Inflation and Multiverses

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1 Inflation

1.1 Arguments for Inflation

There are certain observable properties of the universe, which can not be explained by the standard Big-Bang model. A modification of the model is needed, to explain these properties, without making to detailed assumptions.

- **Flatness Problem**

  The curvature of the observable universe is too small to be explained by a linear accelerated model without making very detailed assumptions.
  This problem can be solved by an inflation model, which predicts an exponential expansion of the universe for a certain time. An expansion model, which results in a huge expansion after a short time, will lead to the currently observable flat curvature of the universe right now.

- **Horizon Problem**

  The horizon problem occurs because different regions in the event horizon of the now observable universe have a highly homogeneous temperature. But these regions are too far away from each other, to be in a causal contact. This results in difficulties in explaining why these areas are in thermal equilibrium, when they didn't have the time to "know",what temperature the other areas have.

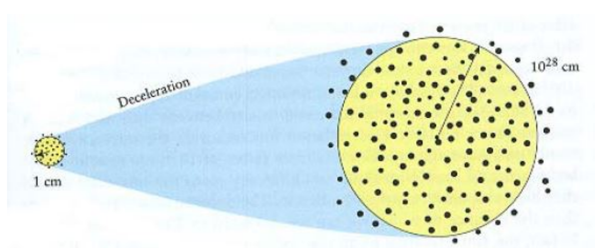

Figure 1: Graphical description how the universe would have evolved without inflation,[2].

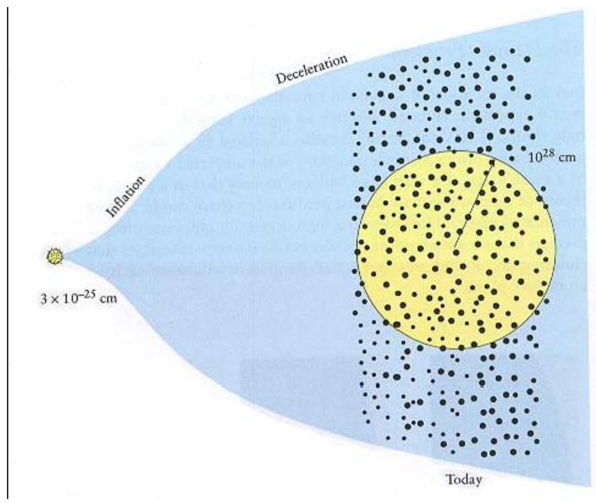

Figure 2: Graphical explanation of the expansion of the universe with inflation,[2].

This can be explained, if again an exponential expansion is assumed. This would mean, that the universe at some point, was small enough for the areas to be in causal contact and after that expanded so drastically, that the possibility of causal contact is now eliminated. However, the before the inflation exchanged information can cause the thermal equilibrium right now.

- **Structure Problem**

  The universe is not completely homogeneous. Galaxies and clusters are density fluctuations in the universe. The origin of these density fluctuations can be explained by an inflation theory which allows quantum fluctuations in the inflation potential.

1.1.1 Slow Roll

For Inflation to be possible the inflation-potential has to dominate the kinetic energy term. Inflation ends, when the kinetic energy term in not anymore dominated. This is called the slow roll condition, which allows us to ignore the time derivative of of the field as well as the second time derivative and makes the following calculation possible.

1.2 Inflation Potential

For inflation to start or even be possible, there has to exist a potential which can cause inflation. We are doing a calculation for the easiest form for an inflation-potential which is

$$V(\Phi) = \frac{1}{2}m^2\Phi^2 \tag{1}$$

with the inflation-field Φ.
With the approach of an equation of motion of the field which has the form of an oscillator equation:

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{\partial V}{\partial \Phi} = 0 = \ddot{\Phi} + 3H\dot{\Phi} + m^2\Phi^2. \tag{2}$$

Another used equation is the Friedmann-Equation:

$$H^2 = \frac{1}{a^2}\frac{da}{dt} = \left(\frac{d}{dt}ln(a)\right)^2 = \frac{8\pi G}{3}\rho_\Phi = \frac{8\pi G}{3}(\frac{1}{2}\dot{\Phi} + V(\Phi)) \tag{3}$$

With the slow-roll-condition we can ignore the terms $\ddot{\phi}$ and $\frac{1}{2}\dot{\Phi}^2$ so we get to the set of equation:

$$-m^2\Phi = 3H\dot{\Phi} \tag{4}$$

and

$$H^2 = \frac{8\pi G}{3}V(\Phi) = \alpha^2\Phi^2 = \left(\frac{d}{dt}ln(a)\right)^2. \tag{5}$$

with $\alpha = \frac{4\pi Gm}{3}$. Now we insert

$$H = \alpha\Phi \tag{6}$$

in equation (4) so we get:

$$3\alpha\Phi\dot{\Phi} + m\Phi = 0 \Leftrightarrow \dot{\Phi} = -\frac{m^2}{3\alpha} \tag{7}$$

With an integration we get over to the solution of Φ and we get:

$$\Phi = \frac{m^2}{3\alpha}(t_0 - t) + \Phi_0. \tag{8}$$

Now we insert our equation for Φ in (6) an we get:

$$H = \alpha\left(\frac{m^2}{3\alpha}(t_0 - t) + \Phi_0\right) \Leftrightarrow H = H_0 - \frac{m^2}{3}t \tag{9}$$

Now we replace H with $\frac{d}{dt}ln(a)$ using equation and get: (3):

$$\frac{dln(a)}{dt} = H_0 - \frac{m^2}{3}t. \tag{10}$$

This is another differential equation and integrating gets us to:

$$ln(\frac{a}{a_0}) = H_t - \frac{m^2}{6}t^2 + C \Leftrightarrow a(t) = a_iDe^{H_0t - \frac{m^2}{6}t^2} \tag{11}$$

1.3 Fluctuations

Quantum fluctuations in the inflation potential which are random events cause a fluctuation in inflation field. These fluctuations again cause different starting energies for the inflation and therefore different inflation durations which results in a different number of e-foldings and causes density fluctuations in the expanded universe.

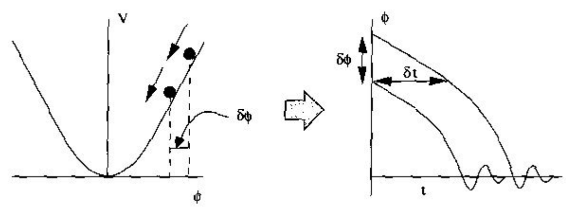

Figure 3: A random walk across the potential and the influence on the inflation field and inflation duration [4]

It is important to know, that the quantum fluctuations completely dominate the motion for high potential energies. The domination decrease as the potential energy decreases until the classical behavior dominates and reheating takes place. This is shown in the following figure:

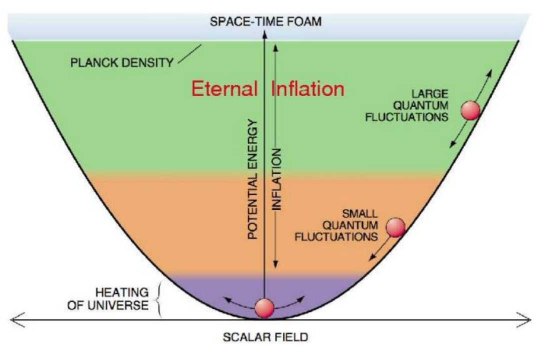

Figure 4: A graphical illustration of the domination of quantum fluctuations at high energies,[2], originally from A.D. Linde.

1.4 Chaotic and Eternal Inflation

With the domination of the quantum fluctuations at high energies, it is possible for inflation to last eternal. At very high energies the influence of the potential energy will remain very high, because the mean value for fluctuations, in other

words the performed random walk, is zero. The potential doesn't have an impact on the duration of inflation anymore. It will only end, if through random events the potential energy gets low enough, so that the quantum fluctuations don't dominate anymore.
Eternal inflation can cause other universes, which are not observable by us, to be created. It can be explained by a bubble model. If our current universe was created by one or rather some bubbles(bubbles inside bubbles are also possible), which had different potential energies, then other bubbles, which were close to us at the beginning, were driven away from us due to the inflation of our universe. However these bubbles also had the possibility to expand exponentially and they eventually did. Causing another universe, very far away from us to be created.

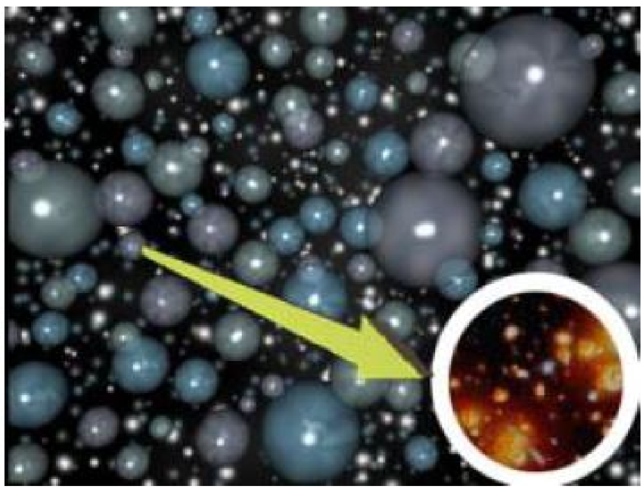

Figure 5: Different bubbles which can expand to different structures or universes,[2]

2 Multiverses

The *multiverse* is a hypothetical set of universes that contains everything that exists, such as our observable universe, physical laws, and constants.
Even though this is a very speculative subject, it doesn't come strictly from science fiction works. Multiverse aspects arise naturally from inflationary and string theory models, and even on some interpretations of quantum mechanics. We will discuss two categorisations for multiverses: *Tegmark's four levels* and *Greene's nine types.*

2.1 Tegmark's four levels

[H]

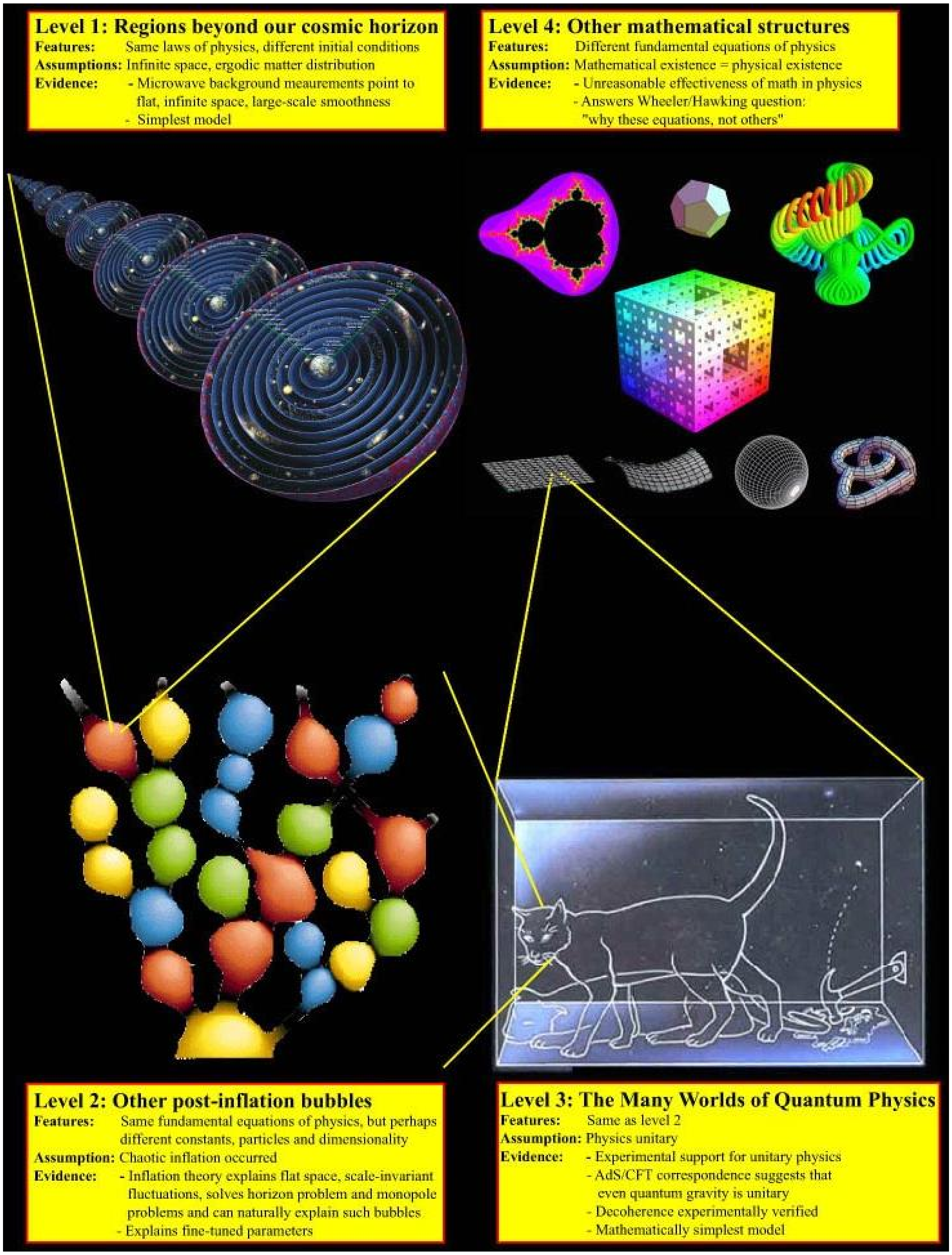

Figure 6: Tegmark's four levels

Tegmark's four levels are a classification of types of universes beyond the observable one. Each level expands and builds upon the previous ones.

2.1.1 Level I: Regions beyond our cosmic horizon

This level represents regions of an infinite universe that are forbidden for being beyond our cosmic horizon. Those regions differ on initial conditions, but have the same physical laws of the observable universe. As in a infinite universe we'd have an infinite number of regions, and the cosmological principle still holds, there are infinite regions similar or even equal to ours.

2.1.2 Level II: Other post inflation bubbles

On this level, there's the assumption that chaotic inflation ocurred, and some regions in space form distinctive 'bubbles'. On different bubbles, different spontaneous symmetry breaking would result in different properties of physics. This model explain fine-tuned parameters.

2.1.3 Level III: The many-worlds interpretation of Quantum Mechanics

On this level, the universes on level II would, in fact, be in quantum branches in infinite-dimensional Hilbert space. This level would be compliant with the many-worlds interpretation of quantum mechanics.

2.1.4 Level IV: Ultimate multiverse

This level was first proposed by Tegmark himself, and considers all universes that can be described by any mathematical structures. As Tegmark says, "abstract mathematics is so general that any Theory Of Everything (TOE) that is definable in purely formal terms (independent of vague human terminology) is also a mathematical structure. For instance, a TOE involving a set of different types of entities (denoted by words, say) and relations between them (denoted by additional words) is nothing but what mathematicians call a set-theoretical model, and one can generally find a formal system that it is a model of." He argues this "implies that any conceivable parallel universe theory can be described at Level IV" and "subsumes all other ensembles, therefore brings closure to the hierarchy of multiverses, and there cannot be say a Level V." [5]

2.2 Greene's nine types

String theorist Brian Greene explores the concept of multiverse and discusses nine types of parallel universes [3].

2.2.1 Quilted multiverse

The quilted multiverse requires an infinite universe, and works basically like Tegmark's level I.

2.2.2 Inflationary multiverse

The inflationary universe is composed of the various regions where the field responsible for inflation collapses and form new universes.

2.2.3 Brane multiverse

The brane multiverse arises from M-theory, where each universe would be a 3-dimensional brane that exists with many others.

2.2.4 Cyclic multiverse

This is like the brane multiverse, but with collisions between branes that would basically be Big Bangs.

2.2.5 Landscape multiverse

The Landscape multiverse relies on Calabi-Yau manifolds in a way that quantum fluctuations lead the Calabi-Yau shapes to lower energy levels, creating a pocket with different physical laws.

2.2.6 Quantum multiverse

As in the many worlds interpretation of quantum mechanics, new universes are created when events with different possible outcomes occur.

2.2.7 Holographic multiverse

The holographic multiverse assumes the holographic principle [1].

2.2.8 Simulated multiverse

A simulated universe would take place at hypothetical complex computers powerful enough to simulate entire universes.

2.2.9 Ultimate multiverse

As Tegmark's level IV, this contains every mathematically possible universe, with any physical laws and constants.

References

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[6] Max Tegmark. http://space.mit.edu/home/tegmark/crazy.html.