soil density and sampling method. Similarly, experimental data from Gault clay are used to derive parameters M_0, k_1, k_2, and k_3. The derived parameters k_1, k_2, and k_3 are then adopted for predicting M_R of London clay, which is in the same category of A-7-5 soils. The parameter values are summarized in Table 3.

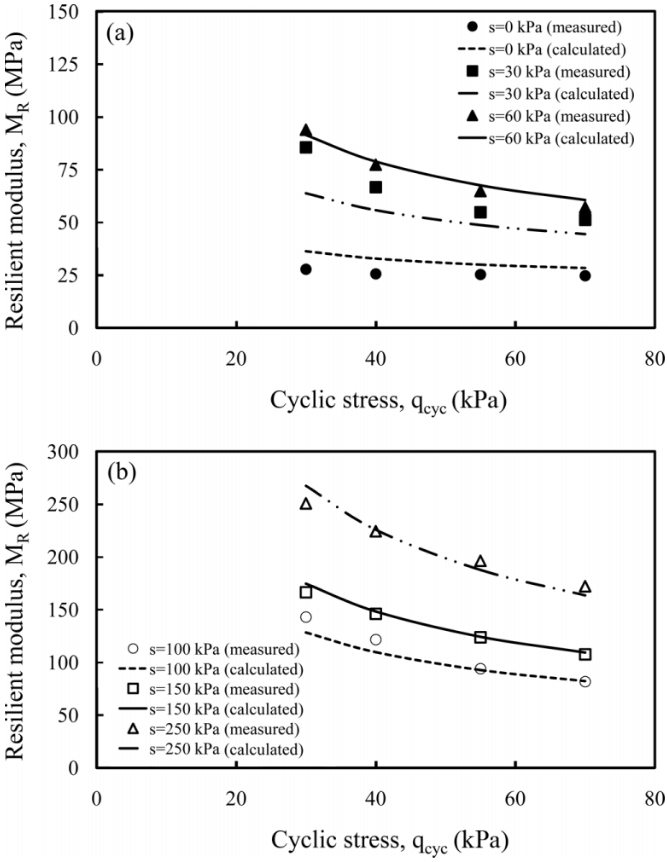
The newly proposed eq. [4] is first applied to fit the measured M_R values of CDT that were measured in series 1 and 2 tests. The parameters M_0, k_2, and k_3 are obtained using the least-square method, while parameter k_1 is assumed equal to 1.0. Considering that this study does not focus on the influence of p on M_R, parameter k_1 is simply determined from previous theoretical and experimental studies. According to the well-known modified Cam clay model, M_R (i.e., the axial Young's modulus during unloading) is proportional to effective mean stress p' (Muir Wood 1990). Viggiani and Atkinson (1995) also carried out triaxial compression tests to study the relationship between shear modulus and p'^n, where n is a regression coefficient. Experimental results have revealed that n is 0.72 at very small strain levels and increases to 1.0 at a strain level of 0.5%. In this study, measured total axial strain, including both permanent plastic strain and recoverable resilient strain, is between 0.1% and 1%. Therefore, it may be reasonable to assume that M_R increases linearly with p (i.e., $k_1 = 1$) for simplicity.

Figures 11*a* and 11*b* compare measured and calculated M_R of CDT along a wetting path and a drying path, respectively. In general, they are closely matched with a maximum difference of less than 25%. The coefficient of determination (R^2) and Se/Sy, which are determined from measured and calculated resilient moduli at 24 different stress and suction conditions, are given in Table 3. Se and Sy are residual standard deviation and sample standard deviation, respectively. R^2 and Se/Sy are found to be 0.98 and 0.14, respectively, suggesting a strong correlation between measured and calculated resilient moduli. The strong correlation implies that eq. [4] can generally capture the variations of resilient moduli with net stress and matric suction. Comparing Figs. 11*a* and 11*b* reveals that the results obtained from eq. [4] are in slightly better agreement with the corresponding experimental data along a drying path than along a wetting path. As illustrated in Fig. 10, the soil specimen tested along a wetting path behaves like an overconsolidated soil due to the effect of wetting and drying history. However, the effects of wetting and drying are not taken into account by eq. [4]. As a result, special attention should be given when eq. [4] is used to predict M_R of a soil specimen along a wetting path, although reasonably accurate predictions can still be made.

To further investigate the validity of eq. [4], it is applied to fit the measured M_R of three other soils (Keuper Marl, Gault clay, and London clay) reported by Brown et al. (1987). Figure 12 compares the measured and calculated M_R of different types of soils. For each soil type, measured and calculated resilient moduli are very well matched. It should be pointed out that measured soil suctions by Brown et al. (1987) range from 15 to 75 kPa. More high-quality experimental data are necessary to verify the validity of eq. [4] over a wider range of suctions. Close inspection of this figure reveals that the measured and calculated results for A-7-5 soil seems to be better matched than those for A-7-6 soil. One possible reason is that all data obtained for A-7-5 are from the same study. Each test is performed using the same apparatus and test standard. Therefore, any experimental uncertainty is expected to be smaller for A-7-5 soil. On the other hand, the relatively larger discrepancy in A-7-6 soils might be caused by different plastic limits between measurements of CDT and Keuper Marl (refer to Table 3).

It can be seen from Fig. 12 and Table 3 that the same values of k_1, k_2, and k_3 can be assumed to estimate M_R for the same type of soils. Parameter k_2 of each soil is negative as expected, considering that M_R decreases with q_{cyc} due to the nonlinearity of the soil stress–strain relationship. The result shows that A-7-6 has a smaller k_2 than does A-7-5. This implies that M_R of A-7-6 soil decreases more significantly with q_{cyc}. This could be explained by the fact that A-7-6 soil has a smaller plasticity index than does A-7-5 soil. Vucetic and Dobry (1991) found that the soil stiffness degradation with strain level is more significant for soils with smaller plasticity indexes. The parameter k_3 of each soil is positive, implying that M_R generally increases with s due to the beneficial effect of the

Fig. 11. Comparisons between measured and calculated resilient modulus using the proposed semi-empirical equation: (*a*) wetting series; (*b*) drying series.



air–water interfaces. Its value is also larger for A-7-5 soils, which consist of smaller particles, implying that the suction effect on M_R is more significant for soils containing more fine particles.

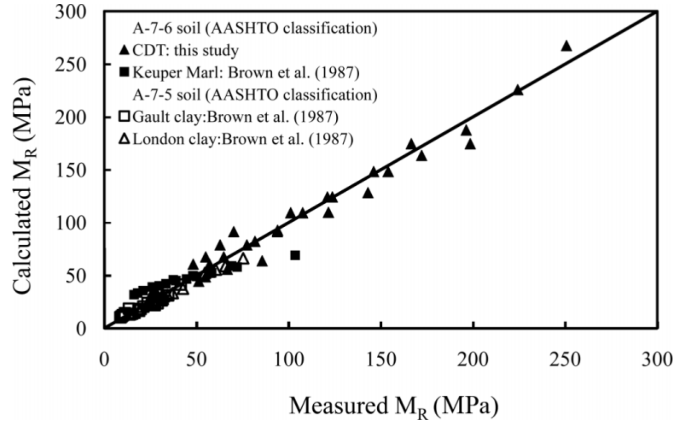
Equation [4] is proposed for estimating stress-dependent M_R of unsaturated subgrade soil, given parameters k_1, k_2, k_3, and M_0. However, it does not consider all the complex soil behaviour, including the influence of wetting and drying history on mechanical behaviour. Besides, two underlying assumptions are made to keep this equation simple. First, parameters k_1, k_2, and k_3 are assumed to be constant, independent of stress level and stress history. Second, the density effect on M_R is incorporated in M_0 in the new equation for simplicity. In this study, all soil specimens were compacted at the same initial water content and dry density. The volumetric strain measured at the stage of suction equalization is smaller than 0.05% in each test. Given such a small volume change in all tests, a single M_0 may be sufficient in calculating M_R. If this equation is used to represent M_R of soil specimens with different initial densities, it may be modified by incorporating a void ratio function.

Summary and conclusions

A suction-controlled cyclic triaxial apparatus adopting the axis-translation technique is used to study M_R of a subgrade soil. Test results are analysed under the framework of two stress state variables, i.e., net stress and suction.

The measured M_R of unsaturated subgrade soil increases with number of load applications when a soil contracts under cyclic loads. On the contrary, M_R decreases slightly with number of load applications when a soil dilates. An unsaturated soil specimen

Fig. 12. Comparison between measured and calculated resilient modulus using the proposed semi-empirical equation of different soils. (AASHTO refers to AASHTO (2000).)



generally achieves a stable resilient response within 100 cycles of loading–unloading.

Measured M_R is found to be dependent on stress and suction level. It decreases with cyclic stress because soil stress–strain behaviour under cyclic loads is highly nonlinear. On the other hand, M_R increases significantly with suction. When suction increases from 0 to 250 kPa, M_R increases by up to one order of magnitude. This is due to the fact that an increase in suction induces an additional interparticle normal force and hence stiffens the soil specimen.

Given the same stress and suction level, M_R measured along a wetting path is larger than that measured along a drying path at low cyclic stress. The observed difference between the two paths becomes less significant with an increase in cyclic stress.

A new semi-empirical equation describing the stress-dependency of M_R for both saturated and unsaturated soils is proposed. This new equation is able to capture the variation of M_R with both net stress and suction. For the four soils verified, the M_R values predicted using this equation are generally consistent with the measured results.

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List of symbols

AEV	air-entry value
a, b, c	regression coefficients
C_{ij}	constant reflecting inherent soil structure
c'	effective cohesion
D_{10}, D_{30}, D_{60}	grain diameter that corresponds to 10%, 30%, and 60% finer by weight, respectively
k	regression parameter
k_1, k_2, k_3	regression exponents
G	shear modulus
G_0	very small strain shear modulus
E	Young's modulus
$F(e)$	void ratio function relating shear modulus to void ratio
M_0	resilient modulus at the reference stress state
M_R	resilient modulus
M_r^N	resilient modulus determined from the Nth cycle
N	number of load applications
n	regression coefficient
OCR	overconsolidation ratio
p	net mean stress
p_{atm}	atmospheric pressure
p_r	reference pressure
q	deviator stress
q_{cyc}	cyclic shear stress
R^2	coefficient of determination
Se	residual standard deviation
Sy	sample standard deviation
s	matric suction
u_a	pore-air pressure
u_w	pore-water pressure
ν	Poisson's ratio
σ	total normal stress
σ_1, σ_2, σ_3	principal total stresses
ϕ'	angle of internal friction
ϕ^b	angle indicating the rate of increase in shear strength relative to matric suction