GABARITO PROVA RECUPERAÇÃO

FEP2196 / 2009

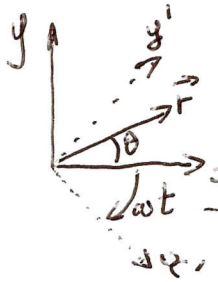
(1.0) a) $r = \sqrt{x^2+y^2} = \sqrt{\frac{3}{4}v_0^2t^2 + \frac{1}{4}v_0^2t^2} = v_0 t$

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\frac{1}{2}v_0 t}{\frac{\sqrt{3}}{2}v_0 t}\right) = \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\boxed{r = v_0 t} \qquad \boxed{\theta = \frac{\pi}{6}}$$

(0.5) b) $x'(t) = r\cos(\omega t + \theta) = v_0 t \cos\left(\omega t + \frac{\pi}{6}\right)$

$y'(t) = r\,\mathrm{sen}(\omega t + \theta) = v_0 t\,\mathrm{sen}\left(\omega t + \frac{\pi}{6}\right)$



em coordenadas polares $r = v_0 t$

$\theta = \left(\omega t + \frac{\pi}{6}\right)$

(1.0) c) $\vec{V} = \frac{dr}{dt}\hat{r} + r\frac{d\theta}{dt}\hat{\theta}$

$\vec{V} = v_0\hat{r} + v_0 t\,\omega\,\hat{\theta}$

Velocidade radial $\boxed{V_r = v_0}$

Velocidade transversal $\boxed{V_\theta = \omega v_0 t}$

(2) $y(x,t) = 2\times10^{-2}\cos(\pi(0{,}5x + 10t))$

Equação Geral onda progressiva

$$y(x,t) = A\cos(kx - \omega t + \phi)$$

(0.5a) $$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0{,}5\times 2\pi} = 2\ m$$

$$f = \frac{\omega}{2\pi} = \frac{2\pi\times 10}{2\pi} = 10\ Hz$$

(0.5)b) Sentido negativo de x

(0.5)c) $$|v| = \frac{\omega}{k} = \frac{2\pi\times 10}{2\pi\times 0{,}5} = 20\ m/s$$

$$\boxed{v = -20\ m/s}$$

(0.5)d) $$v_{Trans} = \frac{\partial y(x,t)}{\partial t} = -2\times10^{-2}\times 2\pi\times 10\ Sen(\pi(0{,}5x + 10t))$$

$$|v_{trans}|_{max} = \frac{4\pi}{10}\ m/s$$

(0.5)e) $$\pi(0{,}5x_1 + 10t) - \pi(0{,}5x_2 + 10t) = \frac{\pi}{6}$$

$$\pi x_1 - \pi x_2 = \pi/6$$

$$x_1 - x_2 = \frac{1}{6} \Rightarrow d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} \cong 0.17\ m$$

(3)

(0.5)a) $$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m} \cos(\Omega t)$$

$$\gamma = \frac{\varphi}{m} = \frac{0,25}{0,05} = \boxed{5\ s^{-1}}$$

$$\omega_0 = 20\ rad/s \Rightarrow \boxed{\omega_0^2 = 400\ rad^2/s^2}$$

$$\frac{F_0}{m} = \frac{0,25}{0,05} = \boxed{5\ m/s^2}$$

$$\Omega = 20\ rad/s$$

(0.5)b) $$x(t) = A(\Omega) \cos(\Omega t + \varphi(\Omega))$$

$$A(\Omega) = \frac{F_0^2/m^2}{(\Omega^2 - \omega_0^2)^2 + \gamma^2 \Omega^2} = \frac{25}{25 \times 400} = \boxed{2,5 \times 10^{-3}\ m}$$

(0.5)c) $$\cos(\Omega t + \varphi(\Omega)) = \pm 1$$

$$\Omega t_{max} + \varphi(\Omega) = n\pi \quad (n = 0, 1, 2 \dots)$$

$$t_{max} = \frac{n\pi - \varphi(\Omega)}{\Omega}$$

$$\varphi(\Omega) = -\arctan\left(\frac{\gamma\Omega}{\omega_0^2 - \Omega^2}\right) = -\arctan\left(\frac{\gamma\Omega}{0}\right) = -\pi/2$$

$$\boxed{t_{max} = \frac{n\pi + \pi/2}{20}}$$

(0.5) d)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 400\,x = 0$$

($5 \to s^{-1}$; $400 \to rad^2/s^2$)

(0.5) e) como $\omega_0^2 > \frac{\gamma^2}{4} \Rightarrow$ regime subcritico

$$\omega' = \sqrt{\omega_0^2 - \gamma^2/4} = \sqrt{400 - \frac{25}{4}} = \sqrt{393{,}75}\ rad/s$$

④ $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0{,}64}} = \frac{5}{3}$$

a) $\Delta t_0(s) = \gamma\, \Delta t'(s') = \frac{5}{3} \times 1 =$ 1 hora 40 min

b) $\Delta x(s) = \Delta t(s)\, v = \Delta t(s)\, 0{,}8c$

$\Delta t_1(s) = \frac{\Delta x(s)}{c} =$ 1 hora 20 min

$\Delta t_{total} = \Delta t_0(s) + \Delta t_1(s) =$ 3 horas

c) $\Delta t_1(s') = \gamma\, \Delta t_{total} = \frac{5}{3} \times 3$ horas $=$ 5 horas

d) $\Delta t_{total}(s') = \Delta t_1(s') +$ 1 hora $=$ 6 horas