Resilient modulus of unsaturated subgrade soil: experimental and theoretical investigations

C.W.W. Ng, C. Zhou, Q. Yuan, and J. Xu

Abstract: The resilient modulus, $M_R$, of subgrade soil is an important stiffness parameter for analysing fatigue cracking in either the asphalt or concrete layer of a pavement. Although subgrade soil is often unsaturated and subject to seasonal variations of moisture content and hence suction in the field, effects of soil suction on the resilient modulus are generally not accounted for in existing testing methods. In this study, $M_R$ values of a subgrade soil under various stress and suction conditions were investigated using a suction-controlled cyclic triaxial apparatus. To enhance the accuracy of measurements, Hall-effect transducers were employed to monitor the local axial and radial deformation of each specimen. It was found that $M_R$ increases with number of load applications when a soil contracts, but decreases slightly when a soil dilates. When suction increases, the soil response tends to change from contractive to dilative due to suction-induced dilatancy. Moreover, the measured $M_R$ is highly dependent on the stress state. It decreases with cyclic stress due to the nonlinearity of the soil stress–strain behaviour, but increases significantly with suction due to the presence of water tension. At the same stress and suction conditions, $M_R$ measured along the wetting path is generally larger than that measured along the drying path. A new semi-empirical equation representing the stress-dependency of $M_R$ is proposed and was verified using experimental results of four different soils.

Key words: unsaturated subgrade soil, cyclic loading–unloading, resilient modulus, net stress, matric suction, wetting and drying.

Introduction

Fatigue cracking in either the asphalt or concrete layer of a pavement is of great concern to pavement designers and users. Its incidence may be caused by a number of factors, such as increase in traffic volume, deterioration of asphalt and concrete, rutting of unbound granular materials, and differential settlement of subgrade soils (Brown 1997). Previous researchers have found that any ununiform deformation of subgrade soils under cyclic traffic loads plays an important role in crack generation and thus the performance of a pavement (Seed et al. 1962; Brown 1996). The resilient modulus ($M_R$), defined by Seed et al. (1962) as the ratio of repeated deviator stress to axial recoverable strain in a cyclic triaxial test, is widely used as a stiffness parameter in pavement engineering to determine soil deformation under cyclic traffic loads (Li and Selig 1994; Brown 1996; Kim and Kim 2007).

Subgrade soil is often unsaturated and subject to seasonal variations of moisture content and hence suction in the field (Jin et al. 1994; Brown 1996; Khoury and Zaman 2004; Yang et al. 2008). It is generally recognized that the behaviour of unsaturated soils is governed by two stress state variables, namely net normal stress ($\sigma - u_a$) and matric suction ($u_a - u_w$), where $\sigma$, $u_a$, and $u_w$ are total normal stress, pore-air pressure, and pore-water pressure, respectively (Coleman 1962; Fredlund and Morgenstern 1977). By controlling these two stress state variables in the cyclic triaxial test, Yang et al. (2008) observed that a change of suction from 50 to 450 kPa at a cyclic stress of 103 kPa results in an increase of 200% in measured $M_R$. Although matric suction is very important for understanding resilient modulus of subgrade soil, it is rarely controlled or measured in current resilient modulus tests. This is possibly due to difficulties in suction control and measurement.
In addition, suction-controlled tests on unsaturated soil are generally time-consuming.

Laboratory tests previously carried out by other researchers have shown that $M_R$ decreases when soil water content increases (Seed et al. 1967; Jin et al. 1994; Lekarp et al. 2000). It is difficult to quantify the relationship between water content and $M_R$ because the relationship is highly soil-type-dependent. Therefore, some researchers have interpreted measured data in terms of matric suction rather than water content. They found that $M_R$ increases with increasing matric suction (Fredlund et al. 1977; Brown et al. 1987; Khoury and Zaman 2004; Yang et al. 2005). In their studies, soil specimens were compacted at different water contents and sample suction were measured using the filter paper technique or deduced from a soil-water characteristic curve (SWCC) under no confinement. This approach has certain limitations. First, specimens re-compacted to the same dry density but at different initial water contents may not be considered as “identical.” Some researchers reported that different compaction water contents could induce different inherent soil structures (Lambe 1958; Ng and Yung 2000; Mancuso et al. 2002). On the other hand, Li and Selig (1994) assumed that soil structure is independent of compaction water content with the same dry density when developing semi-empirical equations for $M_R$. So far, the influence of compaction water content on soil structure is still not fully understood. Second, suction determination in these studies may not be accurate, as the filter paper technique is very user-dependent (Ng and Menzies 2007). Moreover, Ng and Pang (2000) proposed and verified that SWCC is stress-dependent. Soil suction determined under no confinement may not represent the one during the resilient modulus test. More recently, Yang et al. (2008) developed a suction-controlled triaxial apparatus to investigate suction effects on $M_R$. They observed that when cyclic stress increases, $M_R$ decreases at low matric suction but increases at high matric suction. More suction-controlled tests and theoretical considerations are needed to verify and explain resilient modulus characteristics of unsaturated soil.

On the other hand, many experimental results, such as those reported by Mancuso et al. (2002) and Ng and Yung (2008), have demonstrated that matric suction significantly affects the very small–strain shear modulus ($G_0$) of an unsaturated soil. Ng et al. (2009) demonstrated that $G_0$ is also affected by wetting and drying history. As far as the authors are aware, both laboratory and theoretical studies on unsaturated soil stiffness have focused on $G_0$. No systematic investigation of the influence of matric suction on $M_R$ has been reported that is similar to the one presented in this paper.

In this study, a suction-controlled triaxial apparatus adopting the axis-translation technique (Hilf 1956) was used to determine $M_R$ of a subgrade soil. The influence of (i) two stress-state variables (matric suction and net stress) and (ii) wetting and drying history on $M_R$ were investigated. Effects of the number of load applications were also studied. The measured results were interpreted using a newly proposed semi-empirical equation for $M_R$ of saturated and unsaturated soils.

Theoretical considerations

Resilient modulus ($M_R$) is equivalent to secant Young’s modulus following its definition and it is sometimes called resilient Young’s modulus (Brown et al. 1987; Li and Selig 1994). For an isotropic elastic material, Young’s modulus is linked to shear modulus $G$ by $E = 2G(1 + \nu)$, where $\nu$ is Poisson’s ratio. Thus, $M_R$ can be determined from $G$ and $\nu$. Many researchers (Mancuso et al. 2002; Ng and Yung 2008; Ng et al. 2009) measured $G$ of unsaturated soils using binder elements or a resonant column at very small strains (or at elastic state). Ng and Yung (2008) proposed a semi-empirical equation for $G_0$ of unsaturated soil

$$G_0 = C_0 F(e) \left( \frac{P}{P_r} \right)^{2b} \left( 1 + \frac{s}{P_r} \right)^{2b}$$

where $G_0$ is a constant reflecting the inherent soil structure in the $i$ plane (i.e., plane of shear), with units of m/s; $F(e)$ is a void ratio function relating shear modulus to void ratio; $p$ and $s$ are net mean stress and matric suction ($u_c - u_a$), respectively; $P_r$ is reference pressure for normalizing $p$, assumed as 1 kPa for simplicity; $n$ and $k$ are regression parameters. The validity of this equation for representing the variation of $G_0$ with $p$ and $s$ was verified using experimental data (Ng and Yung 2008). For cyclic tests, this equation may be modified to predict $M_R$.

The resilient modulus, $M_R$, of a subgrade soil is dependent on various factors, including grain-size distribution, density, stress level, stress history, loading frequency, and cyclic number. Experimental results obtained by other researchers have demonstrated that stress level is the most important factor (see Lekarp et al. (2000) for a summary). Numerous efforts have been devoted to establish the relationship between $M_R$ and stress level. One of the most widely used formulations was proposed by Uzan (1985) as follows:

$$M_R = a \left( \frac{\sigma_1 + \sigma_2 + \sigma_3}{p_{\text{atm}}} \right)^{b_1} \left( \frac{q_{\text{cyc}}}{p_{\text{atm}}} \right)^{b_2} \left( \frac{1 + s}{p} \right)^{b_3}$$

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are principal stresses; $q_{\text{cyc}}$ and $p_{\text{atm}}$ are cyclic stress (i.e., the amplitude of change in deviator stress during cyclic loading–unloading) and atmospheric pressure, respectively; $a$, $b_1$, and $c$ are regression coefficients. Clearly, this equation does not consider seasonal variations of $M_R$, as it ignores soil suction. It can be modified by adopting two stress state variables.

Equation [1] incorporates net confining pressure and matric suction, whereas eq. [2] considers net confining pressure and cyclic deviator stress. Both equations can be modified to represent $M_R$ of unsaturated soil. To completely describe $M_R$ of unsaturated soil, a new equation is proposed by employing the advantages of each equation

$$M_R = M_0 \left( \frac{P}{P_r} \right)^{b_1} \left( \frac{q_{\text{cyc}}}{P_r} \right)^{b_2} \left( 1 + \frac{s}{P_r} \right)^{b_3}$$

where net mean stress, $p$, is defined as $[\sigma_1 + \sigma_2 + \sigma_3]/3 - u_a$; $k_1$, $k_2$, and $k_3$ are regression exponents. The first term on the right-hand side denotes the resilient modulus at the reference stress state when $P = P_r$, $q_{\text{cyc}} = p_r$, and $s = 0$. The second term quantifies the influence of net mean stress on $M_R$. Numerous experimental studies have demonstrated that soil stiffness including $M_R$ increases with confinement (Houlby and Wroth 1991; Viggiani and Atkinson 1995; Lekarp et al. 2000). Thus, the empirical exponent $k_1$ should be positive. The third term reflects variation of $M_R$ with cyclic stress. For a linearly elastic material, the exponent $k_3$ should be equal to 0. For a soil specimen characterized by the nonlinearity of stress–strain behaviour, the exponent $k_3$ is negative because soil stiffness decreases with increasing strain (Viggiani and Atkinson 1995). The fourth term accounts for the influence of suction on $M_R$. Experimental results such as those reported by Ng and Yung (2008) have shown that $G_0$ of an unsaturated soil increases significantly with matric suction. Similarly, $M_R$ of an unsaturated soil is expected to increase with matric suction. Therefore, parameter $k_2$ should be positive. Equation [3] allows for a smooth transition between an unsaturated soil and a saturated soil. When matric suction is zero, the fourth term reduces to 1.0. Then this equation can be applied to determine $M_R$ of a saturated soil from effective confining pressure and cyclic stress.
When \( q_{\text{cyc}} \) approaches 0 (i.e., at very small strains), \( q_{\text{cyc}}^3 \) becomes a very large value because the exponent \( k_3 \) is negative. Based on eq. [3], a very large \( M_R \) can be predicted. The limitation of this equation can be simply overcome by replacing the term \( q_{\text{cyc}}/p_r \) by \( 1 + q_{\text{cyc}}/p_r \). Then, this equation can be rewritten as

\[
M_R = M_0 \left( \frac{p}{p_l} \right)^{k_1} \left( 1 + \frac{q_{\text{cyc}}}{p_r} \right)^{k_2} \left( 1 + \frac{s}{p} \right)^{k_3}
\]

This new equation is proposed to represent the influence of net stress and suction on \( M_R \) of unsaturated soil. Its validity is verified experimentally in the next section.

**Experimental study**

**Testing apparatus and measuring device**

In this study, a suction-controlled cyclic triaxial system was adopted to investigate \( M_R \) of subgrade soils. Figure 1 shows a schematic diagram of this cyclic triaxial system. To test unsaturated soils and prevent potential cavitation in the water drainage system, the axis-translation technique (Hilf 1956) was employed to control the matric suction of the soil specimen. This technique imposes \( u_s - u_w \) on a soil specimen by controlling \( u_s \) and \( u_w \) independently. \( u_s \) is controlled through a coarse, low air-entry value (AEV) porous stone on top of each soil specimen. \( u_w \) is controlled through a saturated high AEV (i.e., 3 bar (1 bar = 100 kPa)) ceramic disc sealed to the pedestal of the triaxial cell. The saturated high AEV ceramic disc allows the exchange of water across it, but prevents the passage of free air as long as \( u_s - u_w \) is lower than its AEV. However, dissolved air in water may pass through the ceramic disc and accumulate either underneath the ceramic disc or in the water drainage system. In this study, any accumulated air bubble was flushed and collected once every 24 h, using a diffused air volume indicator proposed by Fredlund (1975). The volume of collected air was used to correct measured soil water content change. Readers may refer to Ng and Yung (2008) for details of this cyclic triaxial system.

In addition to the conventional external measurement of axial strain using a linear variable differential transformer (LVDT), the cyclic triaxial system is equipped with three Hall-effect transducers (Clayton et al. 1989) for measuring local soil deformation at the middle-height of each specimen, as shown in Fig. 1. One of these Hall-effect transducers is used to measure radial deformation, while the other two are used to measure axial deformation independently. The resolution and accuracy of each Hall-effect transducer are about 1 and 3 \( \mu \text{m} \), respectively, corresponding to a strain of about 0.001% and 0.003% (Ng and Xu 2012). The percent difference (Wilson and Hernandez-Hall 2004) between measured axial strains using the two Hall-effect transducers is less than 10% in this study, depending on various factors such as soil stiffness and background noise. As expected, the axial strain obtained using an LVDT (external device) is generally larger than that obtained using a Hall-effect transducer (local device), as the former measures the overall deformation of a soil specimen together with bedding errors and compliance of the system (Jardine et al. 1984), whereas the latter records the actual displacement of the soil specimen. To obtain reliable data, axial strain measured using a Hall-effect transducer is used to calculate \( M_R \).

As the Hall-effect transducers measure the local strain at the middle half of each specimen, it is sensible to also measure pore-water pressure within this region. Apart from measuring pore-water pressure at the bottom of each soil specimen using a conventional pore-water pressure transducer; it was also measured at the mid-height of each specimen using a suction probe as illustrated in Fig. 1. The suction probe was a modified Druck PDCR 81 miniature pore-water pressure transducer, which mainly consists of a ceramic tip, small water reservoir, and a diaphragm connected to a transducer. The original low AEV ceramic tip was replaced by a ceramic tip with a higher AEV (500 kPa). After this modification, the transducer is able to measure a negative pore-

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*Fig. 1. Suction-controlled cyclic triaxial apparatus for testing unsaturated soils (modified after Ng and Yung 2008).*
water pressure of up to 480 kPa, close to the AEV of the ceramic tip used in the probe. More details of modification, saturation procedures, and performance of the suction probe are given by Xu (2011).

Soil type, specimen preparation, and SWCC

The material tested is a completely decomposed tuff (CDT) sampled from Hong Kong. It is described as silt (ML) according to the Unified Soil Classification System of the American Society of Testing and Materials (ASTM 2006), and as A-7-6 following the American Association of State Highway and State Highway Officials (AASHTO 2000) classification. Figure 2 shows the particle-size distribution of the CDT as determined by sieve and hydrometer analyses (British Standards Institution 1990). The material is yellowish-brown, slightly plastic, and has very small percentages of fine sand and coarse sand. The physical properties of the CDT are summarized in Table 1.

To obtain soil specimens with identical fabric, all specimens were prepared following the same method. De-aired water was added to oven-dried soil to obtain the desired water content of 16.3%, corresponding to the optimum water content as determined in the Standard Proctor compaction test (British Standards Institution 1990). The soil and water were mixed thoroughly and large aggregates of soil formed during mixing were crushed using a pestle. Thereafter, the mixed soil was sieved through a 2 mm sieve again and any remaining lumps of soil were again crushed with the pestle. To minimize the loss of moisture, the process of sieving and grinding was done as quickly as possible. The soil that passed through the sieve was transferred to a plastic bag, which was then sealed and kept in a temperature- and moisture-controlled room for 48 h for moisture equalization. Then each soil specimen, 76 mm in diameter and 152 mm in height, was statically compacted in 10 layers in a mould at a loading rate of 1.5 mm/min. After compaction, each specimen was set up in the triaxial system. The initial stress state of each specimen is indicated by point A in the figure. First, each specimen was compressed isotropically to a net confining stress of 30 kPa at constant water content (A→B). Then, specimens were subjected to different suction paths at the same net confining pressure of 30 kPa. Finally, cyclic loading-unloading was carried out on each specimen to determine $M_R$. More details of testing conditions are summarized in Table 2.

In series 1 tests, three specimens were subjected to different suction paths at the same net confining pressure of 30 kPa. Finally, cyclic loading-unloading was carried out on each specimen to determine $M_R$. More details of testing conditions are summarized in Table 2.

Test program and procedures

Three series of cyclic triaxial tests were conducted to investigate the effects of (i) two stress state variables (i.e., net stress and matric suction) and (ii) wetting and drying history on $M_R$ of a subgrade soil. Figure 4 shows the stress paths of the three series of tests. After compaction, each specimen was set up in the triaxial system. The initial stress state of each specimen is indicated by point A in the figure. First, each specimen was compressed isotropically to a net confining stress of 30 kPa at constant water content (A→B). Then, specimens were subjected to different suction paths at the same net confining pressure of 30 kPa. Finally, cyclic loading-unloading was carried out on each specimen to determine $M_R$. More details of testing conditions are summarized in Table 2.

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Table 1. Index properties of CDT (data from Ng and Yung 2008).

<table>
<thead>
<tr>
<th>Index test</th>
<th>Measured value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard compaction tests</td>
<td></td>
</tr>
<tr>
<td>Maximum dry density (kg/m³)</td>
<td>1760</td>
</tr>
<tr>
<td>Optimum water content (%)</td>
<td>16.3</td>
</tr>
<tr>
<td>Grain-size distribution</td>
<td></td>
</tr>
<tr>
<td>Percentage of sand (%)</td>
<td>24</td>
</tr>
<tr>
<td>Percentage of silt (%)</td>
<td>72</td>
</tr>
<tr>
<td>Percentage of clay (%)</td>
<td>4</td>
</tr>
<tr>
<td>$D_{10}$ (mm)</td>
<td>0.003</td>
</tr>
<tr>
<td>$D_{30}$ (mm)</td>
<td>0.006</td>
</tr>
<tr>
<td>$D_{60}$ (mm)</td>
<td>0.015</td>
</tr>
<tr>
<td>Coefficient of uniformity, $D_{10}/D_{60}$</td>
<td>4.55</td>
</tr>
<tr>
<td>Coefficient of curvature, $D_{30}^2/(D_{10}D_{60})$</td>
<td>0.61</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>2.73</td>
</tr>
<tr>
<td>Atterberg limits (grain size &lt;425 μm)</td>
<td></td>
</tr>
<tr>
<td>Liquid limit (%)</td>
<td>43</td>
</tr>
<tr>
<td>Plastic limit (%)</td>
<td>29</td>
</tr>
<tr>
<td>Plasticity index (%)</td>
<td>14</td>
</tr>
</tbody>
</table>
specimen DW150 in series 3 was increased from 95 to 300 kPa and equalization stage is smaller than 0.05% in each test. This implies that the difference between the dry densities of specimens is very small, after the stage of suction equalization.

soil suction is imposed on the soil specimen by controlling a pore-water pressure of 50 kPa at the bottom and applying a predefined $u_a$ from the top. After applying predefined $u_a$ and $u_w$ at the top and bottom, respectively, an equalization stage is needed to ensure that the whole specimen reaches the desired suction ($u_a - u_w$). The equalization stage is considered to be completed when water flow of the specimen is less than 0.5 cm$^3$ within 24 h, corresponding to a rate of water content change of less than 0.04% per day (Sivakumar 1993). The equalization time of each specimen, which falls in the range of 4 to 13 days, is given in Table 2. In this study, the volumetric strain measured during the suction equalization stage is smaller than 0.05% in each test. This implies that the difference between the dry densities of specimens is very small, after the stage of suction equalization.

Once a specimen had equalized at a given net stress and matric suction, it was subjected to cyclic loading–unloading to determine its $M_R$. In each cyclic test, net confining pressure was maintained constant at 30 kPa while applied axial stress was varied with time following a haversine form. For clarity, variations of axial stress during cyclic loading–unloading are shown in an figure. It should be noted that the variations of pore-water pressures during the remaining 80 cycles are similar to those in these 20 cycles. It can be seen that pore-water pressures measured at the base and mid-height vary with applied deviator stress in a similar manner. The magnitude of variation of measured pore-water pressure is about 10 kPa at the base and 5 kPa at mid-height. Previous researchers found that pore-water pressure measurement at mid-height is more representative, as it is not affected by end restraint (Hight 1982). On the other hand, pore-air pressure applied from the top of each specimen is maintained constant during cyclic loading–unloading. Considering that the coefficient of air permeability of unsaturated soil is high when the air phase is continuous, it may be reasonable to assume that excess pore-air pressure is negligible throughout the soil specimen.

Table 2. Details of experimental program.

<table>
<thead>
<tr>
<th>Specimen identity</th>
<th>Matric suction (kPa)</th>
<th>Monotonic shear strength (kPa)</th>
<th>Equalization time (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series 1: wetting path</td>
<td>W0 95→90</td>
<td>81</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>W30 95→30</td>
<td>113</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>W60 95→60</td>
<td>146</td>
<td>4</td>
</tr>
<tr>
<td>Series 2: drying path</td>
<td>D100 95→100</td>
<td>189</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>D150 95→150</td>
<td>243</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>D250 95→250</td>
<td>351</td>
<td>13</td>
</tr>
<tr>
<td>Series 3: wetting and drying</td>
<td>WD60 95→60</td>
<td>146</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>DW150 95→300→150</td>
<td>243</td>
<td>17</td>
</tr>
</tbody>
</table>

specimen DW150 in series 3 was increased from 95 to 300 kPa and then decreased to 150 kPa (B→I→G). The measured $M_R$ of this specimen was compared with that of specimen D150 from series 2 (B→G).

Soil suction is imposed on the soil specimen by controlling a pore-water pressure of 50 kPa at the bottom and applying a predefined $u_a$ from the top. After applying predefined $u_a$ and $u_w$ at the top and bottom, respectively, an equalization stage is needed to ensure that the whole specimen reaches the desired suction ($u_a - u_w$). The equalization stage is considered to be completed when water flow of the specimen is less than 0.5 cm$^3$ within 24 h, corresponding to a rate of water content change of less than 0.04% per day (Sivakumar 1993). The equalization time of each specimen, which falls in the range of 4 to 13 days, is given in Table 2. In this study, the volumetric strain measured during the suction equalization stage is smaller than 0.05% in each test. This implies that the difference between the dry densities of specimens is very small, after the stage of suction equalization.

Once a specimen had equalized at a given net stress and matric suction, it was subjected to cyclic loading–unloading to determine its $M_R$. In each cyclic test, net confining pressure was maintained constant at 30 kPa while applied axial stress was varied with time following a haversine form. For clarity, variations of axial stress during cyclic loading–unloading are shown in an figure. It should be noted that the variations of pore-water pressures during the remaining 80 cycles are similar to those in these 20 cycles. It can be seen that pore-water pressures measured at the base and mid-height vary with applied deviator stress in a similar manner. The magnitude of variation of measured pore-water pressure is about 10 kPa at the base and 5 kPa at mid-height. Previous researchers found that pore-water pressure measurement at mid-height is more representative, as it is not affected by end restraint (Hight 1982). On the other hand, pore-air pressure applied from the top of each specimen is maintained constant during cyclic loading–unloading. Considering that the coefficient of air permeability of unsaturated soil is high when the air phase is continuous, it may be reasonable to assume that excess pore-air pressure is negligible throughout the soil specimen.

It should be noted that $M_R$ measured in this study is based on a constant water content condition. This approach is different from those suggested by the AASHTO (2003) standard, in which water drainage is open to the atmosphere. Given the low permeability of unsaturated fine-grained soil and high rate of cyclic loading–unloading, the measured $M_R$ should be quite similar for these two drainage conditions.

Experimental results

Influence of number of load applications on resilient modulus

At each cyclic stress, there are 100 cycles of loading–unloading. The resilient modulus, which is defined as the ratio of repeated deviator stress to axial recoverable strain, is determined from each cycle. To investigate the influence of number of load applications on resilient modulus, the resilient modulus from the Nth cycle $(M_R^N)$ is normalized by the resilient modulus from the first cycle $(M_R^1)$.

Figure 6 shows the relationship between normalized resilient modulus $(M_R^N/M_R^1)$ and number of load applications (N) after wetting to zero suction (A→B→E in Fig. 4). It can be seen that $M_R^N/M_R^1$ increases with N at each cyclic stress (30, 40, 55, and 70 kPa). This is a consequence of progressive densification resulting from the application of repeated cyclic stress (Dahlen 1969). In this study, volume change was determined from axial and radial strain measured using Hall-effect transducers. At zero suction, contractive volumetric strains measured after 100 cycles of loading–unloading are 0.03%, 0.04%, 0.09%, and 0.25%, corresponding to cyclic stresses of 30, 40, 55, and 70 kPa, respectively. The decreasing volume and hence increasing dry density of each specimen results in an increase in $M_R^N/M_R^1$ with N.

Figure 6 also reveals that the rate of increase in $M_R^N/M_R^1$ with N is dependent on cyclic stress. The influence of N on $M_R^N/M_R^1$ is more
significant at higher cyclic stress. When cyclic stress is 30 and 40 kPa, $M_{cyc}^r/M_1^r$ increases by about 10% during the first 20 cycles of loading–unloading. After the first 20 cycles, $M_{cyc}^r/M_1^r$ almost keeps constant. For the cases with cyclic stress of 55 and 70 kPa, $M_{cyc}^r/M_1^r$ increases continuously during the 100 cycles of loading–unloading, but at a decreasing rate after the first 20 cycles. This is due to the fact that accumulated contractive volumetric strain during cyclic loading–unloading increases with cyclic stress at an increasing rate. Due to larger contractive volumetric strain, the densification effect on $M_1$ is more significant at higher cyclic stresses.

**Coupled effects of number of load applications and suction on resilient modulus**

Figure 7 shows the relationship between $M_{cyc}^r/M_1^r$ and $N$ at the same cyclic stress (i.e., 70 kPa), but different suctions (0, 30, 60, 100, 150, and 250 kPa). This figure clearly reveals two types of soil response. At zero suction, $M_{cyc}^r/M_1^r$ increases continuously with $N$. The total increase during the 100 cycles of loading–unloading is up to 20%. On the other hand, when suction is equal to or larger than 30 kPa ($s = 30$, 60, 100, 150 or 250 kPa), $M_{cyc}^r/M_1^r$ varies only slightly with $N$. One reason is that contractive volumetric strain during cyclic loading–unloading is much smaller when suction is equal to or larger than 30 kPa. For example, contractive volumetric strain under cyclic loading–unloading measured at a cyclic stress of 70 kPa decreases from 0.25% to 0.03% when matric suction increases from 0 to 30 kPa. Given such a small volumetric strain, the variation of $M_{cyc}^r/M_1^r$ with $N$ becomes insignificant. At a high suction such as 100 kPa, there is even a slight reduction in $M_{cyc}^r/M_1^r$ with $N$. This is because soil dilation rather than contraction occurs during cyclic loading–unloading. Dilative volumetric strain of ~0.03% is measured during cyclic loading–unloading at a cyclic stress of 70 kPa and matric suction of 100 kPa. Soil density and hence resilient modulus decreases with an increase in number of load applications. Suction effects on soil behaviour under cyclic loads are further discussed in the subsection “Influence of suction on resilient modulus”.

It is revealed in Figs. 6 and 7 that the variation of $M_{cyc}^r/M_1^r$ is negligible when the number of load applications is larger than 20, except when cyclic stress is larger than 55 kPa and the soil is at zero matric suction. An unsaturated CDT specimen generally achieves a stable resilient modulus within 100 loading–unloading cycles.

**Influence of cyclic stress on resilient modulus**

Figure 8 shows the relationship between $M_{cyc}^r$ and $q_{cyc}$ at different suctions (0, 30, 60, 100, 150, and 250 kPa). $M_{cyc}^r$ is the average resilient modulus of the last five cycles at each stress state (i.e., $N = 96–100$). It can be seen from this figure that $M_{cyc}^r$ decreases significantly with increasing $q_{cyc}$ at all suctions except $s = 0$. For instance, $M_{cyc}^r$ decreases by about 40% when $q_{cyc}$ increases from 30 to 70 kPa at a suction of 30 kPa. The observed decrease of $M_{cyc}^r$ with $q_{cyc}$ is due to the nonlinearity of the soil stress–strain relationship. Previous studies have demonstrated that soil stiffness is high at small strains, but it decays with an increase in strain level (Atkinson 2000). In resilient modulus tests, strain level increases with $q_{cyc}$, hence measured $M_{cyc}^r$ decreases with an increase in $q_{cyc}$. The non-linearity of soil stress–strain behaviour is captured by the term $(1 + q_{cyc}/k_3)^{k_2}$ in eq. [4]. As $M_{cyc}^r$ decreases with $q_{cyc}$, the parameter $k_2$ should be negative. For a soil specimen with a larger degree of nonlinearity, the reduction of $M_{cyc}^r$ with $q_{cyc}$ should be more significant and parameter $k_2$ should be smaller.

Most experimental data obtained by other researchers have shown that $M_{cyc}^r$ decreases with $q_{cyc}$ (Fredlund et al. 1977; Loach 1987; Khoury and Zaman 2004). On the contrary, some experimental tests have revealed the possibility that $M_{cyc}^r$ increases with increasing $q_{cyc}$ (Seed et al. 1962; Kim and Kim 2007; Yang et al. 2008). These controversial observations may be explained consistently using eq. [4]. Based on this equation, $M_{cyc}^r$ increases with net mean stress $P$, but decreases with $q_{cyc}$. In these resilient modulus tests, cyclic loading is applied under constant confinement. When axial stress increases, both $P$ and $q_{cyc}$ increase. Thus the measured $M_{cyc}^r$ may either increase or decrease with $q_{cyc}$, depending on the stress level and parameters $k_1$, $k_2$, and $k_3$.

Fig. 8 also reveals that the slope of each curve is generally larger at higher suction. The influence of $q_{cyc}$ on $M_{cyc}^r$ is more significant at

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**Figure 5.** Pore-water pressures measured at the base and mid-height of sample during a cyclic loading–unloading test.

**Figure 6.** Relationship between normalized resilient modulus and number of load applications at zero suction in series 1 tests.

**Figure 7.** Relationship between normalized resilient modulus and number of load applications at a cyclic stress of 70 kPa ($s = 0$, 30, and 60 in series 1 tests; $s = 100$, 150, and 250 in series 2 tests).
higher suction. This is because the degree of nonlinearity of the soil stress–strain relationship is more significant at higher suctions (Xu 2011). Suction effects on the resilient modulus are further discussed in the next subsection.

**Influence of suction on resilient modulus**

Figure 9 shows the relationship between \( M_R \) and \( s \) at different cyclic stresses (30, 40, 55, and 70 kPa). Irrespective of whether it is along a drying or wetting path, \( M_R \) increases significantly with increasing \( s \). At a cyclic stress of 30 kPa, \( M_R \) increases by about 10 times when \( s \) increases from 0 to 250 kPa. The beneficial effects of \( s \) on \( M_R \) arise due to at least two possible reasons. First, when a soil specimen becomes unsaturated, voids are partly filled with water and partly occupied by air, resulting in an air–water interface in each void. When there is an increase in matric suction, the radius of an air–water interface decreases and hence induces a larger normal interparticle contact force (Fisher 1926; Mancuso et al. 2002; Wheeler et al. 2003; Ng and Yung 2008). This normal interparticle contact force provides a stabilizing effect on an unsaturated soil by inhibiting slippage at particle contacts and enhancing the shear resistance of the unsaturated soil (Wheeler et al. 2003). Second, an increase in \( s \) induces shrinkage of the soil specimen (Ng and Pang 2000). Due to the stronger internormal force between particles and higher density, \( M_R \) measured during cyclic loading–unloading is larger at higher suctions. The influence of \( s \) on \( M_R \) is captured by the term \((1 + sj)^{k_3}\) in eq. [4]. As \( M_R \) increases with \( s \), the parameter \( k_3 \) should be positive. In most state-of-the-art standards for testing resilient modulus, seasonal variation of soil moisture is not taken into account. In AASHTO (2003), each soil specimen is prepared and tested at the in situ soil moisture level. The resilient characteristic of an unsaturated subgrade soil is always represented by a single \( M_R \). The usage of a single \( M_R \) cannot reflect its seasonal variation. Observed significant increases in \( M_R \) with \( s \) from this figure demonstrate that \( M_R \) of a subgrade soil is very likely to be underestimated in a dry season and overestimated in a wet season. To appropriately describe resilient characteristics of an unsaturated subgrade soil at a given stress state, a suction equalization stage is necessary prior to cyclic loading–unloading.

Further inspection of this figure reveals that the relationship between \( M_R \) and \( s \) is highly nonlinear along a wetting path (in series 1 tests). Given the same increase in \( s \), the percentage of increase in \( M_R \) is much larger in the lower suction range. At a cyclic stress of 30 kPa, \( M_R \) doubles when \( s \) increases from 0 to 30 kPa. On the other hand, when \( s \) increases from 30 to 60 kPa, the percentage increase is only 10%. In series 2 tests, it can be seen that the increase rate of \( M_R \) with \( s \) is almost constant along the drying path. The different results observed in different suction ranges are probably because the bulk water effects dominate soil behaviour when suction is lower than the AEV of the soil (here about 60 kPa, see Fig. 3) in series 1 tests and meniscus water effects dominate soil behaviour when suction exceeds the AEV in series 2 tests (Ng and Yung 2008).

Comparisons of the slope of each curve in Fig. 9 reveal that suction effects on \( M_R \) are generally more obvious at low cyclic stress levels (i.e., low strain levels). This observation is also illustrated in Fig. 8 and can be explained by experimental results from conventional triaxial compression tests. Nyunt et al. (2009) conducted constant water content triaxial compression tests to investigate the influence of \( s \) on the stiffness–strain relationship of an unsaturated soil. They found that suction effects on the secant axial Young’s modulus decrease with axial strain. It is therefore evident that suction effects on \( M_R \) decrease slightly with increasing strain level and cyclic stress.

**Influence of wetting and drying history on resilient modulus**

Figure 10 illustrates the influence of wetting and drying history on measured \( M_R \) at two suctions (i.e., 60 and 150 kPa) in series 3 tests. At a suction of 60 kPa, \( M_R \) measured along a wetting path is larger than that measured along a drying path. Observed differences between these two paths decrease slightly with an increase in cyclic stress. Similarly, Ng et al. (2009) found that \( G_\alpha \) measured along a wetting path is obviously larger than that measured along a drying path.

At a suction of 150 kPa, \( M_R \) measured along a wetting path is also larger than that measured along a drying path when cyclic stress...
tests to study the relationship between shear modulus and p
could be even larger, when the effects of overconsolidation become
predicted by Vucetic and Dobry (1991) found that the soil stiffness degradation
is low (30 and 40 kPa). The influence of wetting and drying history
and hydraulic behaviour. Experimental studies have concluded that
a larger stiffness, at least in the low cyclic stress range.
result in a larger stiffness, at least in the low cyclic stress range.
plastic shrinkage occurs and soil density increases. The soil specimen
along a wetting path may behave like an overconsolidated soil and
result in a larger stiffness, at least in the low cyclic stress range.
When cyclic stress increases to 50 kPa at a suction of 150 kPa, M_R
measured along a drying path becomes even larger than that mea-
sured along a wetting path. One possible reason is that the wetting
and drying history not only induces effects of overconsolidation, but
also affects the equilibrium soil water content. Under the same stress
and suction condition, the soil specimen along the drying path has a
larger water content as shown in Fig. 3. At higher water content, the
number of air-water interfaces is larger and hence the average ske-
eton force is higher. Therefore, M_R measured along the drying path
could be even larger, when the effects of overconsolidation become
relatively less important at high cyclic stress levels.
A comparison of Figs. 6 to 10 demonstrates that the stress and
suction conditions impose a much more pronounced influence on the M_R
than number of load applications and wetting and drying history.
This observation is taken into account in the new semi-
empirical eq. [4]. Although this equation does not consider the
effects of number of load applications and the wetting and drying history, it can still represent the resilient modulus of an unsatu-
rated subgrade soil with good accuracy.

Verification of the proposed equation
To verify the validity of the newly proposed eq. [4] for resilient modulus at a quantitative level, the measured and calculated re-
silient modulus of four different soils, i.e., CDT, Keuper Marl,
Gault clay, and London clay (Brown et al. 1987), are compared.
According to the AASHTO (2000) soil classification, CDT and Keuper
Marl are classified as A-7-6 soils, while Gault clay and London clay are classified as A-7-5 soils. Resilient moduli measured from CDT
tests are first used to fit eq. [4] to derive parameters
M_0, k_1, k_2, and k_3. The derived parameters k_1, k_2, and k_3 are then adopted to pre-
dict M_R of Keuper Marl, which is in the same category of A-7-6 soils, while M_R is fitted. This is because M_R is affected by various factors such as soil density and sampling method. Similarly, experi-
mental data from Gault clay are used to derive parameters M_R, k_1, k_2,
and k_3. The derived parameters k_1, k_2, and k_3 are then adopted for pre-
dicting M_R of London clay, which is in the same category of A-7-5 soils.
The parameter values are summarized in Table 3.
The newly proposed eq. [4] is first applied to fit the measured M_R values of CDT that were measured in series 1 and 2 tests. The parameters M_R, k_1, and k_2 are obtained using the least-square method, while parameter k_3 is assumed equal to 1.0. Considering that this study does not focus on the influence of p on M_R, param-
eter k_3 is simply determined from previous theoretical and exper-
imental studies. According to the well-known modified Cam clay model, M_R (i.e., the axial Young’s modulus during unloading) is proportional to effective mean stress p’ (Muir Wood 1990).
Viggiani and Atkinson (1995) also carried out triaxial compression tests to study the relationship between shear modulus and p’.

Table 3. Summary of soil properties and regression coefficients of proposed semi-empirical equation.

<table>
<thead>
<tr>
<th>Material</th>
<th>AASHTO (2000) classification</th>
<th>Specific gravity</th>
<th>Plastic limit</th>
<th>Liquid limit</th>
<th>Plasticity index</th>
<th>M_R</th>
<th>k_1</th>
<th>k_2</th>
<th>k_3</th>
<th>R^2</th>
<th>Se/Sy</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDT</td>
<td>A-7-6</td>
<td>2.73</td>
<td>29</td>
<td>43</td>
<td>14</td>
<td>8.32</td>
<td>1.00</td>
<td>−0.65</td>
<td>1.01</td>
<td>0.98</td>
<td>0.14</td>
</tr>
<tr>
<td>Keuper Marl</td>
<td>A-7-6</td>
<td>2.69</td>
<td>18</td>
<td>37</td>
<td>19</td>
<td>6.32</td>
<td>1.00</td>
<td>−0.65</td>
<td>1.01</td>
<td>0.66</td>
<td>0.60</td>
</tr>
<tr>
<td>Gault clay</td>
<td>A-7-5</td>
<td>2.69</td>
<td>25</td>
<td>61</td>
<td>36</td>
<td>0.61</td>
<td>1.00</td>
<td>−0.36</td>
<td>1.31</td>
<td>0.98</td>
<td>0.14</td>
</tr>
<tr>
<td>London clay</td>
<td>A-7-5</td>
<td>2.73</td>
<td>23</td>
<td>71</td>
<td>48</td>
<td>0.53</td>
<td>1.00</td>
<td>−0.36</td>
<td>1.31</td>
<td>0.96</td>
<td>0.21</td>
</tr>
</tbody>
</table>

where n is a regression coefficient. Experimental results have re-
vealed that n is 0.72 at very small strain levels and increases to 1.0
at a strain level of 0.5%. In this study, measured total axial strain,
including both permanent plastic strain and recoverable resilient strain, is between 0.1% and 1%. Therefore, it may be reasonable to
assume that M_R increases linearly with p (i.e., k_n = 1) for simplicity.

Figures 11a and 11b compare measured and calculated M_R of CDT
along a wetting path and a drying path, respectively. In general,
they are closely matched with a maximum difference of less than
25%. The coefficient of determination (R^2) and Se/Sy, which are determined from measured and calculated resilient moduli at 24
different stress and suction conditions, are given in Table 3. Se
and Sy are residual standard deviation and sample standard devi-
ation, respectively. R^2 and Se/Sy are found to be 0.98 and 0.14,
respectively, suggesting a strong correlation between measured and calculated resilient moduli. The strong correlation implies that eq. [4] can generally capture the variations of resilient moduli
with net stress and matric suction. Comparing Figs. 11a and 11b
reveals that the results obtained from eq. [4] are in slightly better
agreement with the corresponding experimental data along a drying
path than along a wetting path. As illustrated in Fig. 10, the soil
specimen tested along a wetting path behaves like an overconsoli-
dated soil due to the effect of wetting and drying history. However,
the effects of wetting and drying are not taken into account by eq. [4].
As a result, special attention should be given when eq. [4] is used to
predict M_R of a soil specimen along a wetting path, although reason-
ably accurate predictions can still be made.

To further investigate the validity of eq. [4], it is applied to fit the
measured M_R of three other soils (Keuper Marl, Gault clay, and
London clay) reported by Brown et al. (1987). Figure 12 compares the measured and calculated M_R of different types of soils. For
each soil type, measured and calculated resilient moduli are very
well matched. It should be pointed out that measured soil suc-
tions by Brown et al. (1987) range from 15 to 75 kPa. More high-
quality experimental data are necessary to verify the validity of
eq. [4] over a wider range of suction. Close inspection of this
figure reveals that the measured and calculated results for A-7-5
soil seems to be better matched than those for A-7-6 soil. One possible reason is that all data obtained for A-7-5 are from the
same study. Each test is performed using the same apparatus and test standard. Therefore, any experimental uncertainty is ex-
pected to be smaller for A-7-5 soil. On the other hand, the rela-
tively larger discrepancy in A-7-6 soils might be caused by
different plastic limits between measurements of CDT and Keuper
Marl (refer to Table 3).

It can be seen from Fig. 12 and Table 3 that the same values of k_1,
k_2, and k_4 can be assumed to estimate M_R for the same type of soils. Parameter k_3 of each soil is negative as expected, considering that
M_R decreases with q_{scyc} due to the nonlinearity of the soil stress-
strain relationship. The result shows that A-7-6 has a smaller k_2
does A-7-5. This implies that M_R of A-7-6 soil decreases more
significantly with q_{scyc} This could be explained by the fact that
A-7-6 soil has a smaller plasticity index than does A-7-5 soil.
Vucetic and Dobry (1991) found that the soil stiffness degradation
with strain level is more significant for soils with smaller plastic-
ity indexes. The parameter k_3 of each soil is positive, implying that
M_R generally increases with s due to the beneficial effect of the
The volumetric strain measured at the stage of suction equalization were compacted at the same initial water content and dry density. However, it does not consider all the complex soil behaviour, including the influence of wetting and drying history on mechanical behaviour. Besides, two underlying assumptions are made to keep this equation simple. First, parameters $k_1$, $k_2$, and $k_3$ are assumed to be constant, independent of stress level and stress history. Second, the density effect on $M_R$ is incorporated in $M_R$ in the new equation for simplicity. In this study, all soil specimens were compacted at the same initial water content and dry density. The volumetric strain measured at the stage of suction equalization is smaller than 0.05% in each test. Given such a small volume effect semiconductors in geotechnical instrumentation. Geotechnical Testing Journal, 12(1): 69–76. doi:10.1520/GTJ10676J.

Summary and conclusions

A suction-controlled cyclic triaxial apparatus adopting the axis-translation technique is used to study $M_R$ of a subgrade soil. Test results are analysed under the framework of two stress state variables, i.e., net stress and suction.

The measured $M_R$ of unsaturated subgrade soil increases with number of load applications when a soil contracts under cyclic loads. On the contrary, $M_R$ decreases slightly with number of load applications when a soil dilates. An unsaturated soil specimen generally achieves a stable resilient response within 100 cycles of loading–unloading.

Measured $M_R$ is found to be dependent on stress and suction level. It decreases with cyclic stress because soil stress–strain behaviour under cyclic loads is highly nonlinear. On the other hand, $M_R$ increases significantly with suction. When suction increases from 0 to 250 kPa, $M_R$ increases by up to one order of magnitude. This is due to the fact that an increase in suction induces an additional interparticle normal force and hence stiffens the soil specimen.

Given the same stress and suction level, $M_R$ measured along a wetting path is larger than that measured along a drying path at low cyclic stress. The observed difference between the two paths becomes less significant with an increase in cyclic stress.

A new semi-empirical equation describing the stress-dependency of $M_R$ for both saturated and unsaturated soils is proposed. This new equation is able to capture the variation of $M_R$ with both net stress and suction. For the four soils verified, the $M_R$ values predicted using this equation are generally consistent with the measured results.

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List of symbols

- $a$, $b$, $c$: regression coefficients
- $C_{un}$: constant reflecting inherent soil structure
- $c$: coefficient of determination
- $D_{w}$, $D_{rgb}$, $D_{so}$: grain diameter that corresponds to 10%, 30%, and 60% finer by weight, respectively
- $k$: regression parameter
- $k_{vc}$, $k_{p}$: regression exponents
- $G$: shear modulus
- $G_{0}$: very small strain shear modulus
- $E$: Young’s modulus
- $F(E)$: void ratio function relating shear modulus to void ratio
- $M_{RC}$: resilient modulus at the reference stress state
- $M_{R}$: resilient modulus
- $M_{Rd}$: resilient modulus determined from the Nth cycle
- $P$: net mean stress
- $p_{atm}$: atmospheric pressure
- $p_{r}$: reference pressure
- $q$: deviator stress
- $q_{sw}$: cyclic shear stress
- $R_{2}$: coefficient of determination
- $S_{d}$: residual standard deviation
- $S_{y}$: sample standard deviation
- $s$: matric suction
- $u_{a}$: pore-air pressure
- $u_{w}$: pore-water pressure
- $v$: Poisson’s ratio
- $\sigma_{1r}$, $\sigma_{2r}$, $\sigma_{3r}$: principal total stresses
- $\theta_{i}$: angle of internal friction
- $\theta_{e}$: angle indicating the rate of increase in shear strength relative to matric suction

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