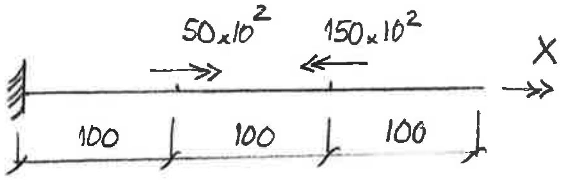
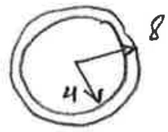
$\boxed{\text{Ex. 1}}$ Lista de exercícios de PEF 2201

Resolução pelo processo dos esforços $(GH = 1)$ X

$\boxed{\text{EIF}}$

$\theta_D = 0$

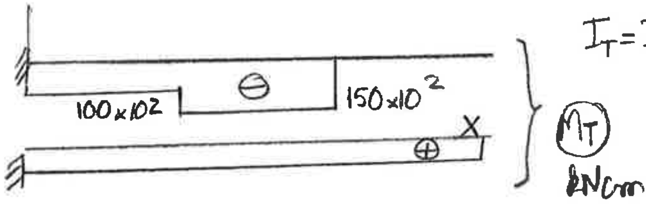




$I_T = I_p = \frac{\pi}{2}(8^4 - 4^4)$

$= 6031{,}86\ cm^4$

• Equilíbrio



• Eq. Constitutiva

$$\theta_D = \sum_{i=1}^{4} \frac{M_{T_i} l_i}{G_i I_{T_i}} = \frac{100}{G I_T}\left[(X - 100\times10^2) + (X - 150\times10^2) + X\right]$$

$$= \frac{100}{G I_T}\left[-250\times10^2 + 3X\right]$$

• Eq. de compatibilid.

$$\theta_D = 0 \implies X = \frac{250\times10^2}{3} = \underline{83{,}3\times10^2\ kNcm}$$

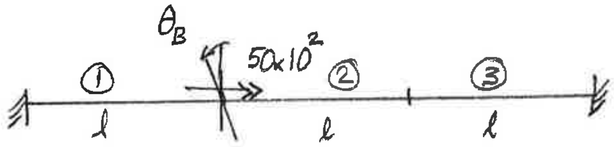
Tensão tangencial máxima:

$$\tau_{máx} = \frac{M_T^{\,máx}}{I_p} R_e = \frac{83{,}3\times10^2}{6031{,}86}\times 8 = 11{,}1\ \frac{kN}{cm^2}$$

$$= \underline{111\ MPa}$$

Resolução pelo processo dos deslocamentos $(GL = 2)$ θ_B e θ_C

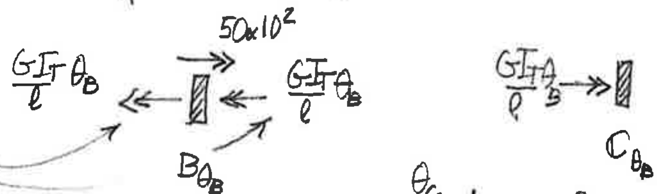
• Compatibilid. e Eqs. Const.



$\boxed{\theta_B}$

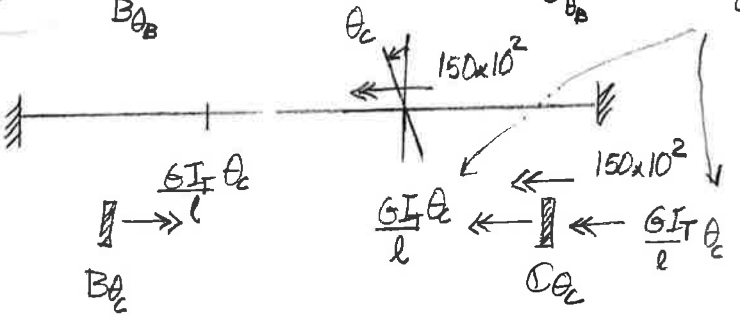
$T = \frac{G I_T}{l}\theta_B$

sentidos contrários ao de θ_B



sentido contrário ao de θ_C

$\boxed{\theta_C}$



colocando os momen. diretamente nos nós.

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(Ex. 1)

- Equilíbrio

Somando os momentos que atuam em cada nó:

$$\Leftarrow \left\{ B: \quad 2\frac{GI_T}{\ell}\theta_B - \frac{GI_T}{\ell}\theta_C = 50\times 10^2 \right.$$

$$\Leftarrow \left\{ C: \quad -\frac{GI_T}{\ell}\theta_B + \frac{2GI_T}{\ell}\theta_C = -150\times 10^2 \right.$$

$$\Rightarrow \begin{cases} 2\theta_B - \theta_C = \dfrac{50\times 10^2 \ell}{GI_T} \\ -\theta_B + 2\theta_C = -\dfrac{150\times 10^2 \ell}{GI_T} \end{cases}$$

Resolvendo o sistema:

$$3\theta_C = -250\times 10^2 \frac{\ell}{GI_T} \Rightarrow \underline{\theta_C = -83{,}3\times 10^2 \frac{\ell}{GI_T}}$$

$$\underline{\theta_B = -16{,}7\times 10^2 \frac{\ell}{GI_T}}$$

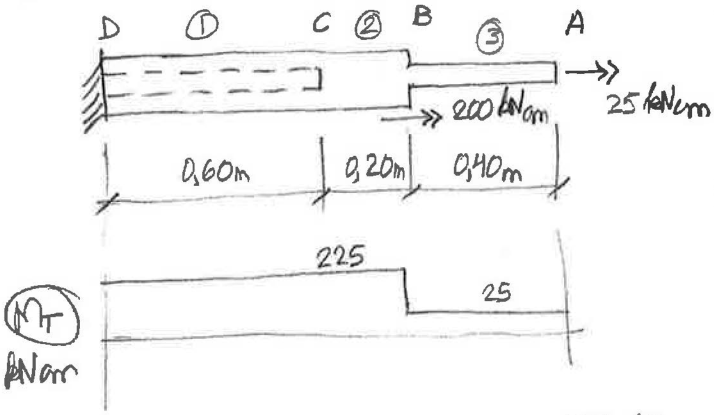
Momentos nas barras:

$$M_{T_1} = \underbrace{+\frac{GI_T}{\ell}\theta_B}_{\text{mom. na extremid. B da barra 1}} = -16{,}7\times 10^2 \text{ kNcm}$$

$$M_{T_2} = \underbrace{-\frac{GI_T}{\ell}\theta_B + \frac{GI_T}{\ell}\theta_C}_{\text{mom. na extremidade B da barra 2}} = (16{,}7 - 83{,}3)\times 10^2 = -66{,}6\times 10^2 \text{ kNcm}$$

$$M_{T_3} = -\frac{GI_T}{\ell}\theta_C = 83{,}3\times 10^2 \text{ kNcm}$$

4.



$$I_{P_1} = \frac{\pi}{2}(3^4 - 2{,}2^4) = 90{,}44\,cm^4$$

$$I_{P_2} = \frac{\pi}{2}\,3^4 = 127{,}23\,cm^4$$

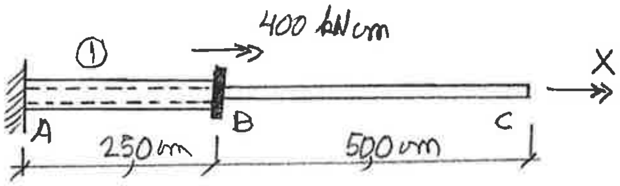
$$I_{P_3} = \frac{\pi}{2}\,1{,}5^4 = 7{,}95\,cm^4$$

$$\theta = \sum_{i=1}^{3} \frac{M_T l_i}{G I_p} = \frac{1}{8\times10^3}\left[\frac{225\times60}{90{,}44} + \frac{225\times20}{127{,}23} + \frac{25\times40}{7{,}95}\right] = \underline{3{,}88\times10^{-2}\ rad}$$

$\boxed{\text{Ex. 6}}$ Lista de exercícios de PEF2201

Resolução pelo processo dos esforços $(GH = 1)$ $\quad X = M_{T_C}$

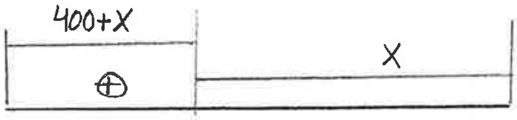
$\boxed{\text{EIF}}$



$$I_1 = \frac{\pi}{2}(3^4 - 2^4) = 102{,}1\ cm^4$$

$$I_2 = \frac{\pi}{2}(2{,}5^4) = 61{,}36\ cm^4$$

· Equilíbrio



· Eq. constitutiva

$$\theta_C = \sum_{i=1}^{2} \frac{M_{T_i} l_i}{G I_{T_i}} = \frac{1}{G}\left[\frac{(400+X)25}{102{,}1} + \frac{X.50}{61{,}36}\right] = \frac{1}{G}\left[97{,}94 + 1{,}060X\right]$$

· Eq. de compatibili// $\boxed{\theta_C = 0}$ $\quad \frac{1}{G}\left[97{,}94 + 1{,}060X\right] = 0 \Rightarrow \underline{X = -92{,}4\ kNcm}$

cálculo das Tensões:

$M_{T_1} = 400 - X = 307{,}6$

$W_{T_1} = I_P/R_e = \frac{102{,}1}{3} = 34{,}03$

$\Big\}\ |\tau_1| = \frac{M_{T_1}}{W_{T_1}} = \frac{307{,}6}{34{,}03} = 9{,}04\ \frac{kN}{cm^2}$

$M_{T_2} = X = -92{,}4$

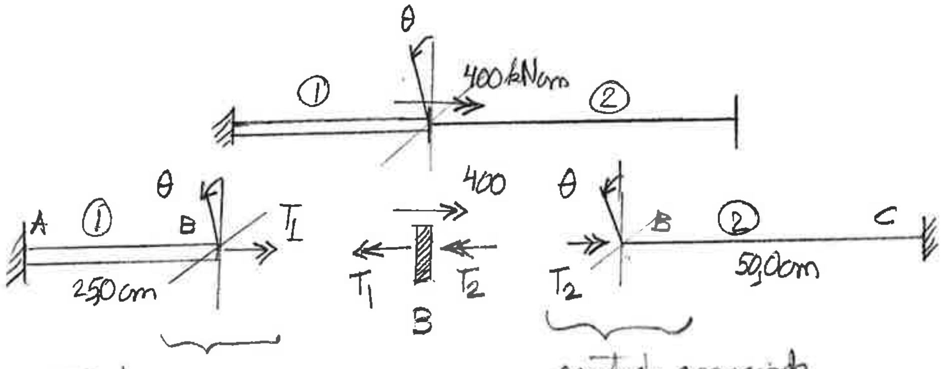
$W_{T_2} = I_{P2}/R = \frac{61{,}36}{2{,}5} = 24{,}54$

$\Big\}\ |\tau_2| = \frac{M_{T_2}}{W_{T_2}} = \frac{92{,}4}{24{,}54} = 3{,}76\ \frac{kN}{cm^2}$

Resolução pelo processo dos deslocamentos $(GL = 1)$ $\quad \theta = \theta_B$

Quando o disco em B sofre uma rotação θ, as extremidades das barras a ele conectadas sofrem a mesma rotação θ

· Compatibilidade



sentido associado à rotação θ.

sentido associado à rotação θ

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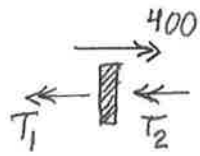
(Ex. 6)

• Eqs. Constitutivas

$$T_1 = \frac{GI_{T_1}}{l_1}\theta = G\,\frac{102{,}1}{25}\,\theta = 4{,}084\,G\theta$$

$$T_2 = \frac{GI_{T_2}}{l_2}\theta = G\cdot\frac{61{,}36}{50}\,\theta = 1{,}227\,G\theta$$

• Equilíbrio



$$\rightarrow \left\{\ 400 = T_1 + T_2 = (4{,}084 + 1{,}227)\,G\theta\right.$$

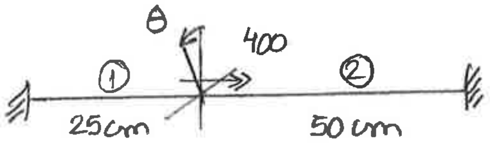
$$\Rightarrow \theta = \frac{400}{5{,}311}\,\frac{1}{G} = \frac{75{,}32}{G}\ (\text{rad})$$

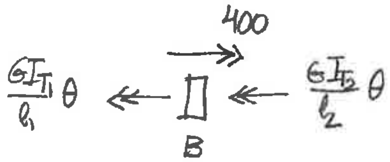
$$M_{T_1} = T_1 = 4{,}084 \times 75{,}32 = 307{,}6\ \text{kNcm}$$

$$M_{T_2} = -T_2 = 1{,}227 \times 75{,}32 = -92{,}4\ \text{kNcm}$$

obs. Como os momentos aplicados pelas barras no nó B têm o sentido contrário ao da rotação podemos condensar a resolução em uma única figura representando apenas os esforços nos nós.

• Compatibilid. e Eqs. constitutivas





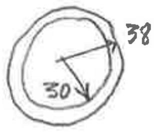
• Equilíbrio

$$\rightarrow B \left\{\ 400 = \frac{GI_{T_1}}{l_1}\theta + \frac{GI_{T_2}}{l_2}\theta = G\left(\frac{102{,}1}{25} + \frac{61{,}36}{50}\right)\theta\right.$$

$$\Rightarrow \theta = \frac{75{,}32}{G}\ (\text{rad}).$$

7.

<u>Tubo de alumínio:</u>



$$I_P = \frac{\pi}{2}(38^4 - 30^4) = 2{,}003 \times 10^6 \text{ mm}^4$$

$$W_T = \frac{I_P}{38} = 52710 \text{ mm}^3$$

$$\tau_{máx} = \frac{M_T^{al}}{W_T} \leq 70 \frac{N}{mm^2} \Rightarrow M_T^{al} \leq 3{,}690 \times 10^6 \text{ Nmm} \quad (1)$$

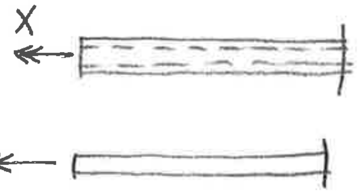
<u>Eixo de aço</u>

$$I_P = \frac{\pi}{2}(25^4) = 0{,}6136 \times 10^6 \text{ mm}^4$$

$$W_T = \frac{I_P}{25} = 24544 \text{ mm}^3$$

$$\tau_{máx} = \frac{M_T^{aço}}{W_T} \leq 120 \frac{N}{mm^2} \Rightarrow M_T^{aço} \leq 2{,}945 \times 10^6 \text{ Nmm} \quad (2)$$

$\boxed{\text{EIF}}$



Eq. compatib.: $\theta_A^{al} = \theta_A^{aço}$.

$$\frac{X \cdot \cancel{500}}{27 \times \cancel{10^3} \times 2{,}003 \times \cancel{10^6}} = \frac{(T_0 - X) \times \cancel{500}}{80 \times \cancel{10^3} \times 0{,}6136 \times \cancel{10^6}} \Rightarrow 3{,}89 \times 10^{-2} X = 2{,}037 \times 10^{-2} T_0$$

$$M_T^{al} = X = 0{,}524\, T_0$$

$$M_T^{aço} = 0{,}476\, T_0$$

$$\text{al } (1) \Rightarrow T_0 \leq \frac{3{,}69 \times 10^6}{0{,}524} \longrightarrow T_0 \leq 7{,}04 \times 10^6 \text{ Nmm}.$$

$$\text{aço } (2) \Rightarrow T_0 \leq \frac{2{,}945 \times 10^6}{0{,}476} \Rightarrow T_0 \leq 6{,}19 \times 10^6 \text{ Nmm}$$

$$\underline{T_0 \leq 6{,}19 \text{ kNm}}$$